Activity Report 2015

Team GALAAD2

Géométrie , Algèbre, Algorithmes

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions).

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Algorithmics, Computer Algebra and Cryptology

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Team GALAAD2

Creation of the Team: 2014 January 01

Keywords:

Computer Science and Digital Science:

2.4. - Reliability, certification
5.5.1. - Geometrical modeling
6.1. - Mathematical Modeling
6.2.6. - Optimization
7.5. - Geometry
7.6. - Computer Algebra

Other Research Topics and Application Domains:

5.1. - Factory of the future
5.2. - Design and manufacturing
9.4.1. - Computer science
9.4.2. - Mathematics

1. Members

Research Scientists

Bernard Mourrain [Team leader, Inria, Senior Researcher, HdR]
Laurent Busé [Inria, Researcher, HdR]
Evelyne Hubert [Inria, Researcher, HdR]

Faculty Member

André Galligo [Univ. Nice, Professor Emeritus]

PhD Students

Elisa Berrini [granted by CIFRE]
Mathieu Collowald [Univ. Nice]

Visiting Scientists

Ibrahim Adamou [Université Dan Dicko DanKoulodo de Maradi, Niger, from December 2015 until January 2016]
Nathan Clement [University of Texas at Austin, USA, from Jun 2015 until Aug 2015]
Alexis Papagiannopoulos [NTUA, Athens, Greece, from May 2015 until Sep 2015]
Meng Wu [Hefei Univ. of Technology, China, Nov 2015]

Administrative Assistant

Sophie Honnorat [Inria]

Others

Jiajun Zhang [Univ. Nice, from Jun 2015 until Sep 2015]
Meiyu Xu [Univ. Nice, from Jun 2015 until Sep 2015]
Marta Abril-Bucero

2. Overall Objectives

2.1. Overall Objectives

There is a shared vision that our day life environment will increasingly interact with a digital world, populated by captors, sensors, or devices used to simplify or improve some of our activities. Digital cameras, positioning systems, mobile phones, internet web interfaces are such typical examples which are nowadays completely standard tools. Interconnected with each other, these devices are producing, exchanging or processing digital data in order to interact with the physical world. Computing is becoming ubiquitous and this evolution raises new challenges to represent, analyze and transform this digital information.

From this perspective, geometry is playing an important role. There is a strong interaction between physical and digital worlds through geometric modeling and analysis. Understanding a physical phenomenon can be done by analyzing numerical simulations on a digital representation of the geometry. Conversely developing digital geometry (as in Computer Aided Geometric Design – CAGD for short) is nowadays used to produce devices to overcome some physical difficulties (car, planes, ...). Obviously, geometry is not addressing directly problems related to storage or transmission of information, but it deals with structured and efficient representations of this information and methods to compute with these models.

Within this context, our research program aims at developing new and efficient methods for modeling geometry with algebraic representations. We don't see shapes just as set of points with simple neighbor information. In our investigations, we use richer algebraic models which provide structured and compact representation of the geometry, while being able to encode their important characteristic features.

The first challenge to be addressed is how to move from the digital world to an algebraic world. Our objective is to develop efficient methods which can transform digital data produced by cameras, laser scanners, observations or simulations into algebraic models involving few parameters. This is a way to structure the digital information and to further exploit its properties. This methodological investigations are connected with practical problems such as compression of data for exchange of geometric information, accurate description and simulation with manufactured objects, shape optimization in computer aided design, ...

A second challenge concerns operations and transformations on these algebraic representations. They require the development of dedicated techniques which fully exploit the algebraic characteristics of these representations. The theoretical foundations of our investigations are in algebraic geometry. This domain deals with the solutions of algebraic equations and its effective aspect concerns algorithms to compute and analyze them. It is an old, important and very active part of mathematics. Its combination with algorithmic developments for algebraic computation leads to new methods to treat effectively geometric problems. These investigations result in new contributions in commutative algebra, new algorithms in computer algebra, complexity analyses and/or software development for practical experimentation.

The third challenge is how to analyze and understand digital geometric data. In this approach, constructing algebraic representation and developing methods to compute with these models are the preliminary steps of our analysis process. The goal is to develop methods to extract some type of information we are searching from this data, such as topological descriptions, subdivisions in smooth components and adjacency relations, decomposition in irreducible components. The interplay between algebraic models and numerical computation is central in this activity. A main issue concerns the approximation of models and the certification of the computation.

3. Research Program

3.1. Introduction

Our scientific activity is structured according to three broad topics:

1. **Algebraic representations for geometric modeling**.
2. **Algebraic algorithms for geometric computing**,
3. **Symbolic-numeric methods for analysis**,

3.2. Algebraic representations for geometric modeling

Compact, efficient and structured descriptions of shapes are required in many scientific computations in engineering, such as "Isogeometric" Finite Elements methods, point cloud fitting problems or implicit surfaces defined by convolution. Our objective is to investigate new algebraic representations (or improve the existing ones) together with their analysis and implementations.

We are investigating representations, based on semi-algebraic models. Such non-linear models are able to capture efficiently complex shapes, using few data. However, they require specific methods to solve the underlying non-linear problems, which we are investigating.

Effective algebraic geometry is a natural framework for handling shape representations. This framework not only provides tools for modeling but it also allows to exploit rich geometric properties.

The above-mentioned tools of effective algebraic geometry make it possible to analyse in detail and separately algebraic varieties. We are interested in problems where collections of piecewise algebraic objects are involved. The properties of such geometrical structures are still not well controlled, and the traditional algorithmic geometry methods do not always extend to this context, which requires new investigations.

The use of piecewise algebraic representations also raises problems of approximation and reconstruction, on which we are working on. In this direction, we are studying B-spline function spaces with specified regularity associated to domain partitions.

Many geometric properties are, by nature, independent from the reference one chooses for performing analytic computations. This leads naturally to invariant theory. We are interested in exploiting these invariant properties, to develop compact and adapted representations of shapes.

3.3. Algebraic algorithms for geometric computing

This topic is directly related to polynomial system solving and effective algebraic geometry. It is our core expertise and many of our works are contributing to this area.

Our goal is to develop algebraic algorithms to efficiently perform geometric operations such as computing the intersection or self-intersection locus of algebraic surface patches, offsets, envelopes of surfaces, ...

The underlying representations behind the geometric models we consider are often of algebraic type. Computing with such models raises algebraic questions, which frequently appear as bottlenecks of the geometric problems.

In order to compute the solutions of a system of polynomial equations in several variables, we analyse and take advantage of the structure of the quotient ring defined by these polynomials. This raises questions of representing and computing normal forms in such quotient structures. The numerical and algebraic computations in this context lead us to study new approaches of normal form computations, generalizing the well-known Gröbner bases.

Geometric objects are often described in a parametric form. For performing efficiently on these objects, it can also be interesting to manipulate implicit representations. We consider particular projections techniques based on new resultant constructions or syzygies, which allow to transform parametric representations into implicit ones. These problems can be reformulated in terms of linear algebra. We investigate methods which exploit this matrix representation based on resultant constructions.

They involve structured matrices such as Hankel, Toeplitz, Bezoutian matrices or their generalization in several variables. We investigate algorithms that exploit their properties and their implications in solving polynomial equations.

We are also interested in the "effective" use of duality, that is, the properties of linear forms on the polynomials or quotient rings by ideals. We undertake a detailed study of these tools from an algorithmic perspective, which yields the answer to basic questions in algebraic geometry and brings a substantial improvement on the complexity of resolution of these problems.

We are also interested in subdivision methods, which are able to efficiently localise the real roots of polynomial equations. The specificities of these methods are local behavior, fast convergence properties and robustness. Key problems are related to the analysis of multiple points.

An important issue while developing these methods is to analyse their practical and algorithmic behavior. Our aim is to obtain good complexity bounds and practical efficiency by exploiting the structure of the problem.

3.4. Symbolic numeric analysis

While treating practical problems, noisy data appear and incertitude has to be taken into account. The objective is to devise adapted techniques for analyzing the geometric properties of the algebraic models in this context.

Analysing a geometric model requires tools for structuring it, which first leads to study its singularities and its topology. In many contexts, the input representation is given with some error so that the analysis should take into account not only one model but a neighborhood of models.

The analysis of singularities of geometric models provides a better understanding of their structures. As a result, it may help us better apprehend and approach modeling problems. We are particularly interested in applying singularity theory to cases of implicit curves and surfaces, silhouettes, shadows curves, moved curves, medial axis, self-intersections, appearing in algorithmic problems in CAGD and shape analysis.

The representation of such shapes is often given with some approximation error. It is not surprising to see that symbolic and numeric computations are closely intertwined in this context. Our aim is to exploit the complementarity of these domains, in order to develop controlled methods.

The numerical problems are often approached locally. However, in many situations it is important to give global answers, making it possible to certify computation. The symbolic-numeric approach combining the algebraic and analytical aspects, intends to address these local-global problems. Especially, we focus on certification of geometric predicates that are essential for the analysis of geometrical structures.

The sequence of geometric constructions, if treated in an exact way, often leads to a rapid complexification of the problems. It is then significant to be able to approximate the geometric objects while controlling the quality of approximation. We investigate subdivision techniques based on the algebraic formulation of our problems which allow us to control the approximation, while locating interesting features such as singularities.

According to an engineer in CAGD, the problems of singularities obey the following rule: less than 20% of the treated cases are singular, but more than 80% of time is necessary to develop a code allowing to treat them correctly. Degenerated cases are thus critical from both theoretical and practical perspectives. To resolve these difficulties, in addition to the qualitative studies and classifications, we also study methods of *perturbations* of symbolic systems, or adaptive methods based on exact arithmetics.

The problem of decomposition and factorisation is also important. We are interested in a new type of algorithms that combine the numerical and symbolic aspects, and are simultaneously more effective and reliable. A typical problem in this direction is the problem of approximate factorization, which requires to analyze perturbations of the data, which enables us to break up the problem.

4. Application Domains

4.1. Shape modeling

Geometric modeling is increasingly familiar for us (synthesized images, structures, vision by computer, Internet, ...). Nowadays, many manufactured objects are entirely designed and built by means of geometric software which describe with accuracy the shape of these objects. The involved mathematical models used to represent these shapes have often an algebraic nature. Their treatment can be very complicated, for example requiring the computations of intersections or isosurfaces (CSG, digital simulations, ...), the detection of singularities, the analysis of the topology, etc. Optimizing these shapes with respect to some physical constraints is another example where the choice of the models and the design process are important to lead to interesting problems in algebraic geometric modeling and computing. We propose the development of methods for shape modeling that take into account the algebraic specificities of these problems. We tackle questions whose answer strongly depends on the context of the application being considered, in direct relationship with the industrial contacts that we are developing in Computer Aided Geometric Design.

4.2. Shape processing

Many problems encountered in the application of computer sciences start from measurement data, from which one wants to recover a curve, a surface, or more generally a shape. This is typically the case in image processing, computer vision or signal processing. This also appears in computer biology where the geometry of distances plays a significant role, for example, in the reconstruction from NMR (Nuclear Magnetic Resonance) experiments, or the analysis of realizable or accessible configurations. In another domain, scanners which tend to be more and more easily used yield large set of data points from which one has to recover a compact geometric model. We are working in collaboration with groups in agronomy on the problem of reconstruction of branching models (which represent trees or plants). We are investigating the application of algebraic techniques to these reconstruction problems. Geometry is also highly involved in the numerical simulation of physical problems such as heat conduction, ship hull design, blades and turbines analysis, mechanical stress analysis. We apply our algebraic-geometric techniques in the isogeometric approach which uses the same (B-spline) formalism to represent both the geometry and the solutions of partial differential equations on this geometry.

5. New Software and Platforms

5.1. AXEL

KEYWORDS: CAO - Algebraic geometric modeler
SCIENTIFIC DESCRIPTION

Axel is an algebraic geometric modeler that aims at providing "algebraic modeling" tools for the manipulation and computation with curves, surfaces or volumes described by semi-algebraic representations. These include parametric and implicit representations of geometric objects. Axel also provides algorithms to compute intersection points or curves, singularities of algebraic curves or surfaces, certified topology of curves and surfaces, etc. A plugin mechanism allows to extend easily the data types and functions available in the plateform.
FUNCTIONAL DESCRIPTION

Axel is a cross platform software to visualize, manipulate and compute 3D objects. It is composed of a main application and several plugins. The main application provides atomic geometric data and processes, a viewer based on VTK, a GUI to handle objects, to select data, to apply process on them and to visualize the results. The plugins provides more data with their reader, writer, converter and interactors, more processes on the new or atomic data. It is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python. The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows.

- Participants: Nicolas Douillet, Anaïs Ducoffe, Valentin Michelet, Bernard Mourrain, Meriadeg Perrinel, Stéphane Chau and Julien Wintz
- Contact: Bernard Mourrain
- URL: http://axel.inria.fr/

Collaboration with Elisa Berrini (MyCFD, Sophia), Tor Dokken (Gotools library, Oslo, Norway), Angelos Mantzaflaris (GISMO library, Linz, Austria), Laura Saini (Post-Doc GALAAD/Missler, TopSolid), Gang Xu (Hangzhou Dianzi University, China).

5.2. Mathemagix

SCIENTIFIC DESCRIPTION

The project aims at building a bridge between symbolic computation and numerical analysis. It is structured by collaborative software developments of different groups in the domain of algebraic and symbolic-numeric computation.

In this framework, we are working more specifically on the following components:

realroot: a set of solvers using subdivision methods to isolate the roots of polynomial equations in one or several variables, continued fraction expansion of roots of univariate polynomials, Bernstein basis representation of univariate and multivariate polynomials and related algorithms, exact computation with real algebraic numbers, sign evaluation, comparison, certified numerical approximation.

shape: tools to manipulate curves and surfaces of different types including parameterized, implicit with different type of coefficients, algorithms to compute their topology, intersection points or curves, self-intersection locus, singularities, ...

These packages are integrated from the former library Synaps (SYmbolic Numeric APplicationS) dedicated to symbolic and numerical computations. There are also used in the algebraic-geometric modeler axel .

FUNCTIONAL DESCRIPTION

Mathemagix is a free computer algebra system which consists of a general purpose interpreter, which can be used for non-mathematical tasks as well, and efficient modules on algebraic objects. It includes the development of standard libraries for basic arithmetic on dense and sparse objects (numbers, univariate and multivariate polynomials, power series, matrices, etc., based on FFT and other fast algorithms). These developments, based on C++, offer generic programming without losing effectiveness, via the parameterization of the code (template) and the control of their instantiations.

- Participants: Bernard Mourrain, Grégoire Lecerf, Philippe Trebuchet and Joris Van Der Hoeven
- Contact: Bernard Mourrain
- URL: http://www.mathemagix.org/

6. New Results

6.1. Certifying isolated singular points and their multiplicity structure

Participant: Bernard Mourrain.

The paper [4] presents two new constructions related to singular solutions of polynomial systems. The first is a new deflation method for an isolated singular root. This construction uses a single linear differential form defined from the Jacobian matrix of the input, and defines the deflated system by applying this differential form to the original system. The advantages of this new deflation is that it does not introduce new variables and the increase in the number of equations is linear instead of the quadratic increase of previous methods. The second construction gives the coefficients of the so-called inverse system or dual basis, which defines the multiplicity structure at the singular root. We present a system of equations in the original variables plus a relatively small number of new variables. We show that the roots of this new system include the original singular root but now with multiplicity one, and the new variables uniquely determine the multiplicity structure. Both constructions are "exact", meaning that they permit one to treat all conjugate roots simultaneously and can be used in certification procedures for singular roots and their multiplicity structure with respect to an exact rational polynomial system.

Joint work with Agnes Szanto, Department of Mathematics, North Carolina State University, Raleigh, USA; Jonathan D. Hauenstein, Department of Applied and Computational Mathematics and Statistics, University of Notre Dame, USA.

6.2. On the construction of general cubature formula by flat extensions

Participants: Marta Abril-Bucero, Bernard Mourrain.

We describe a new method to compute general cubature formulae [5]. The problem is initially transformed into the computation of truncated Hankel operators with flat extensions. We then analyse the algebraic properties associated to flat extensions and show how to recover the cubature points and weights from the truncated Hankel operator. We next present an algorithm to test the flat extension property and to additionally compute the decomposition. To generate cubature formulae with a minimal number of points, we propose a new relaxation hierarchy of convex optimization problems minimizing the nuclear norm of the Hankel operators. For a suitably high order of convex relaxation, the minimizer of the optimization problem corresponds to a cubature formula. Furthermore cubature formulae with a minimal number of points are associated to faces of the convex sets. We illustrate our method on some examples, and for each we obtain a new minimal cubature formula.

This is a joint work with C. Bajaj (Univ. of Austin, Texas, USA).

6.3. A moment matrix approach to computing symmetric cubatures

Participants: Mathieu Collowald, Evelyne Hubert.

A quadrature is an approximation of the definite integral of a function by a weighted sum of function values at specified points, or nodes, within the domain of integration. Gaussian quadratures are constructed to yield exact results for any polynomial of degree $2r-1$ or less by a suitable choice of r nodes and weights. Cubature is a generalization of quadrature in higher dimension. Constructing a cubature amounts to find a linear form

$$\Lambda : \mathbb{R}[x] \to \mathbb{R}, p \mapsto \sum_{j=1}^{r} a_j\, p(\xi_j)$$

from the knowledge of its restriction to $\mathbb{R}[x]_{\leq d}$. The unknowns are the number of nodes r, the weights a_j and the nodes ξ_j.

In [7] we use a basis-free version of an approach to cubatures based on moment matrices in terms of the Hankel operator $\mathcal{H}$ associated to Λ. The existence of a cubature of degree d with r nodes boils down to conditions of ranks and positive semidefiniteness on $\mathcal{H}$. We then recognize the nodes as the solutions of a generalized eigenvalue problem.

Standard domains of integration are symmetric under the action of a finite group. It is natural to look for cubatures that respect this symmetry. Introducing adapted bases obtained from representation theory, the symmetry constraint allows to block diagonalize the Hankel operator $\mathcal{H}$. We then deal with smaller-sized matrices both for securing the existence of the cubature and computing the nodes. The sizes of the blocks are furthermore explicitly related to the orbit types of the nodes with the new concept of the matrix of multiplicities of a finite group. It provides preliminary criteria of existence of a cubature with a given organisation of the nodes in orbit types.

The Maple implementation of the presented algorithms allows to determine, with moderate computational efforts, all the symmetric cubatures of a given degree. We present new relevant cubatures.

6.4. Invariantization of symmetric polynomial systems

Participants: Mathieu Collowald, Evelyne Hubert.

Assuming the variety of a set of polynomials is invariant under a group action, we provide a set of invariants that define the same variety. The contribution is about infinite algebraic groups, the case of finite group being previously known. We introduce for those a new concept of algebraic invariantization. It is based on the construction of rational invariants by Hubert and Kogan [14], a construction for which we provide here new simplified proofs.

6.5. Effective criterions for bigraded birational maps

Participant: Laurent Busé.

A rational map $\mathcal{F} : \mathbb{P}^m \dashrightarrow \mathbb{P}^n$ between projective spaces is defined by a collection of homogeneous polynomials $\mathbf{f} := (f_0, ..., f_n)$ in $m + 1$ variables of the same degree. The problem of deciding or providing sufficient conditions for such a map $\mathcal{F}$ to be birational have attracted a lot of interest in the past and it is still an active area of research. Methods that are based of some properties of the syzygies of $\mathbf{f}$ are definitely the more adapted for computational purposes in the sense that they make the problem of birationality effectively computable in the usual implementation of the Gröbner basis algorithm. The goal of this work is to extend these syzygies-based methods and techniques to the context of rational maps whose source is a product of two projective spaces $\mathbb{P}^r \times \mathbb{P}^s$ instead.

An important motivation for considering bi-graded rational maps comes from the field of geometric modeling. Indeed, the geometric modeling community uses almost exclusively bi-graded rational maps for parameterizing curves, surfaces or volumes under the name of rational tensor-product Bézier parameterizations. It turns out that an important property is to guarantee the birationality of these parameterizations onto their images. An even more important property is to preserve this birationality property during a design process, that is to say when the coefficients of the defining polynomials are continuously modified. As a first attempt to tackle these difficult problems, we analyze in detail birational maps from $\mathbb{P}^1 \times \mathbb{P}^1$ to $\mathbb{P}^2$ in low bi-degree by means of syzygies.

This work is done in the context of the SYRAM project which is funded by the MathAmSud programme. It is a collaboration with N. Botbol (University of Buenos Aires), M. Chardin (University of Paris 6), H. Hassanzadeh (University of Rio de Janeiro), A. Simis (University de Pernambuco) and Q. H. Tran (University of Paris 6). A paper is in preparation.

6.6. Orthogonal projection of points on Bézier curves and surfaces

Participant: Laurent Busé.

In this work, we introduce a new method for computing the orthogonal projections of a point onto a Bézier curve or surface. It is based on the concept of matrix representation we have introduced and developed in some previous works, which is here applied to the parameterizations of the normal planes or lines of a curve or surface, respectively. It consists in the computation of a matrix depending of the ambient space variables, which is done in a pre-processing step, and then the use of tools from numerical linear algebra for a fast and accurate solving of each instance of the problem.

This is an on going work done in the context of the SYRAM project which is funded by the MathAmSud programme. It is a collaboration with N. Botbol (University of Buenos Aires) and M. Chardin (University of Paris 6).

6.7. Extraction of cylinders and cones from minimal point sets

Participants: Laurent Busé, André Galligo, Jiajun Zhang.

The extraction of geometric primitives from 3D point clouds is an important problem in reverse engineering. These 3D point clouds are typically obtained by means of accurate 3D scanners and there exists several methods for performing the 3D geometric primitives extraction. An important category among these method are based on a RANSAC method. For such methods, the primitives are directly extracted for the input point cloud. The basic idea is to extract a particular elementary type of shape, such as planes, spheres, cylinders, cones or tori, from the smallest possible set of points and then to judge if this extracted primitive is relevant to the full point cloud. Therefore, for this category of methods it is very important to compute a particular type of shape through the smallest possible number of points, including normals or not. If the extraction of planes and spheres is easy to treat, the cases of cylinders, cones and tori are more involved. In this work, we aim at developing methods for extracting these geometric primitives from the smaller possible number of points (counting multiplicities if normals are taken into account). Another objective is also to provide methods for extraction without using estimated normals in order to improve the accuracy of the extracted geometric primitive, or to use mixed data depending of the applied context (some points with normals and some other points without normals). A paper is in preparation.

6.8. Discriminant of a complete intersection space curve

Participant: Laurent Busé.

In this work, we develop the formalism of the discriminant of a complete intersection curve in the three dimensional projective space, that is to say a curve which is represented as the zero locus of two homogeneous polynomials in four variables. Our main objective is to provide a new computational approach to this object without relying on the so-called "Cayley trick" for which it is necessary to introduce new variables. We also aim at getting a universal definition of this discriminant over the integers so that it holds under any specialization of the coefficients to an arbitrary commutative ring. Another aspect of this work is to explore properties of this discriminant, typically invariance, covariance and change of basis properties.

This is an on going work which is done in collaboration with Ibrahim Nonkane (University of Ouagadougou, Burkina Faso).

6.9. Resultants, flexes, and the generalization of Salmon's formula

Participant: Laurent Busé.

Given an algebraic variety $S \subset \mathbb{P}^n$ and a point $p \in S$, the osculation order of the point p is the maximum of the multiplicity of intersection at p of S with any line through p. We denote it by μ_p and define $Flex(S) = \{p \in \mathbb{P}^n | \mu_p > n\}$.

If $n = 2$, it is known that if C is a plane algebraic curve of degree d then $Flex(C)$ is the intersection of C with its Hessian, this latter being of degree $3d - 6$. A famous generalization of this result to the case $n = 3$ has been obtained by Salmon in 1860: for a general variety S, $Flex(S)$ is the intersection of S with another hypersurface of degree $11d - 24$. In this work, we are studying the generalization of this formula to arbitrary dimension n. We proved that given $S \subset \mathbb{P}^n$ of degree d, $Flex(S)$ is obtained by intersecting S with another hypersurface of degree

$$d\left(\sum_{k=1}^{n} \frac{n!}{k}\right) - n!$$

We are also looking for an explicit expression of an equation of this latter hypersurface.

This is a work in progress which is done in the context of a PICS collaboration funded by CNRS. It is a joint work with M. Chardin (University Paris 6), C. D'Andrea (University of Barcelona), M. Sombra (University of Barcelona) and M. Weiman (University of Caen).

6.10. Computer Algebra Applied to a Solitary Waves Study

Participant: André Galligo.

In [3], we apply Computer algebra techniques, such as algebraic computations of resultants and discriminants, certified drawing (with a guaranteed topology) of plane curves, to a problem in Fluid dynamics: We investigate "capillary-gravity" solitary waves in shallow water, relying on the framework of the Serre-Green-Naghdi equations. So, we deal with 2 dimensional surface waves, propagating in a shallow water of constant depth. By a differential elimination process, the study reduces to describing the solutions of an ordinary non linear first order differential equation, depending on two parameters. The paper is illustrated with examples and pictures computed with the computer algebra system Maple.

Joint work with Didier Clamond (University of Nice, France) and Denys Dutykh (University of Le Bourget, France).

6.11. H1-parameterizations of plane physical domains with complex topology in Isogeometric analysis

Participants: André Galligo, Bernard Mourrain, Meng Wu.

Isogeometric analysis (IGA) is a method for solving geometric partial differential equations(PDEs). Generating parameterizations of a PDE's physical domain is a basic and important issue within IGA framework. In [13] , we present a global H1-parameterization method for a planar physical domain with complex topology.

Joint work with B. NKonga, Univeristy of Nice - Sophia Antipolis and EPI CASTOR, Inria.

7. Partnerships and Cooperations

7.1. National Initiatives

7.1.1. GEOLMI

GEOLMI - Geometry and Algebra of Linear Matrix Inequalities with Systems Control Applications - is an ANR project working on topics related to the Geometry of determinantal varieties, positive polynomials, computational algebraic geometry, semidefinite programming and systems control applications.

The partners are LAAS-CNRS, Univ. de Toulouse (coordinator), LJK-CNRS, Univ. Joseph Fourier de Grenoble; Inria Sophia Antipolis Méditerranée; LIP6-CNRS Univ. Pierre et Marie Curie; Univ. de Pau et des Pays de l'Adour; IRMAR-CNRS, Univ. de Rennes.

More information available at http://homepages.laas.fr/henrion/geolmi.

7.1.2. ANEMOS

ANEMOS - Advanced Numeric for ELMs (Edge Localized Mode): Modeling and Optimized Schemes - is an ANR project devoted to the numerical modelling study of such ELM control methods as Resonant Magnetic Perturbations (RMPs) and pellet ELM pacing both foreseen in ITER. The goals of the project are to improve understanding of the related physics and propose possible new strategies to improve effectiveness of ELM control techniques. The study of spline spaces for isogemetric finite element methods is proposed in this context.

The partners are IRFM, CEA, Cadarache; JAD, University of Nice - Sophia Antipolis; Inria, Bacchus; Maison de la Simulation CEA-CNRS-Inria-University of Orsay- University of Versailles St Quentin.

7.2. International Initiatives

7.2.1. Participation In other International Programs

We have a bilateral collaboration between Galaad and the University of Athens-DIT team ERGA, headed by Ioannis Emiris for the period August 2014-August 2015. It is supported by both Inria and the University of Athens.

Title: Algebraic algorithms in optimization

Abstract: In the past decade, algebraic approaches to optimization problems defined in terms of multivariate polynomials have been intensively explored and studied in several directions. One example is the work on semidefinite optimization and, more recently, convex algebraic geometry. This project aims to focus on algebraic approaches for optimization applications in the wide sense. We concentrate on specific tools, namely root counting techniques, the resultant, the discriminant and non-negative polynomials, on which the two teams have extensive collaboration and expertise. We examine applications in convex algebraic geometry as well as to a newer topic for the two teams, namely game theory. A common thread to these approaches is to exploit any (sparse) structure.

We participate to a bilateral collaboration between France and Spain which is supported as a PICS from CNRS. The Spanish partner is the University of Barcelona (J. Burgos, C. D'Andrea, Martin Sombra) and the French partners are The university of Caen (F. Amoroso, M. Weimann), the University of Paris 6 (M. Chardin, P. Philippon) and GALAAD.

Title: Diophantine Geometry and Computer Algebra

Abstract: This project aims at exploring interactions between diophantine geometry and computer algebra by stimulating collaborations between experts in both domains. The research program focus on five particular topics: toric varieties and height, equidistribution, Diophantine geometry and complexity, Factorization of multivariate polynomials by means of toric geometry and study of singularities of toric parameterizations.

We coordinate a research project which is funded by the regional program Math-AmSud for two years: 2015-2016. This project is composed by research teams from Argentina, Universidad de Buenos Aires (Nicolás Botbol, Alicia Dickenstein), Brazil, Universidade Federal de Rio de Janeiro, de Pernambuco e de Sergipe (Sayed Hamid Hassanzadeh, Aron Simis) and France, Institut de Mathématiques de Jussieu (Marc Chardin) and Galaad.

Title: Geometry of SYzygies of RAtional Maps with applications to geometric modeling (SYRAM)

Abstract: The study of rational maps is of theoretical interest in algebraic geometry and commutative algebra, and of practical importance in geometric modeling. This research proposal focus on rational maps in low dimension, typically parameterizations of curves and surfaces embedded in the projective space of dimension 3, but also dominant rational maps in dimension two and three. The two main objectives amount to unravel geometric properties of these rational maps from the syzygies of their projective coordinates. The first one aims at extending and generalizing the determination of the closed image of a rational map, as well as its geometric features, whereas the second one will focus on the study of dominant rational maps, in particular on the characterization of those that are generically one-to-one.

7.3. International Research Visitors

7.3.1. Visits of International Scientists

7.3.1.1. Internships

Ibrahim Adamou (Université Dan Dicko DanKoulodo de Maradi, Niger), *Voronoï diagram of half-lines*, December 2015 - January 2016.

Nathan Clement (University of Texas at Austin, USA), *Offset of parametric curves*, Jun 2015-Aug 2015

Alexis Papagiannopoulos (NTUA, Athens, Greece), *Isogeometric analysis and parameterization of computational domains*, May 2015- September 2015.

Meng Wu (Hefei Univ. of Technology, China), *Splines over domain with arbitrary topology and isogeometric applications*, October 2015 - November 2015.

7.3.2. Visits to International Teams

7.3.2.1. Sabbatical programme

Hubert Evelyne

Date: Sep 2015 - Feb 2016

Institution: Fields Institute, Toronto, Canada.

8. Dissemination

8.1. Promoting Scientific Activities

8.1.1. Scientific events selection

8.1.1.1. Member of the conference program committees

Evelyne Hubert, André Galligo and Bernard Mourrain were part of the selection comittee for the conference MEGA that took place in Trento (Italy) in June.

8.1.1.2. Reviewer

Laurent Busé wrote reviews for the conference ISSAC (International Symposium in Symbolic and Algebraic Computation), Bath, UK, July.

Bernard Mourrain was reviewer for the conferences MEGA (Effective Methods in Algebraic Geometry), Trento, Italy, June, ISSAC (International Symposium in Symbolic and Algebraic Computation), Bath, UK, July.

8.1.2. Journal

8.1.2.1. Member of the editorial boards

Evelyne Hubert and Bernard Mourrain are associate editors of the Journal of Symbolic Computation (since 2007).

8.1.2.2. Reviewer - Reviewing activities

Laurent Busé wrote reviews for the following journals: Graphical Models, IEEE Transactions on Visualization and Computer Graphics, Computer Aided Geometric Design, Journal of Algebra, Journal of Symbolic Computation, Math. Zeitschrift, Mathematical Problems in Engineering, Applicable Algebra in Engineering Communication and Computing, Applied Mathematics and Computation. He also reviewed a research proposal for the Austrian Science Fund (FWF).

Evelyne Hubert reviewed articles for the Journal of Symbolic Computation, Mathematics of Computation, Acta Applicandae Mathematicae, and the Journal of Mathematical Analysis and Applications.

Bernard Mourrain reviewed articles for Applied Numerical Mathematics journal, Computer Aided Geometric Design journal, Computing Surveys, Journal of Symbolic Computation, Journal of Pure and Applied Algebra, Reliable Computing, SIAM Journal on Matrix Analysis and Applications, SIAM Journal on Optimization, Transactions on Mathematical Software.

8.1.3. Invited talks

Evelyne Hubert was invited to give colloquium talks at the Aalto Science Institute (Finland) in April and at the department of mathematics of the University of Western Ontario (Canada) in november. She was also invited to give a talk at the *Workshop on Symbolic Combinatorics and Computational Differential Algebra* that took place in September at the Fields institute in Toronto (Canada).

Bernard Mourrain was an invited speaker of the Algebra and Geometry Meeting, Universitat de Barcelona, 2-5 December.

8.2. Teaching - Supervision - Juries

8.2.1. Teaching

Master: Laurent Busé, Curves and Surfaces, 66h ETD, M1, EPU of the university of Nice-Sophia Antipolis.

Master: Bernard Mourrain, Real effective algebraic geometry, 30h, M2 Mathématiques, Univ. Nice Sopha Antipolis,

8.2.2. Supervision

PhD in progress: Elisa Berrini, Parametric modeling for ship hull deformation and optimization. CIFRE with MyCFD, started in January 2014, supervised by Bernard Mourrain.

PhD in progress: Mathieu Collowald, Integral representation of shapes for feature conservation or extraction, started in 2011, supervised by Evelyne Hubert.

8.2.3. Juries

Laurent Busé was part of the PhD jury of José Naéliton da Silva (residual intersections and annihilators of Koszul homologies) at IMPA, Rio de Janeiro, April 13.

Evelyne Hubert was part of the PhD juries of Romain Casati (modélisation numérique de structures élancées pour l'informatique graphique) from University of Grenoble, of Romain Basson (Arithmétique des espaces de modules des courbes hyperelliptiques de genre 3 en caracteéristique positive) from University of Rennes, of Mathieu Collowald (Multivariate moment problems: applications of the reconstruction of linear forms on the polynomial ring) Inria, Sophia Antipolis, defended in December.

Bernard Mourrain was member of the PhD juries of Esteban Segura Ugalde (Computation of Invariant Pairs and Matrix Solvents) University of Limoges, defended in June; of Simone Naldi (Exact algorithms for determinantal varieties and semidefinite programming) LAAS, Toulouse, defended in September; of Mathieu Collowald (Multivariate moment problems: applications of the reconstruction of linear forms on the polynomial ring) Inria, Sophia Antipolis, defended in December.

Bernard Mourrain was member of the Jury for Inria CR2 positions at the center of Inria Rôhne-Alpes, Grenoble.

9. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

[1] M. ABRIL BUCERO, B. MOURRAIN. *Border Basis relaxation for polynomial optimization*, in "Journal of Symbolic Computation", August 2015, vol. 74, pp. 378-399 [*DOI :* 10.1016/J.JSC.2015.08.004], https://hal.inria.fr/hal-00981546

[2] M. COLLOWALD, A. CUYT, E. HUBERT, W.-S. LEE, O. SALAZAR CELIS. *Numerical Reconstruction of Convex Polytopes from Directional Moments*, in "Advances in Computational Mathematics", December 2015, vol. 41, n^{o} 6, 21 p. [*DOI :* 10.1007/s10444-014-9401-0], https://hal.inria.fr/hal-00926357

International Conferences with Proceedings

[3] D. CLAMOND, D. DUTYKH, A. GALLIGO. *Computer Algebra Applied to a Solitary Waves Study*, in "ISSAC'2015", Bath, United Kingdom, ACM, July 2015 [*DOI :* 10.1145/2755996.2756659], https://hal.inria.fr/hal-01255412

[4] J. D. HAUENSTEIN, B. MOURRAIN, A. SZANTO. *Certifying isolated singular points and their multiplicity structure*, in "ISSAC'15", Bath, United Kingdom, ACM, July 2015, pp. 213-220 [*DOI :* 10.1145/2755996.2756645], https://hal.inria.fr/hal-01107541

Other Publications

[5] M. ABRIL BUCERO, C. BAJAJ, B. MOURRAIN. *On the construction of general cubature formula by flat extensions*, January 2015, working paper or preprint, https://hal.inria.fr/hal-01158099

[6] A. BERNARDI, N. S. DALEO, J. D. HAUENSTEIN, B. MOURRAIN. *Tensor decomposition and homotopy continuation*, January 2016, working paper or preprint, https://hal.inria.fr/hal-01250398

[7] M. COLLOWALD, E. HUBERT. *A moment matrix approach to computing symmetric cubatures*, November 2015, working paper or preprint, https://hal.inria.fr/hal-01188290

[8] I. EMIRIS, B. MOURRAIN, E. TSIGARIDAS. *Separation bounds for polynomial systems*, December 2015, working paper or preprint, https://hal.inria.fr/hal-01105276

[9] J. D. HAUENSTEIN, B. MOURRAIN, A. SZANTO. *On deflation and multiplicity structure*, January 2016, working paper or preprint, https://hal.inria.fr/hal-01250388

[10] E. HUBERT. *Invariantization and Polynomial Systems with Symmetry*, January 2016, working paper or preprint, https://hal.inria.fr/hal-01254954

[11] B. MOURRAIN, R. VIDUNAS, N. VILLAMIZAR. *Geometrically continuous splines for surfaces of arbitrary topology*, August 2015, working paper or preprint, https://hal.inria.fr/hal-01196996

[12] M. WU, B. MOURRAIN, A. GALLIGO, B. NKONGA. *Bicubic Spline spaces over rectangular meshes with arbitrary topologies*, November 2015, working paper or preprint, https://hal.inria.fr/hal-01196428

[13] M. WU, B. MOURRAIN, A. GALLIGO, B. NKONGA. *H 1 -parameterizations of plane physical domains with complex topology in Isogeometric analysis*, November 2015, working paper or preprint, https://hal.inria.fr/hal-01196435

References in notes

[14] E. HUBERT, I. KOGAN. *Rational Invariants of a Group Action. Construction and Rewriting*, in "Journal of Symbolic Computation", 2007, vol. 42, n^{o} 1-2, pp. 203-217 [*DOI :* 10.1016/J.JSC.2006.03.005], https://hal.inria.fr/inria-00198847