



IN PARTNERSHIP WITH:
Université de Technologie de
Troyes

Activity Report 2015

Project-Team GAMMA3

Automatic mesh generation and advanced methods

RESEARCH CENTER
Paris - Rocquencourt

THEME
Numerical schemes and simulations

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Project-Team GAMMA3

Creation of the Project-Team: 2010 January 01

Keywords:

Computer Science and Digital Science:

- 2.5. - Software engineering
- 6.1. - Mathematical Modeling
- 6.2. - Scientific Computing, Numerical Analysis & Optimization
- 7.1. - Parallel and distributed algorithms
- 7.5. - Geometry

Other Research Topics and Application Domains:

- 5.2.3. - Aviation
- 5.2.4. - Aerospace

1. Members

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2. Overall Objectives

2.1. Introduction

Une branche importante des sciences de l'ingénieur s'intéresse aux calculs des solutions d'équations aux dérivées partielles très variées (en mécanique du solide, en mécanique des fluides, en modélisation de problèmes thermiques, ...) par la méthode des éléments ou des volumes finis. Ces méthodes utilisent comme support spatial des calculs un maillage du domaine sur lequel les équations sont formulées. Par suite, les algorithmes (de construction) de maillages occupent un rôle important dans toute simulation par la méthode des éléments ou des volumes finis d'un problème modélisé en équations aux dérivées partielles. En particulier, la précision, voire la validité, des solutions calculées est liée aux propriétés du maillage utilisé [27].

L'équipe-projet GAMMA3 a été créé en 2010 à la suite du projet GAMMA. L'équipe est bilocalisée avec une partie à l'UTT (Troyes) et l'autre à Rocquencourt. Les thèmes du projet regroupent un ensemble d'activités concernant les points indiqués ci-dessus, en particulier, l'aspect génération automatique de maillages afin de construire les supports utilisés par les méthodes d'éléments ou de volumes finis. Sont également étudiés les aspects de modélisation géométrique, de post-traitement et de visualisation des résultats issus de tels calculs [28].

L'évolution de la demande en termes de génération automatique de maillages implique une évolution des méthodes classiques de création de maillages vers des méthodes permettant de construire des maillages contrôlés. Les maillages doivent donc être soit isotropes, le contrôle portant sur des tailles souhaitées, soit anisotropes, le contrôle portant à la fois sur des directions et des tailles selon ces dernières.

Le développement d'algorithmes de maillages gouvernés sert de support naturel à la conception de boucles de maillages adaptatifs qui, via un estimateur d'erreurs *a posteriori*, permettent de contrôler la qualité des solutions.

Ces préoccupations amènent à considérer le problème du maillage des domaines de calculs en eux-mêmes tout comme celui du maillage ou du remaillage des courbes et surfaces, frontières de ces domaines.

La taille, en termes de nombre de noeuds, des maillages nécessaires pour certaines simulations, amène à travailler sur la parallélisation des processus de calculs. Cette problématique conduit également à s'intéresser à l'aspect multi-cœurs au niveau des algorithmes de maillages proprement dits.

Simultanément, le volume des résultats obtenus dans de telles simulations, nécessite d'envisager le post-traitement de ces résultats en parallèle ou par des méthodes appropriées.

Par ailleurs, de nombreux problèmes partent de saisies *scanner* (ou autre système discret) des géométries à traiter et demandent d'en déduire des maillages de surfaces aptes à être, par la suite, traités par les méthodes classiques (de remaillage, d'optimisation, de calculs).

Enfin, la maturité de certaines méthodes (victimes de leur succès) conduit les utilisateurs à demander plus et à considérer des problèmes de maillage ou des conditions d'utilisations extrêmes induisant des algorithmes *a priori* inattendus.

Les objectifs du projet GAMMA3 consistent à étudier l'ensemble des points mentionnés ci-dessus afin de rendre automatique le calcul de la solution d'un problème donné avec une précision imposée au départ. Par ailleurs, certaines des techniques utilisées dans les problématiques de maillage sont utilisables dans d'autres disciplines (compression d'images pour ne citer qu'un seul exemple).

3. Highlights of the Year

3.1. Highlights of the Year

3.1.1. Awards

BEST PAPER AWARD:

[19]

A. LOSEILLE, V. MENIER, F. ALAUZET. *Parallel generation of large-size adapted meshes*, in "Procedia Engineering", 2015, vol. 124, n° 57 – 69 [DOI : 10.1016/j.proeng.2015.10.122], <https://hal.inria.fr/hal-01270708>

4. New Software and Platforms

4.1. ABL4FLO

FUNCTIONAL DESCRIPTION

KEYWORDS: Boundary layer, Hybrid meshes

SCIENTIFIC DESCRIPTION

Automatic boundary layer mesh generation for complex geometries

FUNCTIONAL DESCRIPTION

ABL4FLO is designed to generate 3D adapted boundary layer meshes by using a cavity-based operator.

- Participant: Adrien Loseille
- Contact: Adrien Loseille
- URL: <https://www.rocq.inria.fr/gamma/Adrien.Loseille/index.php?page=softwares>

4.2. AMA4FLO

FUNCTIONAL DESCRIPTION

KEYWORDS: Anisotropic mesh adaptation, Surface and volume remeshing, Non manifold geometries

SCIENTIFIC DESCRIPTION

Robust and automatic generation of anisotropic meshes in 3D

FUNCTIONAL DESCRIPTION

AMA4FLO is designed to generate adapted meshes with respect to a provided anisotropic sizing field. The surface and the volume mesh is adapted simultaneously to guarantee that a 3D valid mesh is provided on output.

- Participant: Adrien Loseille
- Contact: Adrien Loseille
- URL: <https://www.rocq.inria.fr/gamma/Adrien.Loseille/index.php?page=softwares>

4.3. BL2D

KEYWORDS: Automatic mesher - Delaunay - Anisotropic - Planar domain

SCIENTIFIC DESCRIPTION

The meshing method is of controlled Delaunay type, isotropic or anisotropic. The internal point generation follows an advancing-front logic, and their connection is realised as in a classical Delaunay approach. Quadrilaterals are obtained by a pairing process. The direct construction of degree 2 elements has been made possible via the control of the domain boundary mesh, in order to ensure the desired compatibility.

FUNCTIONAL DESCRIPTION

Planar mesh generator (isotropic or anisotropic, adaptive).

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: <https://www.rocq.inria.fr/gamma/Patrick.Laug/logiciels/logiciels.html>

4.4. BL2D-ABAQ

KEYWORDS: Automatic mesher - Delaunay - Anisotropic - Planar domain - error estimation - interpolation

SCIENTIFIC DESCRIPTION

The meshing method is the same as BL2D (see above) in an adaptive process. An error estimation (*a posteriori*) of a solution at the nodes of the current mesh results in a size map. A new mesh satisfying these size specifications (made continuous) is built, and the solution is interpolated on the new mesh.

FUNCTIONAL DESCRIPTION

Planar mesh generator (isotropic or anisotropic, adaptive) for deformable domains, interacting with the ABAQUS solver.

- Participants: Houman Borouchaki, Patrick Laug and Abel Cherouat
- Contact: Patrick Laug
- URL: <https://www.rocq.inria.fr/gamma/Patrick.Laug/logiciels/logiciels.html>

4.5. BLGEOL

KEYWORDS: Automatic mesher - Hex-dominant - Geologic structures

SCIENTIFIC DESCRIPTION

The aim is to generate hex-dominant meshes of geologic structures complying with different geometric constraints: surface topography (valleys, reliefs, rivers), geologic layers and underground workings. First, a reference 2D domain is obtained by projecting all the line constraints into a horizontal plane. Different size specifications are given for rivers, outcrop lines and workings. Using an adaptive methodology, the size variation is bounded by a specified threshold in order to obtain a high quality quad-dominant mesh. Secondly, a hex-dominant mesh of the geological medium is generated by a vertical extrusion, taking into account the surfaces found (interfaces between two layers, top or bottom faces of underground workings). The generation of volume elements follows a global order established on the whole set of surfaces to ensure the conformity of the resulting mesh.

FUNCTIONAL DESCRIPTION

Hex-dominant mesher of geologic structures and storage facilities.

- Participants: Patrick Laug and Houman Borouchaki
- Contact: Patrick Laug
- URL: <https://www.rocq.inria.fr/gamma/Patrick.Laug/logiciels/logiciels.html>

4.6. BLMOL

KEYWORDS: Automatic mesher - Molecular surface

SCIENTIFIC DESCRIPTION

To model a molecular surface, each constituting atom is idealized by a simple sphere. First, a boundary representation (B-rep) of the surface is obtained, i.e. a set of patches and the topological relations between them. Second, an appropriate parameterization and a metric map are computed for each patch. Third, meshes of the parametric domains are generated with respect to an induced metric map, using a combined advancing-front generalized-Delaunay approach. Finally these meshes are mapped onto the entire surface.

FUNCTIONAL DESCRIPTION

Molecular surface mesher.

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: <https://www.rocq.inria.fr/gamma/Patrick.Laug/logiciels/logiciels.html>

4.7. BLSURF

KEYWORDS: Automatic mesher - parametric surface - CAD surface

SCIENTIFIC DESCRIPTION

An indirect method for meshing parametric surfaces conforming to a user-specifiable size map is used. First, from this size specification, a Riemannian metric is defined so that the desired mesh is one with unit length edges with respect to the related Riemannian space (the so-called ,unit mesh,). Then, based on the intrinsic properties of the surface, the Riemannian structure is induced into the parametric space. Finally, a unit mesh is generated completely inside the parametric space such that it conforms to the metric of the induced Riemannian structure. This mesh is constructed using a combined advancing-front Delaunay approach applied within a Riemannian context.

FUNCTIONAL DESCRIPTION

CAD surface mesher.

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: <https://www.rocq.inria.fr/gamma/Patrick.Laug/logiciels/logiciels.html>

4.8. FEFLOA-REMESH

KEYWORDS: Anisotropic mesh adaptation, Surface remeshing, Cavity-based operator

SCIENTIFIC DESCRIPTION

Automatic generation of metric-aligned and metric-orthogonal anisotropic meshes in 3D

FUNCTIONAL DESCRIPTION

FEFLOA-REMESH is intended to generate adapted 2D, surface and volume meshes by using a unique cavity-based operator. The metric-aligned or metric-orthogonal approach is used to generate high quality surface and volume meshes independently of the anisotropy involved.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Adrien Loseille
- URL: <https://www.rocq.inria.fr/gamma/Adrien.Loseille/index.php?page=softwares>

4.9. GAMANIC 3D

KEYWORDS: Tetrahedral mesh - Delaunay - Anisotropic size and direction control - Automatic Mesher

SCIENTIFIC DESCRIPTION

Automatic tetrahedral mesher based on an anisotropic Delaunay type point insertion method. A metric field is provided specifying the desired size (edge length) and directional properties.

FUNCTIONAL DESCRIPTION

GAMANIC3D is a volume mesher governed by a (anisotropic) size and directional specification metric field.

- Participants: Houman Borouchaki, Paul Louis George, Frederic Hecht, \sqrt{eric} Saltel, Frédéric Alauzet and Adrien Loseille
- Contact: Paul Louis George
- URL: <http://www.meshgems.com/volume-meshing.html>

4.10. GAMHIC 3D

KEYWORDS: Tetrahedral mesh - Delaunay - Isotropic size control - Automatic Mesher

SCIENTIFIC DESCRIPTION

Automatic tetrahedral mesher based on the Delaunay point insertion method. A metric field is provided specifying the desired size (edge length).

FUNCTIONAL DESCRIPTION

GAMHIC3D is a volume mesher governed by a (isotropic) size specification metric field.

- Participants: Houman Borouchaki, Paul Louis George, Frederic Hecht, \sqrt{eric} Saltel, Frédéric Alauzet and Adrien Loseille
- Contact: Paul Louis George
- URL: <http://www.meshgems.com/volume-meshing.html>

4.11. GHS3D

KEYWORDS: Tetrahedral mesh - Delaunay - Automatic Mesher

SCIENTIFIC DESCRIPTION

Automatic tetrahedral mesher based on the Delaunay point insertion method.

FUNCTIONAL DESCRIPTION

GHS3D is an automatic volume mesher

- Participants: Paul Louis George, Houman Borouchaki, \sqrt{eric} Saltel, Frédéric Alauzet, Adrien Loseille and Frederic Hecht
- Contact: Paul Louis George
- URL: <http://www.meshgems.com/volume-meshing.html>

4.12. HEXOTIC

KEYWORDS: Hexahedral mesh - Octree - Automatic mesher

SCIENTIFIC DESCRIPTION

Automatic full hexahedral mesher primarily based on an octree.

FUNCTIONAL DESCRIPTION

HEXOTIC is an automatic hexahedral mesher

- Contact: Loïc Maréchal
- URL: <https://www.rocq.inria.fr/gamma/gamma/Membres/CIPD/Loic.Marechal/Research/Hexotic.html>

4.13. Metrix

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Compute a metric field from a given solution field using various error estimates.

FUNCTIONAL DESCRIPTION

Metrix computes metric field from a given solution field using various error estimates. Available error estimates are feature-based and goal-oriented based error estimates for steady or unsteady fields. Metrix also performs operations on metrics: gradation, intersection, natural metric of a mesh.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.14. Nimbus 3D

KEYWORDS: Surface reconstruction - Point cloud

SCIENTIFIC DESCRIPTION

Given a point cloud, a surface is constructed primarily based on a Delaunay approach.

FUNCTIONAL DESCRIPTION

Nimbus3D is a surface reconstruction method piece of software

- Participants: Paul Louis George and Houman Borouchaki
- Contact: Paul Louis George
- URL: <http://www.meshgems.com/volume-meshing.html>

4.15. VIZIR

KEYWORDS: Mesh and solution visualization

SCIENTIFIC DESCRIPTION

Interactive mesh and solution visualization for linear, and high order curved elements

FUNCTIONAL DESCRIPTION

VIZIR is intended to visualize and modify interactively simplicial, hybrid and high order curved meshes.

- Participants: Julien Castelneau, Adrien Loseille and Alexis Loyer
- Contact: Adrien Loseille
- URL: <http://www-roc.inria.fr/gamma/gamma/vizir/>

4.16. Wolf

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

General solver platform containing all the Wolf modules: Wolf-Bloom, Wolf-Elast, Wolf-Interpol, Wolf-MovMsh, Wolf-Nsc, Wolf-Shrimp, Wolf-Spyder and Wolf-Xfem.

FUNCTIONAL DESCRIPTION

Wolf is a general solver platform containing all the Wolf modules: Wolf-Bloom, Wolf-Elast, Wolf-Interpol, Wolf-MovMsh, Wolf-Nsc, Wolf-Shrimp, Wolf-Spyder and Wolf-Xfem.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.17. Wolf-Bloom

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Structured boundary layer mesh generator using a pushing approach.

FUNCTIONAL DESCRIPTION

Wolf-Bloom is a structured boundary layer mesh generator using a pushing approach. It starts from an existing volume mesh and inserts a structured boundary layer by pushing the volume mesh. The volume mesh deformation is solved with an elasticity analogy. Mesh-connectivity optimizations are performed to control volume mesh element quality.

- Participants: Frédéric Alauzet, Adrien Loseille and Dave Marcum
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.18. Wolf-Elast

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Linear elasticity solver using a P1 Finite-Element method.

FUNCTIONAL DESCRIPTION

Wolf-Elast is a linear elasticity solver using the P1 Finite-Element method. The Young and Poisson coefficient can be parametrized. The linear system is solved using the Conjugate Gradient method with the LUSGS preconditioner.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.19. Wolf-Interpol

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Software transferring scalar, vector and tensor fields from one mesh to another one.

FUNCTIONAL DESCRIPTION

Wolf-Interpol is a tool to transfer scalar, vector and tensor fields from one mesh to another one. Polynomial interpolation (from order 2 to 4) or conservative interpolation operators can be used. Wolf-Interpol also extracts solutions along lines or surfaces.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.20. Wolf-MovMsh

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Moving mesh algorithm coupled with mesh-connectivity optimization.

FUNCTIONAL DESCRIPTION

Wolf-MovMsh is a moving mesh algorithm coupled with mesh-connectivity optimization. Mesh deformation is computed by means of a linear elasticity solver or a RBF interpolation. Smoothing and swapping mesh optimization are performed to maintain good mesh quality. It handles rigid bodies or deformable bodies, and also rigid or deformable regions of the domain.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.21. Wolf-Nsc

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Numerical flow solver solving the compressible Navier-Stokes equations.

FUNCTIONAL DESCRIPTION

Wolf-Nsc is numerical flow solver solving steady or unsteady turbulent compressible Euler and Navier-Stokes equations. The available turbulent models are the Spalart-Almaras and the Menter SST k-omega. A mixed finite volume - finite element numerical method is used for the discretization. Second order spatial accuracy is reached thanks to MUSCL type methods. Explicit or implicit time integration are available. It also resolved dual (adjoint) problem and compute error estimate for mesh adaptation.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.22. Wolf-Shrimp

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Mesh partitioner for parallel mesh generation and parallel computation.

FUNCTIONAL DESCRIPTION

Wolf-Shrimp is a generic mesh partitioner for parallel mesh generation and parallel computation. It can partition planar, surface (manifold and non manifold), and volume domain. Several partitioning methods are available: Hilbert-based, BFS, BFS with restart. It can work with or without weight function and can correct the partitions to have only one connected component.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.23. Wolf-Spyder

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Metric-based mesh quality optimizer using vertex smoothing and edge/face swapping.

FUNCTIONAL DESCRIPTION

Wolf-Spyder is a metric-based mesh quality optimizer using vertex smoothing and edge/face swapping.

- Participants: Frédéric Alauzet and Adrien Loseille
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

4.24. Wolf-Xfem

KEYWORD: Scientific calculation

SCIENTIFIC DESCRIPTION

Tool providing the mesh of the intersection between a surface mesh and a volume mesh.

FUNCTIONAL DESCRIPTION

Wolf-Xfem is a tool providing the mesh of the intersection between a surface mesh and a volume mesh.

- Participants: Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

5. New Results

5.1. Serendipity and reduced elements

Participants: Paul Louis George [correspondant], Houman Borouchaki, Nicolas Barral.

We give a method to constructing Serendipity elements for quads and hexes with full symmetry properties and indicate the reading of their shape functions. We show that, since the degree 5, the Serendipity elements are no longer symmetric but we propose a method resulting in a Lagrange element of degree 5 with full symmetry properties after adding an adequate number of additional nodes.

On the other hand, we show how to guarantee the geometric validity of a given curved element (seen as a patch) of a mesh. This is achieved after writing the patch in a Bézier setting (Bernstein polynomials and control points). In addition, we discuss the case of patch derived from a transfinite interpolation and it is proved that only some of them are Serendipity elements indeed, we return to the same elements as above

We also give a method to constructing Lagrange Serendipity (or reduced) simplices with a detailed description of the triangles of degree 3 and 4. We indicate that higher order triangles are not candidate apart if we impose a restricted polynomial space. We show that a tetrahedron of degree 3 is a candidate while high order elements are not candidate even if a restriction in the polynomial space is considered. In addition, we propose a method for the validation of such elements, in a given mesh, where the validation means the positiveness of the jacobian.

5.2. Validity of transfinite and Bézier-Serendipity patches

Participants: Paul Louis George [correspondant], Houman Borouchaki, Nicolas Barral.

We define generalized transfinite patches for quads and hexes with full symmetry properties. We give a way of constructing those patches by considering the Bézier setting using linear combinations of tensor-product patches of various degree. Those patches are exactly the Bézier-Serendipity patches recently introduced

ASsfor reduced quadrilateral patches, we introduce the so called "Bézier-Serendip" patches. After some recalls about standard Bézier patches, we propose a method to constructing those reduced patches. The corresponding Bernstein polynomials are written by means of linear combinations of the standard Bernstein polynomials. We give a full description of the patches of degree 2, 3, 4 and 5. Since degree 5, the location of the control points is no longer symmetric and to remedy this problem, we propose adding a number of control points which results in *extended* Bézier-Serendip patches. Those reduced patches are in the Bézier framework what the Serendipity elements are in the finite element framework.

A technical report and a paper have been published [16].

5.3. Meshing Strategies and the Impact of Finite Element Quality on the Velocity Field in Fractured Media

Participants: Patrick Laug [correspondant], Géraldine Pichot.

For calculating flow in a fracture network, the mixed hybrid finite element (MHFE) method is a method of choice as it yields a symmetric, positive definite linear system. However, a drawback to this method is its sensitivity to bad aspect ratio elements. For poor-quality triangles, elementary matrices are ill-conditioned, and inconsistent velocity vectors are obtained by inverting these local matrices. In this work, different strategies have been proposed for better reconstruction of the velocity field.

5.4. Automatic Mesh Generation of Multiface Models on Multicore Processors

Participant: Patrick Laug [correspondant].

This work started in September 2014, as part of a sabbatical year at Polytechnique Montréal. In a previous study, a parallel version of an indirect approach for meshing composite surfaces – also called multiface models – was developed. However, this methodology could be inefficient in practice, as the memory management of most existing CAD (computer aided design) systems use static global caches to save information. In a first approach, CAD queries are fully parallelized, using the Pirate library from Polytechnique Montréal (this library provides a set of C++ classes that implement STEP-compliant B-Rep geometric and topological entities, as well as classes to represent meshes and solutions). In a second approach, the CAD system is completely disconnected from the mesh generator, using a discrete geometric support.

5.5. Applications du maillage et développements de méthodes avancées pour la cryptographie

Participants: Thomas Grosge [correspondant], Dominique Barchiesi, Michael François.

L'utilisation des nombres (pseudo)-aléatoires a pris une dimension importante ces dernières décennies. De nombreuses applications dans le domaine des télécommunications, de la cryptographie, des simulations numériques ou encore des jeux de hasard, ont contribué au développement et à l'usage de ces nombres. Les méthodes utilisées pour la génération de tels nombres (pseudo)-aléatoires proviennent de deux types de processus : physique et algorithmique. Ce projet de recherche a donc pour objectif principal le développement de nouveaux procédés de génération de clés de chiffrement, dits "exotiques", basés sur des processus physiques, multi-échelles, multi-domaines assurant un niveau élevé de sécurité. Deux classes de générateurs basés sur des principes de mesures physiques et des processus mathématiques ont été développé.

La première classe de générateurs exploite la réponse d'un système physique servant de source pour la génération des séquences aléatoires. Cette classe utilise aussi bien des résultats de simulation que des résultats de mesures interférométriques pour produire des séquences de nombres aléatoires. L'application du maillage adaptatif sert au contrôle de l'erreur sur la solution des champs physiques (simulés ou mesurés). A partir de ces cartes physiques, un maillage avec estimateur d'erreur sur l'entropie du système est appliqué. Celui-ci permet de redistribuer les positions spatiales des noeuds. L'étude (locale) de la réduction d'entropie des clés tout au long de la chaîne de création et l'étude (globale) de l'entropie de l'espace des clés générées sont réalisées à partir de tests statistiques.

La seconde classe de générateurs porte sur le développement de méthodes avancées et est basée sur l'exploitation de fonctions chaotiques en utilisant les sorties de ces fonctions comme indice de permutation sur un vecteur initial. Ce projet s'intéresse également aux systèmes de chiffrement pour la protection des données et deux algorithmes de chiffrement d'images utilisant des fonctions chaotiques sont développés et analysés. Ces Algorithmes utilisent un processus de permutation-substitution sur les bits de l'image originale. Une analyse statistique approfondie confirme la pertinence des cryptosystèmes développés.

5.6. Développement de méthodes avancées et maillages appliqués à l'étude de la nanomorphologie des nanotubes-fils en suspension liquide

Participants: Thomas Grosges [correspondant], Dominique Barchiesi, Abel Cherouat, Houman Borouchaki, Laurence Giraud-Moreau, Anis Chaari.

Ce projet de recherche (NANOMORPH) a pour objet principal le développement et la mise au point d'une instrumentation optique pour déterminer la distribution en tailles et le coefficient de forme de nanofils (NF) ou de nanotubes (NT) en suspension dans un écoulement. Au cours de ce projet, deux types de techniques optiques complémentaires sont développées. La première, basée sur la diffusion statique de la lumière, nécessite d'étudier au préalable la physico-chimie de la dispersion, la stabilisation et l'orientation des nanofils dans les milieux d'étude. La seconde méthode, basée sur une méthode opto-photothermique pulsée, nécessite en sus, la modélisation de l'interaction laser/nanofils, ainsi que l'étude des phénomènes multiphysiques induits par ce processus. L'implication de l'équipe-projet GAMMA3 concerne principalement la simulation multiphysique de l'interaction laser-nanofils et l'évolution temporelle des bulles et leurs formations. L'une des principales difficultés de ces problématiques est que la géométrie du domaine est variable (à la fois au sens géométrique et topologique). Ces simulations ne peuvent donc être réalisées que dans un schéma adaptatif de calcul nécessitant le remaillage tridimensionnel mobile, déformable avec topologie variable du domaine (formation et évolution des bulles au cours du temps et de l'espace).

5.7. Applications du maillage à des problèmes multi-physiques, développement de méthodes de résolutions avancées et modélisation électromagnétique-thermique-mécanique à l'échelle mesoscopique

Participants: Dominique Barchiesi [correspondant], Abel Cherouat, Thomas Grosges, Houman Borouchaki, Laurence Giraud-Moreau, Sameh Kessentini, Anis Chaari, Fadhil Mezghani.

Le contrôle et l'adaptation du maillage lors de la résolution de problèmes couplés ou/et non linéaires reste un problème ouvert et fortement dépendant du type de couplage physique entre les EDP à résoudre. Notre objectif est de développer des modèles stables afin de calculer les dilatations induites par l'absorption d'énergie électromagnétique, par des structures matérielles inférieures au micron. Les structures étudiées sont en particulier des nanoparticules métalliques en condition de résonance plasmon. Dans ce cas, un maximum d'énergie absorbée est attendu, accompagné d'un maximum d'élévation de température et de dilatation. Il faut en particulier développer des modèles permettant de simuler le comportement multiphysique de particules de formes quelconques, pour une gamme de fréquences du laser d'éclairage assez étendue afin d'obtenir une étude spectroscopique de la température et de la dilatation. L'objectif intermédiaire est de pouvoir quantifier la dilatation en fonction de la puissance laser incidente. Le calcul doit donc être dimensionné et permettre finalement des applications dans les domaines des capteurs et de l'ingénierie biomédicale. En effet, ces nanoparticules métalliques sont utilisées à la fois pour le traitement des cancers superficiels par nécrose de tumeur sous éclairage adéquat, dans la fenêtres de transparence cellulaire. Déposées sur un substrat de verre, ces nanoparticules permettent de construire des capteurs utilisant la résonance plasmon pour être plus sensibles (voir projet européen *Nanoantenna* et l'activité génération de nombres aléatoires). Cependant, dans les deux cas, il est nécessaire, en environnement complexe de déterminer la température locale, voire la dilatation de ces nanoparticules, pouvant conduire à un désaccord du capteur, la résonance plasmon étant très sensible aux paramètres géométriques et matériels des nanostructures. Dans ce sens, l'étude permet d'aller plus loin que la "simple" interaction électromagnétique avec la matière du projet européen *Nanoantenna*.

Le travail de l'année 2014 a constitué en la poursuite de l'étude des spécificités de ce type de problème multiphysique pour des structures de forme simple et la mise en place de fonctions test, de référence, pour les développements de maillage adaptatifs pour les modèles multiphysiques éléments finis. Nous espérons pouvoir proposer un projet ANR couplant les points de vue microscopiques et macroscopiques dans les deux années qui viennent.

5.8. Visualization and modification of high-order curved meshes

Participants: Alexis Loyer, Adrien Loseille [correspondant].

During the partnership between Inria and Distene, a new visualization software has been designed. It address the typical operations that are required to quickly assess the newly algorithm developed in the team. In particular, interactive modifications of high-order curved mesh and hybrid meshes has been addressed. The software VIZIR is freely available at <https://www.rocq.inria.fr/gamma/gamma/vizir/>.

5.9. Mesh adaptive ALE numerical simulation

Participants: Frédéric Alauzet [correspondant], Nicolas Barral, Adrien Loseille.

Running highly accurate numerical simulations with moving geometries is still a challenge today due to their prohibitive cost in CPU time. Using anisotropic mesh adaptation is one way to drastically reduce the size of the problem and to reach the desired accuracy. Previously, we have developed an ALE formulation using mesh connectivity change in order to achieve any complex displacement. Then, this method has been coupled with the unsteady anisotropic mesh adaptation using the fixed-point algorithm. The key point of this work is the use of an ALE metric that takes into account the mesh motion in the metric field definition.

5.10. Mesh adaptation for Navier-Stokes Equations

Participants: Frédéric Alauzet, Victorien Menier, Adrien Loseille [correspondant].

Adaptive simulations for Navier-Stokes equations require to propose accurate error estimates and design robust mesh adaptation algorithms (for boundary layers).

For error estimates, we design new estimates suited to accurately capture the speed profile in the boundary layers. For mesh adaptation, we design a new method to generate structured boundary layer meshes which are mandatory to accurately compute compressible flows a high Reynolds number (several millions). It couple the specification of the optimal boundary layer from the geometry boundary and moving mesh techniques to extrude the boundary layer in an already existing mesh. The main advantage of this approach is its robustness, *i.e.*, at each step of the algorithm we have always a valid mesh [24].

5.11. Adaptive multigrid strategies

Participants: Frédéric Alauzet [correspondant], Victorien Menier, Adrien Loseille.

Multigrid is a well known technique used to accelerate the convergence of linear system solutions. Using a multigrid strategy to solve non-linear problems improves the robustness and the convergence of each Newton step, the accelerating overall the whole process. In particular, larger time step can be considered. This of main importance when solving turbulent Navier-Stokes equations on complex geometries. First, we developed the classical multigrid method on non-nested meshes. Then, we have pointed out the similarity between the Full MultiGrid (FMG) algorithm and the mesh adaptation algorithm. We have proposed a new Adaptive Full MultiGrid algorithm which improve the overall robustness of the adaptive process and its overall efficiency [24].

5.12. Metric-orthogonal and metric-aligned mesh adaptation

Participants: Frédéric Alauzet, Victorien Menier, Adrien Loseille [correspondant].

A new algorithm to derive adaptive meshes has been introduced through new cavity-based algorithms. It allows to generate anisotropic surface and volume mesh that are aligned along the eigenvector directions. This allows us to improv the quality of the meshes and to deal naturally with boundary layer mesh generation.

5.13. Parallel mesh adaptation

Participants: Frédéric Alauzet, Victorien Menier, Adrien Loseille [correspondant].

We devise a strategy in order to generate large-size adapted anisotropic meshes $O(10^8 - 10^9)$ as required in many fields of application in scientific computing. We target moderate scale parallel computational resources as typically found in R&D units where the number of cores ranges in $O(10^2 - 10^3)$. Both distributed and shared memory architectures are handled. Our strategy is based on typical domain splitting algorithm to remesh the partitions in parallel. Both the volume and the surface mesh are adapted simultaneously and the efficiency of the method is independent of the complexity of the geometry. The originality of the method relies on (i) a metric-based static load-balancing, (ii) dedicated mesh partitioning techniques to (re)split the (complex) interfaces meshes, (iii) anisotropic Delaunay cavity to define the interface meshes, (iv) a fast, robust and generic sequential cavity-based mesh modification kernel, and (v) out-of-core storing of completing parts to reduce the memory footprint. We are able to generate (uniform, isotropic and anisotropic) meshes with more than 1 billion tetrahedra in less than 20 minutes on 120 cores.

5.14. Unsteady adjoint computation on dynamic meshes

Participants: Eléonore Gauci, Frédéric Alauzet [correspondant].

Adjoint formulations for unsteady problems are less common in unsteady methodologies due to the extra complexity inherent in the numerical solution and storage but these methods are a great option in engineering because it takes more into account the cost function we want to minimize. Moreover the engineering applications involve moving elements and this motion must be taken into account by the governing flow equations. We develop a model of unsteady adjoint solver on moving mesh problems. The derivation of the adjoint formulation based on the ALE form of the equations requires consideration of the dynamic meshes. Our model takes into account the DGCL.

5.15. Line solver for efficient stiff parse system resolution

Participants: Loïc Frazza, Frédéric Alauzet [correspondant].

Afin d'accélérer la résolution des problèmes raides, un line-solver a été développé. Cette méthode extrait tout d'abord des lignes dans le maillage du problème selon des critères géométriques ou physiques. Le problème peut alors être résolu exactement le long des ces lignes à moindre cout. Cette méthode est particulièrement bien adaptée aux cas où l'information se propage selon une direction privilégiée tels que les chocs, les couches limites ou les sillages. Ces cas sont généralement associés à des maillages très étirés ce qui conduit à des problèmes raides mais quasi-unidimensionnels. Ils peuvent donc être résolu efficacement par un line-solver, réduisant ainsi les temps de calculs tout en gagnant en robustesse.

5.16. Error estimate for high-order solution field

Participants: Olivier Coulaud, Adrien Loseille [correspondant].

Afin de produire des solveurs d'ordre élevé, et ainsi répondre aux exigences inhérentes à la résolution de problèmes physiques complexes, nous développons une méthode d'adaptation de maillage d'ordre élevé. Celle-ci est basée sur le contrôle par une métrique de l'erreur d'interpolation induite par le maillage du domaine. Plus précisément, pour une solution donnée, l'erreur d'interpolation d'ordre k est paramétrée par la différentielle $k\mathbf{e}$ de cette solution, et le problème se réduit à trouver la plus grande ellipse incluse dans une ligne de niveau de cette différentielle. S'il reste encore quelques difficultés techniques à résoudre avant l'exploitation numérique de notre méthode, les résultats sont très encourageants, tant en terme d'optimalité de la métrique obtenue que de temps de calcul. Il n'y a que peu de doutes sur le fait que ce projet aboutisse prochainement.

6. Bilateral Contracts and Grants with Industry

6.1. Bilateral Contracts with Industry

- Dassault Aviation, *Extraction de la topologie et simplification des détails géométriques*, P. Laug et H. Borouchaki, 66 k-euros, 2013-2015.

7. Partnerships and Cooperations

7.1. National Initiatives

7.1.1. ANR

F. Alauzet, N. Barral, V. Menier and A. Loseille are part of the MAIDESC ANR (2013-2015) on mesh adaptation for moving interfaces in CFD.

7.2. European Initiatives

7.2.1. FP7 & H2020 Projects

P. Laug participates in the GEOPRISM (GEOlogical resources PROtection and exploitation using Innovative Simulation Methods - Towards new generations of simulation technologies) project, submitted to H2020-FETOPEN-2014-2015-RIA. This project involves several Inria teams (Sage, Gamma3, Pomdapi, Coffee) and several European research centers and universities.

7.3. International Initiatives

7.3.1. Inria Associate Teams not involved in an Inria International Labs

7.3.1.1. AM2NS

Title: Advanced Meshing Methods for Numerical Simulations

International Partner (Institution - Laboratory - Researcher):

Mississippi State University (United States) - Center for Advanced Vehicular Systems - Computational Fluid Dynamics Dept. (CAVS-CFD) - Marcum David

Start year: 2014

See also: https://www.rocq.inria.fr/gamma/gamma/Membres/CIPD/Frederic.Alauzet/AssociateTeam_AM2NS/AT_am2ns.html

Numerical simulation is now mature and has become an integral part of design in science and engineering applications. Meshing, i.e., discretizing the computational domain, is at the core of the computational pipeline and a key element to significant improvements. The AM2NS Associate Team focus on developing the next generation of automated meshing methods to improve their robustness and the mesh quality to solve the ever increasing complexity of numerical simulations. Four major meshing issues are targeted: (i) more robustness for mesh generation methods in recovering a given data set, (ii) higher quality for anisotropic adapted meshes via constraint alignment, (iii) higher quality for boundary layer meshes near geometry singularities, and (iv) more robustness in handling complex displacement for moving mesh methods. The impact of this collaborative research will be to provide more reliable solution output predictions in an automated manner by using these new meshing methods.

7.4. International Research Visitors

7.4.1. Visits to International Teams

7.4.1.1. Sabbatical programme

Laug Patrick

Date: Sep 2014 - Aug 2015

Institution: Polytechnique Montréal (Canada)

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