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Activity Report 2015

Project-Team GECO

Geometric Control Design

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team GECO

Creation of the Team: 2011 May 01, updated into Project-Team: 2013 January 01

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1. Members

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2. Overall Objectives

2.1. Overall Objectives

Motion planning is not only a crucial issue in control theory, but also a widespread task of all sort of human activities. The aim of the project-team is to study the various aspects preceding and framing *motion planning*: accessibility analysis (determining which configurations are attainable), criteria to make choice among possible trajectories, trajectory tracking (fixing a possibly unfeasible trajectory and following it as closely as required), performance analysis (determining the cost of a tracking strategy), design of implementable algorithms, robustness of a control strategy with respect to computationally motivated discretizations, etc. The viewpoint that we adopt comes from geometric control: our main interest is in qualitative and intrinsic properties and our focus is on trajectories (either individual ones or families of them).

The main application domain of GECO is *quantum control*. The importance of designing efficient transfers between different atomic or molecular levels in atomic and molecular physics is due to its applications to photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing.

A second application area concerns the control interpretation of phenomena appearing in *neurophysiology*. It studies the modeling of the mechanisms supervising some biomechanics actions or sensorial reactions such as image reconstruction by the primary visual cortex, eyes movement and body motion. All these problems can be seen as motion planning tasks accomplished by the brain.

As a third applicative domain we propose a system dynamics approach to *switched systems*. Switched systems are characterized by the interaction of continuous dynamics (physical system) and discrete/logical components. They provide a popular modeling framework for heterogeneous aspects issuing from automotive and transportation industry, energy management and factory automation.

3. Research Program

3.1. Geometric control theory

The main research topic of the project-team will be **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [30], [64] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;
- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([84], [71], [77]), those exploiting the possible flatness of the system ([58]) and those based on the continuation method ([96]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([83]), geometric measure theory ([59], [34]) and hypoelliptic operator theory ([46]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [51]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [31] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [89], [94]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [50]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [32],
- bounds on the controllability time [28],
- STIRAP processes [99],
- simultaneous control [72],
- optimal control ([68], [41], [52]),
- numerical simulations [78].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [33], [69], [90], [47]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [37], [48], [65], [38]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [98]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [91], [81]. These negative results have been more recently completed by positive ones. In [39], [40] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [54]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [80], [85], [66]. While [80] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [85], [66] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [57], [44].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [63]. Optimal control approaches have also been considered [36], [49]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [87]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word “geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([88]) and then modified by Citti and Sarti ([53]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [62] who received the Nobel prize in 1981.

In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in robotics and neurophysiology. It could help to design better control strategies for robots and artificial limbs, yielding smoother and more progressive movements. Another underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson’s disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain’s choices. For instance, for the problem of human locomotion (see [35]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

  A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [97] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

  Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [67] or for Markov processes [86]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([74]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [95], [75]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [76]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [82] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [42], [43], [55]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [56] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [73], [29].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [61].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [70]. It is known that, in this context, stability cannot be tested on periodic signals alone [45].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([79], [92], [93]) and some recent papers on optimal control ([60], [100]).

5. Highlights of the Year

5.1. Highlights of the Year

- GECO is one of one of the partners of the ANR SRGI, which has been funded in 2015. SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.
- In the recent preprint [23] we answer an open problem proposed by J.P. Hespanha in 2003 in the volume “Unsolved Problems in Mathematical Systems & Control Theory”. The problem deals with the characterization of the finiteness of the L_2-gain of a switched linear control systems, in dependence of the value of the minimal dwell-time of its switching laws.

6. New Results

6.1. New results: geometric control

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- In [12] and [20] we study the sub-Finsler geometry as a time-optimal control problem. In particular, we consider non-smooth and non-strictly convex sub-Finsler structures associated with the Heisenberg, Grushin, and Martinet distributions. Motivated by problems in geometric group theory, we characterize extremal curves, discuss their optimality, and calculate the metric spheres, proving their Euclidean rectifiability.
- In [18] we compare different notions of curvature on contact sub-Riemannian manifolds. In particular we introduce canonical curvatures as the coefficients of the sub-Riemannian Jacobi equation. The main result is that all these coefficients are encoded in the asymptotic expansion of the horizontal derivatives of the sub-Riemannian distance. We explicitly compute their expressions in terms of the standard tensors of contact geometry. As an application of these results, we obtain a sub-Riemannian version of the Bonnet-Myers theorem that applies to any contact manifold.
- In sub-Riemannian geometry the coefficients of the Jacobi equation define curvature-like invariants. We show in [21] that these coefficients can be interpreted as the curvature of a canonical Ehresmann connection associated to the metric, first introduced by Zelenko and Li. We show why this connection is naturally nonlinear, and we discuss some of its properties.
- On a sub-Riemannian manifold we define in [22] two type of Laplacians. The macroscopic Laplacian, as the divergence of the horizontal gradient, once a volume is fixed, and the microscopic Laplacian, as the operator associated with a geodesic random walk. We consider a general class of random walks, where all sub-Riemannian geodesics are taken in account. This operator depends only on the choice of a complement to the sub-Riemannian distribution. We address the problem of equivalence of the two operators. This problem is interesting since, on equiregular sub-Riemannian manifolds, there is always an intrinsic volume (e.g. Popp's one) but not a canonical choice of complement. The result depends heavily on the type of structure under investigation: we describe the relationship between the two approaches in the case of contact structures, Carnot groups, quasi-contact structures.
- In [2] we study 3D almost-Riemannian manifolds, that is, generalized Riemannian manifolds defined locally by 3 vector fields that play the role of an orthonormal frame, but could become collinear on some singular set. Almost-Riemannian manifolds were deeply studied in dimension 2. In this paper we start the study of the 3D case which appear to be reacher with respect to the 2D case, due to the presence of abnormal extremals which define a field of directions on the singular set. We study the type of singularities of the metric that could appear generically, we construct local normal forms and we study abnormal extremals. We then study the nilpotent approximation and the structure of the corresponding small spheres. We finally give some preliminary results about heat diffusion on such manifolds.

New results on motion planning are the following.

- In [7] (written while D. Prandi was PhD student in the team) we study the complexity of the motion planning problem for control-affine systems. Such complexities are already defined and rather well-understood in the particular case of nonholonomic (or sub-Riemannian) systems. Our aim is to generalize these notions and results to systems with a drift. Accordingly, we present various definitions of complexity, as functions of the curve that is approximated, and of the precision of the approximation. Due to the lack of time-rescaling invariance of these systems, we consider geometric and parametrized curves separately. Then, we give some asymptotic estimates for these quantities. As a byproduct, we are able to treat the long-time local controllability problem, giving quantitative estimates on the cost of stabilizing the system near a non-equilibrium point of the drift.

- In [11] and [1] we propose new conditions guaranteeing that the trajectories of a mechanical control system can track any curve on the configuration manifold. We focus on systems that can be represented as forced affine connection control systems and we generalize the sufficient conditions for tracking known in the literature. The new results are proved by a combination of averaging procedures by highly oscillating controls and the notion of kinematic reduction.
- In
- [17] we introduce the concept of Developmental Partial Differential Equation (DPDE), which consists of a Partial Differential Equation (PDE) on a time-varying manifold with complete coupling between the PDE and the manifold's evolution. In other words, the manifold's evolution depends on the solution to the PDE, and vice versa the differential operator of the PDE depends on the manifold's geometry. DPDE is used to study a diffusion equation with source on a growing surface whose growth depends on the intensity of the diffused quantity. The surface may, for instance, represent the membrane of an egg chamber and the diffused quantity a protein activating a signaling pathway leading to growth. Our main objective is to show controllability of the surface shape using a fixed source with variable intensity for the diffusion. More specifically, we look for a control driving a symmetric manifold shape to any other symmetric shape in a given time interval. For the diffusion we take directly the Laplace-Beltrami operator of the surface, while the surface growth is assumed to be equal to the value of the diffused quantity. We introduce a theoretical framework, provide approximate controllability and show numerical results. Future applications include a specific model for the oogenesis of Drosophila melanogaster.

6.2. New results: quantum control

New results have been obtained for the control of the bilinear Schrödinger equation.

- In [4] we study the so-called spin-boson system, namely a two-level system in interaction with a distinguished mode of a quantized bosonic field. We give a brief description of the controlled Rabi and Jaynes–Cummings models and we discuss their appearance in the mathematics and physics literature. We then study the controllability of the Rabi model when the control is an external field acting on the bosonic part. Applying geometric control techniques to the Galerkin approximation and using perturbation theory to guarantee non-resonance of the spectrum of the drift operator, we prove approximate controllability of the system, for almost every value of the interaction parameter.
- The main result in [9] is the approximate controllability of a bilinear Schrödinger equation modeling a system of two ions trapped in a cavity. A new spectral decoupling technique is introduced, which allows to analyze the controllability of the infinite-dimensional system through finite-dimensional considerations. The controllability of a simplified version of the model has been obtained in [16].
- In [13] and [3] we study the controllability of a closed control-affine quantum system driven by two or more external fields. We provide a sufficient condition for controllability in terms of existence of conical intersections between eigenvalues of the Hamiltonian in dependence of the controls seen as parameters. Such spectral condition is structurally stable in the case of three controls or in the case of two controls when the Hamiltonian is real. The spectral condition appears naturally in the adiabatic control framework and yields approximate controllability in the infinite-dimensional case. In the finite-dimensional case it implies that the system is Lie-bracket generating when lifted to the group of unitary transformations, and in particular that it is exactly controllable. Hence, Lie algebraic conditions are deduced from purely spectral properties. We conclude the analysis by proving that approximate and exact controllability are equivalent properties for general finite-dimensional quantum systems.

In [26], written with the members of the European project QUAINT, state-of-the-art quantum control techniques are reviewed and put into perspective by a consortium uniting expertise in optimal control theory and applications to spectroscopy, imaging, quantum dynamics of closed and open systems. Key challenges are addressed and a roadmap to future developments is sketched.

6.3. New results: neurophysiology

In [27] we present a new version of the image inpainting algorithm that GECO developed in the recent years. This new version is called the Averaging and Hypoelliptic Evolution (AHE) algorithm, and is based upon a semi-discrete variation of the Citti–Petitot–Sarti model of the primary visual cortex V1. In particular, we focus on reconstructing highly corrupted images (i.e. where more than the 80% of the image is missing).

6.4. New results: switched systems

- In [5] we consider a continuous-time linear switched system on $\mathbb{R}^n$ associated with a compact convex set of matrices. When it is irreducible and its largest Lyapunov exponent is zero there always exists a Barabanov norm associated with the system. We look at two types of issues: (a) properties of Barabanov norms such as uniqueness up to homogeneity and strict convexity; (b) asymptotic behaviour of the extremal solutions of the linear switched system. Regarding Issue (a), we provide partial answers and propose four related open problems. As for Issue (b), we establish, when $n = 3$, a Poincaré–Bendixson theorem under a regularity assumption on the set of matrices. We then revisit a noteworthy result of N.E. Barabanov describing the asymptotic behaviour of linear switched system on $\mathbb{R}^3$ associated with a pair of Hurwitz matrices $\{A, A + bc^T\}$.
- Motivated by an open problem posed by J.P. Hespanha, in [23] we extend the notion of Barabanov norm and extremal trajectory to classes of switching signals that are not closed under concatenation. We use these tools to prove that the finiteness of the L2-gain is equivalent, for a large set of switched linear control systems, to the condition that the generalized spectral radius associated with any minimal realization of the original switched system is smaller than one.
- In [14] in the totally observed case and in [23] in the general case, we answer an open problem posed by J.P. Hespanha in 2003. We first extend the notion of Barabanov norm and extremal trajectory to classes of switching signals that are not closed under concatenation. We use these tools to prove that the finiteness of the L_2-gain is equivalent, for a large set of switched linear control systems, to the condition that the generalized spectral radius associated with any minimal realization of the original switched system is smaller than one.
- In [24] we address the stability of non-autonomous difference equations by providing a suitable representation of the solution at time t in terms of the initial condition and time-dependent matrix coefficients. This enables us to characterize the asymptotic behavior of solutions in terms of that of such coefficients. As a consequence, we obtain necessary and sufficient stability criteria for non-autonomous linear difference equations. In the case of difference equations with arbitrary switching, we obtain a generalization of the well-known criterion for autonomous systems due to Hale and Silkowski, which, as the latter, is delay-independent. These results are applied to transport and wave propagation on networks. In particular, we show that the wave equation on a network with arbitrarily switching damping at external vertices is exponentially stable if and only if the network is a tree and the damping is bounded away from zero at all external vertices but one.
- For linear systems in continuous time with random switching, we characterize in [25] the Lyapunov exponents using the Multiplicative Ergodic Theorem for an associated system in discrete time. An application to control systems shows that here a controllability condition implies that arbitrary exponential decay rates for almost sure stabilization can be obtained.

A result related to switched system is the one obtained in [6] and [15]: we study the stability of linear time-varying delay differential equations where the delay enters as a switching parameter. In [6] we give a collection of converse Lyapunov–Krasovskii theorems for uncertain retarded differential equations. We show that the existence of a weakly-degenerate Lyapunov–Krasovskii functional is a necessary and sufficient condition for the global exponential stability of linear retarded functional differential equations. In [15] the fundamental question that we consider is the following: assuming that every individual (constant-delay) subsystem is exponentially stable, can we characterize the cases when the system is not exponentially stable? This is nothing else than the so-called Markus-Yamabe instability and we give new conditions ensuring it.

7. Partnerships and Cooperations

7.1. Regional Initiatives

- Project *Stabilité des systèmes à excitation persistante*, Program MathIng, Labex LMH, 2013-2016. This project is about different stability properties for systems whose damping is intermittently activated. The coordinator is Mario Sigalotti. The other members are Yacine Chitour and Guilherme Mazanti.
- iCODE is the Institute for Control and Decision of the Idex Paris Saclay. It was launched in March 2014 for two years until June 2016. We are involved in three actions funded by iCODE:
  - one action on control of quantum systems, in collaboration with Nicoals Boulant of Neurospin. The action is coordinated by Ugo Boscain;
  - one action on control of wave propagation on networks. The action is coordinated by Mario Sigalotti;
  - one action on switched system. The action is coordinated by Marianne Akian (and handled by MAXPLUS).
- Starting from the end of 2015, we obtained a grant by PGMO (Gaspard Monge Program for Optimisation and operational research) on Geometric Optimal Control. The grant duration is one year and is renewable for up to three years. The grant is coordinated by Luca Rizzi and Mario Sigalotti.

7.2. National Initiatives

7.2.1. ANR

The ANR SRGI starts at the end of 2015, for a duration of four years. GECO is one of one of the partners of the ANR. The national coordinator is Emmanuel Trélat (UPMC) and the local one Ugo Boscain.

SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.

7.3. European Initiatives

7.3.1. FP7 & H2020 Projects

Program: ERC Starting Grant

Project acronym: GeCoMethods

Project title: Geometric Control Methods for the Heat and Schroedinger Equations

Duration: 1/5/2010 - 1/5/2015

Coordinator: Ugo Boscain

Abstract: The aim of this project is to study certain PDEs for which geometric control techniques open new horizons. More precisely we plan to exploit the relation between the sub-Riemannian distance and the properties of the kernel of the corresponding hypoelliptic heat equation and to study controllability properties of the Schroedinger equation.

All subjects studied in this project are applications-driven: the problem of controllability of the Schroedinger equation has direct applications in Laser spectroscopy and in Nuclear Magnetic Resonance; the problem of nonisotropic diffusion has applications in cognitive neuroscience (in particular for models of human vision).

Participants. Main collaborator: Mario Sigalotti. Other members of the team: Andrei Agrachev, Riccardo Adami, Thomas Chambrion, Grégoire Charlot, Yacine Chitour, Jean-Paul Gauthier, Frédéric Jean.

7.4. International Initiatives

7.4.1. Inria International Partners

7.4.1.1. Informal International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

7.4.2. Participation In other International Programs

- Laboratoire Euro Maghrébin de Mathématiques et de leurs Interactions (LEM2I) http://www.lem2i.cnrs.fr/
- GDRE Control of Partial Differential Equations (CONEDP) http://www.ceremade.dauphine.fr/~glass/GDRE/

8. Dissemination

8.1. Promoting Scientific Activities

8.1.1. Scientific events organisation

8.1.1.1. General chair, scientific chair

Ugo Boscain was member of the scientific committee of the workshop Nonlinear Control and Geometry, 23-29/08/2015, Banach Center Conferences, Bedlewo. Poland.

8.1.2. Journal

8.1.2.1. Member of the editorial boards

- Ugo Boscain is Associate Editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing Editor of Journal of Dynamical and Control Systems
- Mario Sigalotti is Associate Editor of Journal of Dynamical and Control Systems
- Ugo Boscain is Associate Editor of ESAIM Control, Optimisation and Calculus of Variations
- Ugo Boscain is Associate Editor of Mathematical Control and Related Fields
- Ugo Boscain is Associate editor of Analysis and Geometry in Metric Spaces

8.1.3. Invited talks

- Mario Sigalotti gave an invited talk at the Journée McTAO, Dijon, Jan 2015;
- Ugo Boscain gave an invited talk at the Workshop on Infinite-dimensional Riemannian geometry, Wien Austria, Jan 2015;
- Mario Sigalotti gave an invited talk at the conference on Persistence of population models in temporally fluctuating environments, Lausanne, Switzerland, Feb 2015;
- Mario Sigalotti gave an invited talk at the GSSI-GDRE CONEDP workshop in L'Aquila, Italy, Apr 2015;
- Ugo Boscain gave an invited talk at the INDAM meeting "The Hamilton-Jacobi Equation: at the crossroads of PDE, Dynamical Systems and Geometry", Cortona, Italy, Jun 2015;

- Ugo Boscain gave an invited talk at the workshop “From Open to Closed Loop Control”, Mariatrost, Graz, Austria Jun 2015;
- Mario Sigalotti gave an invited talk at the Workshop on Nonlinear Control and Geometry, Bedlewo, Poland, Aug 2015;
- Ugo Boscain gave an invited talk at the Workshop on Analysis and PDE, Leibniz Universität, Hannover, Germany, Sep 2015;
- Ugo Boscain gave an invited talk at the workshop “Stochastic Analysis and Numerical Perspectives”, Inria Sophia Antipolis, Sep 2015;
- Ugo Boscain has been plenary speaker at the “Journées Annuelles 2015 du GdR MOA”, Dijon, Dec 2015.

8.2. Teaching - Supervision - Juries

8.2.1. Supervision

- PhD in progress: Guiherme Mazanti, “Stabilité et taux de convergence pour les systèmes à excitation persistante", started in 1/9/2013, supervisors: Yacine Chitour, Mario Sigalotti.
- PhD in progress: Ludovic Sacchelli, “Sub-Riemannian geometry, hypoelliptic operators, geometry of vision", started in 1/9/2015, supervisors: Ugo Boscain, Mario Sigalotti.
- PhD in progress: Leonardo Suriano, “Conception d'un cortex visuel primaire artificiel", started in 1/10/2015, supervisors: Ugo Boscain, Mario Sigalotti.

8.2.2. Juries

Mario Sigalotti was member of the commission for the PhD defense of Pierre-Olivier Lamare, University of Grenoble, September 2015.

8.3. Popularization

Ugo Boscain gave a popularization conference at the “Rencontres Mathématiques de Rouen” with a genral introduction to sub-Riemannian geometry and its applications, Jun 2015.

9. Bibliography

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