IN PARTNERSHIP WITH:
CNRS

Ecole des Ponts ParisTech

Université Paris-Est Marne-la-Vallée

Activity Report 2015

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris - Rocquencourt

THEME
Stochastic approaches

Table of contents

Project-Team MATHRISK

Creation of the Team: 2012 January 01, updated into Project-Team: 2013 January 01

Keywords:

Computer Science and Digital Science:

6. - Modeling, simulation and control
6.1. - Mathematical Modeling
6.1.2. - Stochastic Modeling (SPDE, SDE)
6.2.1. - Numerical analysis of PDE and ODE
6.2.2. - Numerical probability
6.2.3. - Probabilistic methods
6.4.2. - Stochastic control

Other Research Topics and Application Domains:

3.1. - Sustainable development
9.5.3. - Economy, Finance
9.9. - Risk management

1. Members

Research Scientist

Agnès Sulem [Team leader, Inria, Senior Researcher, HdR]

Faculty Members

Aurélien Alfonsi [ENPC, Professor, HdR]
Vlad Bally [Univ. Paris Est, Professor, HdR]
Damien Lamberton [Univ. Paris Est, Professor, HdR]
Romuald Elie [Univ. Paris Est Marne la Vallée, Professor, HdR]
Benjamin Jourdain [ENPC, Professor, HdR]
Bernard Lapeyre [ENPC, Professor, HdR]

PhD Students

Rui Chen [Ceremade, from October 2014]
Roxana-Larisa Dumitrescu [Ceremade, Univ. Paris-Dauphine, until Sep 2015]

Post-Doctoral Fellows

Arnaud Lionnet [Inria]
Paolo Pigato [Inria, until Jun 2015]

Administrative Assistant

Martine Verneuille [Inria]

Others

Ahmed Kebaier [Univ. Paris XIII, Associate Professor, Partner Researcher]
Céline Labart [Université de Savoie, Associate Professor, Partner Researcher]
Jérôme Lelong [ENSIMAG, Associate Professor, Partner Researcher]
Houzhi Li [Inria, Trainee, from Mar 2015 until Jul 2015]
Ricardo Rincon [Inria, Trainee, from Apr 2015 until Oct 2015]
Jean-Philippe Chancelier [ENPC, Professor, Partner Researcher]
Oleg Kudryavtsev [invited professor, Rostov University, July-August 2015]
Xiao Wei [invited professor, Beijing University, July-August 2015]

Antonino Zanette [Professor, University of Udine, Italy, Partner Researcher]

2. Overall Objectives

2.1. Overall Objectives

MathRisk is a joint Inria project-team with ENPC (CERMICS Laboratory) and the University Paris Est Marne-la-Vallée (UPEMLV, LAMA Laboratory), located in Rocquencourt and Marne-la-Vallée. http://www.inria.fr/en/teams/mathrisk. Mathrisk is based on the former Mathfi project team. Mathfi was founded in 2000, and was devoted to financial mathematics. The project was focused on advanced stochastic analysis and numerical techniques motivated by the development of increasingly complex financial products. Main applications concerned evaluation and hedging of derivative products, dynamic portfolio optimization in incomplete markets, and calibration of financial models.

2.1.1. Crisis, deregulation, and impact on the research in finance

The starting point of the development of modern finance theory is traditionally associated to the publication of the famous paper of Black and Scholes in 1973 [58]. Since then, in spite of sporadic crises, generally well overcome, financial markets have grown in a exponential manner. More and more complex exotic derivative products have appeared, on equities first, then on interest rates, and more recently on credit markets. The period between the end of the eighties and the crisis of 2008 can be qualified as the “golden age of financial mathematics”: finance became a quantitative industry, and financial mathematics programs flourished in top universities, involving seminal interplays between the worlds of finance and applied mathematics. During its 12 years existence, the Mathfi project team has extensively contributed to the development of modeling and computational methods for the pricing and hedging of increasingly complex financial products.

Since the crisis of 2008, there has been a critical reorientation of research priorities in quantitative finance with emphasis on risk. In 2008, the “subprime” crisis has questioned the very existence of some derivative products such as CDS (credit default swaps) or CDOs (collateralized debt obligations), which were accused to be responsible for the crisis. The nature of this crisis is profoundly different from the previous ones. It has negatively impacted the activity on the exotic products in general, - even on equity derivative markets-, and the interest in the modeling issues for these products. The perfect replication paradigm, at the origin of the success of the Black and Scholes model became unsound, in particular through the effects of the lack of liquidity. The interest of quantitative finance analysts and mathematicians shifted then to more realistic models taking into account the multidimensional feature and the incompleteness of the markets, but as such getting away from the “lost paradi(gm)” of perfect replication. These models are much more demanding numerically, and require the development of hedging risk measures, and decision procedures taking into account the illiquidity and various defaults.

Moreover, this crisis, and in particular the Lehman Brothers bankruptcy and its consequences, has underlined a systemic risk due to the strong interdependencies of financial institutions. The failure of one of them can cause a cascade of failures, thus affecting the global stability of the system. Better understanding of these interlinkage phenomena becomes crucial.

At the same time, independently from the subprime crisis, another phenomenon has appeared: deregulation in the organization of stock markets themselves. This has been encouraged by the Markets in Financial Instruments Directive (MIFID) which is effective since November, 1st 2007. This, together with the progress of the networks, and the fact that all the computers have now a high computation power, have induced arbitrage opportunities on the markets, by very short term trading, often performed by automatic trading. Using these high frequency trading possibilities, some speculating operators benefit from the large volatility of the markets. For example, the flash crash of May, 6 2010 has exhibited some perverse effects of these automatic speculating trading strategies. These phenomena are not well understood and the theme of high frequency trading needs to be explored.

To summarize, financial mathematics is facing the following new evolutions:

- the complete market modeling has become unsatisfactory to provide a realistic picture of the market and is replaced by incomplete and multidimensional models which lead to new modeling and numerical challenges.
- quantitative measures of risk coming from the markets, the hedging procedures, and the lack of liquidity are crucial for banks,
- uncontrolled systemic risks may cause planetary economic disasters, and require better understanding,
- deregulation of stock markets and its consequences lead to study high frequency trading.

The project team MathRisk is designed to address these new issues, in particular dependence modeling, systemic risk, market microstructure modeling and risk measures. The research in modeling and numerical analysis remain active in this new context, motivated by new issues.

The MathRisk project team develops the software Premia dedicated to pricing and hedging options and calibration of financial models, in collaboration with a consortium of financial institutions. https://www.rocq.inria.fr/mathfi/Premia/index.html.

The MathRisk project is part of the Université Paris-Est **"Labex" BÉZOUT**.

3. Research Program

3.1. Dependence modeling

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Bernard Lapeyre.

The volatility is a key concept in modern mathematical finance, and an indicator of the market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling has thus become a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling. The calibration of models for a single asset can now be well managed by banks but modeling of dependence is the bottleneck to efficiently aggregate such models. A typical issue is how to go from the individual evolution of each stock belonging to an index to the joint modeling of these stocks. In this perspective, we want to model stochastic volatility in a *multidimensional* framework. To handle these questions mathematically, we have to deal with stochastic differential equations that are defined on matrices in order to model either the instantaneous covariance or the instantaneous correlation between the assets. From a numerical point of view, such models are very demanding since the main indexes include generally more than thirty assets. It is therefore necessary to develop efficient numerical methods for pricing options and calibrating such models to market data. As a first application, modeling the dependence between assets allows us to better handle derivatives products on a basket. It would give also a way to price and hedge consistenly single-asset and basket products. Besides, it can be a way to capture how the market estimates the dependence between assets. This could give some insights on how the market anticipates the systemic risk.

3.2. Liquidity risk

Participants: Aurélien Alfonsi, Agnès Sulem, Antonino Zanette.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the liquidity risk. Loosely speaking, liquidity risk is the risk that comes from the difficulty of selling (or buying) an asset. At the extreme, this may be the impossibility to sell an asset, which occurred for “junk assets” during the subprime crisis. Hopefully, it is in general possible to sell assets, but this may have some cost. Let us be more precise. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price. Instead, if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), this may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e. how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

3.2.1. Long term liquidity risk.

On a long-term perspective, illiquidity can be approached via various ways: transactions costs [50], [51], [57], [64], [67], [83], [79], delay in the execution of the trading orders [84], [82], [60], trading constraints or restriction on the observation times (see e.g. [66] and references herein). As far as derivative products are concerned, one has to understand how delta-hedging strategies have to be modified. This has been considered for example by Cetin, Jarrow and Protter [81]. We plan to contribute on these various aspects of liquidity risk modeling and associated stochastic optimization problems. Let us mention here that the price impact generated by the trades of the investor is often neglected with a long-term perspective. This seems acceptable since the investor has time enough to trade slowly in order to eliminate its market impact. Instead, when the investor wants to make significant trades on a very short time horizon, it is crucial to take into account and to model how prices are modified by these trades. This question is addressed in the next paragraph on market microstructure.

3.2.2. Market microstructure.

The European directive MIFID has increased the competition between markets (NYSE-Euronext, Nasdaq, LSE and new competitors). As a consequence, the cost of posting buy or sell orders on markets has decreased, which has stimulated the growth of market makers. Market makers are posting simultaneously bid and ask orders on a same stock, and their profit comes from the bid-ask spread. Basically, their strategy is a “round-trip” (i.e. their position is unchanged between the beginning and the end of the day) that has generated a positive cash flow.

These new rules have also greatly stimulated research on market microstructure modeling. From a practitioner point of view, the main issue is to solve the so-called “optimal execution problem”: given a deadline T, what is the optimal strategy to buy (or sell) a given amount of shares that achieves the minimal expected cost? For large amounts, it may be optimal to split the order into smaller ones. This is of course a crucial issue for brokers, but also market makers that are looking for the optimal round-trip.

Solving the optimal execution problem is not only an interesting mathematical challenge. It is also a mean to better understand market viability, high frequency arbitrage strategies and consequences of the competition between markets. For example when modeling the market microstructure, one would like to find conditions that allow or exclude round trips. Beyond this, even if round trips are excluded, it can happen that an optimal selling strategy is made with large intermediate buy trades, which is unlikely and may lead to market instability.

We are interested in finding synthetic market models in which we can describe and solve the optimal execution problem. A. Alfonsi and A. Schied (Mannheim University) [52] have already proposed a simple Limit Order Book model (LOB) in which an explicit solution can be found for the optimal execution problem. We are now interested in considering more sophisticated models that take into account realistic features of the market such as short memory or stochastic LOB. This is mid term objective. At a long term perspective one would like to bridge these models to the different agent behaviors, in order to understand the effect of the different quotation mechanisms (transaction costs for limit orders, tick size, etc.) on the market stability.

3.3. Contagion modeling and systemic risk

Participants: Benjamin Jourdain, Agnès Sulem.

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these. Ideally, one would like to be able to find capital requirements (such as the one proposed by the Basel committee) that ensure that the probability of multiple defaults is below some level.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [53] and A. Minca [73]) or mean field interaction models (Garnier-Papanicolaou-Yang [65]). The recent approach in [53] seems very promising. It describes the financial network approach as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. Distress propagation in a financial system may be modeled as an epidemics on this graph. In the case of incomplete information on the structure of the interbank network, cascade dynamics may be reduced to the evolution of a multi-dimensional Markov chain that corresponds to a sequential discovery of exposures and determines at any time the size of contagion. Little has been done so far on the *control* of such systems in order to reduce the systemic risk and we aim to contribute to this domain.

3.4. Stochastic analysis and numerical probability

3.4.1. Stochastic control

Participants: Vlad Bally, Jean-Philippe Chancelier, Marie-Claire Quenez, Agnès Sulem.

The financial crisis has caused an increased interest in mathematical finance studies which take into account the market incompleteness issue and the default risk modeling, the interplay between information and performance, the model uncertainty and the associated robustness questions, and various nonlinearities. We address these questions by further developing the theory of stochastic control in a broad sense, including stochastic optimization, nonlinear expectations, Malliavin calculus, stochastic differential games and various aspects of optimal stopping.

3.4.2. Optimal stopping

Participants: Aurélien Alfonsi, Benjamin Jourdain, Damien Lamberton, Agnès Sulem, Marie-Claire Quenez.

The theory of American option pricing has been an incite for a number of research articles about optimal stopping. Our recent contributions in this field concern optimal stopping in models with jumps, irregular obstacles, free boundary analysis, reflected BSDEs.

3.4.3. Simulation of stochastic differential equations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Vlad Bally, Damien Lamberton, Bernard Lapeyre, Jérôme Lelong, Céline Labart.

Effective numerical methods are crucial in the pricing and hedging of derivative securities. The need for more complex models leads to stochastic differential equations which cannot be solved explicitly, and the development of discretization techniques is essential in the treatment of these models. The project MathRisk addresses fundamental mathematical questions as well as numerical issues in the following (non exhaustive) list of topics: Multidimensional stochastic differential equations, High order discretization schemes, Singular stochastic differential equations, Backward stochastic differential equations.

3.4.4. Monte-Carlo simulations

Participants: Benjamin Jourdain, Aurélien Alfonsi, Damien Lamberton, Vlad Bally, Bernard Lapeyre, Ahmed Kebaier, Céline Labart, Jérôme Lelong, Antonino Zanette.

Monte-Carlo methods is a very useful tool to evaluate prices especially for complex models or options. We carry on research on *adaptive variance reduction methods* and to use *Monte-Carlo methods for calibration* of advanced models.

This activity in the MathRisk team is strongly related to the development of the Premia software.

3.4.5. Malliavin calculus and applications in finance

Participants: Vlad Bally, Arturo Kohatsu-Higa, Agnès Sulem, Antonino Zanette.

The original Stochastic Calculus of Variations, now called the Malliavin calculus, was developed by Paul Malliavin in 1976 [71]. It was originally designed to study the smoothness of the densities of solutions of stochastic differential equations. One of its striking features is that it provides a probabilistic proof of the celebrated Hörmander theorem, which gives a condition for a partial differential operator to be hypoelliptic. This illustrates the power of this calculus. In the following years a lot of probabilists worked on this topic and the theory was developed further either as analysis on the Wiener space or in a white noise setting. Many applications in the field of stochastic calculus followed. Several monographs and lecture notes (for example D. Nualart [74], D. Bell [56] D. Ocone [76], B. Øksendal [85]) give expositions of the subject. See also V. Bally [54] for an introduction to Malliavin calculus.

From the beginning of the nineties, applications of the Malliavin calculus in finance have appeared : In 1991 Karatzas and Ocone showed how the Malliavin calculus, as further developed by Ocone and others, could be used in the computation of hedging portfolios in complete markets [75].

Since then, the Malliavin calculus has raised increasing interest and subsequently many other applications to finance have been found [72], such as minimal variance hedging and Monte Carlo methods for option pricing. More recently, the Malliavin calculus has also become a useful tool for studying insider trading models and some extended market models driven by Lévy processes or fractional Brownian motion.

We give below an idea why Malliavin calculus may be a useful instrument for probabilistic numerical methods.

We recall that the theory is based on an integration by parts formula of the form $E(f'(X)) = E(f(X)Q)$. Here X is a random variable which is supposed to be "smooth" in a certain sense and non-degenerated. A basic example is to take $X = \sigma\Delta$ where Δ is a standard normally distributed random variable and σ is a strictly positive number. Note that an integration by parts formula may be obtained just by using the usual integration by parts in the presence of the Gaussian density. But we may go further and take X to be an aggregate of Gaussian random variables (think for example of the Euler scheme for a diffusion process) or the limit of such simple functionals.

An important feature is that one has a relatively explicit expression for the weight Q which appears in the integration by parts formula, and this expression is given in terms of some Malliavin-derivative operators.

Let us now look at one of the main consequences of the integration by parts formula. If one considers the *Dirac* function $\delta_x(y)$, then $\delta_x(y) = H'(y-x)$ where H is the *Heaviside* function and the above integration by parts formula reads $E(\delta_x(X)) = E(H(X-x)Q)$, where $E(\delta_x(X))$ can be interpreted as the density of the random variable X. We thus obtain an integral representation of the density of the law of X. This is the starting point of the approach to the density of the law of a diffusion process: the above integral representation allows us to prove that under appropriate hypothesis the density of X is smooth and also to derive upper and lower bounds for it. Concerning simulation by Monte Carlo methods, suppose that you want to compute $E(\delta_x(y)) \sim \frac{1}{M}\sum_{i=1}^{M}\delta_x(X^i)$ where $X^1, ..., X^M$ is a sample of X. As X has a law which is absolutely continuous with respect to the Lebesgue measure, this will fail because no X^i hits exactly x. But if you are able to simulate the weight Q as well (and this is the case in many applications because of the explicit form mentioned above) then you may try to compute $E(\delta_x(X)) = E(H(X-x)Q) \sim \frac{1}{M}\sum_{i=1}^{M}E(H(X^i-x)Q^i)$. This basic remark formula leads to efficient methods to compute by a Monte Carlo method some irregular quantities as derivatives of option prices with respect to some parameters (the *Greeks*) or conditional expectations, which appear in the pricing of American options by the dynamic programming). See the papers by Fournié et al [63] and [62] and the papers by Bally et al., Benhamou, Bermin et al., Bernis et al., Cvitanic et al., Talay and Zheng and Temam in [70].

L. Caramellino, A. Zanette and V. Bally have been concerned with the computation of conditional expectations using Integration by Parts formulas and applications to the numerical computation of the price and the Greeks (sensitivities) of American or Bermudean options. The aim of this research was to extend a paper of Reigner and Lions who treated the problem in dimension one to higher dimension - which represent the real challenge in this field. Significant results have been obtained up to dimension 5 [55] and the corresponding algorithms have been implemented in the Premia software.

Moreover, there is an increasing interest in considering jump components in the financial models, especially motivated by calibration reasons. Algorithms based on the integration by parts formulas have been developed in order to compute Greeks for options with discontinuous payoff (e.g. digital options). Several papers and two theses (M. Messaoud and M. Bavouzet defended in 2006) have been published on this topic and the corresponding algorithms have been implemented in Premia. Malliavin Calculus for jump type diffusions - and more general for random variables with locally smooth law - represents a large field of research, also for applications to credit risk problems.

The Malliavin calculus is also used in models of insider trading. The "enlargement of filtration" technique plays an important role in the modeling of such problems and the Malliavin calculus can be used to obtain general results about when and how such filtration enlargement is possible. See the paper by P. Imkeller in [70]). Moreover, in the case when the additional information of the insider is generated by adding the information about the value of one extra random variable, the Malliavin calculus can be used to find explicitly the optimal portfolio of an insider for a utility optimization problem with logarithmic utility. See the paper by J.A. León, R. Navarro and D. Nualart in [70]).

A. Kohatsu Higa and A. Sulem have studied a controlled stochastic system whose state is described by a stochastic differential equation with anticipating coefficients. These SDEs can be interpreted in the sense of *forward integrals*, which are the natural generalization of the semimartingale integrals, as introduced by Russo and Valois [78]. This methodology has been applied for utility maximization with insiders.

4. Application Domains

4.1. Application Domains

Risk management, Quantitative finance, Computational Finance, Market Microstructure analysis, Systemic risk, Portfolio optimization, Risk modeling, Option pricing and hedging in incomplete markets, insurance.

5. Highlights of the Year

5.1. Highlights of the Year

Conference in honor of Vlad Bally for his 60th birthday, Le Mans, October 6-9 2015 http://www.cmap.polytechnique.fr/~demarco/files/pageWebConfV/ConferenceVladBally.html

5.1.1. Awards

J. Reygner received the 2014 Jacques Neveu prize for his thesis entitled "Longtime behaviour of particle systems : applications in physics, finance and PDEs" co-supervised by B.Jourdain and L. Zambotti

6. New Software and Platforms

6.1. PREMIA

KEYWORDS: Computational finance - Option pricing

SCIENTIFIC DESCRIPTION

Premia is a software designed for option pricing, hedging and financial model calibration. It is provided with it's C/C++ source code and an extensive scientific documentation. The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research & Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: 1 - to be a powerful testing platform for comparing different numerical methods between each other, 2 - to build a link between professional financial teams and academic researchers, 3 - to provide a useful teaching support for Master and PhD students in mathematical finance.

FUNCTIONAL DESCRIPTION

- Participants: Mathrisk project team and contributors
- Partners: Inria - Ecole des Ponts ParisTech - Université Paris-Est - Consortium Premia
- Contact: Agnès Sulem
- URL: http://www.premia.fr
- AMS: 91B28;65Cxx;65Fxx;65Lxx;65Pxx
- License: Licence Propriétaire (genuine license for the Consortium Premia)
- Type of human computer interaction: Console, interface in Nsp, Web interface
- OS/Middelware: Linux, Mac OS X, Windows
- APP: The development of Premia started in 1999 and 16 are released up to now and registered at the APP agency. Premia 16 has been registered on 0303/2015 under the number IDDN.FR.001.190010.013.S.C.2001.000.31000
- Programming language: C/C++ librairie Gtk
- Documentation: the PNL library is interfaced via doxygen
- Size of the software: 280580 lines for the Src part only, that is 11 Mbyte of code, 130400 lines for PNL, 105 Mbyte of PDF files of documentation.
- interfaces : Nsp for Windows/Linux/Mac, Excel, binding Python, and a Web interface.
- Publications: [12], [61], [69], [77], [80], [49], [59].

6.1.1. Content of Premia

Premia contains various numerical algorithms (Finite-differences, trees and Monte-Carlo) for pricing vanilla and exotic options on equities, interest rate, credit and energy derivatives.

1. **Equity derivatives:**

   The following models are considered:

   Black-Scholes model (up to dimension 10), stochastic volatility models (Hull-White, Heston, Fouque-Papanicolaou-Sircar), models with jumps (Merton, Kou, Tempered stable processes, Variance gamma, Normal inverse Gaussian), Bates model.

For high dimensional American options, Premia provides the most recent Monte-Carlo algorithms: Longstaff-Schwartz, Barraquand-Martineau, Tsitsklis-Van Roy, Broadie-Glassermann, quantization methods and Malliavin calculus based methods.

Dynamic Hedging for Black-Scholes and jump models is available.

Calibration algorithms for some models with jumps, local volatility and stochastic volatility are implemented.

2. **Interest rate derivatives**

   The following models are considered:

   HJM and Libor Market Models (LMM): affine models, Hull-White, CIR++, Black-Karasinsky, Squared-Gaussian, Li-Ritchken-Sankarasubramanian, Bhar-Chiarella, Jump diffusion LMM, Markov functional LMM, LMM with stochastic volatility.

   Premia provides a calibration toolbox for Libor Market model using a database of swaptions and caps implied volatilities.

3. **Credit derivatives: Credit default swaps (CDS), Collateralized debt obligations (CDO)**

   Reduced form models and copula models are considered.

   Premia provides a toolbox for pricing CDOs using the most recent algorithms (Hull-White, Laurent-Gregory, El Karoui-Jiao, Yang-Zhang, Schönbucher)

4. **Hybrid products**

   A PDE solver for pricing derivatives on hybrid products like options on inflation and interest or change rates is implemented.

5. **Energy derivatives: swing options**

   Mean reverting and jump models are considered.

   Premia provides a toolbox for pricing swing options using finite differences, Monte-Carlo Malliavin-based approach and quantization algorithms.

6.1.2. Premia design

To facilitate contributions, a standardized numerical library (PNL) has been developed by J. Lelong under the LGPL since 2009, which offers a wide variety of high level numerical methods for dealing with linear algebra, numerical integration, optimization, random number generators, Fourier and Laplace transforms, and much more. Everyone who wishes to contribute is encouraged to base its code on PNL and providing such a unified numerical library has considerably eased the development of new algorithms which have become over the releases more and more sophisticated. J. Ph Chancelier, B. Lapeyre and J. Lelong are using Premia and Nsp for Constructing a Risk Management Benchmark for Testing Parallel Architecture [59].

Development of the PNL in 2015 (J. Lelong) . Release 1.70 and 1.71, PNL Library (http://pnl.gforge.inria.fr).

1. Release 1.72. of the *PNL* library (http://pnl.gforge.inria.fr/).
   1. Addition of a CMake module to include the library in other projects.
   2. Improvement of the `pnl_basis` module.
   3. Addition of the non central chi squared distribution to the random number generation toolbox.
   4. Addition of new functions in the linear algebra toolbox to build views.

6.1.3. Algorithms implemented in Premia in 2015

Premia 17 has been delivered to the consortium members in March 2015.

It contains the following new algorithms:

6.1.3.1. Commodities, FX, Insurance, Credit Risk

- Variables Annuities GLWB pricing in the Heston and Black-Scholes/Hull-White models with finite difference techniques.
- Variables Annuities GMAB, GMDB, GMMB pricing with Fourier-cosine techniques.
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- An Efficient Binomial Lattice Method for Step Double Barrier Options. E.Appolloni, M.Gaudenzi A.Zanette.
  International Journal of Applied and Theoretical Finance Vol.17, Issue No. 6, 2014.

The algorithms

- “Pricing American-Style Options by Monte Carlo Simulation : Alternatives to Ordinary Least Squares” by Stathis Tompaidis and Chunyu Yang
- “Value Function Approximation or Stopping Time Approximation : A comparison of Two Recent Numerical Methods for American Option Pricing using Simulation and Regression” by Lars Stentoft

implemented in 2015 by CélineLabart will be included in the following release.

Moreover, Jérome Lelong has performed the following tasks:

1. Add an importance sampling based code for jump diffusion models.
2. Improve the internal enumeration mechanism (PremiaEnum).
3. Update gnuplot files generation for reports.
4. Everyday maintenance to fix various bugs.
5. Clean the generation of the documentation process.

7. New Results

7.1. Liquidity risk

Participants: Aurélien Alfonsi, Pierre Blanc.

A. Alfonsi and P. Blanc are working on the optimal execution problem when many large traders who modify the market prices. In a previous study, they have developed a price impact model that takes into account an exogeneous flow of market orders, in which the optimal execution strategy is known explicitly. This year,they have worked on the practical implementation of this model. Namely, they have proposed an estimation procedure to estimate the model parameters (decay kernel of the price impact and Hawkes kernel for the self excitation of the order flow). They have run this estimation on market data and backtested the optimal execution strategy.

7.2. Backward stochastic (partial) differential equations with jumps, optimal stopping and stochastic control with nonlinear expectation, risk minimization

Participants: Roxana Dumitrescu, Marie-Claire Quenez [(Univ Paris 7)], Arnaud Lionnet, Agnès Sulem.

R. Dumitrescu, M.C. Quenez and A. Sulem have provided a weak dynamic principle for Combined Optimal Stopping/Stochastic Control with $\mathcal{E}^f-$conditional Expectation . They have investigated the links between generalized Dynkin games and double barriers reflected BSDEs with jumps and also studied mixed generalized Dynkin games in a Markovian framework and associated nonlinear HJB equations with barriers .

In the recent paper [43], they study game options in an imperfect market with default. They extend the results obtained by Kifer [68] in a perfect market model to the case of imperfections taken into account via the nonlinearity of the wealth dynamics. In this framework, the pricing system is expressed as a nonlinear g-expectation/evaluation induced by a nonlinear BSDE with jump. They prove that the superhedging price of a game option coincides with the value function of a corresponding *generalized* Dynkin game expressed in terms of the g-evaluation. They also address the case of ambiguity on the model, - for example an ambiguity on the default probability -, and characterize the superhedging price of a game option as the value function of a *mixed generalized* Dynkin game. They prove the existence of a cancellation time and a trading strategy which allows the seller to be super-hedged, whatever the model is. This study is introduced by the analysis of the simpler case of American options.

In collaboration with Jane Bielagt (Humbold Univ.) and Gonąlo Dos Reis (Univ. of Edimburgh), Arnaud Lionnet investigates in the effects of the social interactions of a finite set of agents on an equilibrium pricing mechanism. They consider an incomplete market where agents invest so as to minimize their risk measure. Here, agents assess risk using convex dynamic risk measures expressed by Backward Stochastic Differential Equations (BSDE). Beside the risk associated with their own economic activity, the agents compare their trading gains to that of the others, and factor this relative performance in the evaluation of their risk/satisfaction. When a derivative product is introduced to complete the market and allow agents to trade a non-financial risk factor (such as temperature), the risk of each agent is lowered, as expected. However, agents then find it in their interest to be more concerned with their relative performance. This leads them to behave more like a herd and this destabilizes the previously stable, purely financial market.

7.3. Systemic risk

Participants: Hamed Amini [EPFL], Andreea Minca [Cornell University)], Agnès Sulem, Rui Chen, Romuald Elie.

We study the issue of control of systemic risk in the framework of random graph models. The paper [16] by H. Amini, A. Minca and A. Sulem, provides important insight on the relation between the value of a financial system, connectivity and optimal intervention. More precisely, we consider a core-periphery random financial network in which links lead to the creation of projects in the outside economy but make banks prone to contagion risk. The controller seeks to maximize, under budget constraints, the value of the financial system, defined as the total value of the projects funded. Under partial information on interbank links, revealed in conjunction with the spread of contagion, the optimal control problem is shown to become a Markov decision problem. Our results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. This insight shows that it is far from obvious that connectivity of a core bank should always be brought forward as an argument for priority intervention and it may be sometimes preferable to invest in non-core banks that lend directly to the economy. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity and how to design a guarantee fund that would represent an intervention fund that can be used to maximize the benefits of connectivity. This is under study with the PhD student Rui Chen.

Moreover R. Elie obtained a CVRS PEPS grant on systemic risk modeling with graphs in collaboration with the Inria team COATI and the economic department of Université de Nice.

7.4. Dependence modeling

7.4.1. Estimation of the parameters of a Wishart process

A.Alfonsi with A. Kebaier and C. Rey have studied the Maximum Likelihood Estimator for the Wishart processes and in particular its convergence in the ergodic and in some non ergodic cases. In the non ergodic cases, their analysis rely on refined results on the Laplace transform for Wishart processes. This work also extends a recent paper by Ben Alaya and Kebaier on the maximum likelihood estimation for the CIR process.

7.5. Interest rate modeling

A. Alfonsi, E. Palidda and A. Ahdida extend the Linear Gaussian Model (LGM) by replacing the constant convariation matrix by some Wishart dynamics. This extension allows them to generate smile while keeping the affine structure of the model. They have obtained a price expansion around the LGM for Caplet and Swaption prices. They also present a second order discretization scheme that allow them to compute exotic prices within this model.

7.6. Numerical Probability

A. Alfonsi with A. Kohatsu-Higa and M. Hayashi are investigating how to apply the parametric method recently proposed by V. Bally and A. Kohatsu-Higa for reflected SDEs. This method allows them to obtain an unbiased estimator for expectations of general functions of the process.

7.7. Optimal transport

Participant: Benjamin Jourdain.

With J. Corbetta (postdoc financed by the chair financial risks), A. Alfonsi and B. Jourdain study a general formula for the time-derivative of the wasserstein distance between the time-marginals of two Markov processes. They have checked the validity of this formula for pure-jump Markov processes with a bounded intensity of jumps. They now study the extension to piecewise deterministic Markov processes.

7.8. Multitype sticky particle systems

Participant: Benjamin Jourdain.

B. Jourdain and J. Reygner study multitype sticky particle systems which can be obtained as vanishing noise limits of multitype rank-based diffusions. Rank-based diffusion processes and their multitype generalization permit to reproduce empirical features of stock markets. B. Jourdain and J. Reygner have obtained the optimal rate of convergence as the number of particles grows to infinity of the approximate solution to a diagonal system of hyperbolic conservation laws based on multitype sticky particles.

7.9. Numerical Probability

7.9.1. American option pricing.

Damien Lamberton with M. Pistorius has worked on the approximation of American options by Canadian options, which originated from the work of Peter Carr. This lead them to revise old results on the binomial approximation of the American put. D. Lamberton is also working with M. Zervos on American options involving the maximum of the underlying.

7.9.2. Convergence in total variation of approximation schemes for Markov processes

(V. Bally and PhD student C. Rey [40])

The main issue was to consider very general approximation schemes and to estimate the approximation error for test functions which are just measurable and bounded. It is worth to mention that the input of noise in the approximation schemes is allowed to be quite general, while in the standard approximation schemes for diffusion processes one considers Gaussian input only. In some sense this means that we treat invariance principles as well. We also considered approximation schemes of higher order, as the Victoir Nynomia scheme for example. An important ingredient is an abstract Malliavin calculus for general random variables (which has been settled in previous papers of V. Bally and Lucia Caramellino.

7.9.3. Approximation schemes for Piecewise Deterministic Markov Processes

(V. Bally and PhD student V.Rabiet [39]).

PDMP processes are very popular in many practical fields as biology, chemistry or fiability theory. The main idea is that such a model may present different scales: slow ones and rapid onces. And from a numerical point of view it is extremely difficult to implement algorithms which take care of rapid scales in details. Then the idea is to average the rapid scales (in the spirit of the Central Limit Theorem) and consequently to replace small (and rapid) jumps by a Brownien component. This procedure is already widely used by practitioners. Our work was to derive estimates of the error which is done by this procedure.

7.9.4. Convergence in distribution norms in the Central Limit Theorem

(V. Bally with Lucia Caramellino and Guillaume Poly)

In the classical theory, the convergence which has already been studied is the convergence in total variation (measurable test functions). The main result is the theorem of Prohorov, in the fifties. We have proved that under similar hypothesis (with more finite moments however) one may obtain a much more accurate estimate of the error, in some norms which are close to distribution norms. As a remarkable consequence, we obtained a CLT for the zeros of trigonometric polynomials with random coefficients.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

- Consortium PREMIA, Natixis - Inria
- Consortium PREMIA, Crédit Agricole CIB - Inria

8.2. Bilateral Grants with Industry

- Chaire X-ENPC-UPMC-Société Générale "Financial Risks" of the Risk fondation : A. Alfonsi, B. Jourdain, B. Lapeyre
- A. Sulem: Research Grant for the project "*Stochastic Control of Systemic Risk*", awarded by the scientific council and Professional Fellows of Institut Europlace de Finance (EIF) and Labex Louis Bachelier with A. Minca (Cornell University).
- R. Elie with Arthur Charpentier:

  Chaire with COVEA on digital impact in actuarial activities (2015-2018).

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR

- ANR Stab 2013-2016, Participant : B. Jourdain, Partners : Lyon 1, Paris-Dauphine
- ANR Cosmos 2015-2018, Participant: B. Jourdain ; Partners : Ecole des Ponts, Telecom, INIRIA Rennes and IBPC

9.1.2. Competitivity Clusters

Pôle Finance Innovation.

9.2. International Initiatives

9.2.1. Inria International Partners

9.2.1.1. Informal International Partners

- Center of Excellence program in Mathematics and Life Sciences at the Department of Mathematics, University of Oslo, Norway, (B. Øksendal).
- Department of Mathematics, University of Manchester (Tusheng Zhang, currently in charge of an EU-ITN program on BSDEs and Applications).
- Kensas University (Yaozhong Hu)
- Mannheim University (Alexander Schied, Chair of Mathematics in Business and Economics, Department of Mathematics)
- Roma Tor Vergata University (Lucia Caramellino)
- Ritsumeikan University (A. Kohatsu-Higa).

9.3. International Research Visitors

9.3.1. Visits of International Scientists

- Oleg Kudryavtsev, Rostov University (Russia), 2 months
- Xiao Wei, Beijing university, 2 months

9.3.1.1. Internships

- Houzhi Li (April to July 2015): Study and implementation in Premia of the 4/2 stochastic volatility model proposed by M. Grasselli; adviser A. Alfonsi

9.3.2. Visits to International Teams

9.3.2.1. Research stays abroad

- A. Alfonsi IPAM, UC Los Angeles , invited by René Carmona (April 13- 23)
- A. Sulem: Participation to the "Stochastics in Environmental and Financial Economics" program, Centre of Advanced Studies of the Norwegian Academy of Sciences and Letters, Oslo,Spring 2015.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific events organisation

- A. Alfonsi: Co-organizer of the working group seminar of MathRisk “Méthodes stochastiques et finance”. http://cermics.enpc.fr/~alfonsi/GTMSF.html
- R. Dumitrescu: Co-organizer of the young researchers in Mathematics Seminar of Université Paris Dauphine.
- B. Jourdain (with B. Bouchard and E. Gobet): organization of the 2015-2016 thematic semester on Monte Carlo methods financed by the Institute Louis Bachelier.

10.1.2. Journal

10.1.2.1. Member of the editorial boards

- R. Elie

  Associate editor of *SIAM Journal on Financial Mathematics (SIFIN)* (since November 2014)
- D. Lamberton

  Associate editor of
  - Mathematical Finance,
  - Associate editor of ESAIM Probability & Statistics
- A. Sulem

  Associate editor of
  - 2011- Present: *Journal of Mathematical Analysis and Applications (JMAA)*
  - 2009- Present: *International Journal of Stochastic Analysis (IJSA)*
  - 2008- Present: *SIAM Journal on Financial Mathematics (SIFIN)*

10.1.2.2. Reviewer - Reviewing activities

The members of the team reviewed numerous papers for numerous journals.

10.1.3. Invited talks

- A. Alfonsi

  - "Dynamic optimal execution in a mixed–market–impact Hawkes price model", Workshop on "The Mathematics of High Frequency Financial Markets", IPAM, Los Angeles, April 13 2015:

  - "Optimal transport bounds between the time-marginals of a multidimensional diffusion and its Euler scheme", AMS-EMS-SPM conference, Porto, June 10 2015:

  - "Dynamic optimal execution in a mixed–market–impact Hawkes price model", séminaire de la chaire Risques financiers, June 12 2015:

  - "Optimal execution in a Hawkes price model", London-Paris Bachelier Workshop on Mathematical Finance 2015, Sept 26 2015

- R. Dumitrescu

  - Financial Mathematics Seminar, Univ Marne-la-Vallée, ENPC and Inria, 12 January

  - Stochastic Analysis Seminar, Imperial College London, 17 March

  - Financial Mathematics Seminar, Univ Evry, 26 March.
- B. Jourdain

  - Conference in honor of Vlad Bally, le Mans, 6-9 October 2015 : Strong convergence properties of the Ninomiya Victoir scheme and applications to multilevel Monte Carlo

  - Workshop on NASPDE 2015, Inria Sophia, 22-23 September 2015 : Multitype sticky particles and diagonal hyperbolic systems of PDEs

  - Workshop BIRS Free-energy calculations, a mathematical perspective, Oaxaca 19-24 july 2015 : Analysis of discrete space versions of the self-healing umbrella sampling and well-tempered metadynamics algorithms

  - Workshop Probabilistic Numerical Methods for Non-Linear PDEs, Imperial College London , 29 june-1st july 2015 : On a stochastic particle approximation of the Keller-Segel equation - Workshop Numerical Probability and Applications to Finance, Enit Tunis, 30 april 2015 : Strong convergence properties of the Ninomiya Victoir scheme and applications to multilevel Monte Carlo

  - Maxwell Colloquium, Edimbourg, 6 february 2015 : Multitype sticky particles and diagonal hyperbolic systems of PDEs
- C. Labart : **MCM2015**, Linz, Austria, July 2015.
- J. Lelong : **MCM2015**, Linz, Austria, July 2015.
- D. Lamberton:

  - Workshop Numerical Probability and Applications to Finance, Tunis, April 2015. "On the Canadian approximation of the American put".

  - Workshop in honor of Vlad Bally, Stochastic Calculus, Monte Carlo Methods and Mathematical Finance, Le Mans October 2015. "On the binomial approximation of the American put".
- R. Elie

  - 7th general AMAMEF and Swissquote conference, Lausanne, 2015

  - Workshop Advanced methods in Mathematical Finance, Angers, France, 2015

  - Workshop on current challenges in financial mathematics and economics, London, 2015

  - Workshop on smart energy and stochastic optimization, ENPC, Paris, 2015

  - South East Asia conference in Mathematical Finance, Siem Reap, Cambodia, 2015

  - 9^{th} Bachelier Colloquium, Metabief, France, 2015

  - seminar, Imperial College, London, 2015
- A. Lionnet - MS-EMS-SPM International Meeting, special session on Stochastic Numerical Methods for Non-linear Equations. Porto, June 10-13.

  - 2nd IMA conference on mathematics in finance. Manchester, 18–19 June.

  - Workshop on Probabilistic numerical methods for nonlinear PDEs at Imperial College. London, June 29 – July 1.

  - The 2015 conference on Stochastic Processes and Applications. Oxford, July 13–17.
- A. Sulem

  - International Conference "Stochastics and Computational Finance 2015- from academia to industry", July 6-10, 2015, ISEG - University of Lisbon, keynote speaker, http://www.scf2015.com

- Conference on Mathematical Finance and Partial Differential Equations, Rutgers University, New Brunswick, New Jersey, May 2015, Plenary speaker, http://www.finmath.rutgers.edu/mfpde2015/

- Second conference on " Stochastics of Environmental and Financial Economics", April 20-24, 2015, Academy of Science in Oslo, Plenary speaker,
http://www.mn.uio.no/math/english/research/groups/stochastic-analysis/events/conferences/StochEnviron-april2015/index.html

- Conference in honor of Prof. V. Bally, "Stochastic Calculus and Malliavin Calculus, Monte-Carlo Methods and Mathematical Finance", Université du Mans, 6-9 Octobre 2015. http://lmm.univ-lemans.fr/spip.php?article129

10.1.4. Research administration

- A. Alfonsi: In charge of the Master “Finance and Application” at the Ecole des Ponts.
- V. Bally: Member of the scientific committee of Université Paris-Est Marne-la-Vallée; member of the scientific committee and responsible of the theme "Stochastic equations and PDEs" of ; Responsible of the Probability team of LAMA (until April) and organizer of the seminar of the LAMA laboratory. Labex Bezout;
- R. Dumitrescu : creation of the ALUMNI of *Fondation des Sciences Mathematiques de Paris*
- B. Jourdain: Head of the doctoral school MSTIC, University Paris-Est
- D. Lamberton: Vice-president for research at Université Paris-Est Marne-la-Vallée
- A. Sulem: member of the Committee for technology development , Inria Paris-Rocquencourt

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Undergraduate programs

A. Alfonsi: “Modéliser, Programmer et Simuler”, second year course at the Ecole des Ponts.

R. Dumitrescu, Applied courses (Travaux Dirigés) "introduction to finance", L2, Université Paris Dauphine

R. Elie

- Machine learning for actuarial sciences , Institut des actuaires français

- Arbitrage et valorisation d’options, ENSAE

- Introduction to Python, TD L2 Université Paris-Est

B. Jourdain : - course "Probability theory and statistics", first year ENPC

- "Introduction to probability theory", 1st year, Ecole Polytechnique

- "Stochastic numerical methods", 3rd year, Ecole Polytechnique

- projects in finance and numerical methods, 3rd year, Ecole Polytechnique

A. Lionnet

Stochastic Processes, ESIEA, Paris , 18h .

Graduate programs

A. Alfonsi:

- “Calibration, Volatilité Locale et Stochastique”, third-year course at ENSTA (Master with University Paris I).

- “Traitement des données de marché : aspects statistiques et calibration”, lecture for the Master at UPEMLV.

- A. Alfonsi: “Mesures de risque”, Master course of UPEMLV and Paris VI.

V. Bally,

- Advanced Probability (Master 1 Recherche Mathématiques et Application, University of Marne-la-Vallée

- "The Malliavin calculus and applications in finance", 30h, Master 2 Mathematical Finance, Université Marne la Vallée

- V. Bally, " Interest Rates", 20h, Master 2 Mathematical Finance , Université Marne la Vallée

- V. Bally, " Risk methodes in actuarial science", 36h, Master IMIS, Université Marne la Vallée

R. Dumitrescu, Applied courses (Travaux Dirigés) in " Asset pricing by absence of arbitrage opportunity", Université Paris Dauphine

R. Elie - Imperfect markets modeling M2 Master MASEF (Paris-Dauphine)

- Machine learning and applications, Master 2 Mathematics, , ENPC

- Portfolio management and risk measures, Master Actuariat, Université Paris-Est

- Arbitrage, volatility and portfolio management, Master 2 Math. Finance, Université Paris-Est

B. Jourdain, B. Lapeyre "Monte-Carlo methods", 3rd year ENPC and Master Recherche Mathématiques et Application, University of Marne-la-Vallée

A. Sulem

- "Finite difference for PDEs in Finance", Master 2 MASEF, Université Paris IX-Dauphine, Département Mathématiques et Informatique de la Décision et des Organisations (MIDO), 18h.

- Master of Mathematics, University of Luxembourg, 22 h lectures and responsible of the module "Numerical Methods in Finance".

10.2.2. Supervision

Anis Al Gerbi (started november 2013) "Discretization of stochastic differential equations and systemic risk modeling", supervised by B. Jourdain and E. Clément

Pierre Blanc, "Price impact on marker orders and limit order books (from Nov. 2012), Ecole des Ponts, Adviser : A. Alfonsi defended on October 9th 2015.

Rui Chen (Fondation Sciences Mathématiques de Paris grant), "Stochastic Control of mean field systems and applications to systemic risk, from September 2014, Université Paris-Dauphine, Adviser A. Sulem

Roxana Dumitrescu, (Fondation Sciences Mathématiques de Paris grant, RDMath Ile de France), *Contributions au contrôle stochastique avec des espérances non linéaires et aux équations stochastiques rrétrogrades* , Advisers: A. Sulem with B. Bouchard and R. Elie, defended on September 28 2015, Université Paris-Dauphine.

Antoine Ly, CIFRE agreement, Partner: Miliman, advisers: R. Elie and Arthur Charpentier, Applications du machine learning en Actuariat, started 2015, Advisers: R. Elie and Arthur Charpentier

Sebastien Mollaret, CIFRE agreement, Partner: Natixis, Valorisation et couverture dans les modèles à changement de régime started 2015, Advisers: R. Elie.

Ernesto Palidda, Advisers: A. Alfonsi and Bernard Lapeyre, Ecole des Ponts, defended on May 29 2015.

- Paulo Pigato, "Lower bounds for the density of the solution of SDE's under the weak Hörmander condition, and applications in finance", Advisers: V. Bally and A. Dai Pra, University of Padova, defended October 2015, Université Paris-Est Marne la Vallée, Victor Rabiet : "On a class of jump type stochastic equations", Université Paris-Est Marne la Vallée, Advisers: V. Bally (75 %) and E. Locherbach, defended June 2015.

Clément Rey (from Oct. 2012), " High order discretization schemes for singular diffusions", Ecole des Ponts, Advisers : A. Alfonsi and Vlad Bally, defended on December 4th 2015.

Giulia Terenzi (from October 2015) "American options in complex financial models", Advisers : D. Lamberton and Lucia Caramellino, from University Tor Vergata, Rome.

Alexandre Zhou (started November 2015) "Analysis of stochastic particle methods applied to finance", supervised by B.Jourdain

10.2.3. Juries

- B.Jourdain

  - Head of the HCERES visit committee of the Laboratoire de Mathématiques Raphael Salem, Rouen, November 20 2015

  - PhD of Victor Rabiet, defended on June 23 2015, University Paris-Est

  - PhD of Thi Quynh Giang NGUYEN, defended on October 19 2015, University Paris-Est

- A. Sulem

  - PhD Roxana Dumitrescu, September 2015, Université Paris-Dauphine

  - Referee (Rapporteur) HdR *"Quelques contributions en finance mathématique, risque de liquidité et finance d'entreprise"*, Ly Vath Vathana, Université Evry Val d'Essonne, November 23 2015

  - Faculty opponent, PhD thesis "*Aspects of Waiting and Contracting in Game Theory*", Peter Helgesson, Chalmers University of Technology and University of Gothenburg, Sweden, June 5th 2015.

  - Sulem : Committe for the recrutment of a Professor of Insurance Mathematics, ETH Zurich January 2015

  - Committe for the recrutment of a Professor (in Financial Mathematics and numerical probability, Laboratoire de probabilités, Université Paris-Diderot, May 2015

  - Committe for the recrutment of a researcher (chargé de recherche) Inria Paris-Rocquencourt, June 2015

10.3. Popularization

- A. Lionnet:

  - article on Financial Mathematics published in the online popularization magazine Interstices https://interstices.info/jcms/p_82123/les-mathematiques-financieres-des-quants

  - Conference given for middle-schoolers at University Paris 8 during the "Week of Mathematics 2015".

  - Participation to the colloquium "Science and You", Nancy, June 1–6.

  - The two entries of A. Lionnet to the contest SIAM Math Matters, Apply It (https://www.siam.org/careers/matters.php were selected to be turned into a poster. One was on financial derivatites (https://www.siam.org/careers/pdf/deriviatives.pdf) the other on systemic risk (https://www.siam.org/careers/pdf/risk.pdf).

- A. Sulem:

  - "Applications of stochastic analysis", (2015), invited contribution for the *Princeton Companion to Applied Mathematics*, http://press.princeton.edu/titles/10592.html

  - Responsible for the partnership of Inria with Institut Henri Poincaré for the creation of a museum of Mathematics.

11. Bibliography

Major publications by the team in recent years

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[2] A. AHDIDA, A. ALFONSI. *Exact and high order discretization schemes for Wishart processes and their affine extensions*, in "Annals of Applied Probability", 2013, vol. 23, n° 3, pp. 1025-1073 [*DOI :* 10.1214/12-AAP863], http://hal.inria.fr/hal-00491371

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[10] D. LAMBERTON, M. ZERVOS. *On the optimal stopping of a one-dimensional diffusion*, in "Electronic Journal of Probability", 2013, vol. 18, n° 34, pp. 1-49

[11] M.-C. QUENEZ, A. SULEM. *BSDEs with jumps, optimization and applications to dynamic risk measures*, in "Stochastic Processes and their Applications", March 2013, vol. 123, n° 8, pp. 3328-3357 [*DOI :* 10.1016/J.SPA.2013.02.016], http://hal.inria.fr/hal-00709632

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Publications of the year

Doctoral Dissertations and Habilitation Theses

[15] E. PALIDDA. *Multi-dimensional stochastic volatility for Interest Rates*, Paris-Est ; Ecole des Ponts, May 2015, https://tel.archives-ouvertes.fr/tel-01217655

Articles in International Peer-Reviewed Journals

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