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Project-Team REALOPT

Reformulation and Algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB), Laboratoire Bordelais de Recherche en Informatique (LaBRI)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, machine learning and statistical methods

Table of contents

1. Members **1**
2. Overall Objectives **2**
3. Research Program **3**
3.1. Introduction 3
3.2. Polyhedral approaches for MIP 3
3.3. Decomposition and reformulation approaches 4
3.4. Integration of Artificial Intelligence Techniques in Integer Programming 4
3.5. Robust Optimization 5
3.6. Polyhedral Combinatorics and Graph Theory 5
4. Application Domains **5**
4.1. Introduction 5
4.2. Network Design and Routing Problems 5
4.3. Packing and Covering Problems 7
4.4. Planning, Scheduling, and Logistic Problems 7
5. Highlights of the Year **8**
6. New Software and Platforms **9**
7. New Results **9**
7.1. Improving Branch-and-Price Methods 9
7.2. Dual feasible functions 10
7.3. Allocation algorithms in Cloud platforms 10
7.4. Scheduling and placement for HPC 11
7.5. Production scheduling 11
7.6. Clustering problems 11
7.7. Tour scheduling with multi-skill heterogeneous workforce 12
7.8. Traffic routing in optical networks 12
7.9. Dense sphere packing 13
8. Bilateral Contracts and Grants with Industry **13**
8.1. Contract with EDF on robust maintenance planning 13
8.2. Collaboration with ERTUS on phytosanitary treatment planning 13
8.3. Collaboration with B-Travel on a yield management problem 14
8.4. Collaboration with Asys on an employee-scheduling problem 14
8.5. Collaboration with Renault S.A. on truck loading problem 14
8.6. Collaboration with St-Gobain Recherche on glass cutting 14
9. Partnerships and Cooperations **14**
9.1. Regional Initiatives 14
9.2. National Initiatives 14
9.2.1.1. ANR Solhar (ANR-13-MONU-0007) 14
9.2.1.2. ANR SONGS (ANR 11 INFRA 13) 15
9.3. International Initiatives 15
9.4. International Research Visitors 16
9.4.1. Visits of International Scientists 16
9.4.2. Visits to International Teams 16
10. Dissemination **16**
10.1. Promoting Scientific Activities 16
10.1.1. Scientific events organisation 16
10.1.2. Scientific events selection 16
10.1.3. Journal 17
10.1.3.1. Member of the editorial boards 17
10.1.3.2. Reviewer - Reviewing activities 17

10.1.4. Invited talks 17
10.1.5. Research administration 17
10.2. Teaching - Supervision - Juries 17
10.2.1. Teaching 17
10.2.2. Supervision 18
10.2.3. Juries 18
10.3. Popularization 19
11. Bibliography **19**

Project-Team REALOPT

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7.3. - Operations research, optimization, game theory
7.9. - Graph theory

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6.5. - Information systems

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2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.
Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications, Lagrangian and Bender's decomposition methods to produce powerful relaxations, multi-level programming to model a hierarchy of decision levels or recourse decision in the case of data adjustment, heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random data by way of modeling expectation (stochastic approaches) or worst-case bahavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

(i) Methodologies: To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. More broadly, to contribute to theoretical and methodological developments of exact approaches in combinatorial optimization, while extending the scope of applications.

(ii) Problem solving: To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact methods and heuristics.

(iii) Software platform: To provide prototypes of specific model solvers and generic software tools that build on our research developments, writing proof-of-concept code, while transferring our research findings to internal and external users.

3. Research Program

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, Dantzig-Wolfe decomposition, mixed integer programing, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [61]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\,x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\tilde{x} \notin conv(X)$, find a linear inequality $\pi\,x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\tilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition and reformulation approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition [5], [8], [9], [7]. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...).

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables.

Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization account for the expected realization of random data and optimize an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behavior of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

3.6. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [60], [59], [55], [57], in production planning [77] and inventory control [55], [57], in network design and traffic routing [38], [47], [53], [80], [35], [48], [66], [73], in cutting and placement problems [63], [64], [74], [75], [76], [78], and in scheduling [6], [67], [33].

4.2. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form "at least two paths link each pair of terminals". We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [47]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today's optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. We developed and compared several decomposition approaches [82], [81], [80] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [79]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [53], [54].

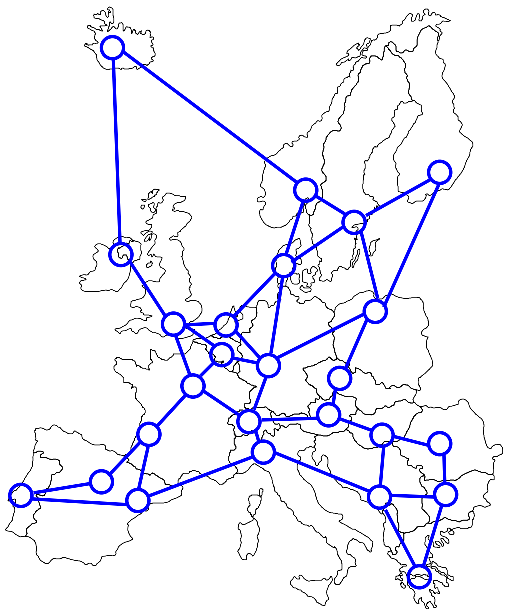

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

We studied several time dependent formulations for the unit demand vehicle routing problem [40], [39]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [72] simplifying results from Baiou and Barahona.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithmes based on the column generation approach. These algorithmes have been testes on a set of real-life instances coming from a Russian freight real transportation company. Our algorithmes have been faster on these instances than the current solution approach being used by the company.

4.3. Packing and Covering Problems

Realopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [71]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developped a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [31] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem [57]: we faced a multiple-class integer knapsack problem with setups [56] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [50], [49], [52], [51]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2d knapsack problem that uses our 2D packing feasibility check. We are currently developping exacti optimization tools for glass-cutting problems in a collaboration with Saint-Gobain. This 2D-3stage-Guillotine cut problems are very hard to solve given the scale of the instance we have to deal with. Moreover one has to issue cutting patterns that avoid the defaults that are present in the glass sheet that are used as raw material. They are extra sequencing constraints regarding the production that make the problem even more complex.
Finally, let us add that we are now organizing a european challenge on packing with society Renault: see http://challenge-esicup-2015.org/. This challenge is about loading trucks under practical constraints. The final results will be announced in March 2015.

4.4. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a branch-and-price algorithm where periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem, and the global planning of customer visits is coordinated by the master program. [58]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [60].

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [65] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximum the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [46] and exact algorithms [33]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [32]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [67]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [69]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [70], [68].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [30], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [29] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [34] provide demonstrably stronger formulations for some problem classes than any previously proposed. We are now working on planning phytosanitary treatments in vineries.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [42], [41], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [36]. We considered train timetabling problems and their re-optimization after a perturbation in the network [44], [43]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem [45]. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge adressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem [37].

5. Highlights of the Year

5.1. Highlights of the Year

The international society in Mathematical Optimisation (MOS) has selected the bid of Realopt for the organization of the next triennial international congress of mathematical optimization. Hence, the 23rd International Symposium on Mathematical Programming (ISMP 2018) shall take place in Bordeaux. The web site is in construction http://ismp2018.sciencesconf.org. This symposium is the most prestigious scientific event in the field of optimization by the quality of its program and its size (it can gather close to 2000 participants). This event has received strong support from the University of Bordeaux, Inria and CNRS, along side national scientific societies: Roadef and SMAI.
The team is tightening its links with industrial partners: our Inria Innovation Lab with Ertus-consulting has been launched; we have had two recruitements (a PhD and a Post-doc) this year on our production planning project with EDF; Saint Gobain is very enthousiastic about our progress in solving glass cutting problems, and Renault was quite happy with the challenge on logistic issues that we organized for them.
We are making progress on methodologic developments of algorithms for large scale optimization (convergence acceleration, filtering to reduce problem size, math heuristics, approximation algorithms) and their application (in cloud computing, sheduling, and planning). In particular, two of our papers were accepted at the prestigious conference IPDPS'16. Our research collaborations are being tightened in particular through the SAMBA associated team project: Ruslan Sadykov is spending a sabbatical year in Brasil in our associated team. We established a new partnership with KEDGE business school.

6. New Software and Platforms

6.1. BaPCod : a generic Branch-And-Price Code

KEYWORDS: Column Generation - Branch-and-Price - Branch-and-Cut - Mixed Integer Programming - Mathematical Optimization - Benders Decomposition - Dantzig-Wolfe Decomposition - Extended Formulation
FUNCTIONAL DESCRIPTION: BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithm, Benders approaches, or network flow algorithms.

- Participants: Francois Vanderbeck, Ruslan Sadykov, Issam Tahiri, Artur Alves Pessoa, Boris Detienne, François Clautiaux, Pierre Pesneau, Eduardo Uchoa Barboza and Michael Poss
- Partners: Université de Bordeaux - CNRS - IPB - Universidade Federal Fluminense
- Contact: Francois Vanderbeck
- URL: https://realopt.bordeaux.inria.fr/?page_id=2

7. New Results

7.1. Improving Branch-and-Price Methods

We have made progress on stabilization techniques and math-heuristics that have become essential components for Branch-and-Price methods.
Smoothing and proximal methods based on penalizing the deviation from the incumbent dual solution have become standards of the domain. Interpreting column generation as cutting plane strategies in the dual problem, we analyze in [26] the mechanisms on which stabilization relies. In particular, the link is established between smoothing and in-out separation strategies to derive generic convergence properties. For penalty function methods as well as for smoothing, we describe proposals for parameter self-adjusting schemes. Such schemes make initial parameter tuning less of an issue as corrections are made dynamically. Such adjustments also allow to adapt the parameters to the phase of the algorithm. We provide extensive test reports that validate our self-adjusting parameter scheme and highlight their performances. Our results also show that using smoothing in combination with penalty function yields a cumulative effect on convergence speed-ups.

Effects of stabilization techniques can be seen in practice. Routing and logistics applications are often viewed as intractable for exact optimization tools. Although such problems are naturally suited for a decomposition approach, branch-and-price-and-cut algorithms of the literature typically do not scale to the size of real-life instances. Some recent progress in stabilization techniques amongst other advances (such as diving heuristics, strong branching, and the combination with cutting plane approaches) generate new ambitions for column generation approach in solving approximately very large scale instances. Let us for instance point to the new benchmarks for the Capacitated Vehicle Routing Problem (CVRP) in [62]. The paper [24] illustrates this trend, showing exact results for freight transportation instances of a scale never considered before. Our column generation algorithm yields dual bounds and serves as the core procedure for a primal heuristic. The overal procedure is quite competitive in great part due to the convergence speed-ups resulting from efficient stabilization schemes. It typically provides optimal solutions as primal and dual bounds tend to be equal. The very large scale freight transportation instances (with up to 1,025 stations, 5,300 demands, and 12,651 rail cars) were submitted to us by our Russian partner Freight-One.
Math-heuristics have become an essential component in mixed integer programming (MIP) solvers. Extending generic MIP heuristics, our study in [28] outlines generic procedures to build primal solutions in the context of a Branch-and-Price approach and reports on their performance. Rounding the linear relaxation solution of the Dantzig-Wolfe reformulation, which is typically tighter than that of the original compact formulation, sometimes produces better solutions than state-of-the-art specialised heuristics as revealed by our numerical experiments. We focus on the so-called diving methods and their combination with diversification-intensification paradigms such as Limited Discrepancy Search, sub-MIPing, relaxation induced neighbourhood search, local branching, and strong branching. The dynamic generation of variables inherent to a column generation approach requires specific adaptation of heuristic paradigms. Our contribution lies in proposing simple strategies to get around these technical issues. Our numerical results on Generalized Assignment, Cutting Stock, and Vertex Coloring problems sets new benchmarks, highlighting the performance of diving heuristics as generic procedures in a column generation context.

7.2. Dual feasible functions

Dual-feasible functions have proved to be very effective for generating fast lower bounds and valid inequalities for integer linear programs with knapsack constraints. However, a significant limitation is that they are defined only for positive arguments. Extending the concept of dual-feasible function to the general domain and range R is not straightforward. In [10], we propose the first construction principles to obtain general functions with domain and range R, and we show that they lead to non-dominated maximal functions.

7.3. Allocation algorithms in Cloud platforms

In the context of service hosting in large-scale datacenters, we provide [11] a deep analysis of a cluster data trace recently released by Google and we focus on a number of questions which have not been addressed in previous studies. In particular, we describe the characteristics of job resource usage in terms of dynamics (how it varies with time), of correlation between jobs (identify daily and/or weekly patterns), and correlation inside jobs between the different resources (dependence of memory usage on CPU usage). From this analysis, we derive scalable formalizations of the allocation problem which encompass most job features. In [19], [22], we study one such model, where long-running services experience demand variations with a periodic (daily) pattern. Such services account for most of the overall CPU demand. This leads to an allocation problem where the classical Bin-Packing issue is augmented with the possibility to co-locate jobs whose peaks occur at different times of the day, which is bound to be more efficient than the usual approach that consists in over-provisioning for the maximum demand. We propose mathematical formulations, column generation approaches, and analyze their performance compared to standard packing heurisics (such as Best-Fit or First-Fit Decreasing). We show that taking periodicity of demand into account allows for a substantial improvement on machine utilization in the context of large-scale, state-of-the-art production datacenters, and that column generation allows to obtain quasi-optimal solutions in reasonable time.

7.4. Scheduling and placement for HPC

With the complexification of the architecture of HPC nodes (multicores, non uniform memory access, GPU and accelerators), a recent trend in application development is to explicitely express the computations as a task graph, and rely on a specialized middleware stack to make scheduling decisions and implement them. Traditional algorithms used in this community are dynamic heuristics, to cope with the unpredictability of execution times. In [17], [18] we analyze the performance of static and hybrid strategies, obtained by adding more static (resp. dynamic) features into dynamic (resp. static) strategies. Our conclusions are somehow unexpected in the sense that we prove that static-based strategies are very efficient, even in a context where performance estimations are not very good.

Another study [13] focuses on the memory-constrained case, where tasks may produce large data. A task can only be executed if all input and output data fit into memory, and a data can only be removed from memory after the completion of the task that uses it as an input data. Trees of such tasks arise in the multifrontal method of sparse matrix factorization. Minimizing the peak memory required on a single processor is well studied, [13] extends the problem to multiple processors, where both makespan and memory need to be minimized. We study the computational complexity of this problem and provide inapproximability results even for unit weight trees. We design a series of practical heuristics achieving different trade-offs between the minimization of peak memory usage and makespan. Some of these heuristics are able to process a tree while keeping the memory usage under a given memory limit. The different heuristics are evaluated in an extensive experimental evaluation using realistic trees.

In [20], we perform another study of static, dynamic and hybrid strategies in the context of load balancing and data placement for matrix multiplication in heterogeneous machines. Through a set of extensive simulations, we analyze the behavior of static, dynamic, and hybrid strategies, and we assess the possible benefits of introducing more static knowledge and allocation decisions in runtime libraries. In [21], we consider the purely static problem, modeled as a partitioning of a square into a set of zones of prescribed areas, while minimizing the overall size of their projections onto horizontal and vertical axes. We combine two ideas from the literature (recursive partitioning, and optimal solution structure for low number of processors) to obtain a non-rectangular recursive partitioning (NRRP), whose approximation ratio is $\frac{2}{\sqrt{3}} \simeq 1.15$, improving over the previous 1.25 ratio. Moreover, we observe on a large set of realistic platforms built from CPUs and GPUs that this proposed NRRP algorithm allows to achieve very efficient partitionings on all considered cases.

7.5. Production scheduling

Together with Shunji Tanaka, from Kyoto University, we developed Lagrangian relaxation-based methods for solving min-sum shop scheduling problems. In our studies, large scale network flow formulations of the problems are suggested together with strong Lagrangian bounds based on these formulations.

In [23], we consider the flow-shop problem on two machines with sequence-independent setup times to minimize total completion time. To cope with the size of the network, filtering procedures are developed. To solve the problem to optimality, we embed the Lagrangian bounds into two branch-and-bound algorithms. The best algorithm is able to solve all 100-jobs instances of our testbed with and without setup times, thus significantly outperforming the best algorithms in the literature, which were limited to instances with 30 and 45 jobs respectively.

In [25], we propose a new dual bound for the job-shop problem with the objective of minimizing the sum of completion costs of the operations. The bound is obtained by a Lagrangian relaxation that decomposes the problem into two types of large network flow problems: one dealing with the precedence constraints among operations of a same job, and the other one satisfying the disjunctive constraints related to the machines. Numerical experiments on the just-in-time job-shop problem show that the method is able to improve the existing lower bounds significantly.

7.6. Clustering problems

Clustering problems, and in particular partitionning problems, are widespread in combinatorial optimization. The goal is to partition a set of items in subset satisfying various constraints such as knapsack constraints, cardinality constraints, connectivity constraints, and so on. Beside the PhD thesis of Jérémy Guillot that aims to develop aggregating techniques to handles large scale instances for partitionning problems, the team also study some particular versions.

In [15] we present the application of branch-and-price approaches to the automatic version of the Software Clustering Problem. To tackle this problem, we apply the Dantzig-Wolfe decomposition to a formulation from literature. Given this, we present two Column Generation (CG) approaches to solve the linear programming relaxation of the resulting reformulation: the standard CG approach, and a new approach, which we call Staged Column Generation (SCG). Also, we propose a modification to the pricing subproblem that allows to add multiple columns at each iteration of the CG. We test our algorithms in a set of 45 instances from the literature. The proposed approaches were able to improve the literature results solving all these instances to optimality. Furthermore, the SCG approach presented a considerable performance improvement regarding computational time, number of iterations and generated columns when compared with the standard CG as the size of the instances grows.

In collaboration with researchers from University Paris 6 and Paris 13, we also study the problem of partitionning a geographical area in connected parcels. A first step of this study was to cut the area in two connected parcels while minimizing the dissimilarities inside each parcels. Such partitionning is also called a bond. It happens that in series-parallel garph, a bond correspond to a circuit in the dual graph. In [12], we give a full description of the circuit polytope on series–parallel graphs. We first show the existence of a compact extended formulation. Though not being explicit, its construction process helps us to inductively provide the description in the original space. As a consequence, using the link between bonds and circuits in planar graphs, we also describe the bond polytope on series–parallel graphs.

7.7. Tour scheduling with multi-skill heterogeneous workforce

In [14], we address a multi-activity tour scheduling problem with time varying demand. The objective is to compute a team schedule for a fixed roster of employees in order to minimize the over-coverage and the under-coverage of different parallel activity demands along a planning horizon of one week. Numerous complicating constraints are present in our problem: all employees are different and can perform several different activities during the same day-shift, lunch breaks and pauses are flexible, demand is given for 15 minutes periods. Employees have feasibility and legality rules to be satisfied, but the objective function does not account for any quality measure associated with each individual's schedule. More precisely, the problem mixes simultaneously days-off scheduling, shift scheduling, shift assignment, activity assignment, pause and lunch break assignment. To solve this problem, we developed four methods: a compact Mixed Integer Linear Programming model, a branch-and-price like approach with a nested dynamic program to solve heuristically the subproblems, a diving heuristic and a greedy heuristic based on our subproblem solver. The computational results, based on both real cases and instances derived from real cases, demonstrate that our methods are able to provide good quality solutions in a short computing time. Our algorithms are now embedded in a commercial software, which is already in use in a mini-mart company.

7.8. Traffic routing in optical networks

In [16], we consider a multi-layer network design model arising from a real-life telecommunication application where traffic routing decisions imply the installation of expensive nodal equipment. Customer requests come in the form of bandwidth reservations for a given origin destination pair. Bandwidth demands are expressed as multiples of nominal granularities. Each request must be single-path routed. Grooming several requests on the same wavelength and multiplexing wavelengths in the same optical stream allow a more efficient use of network capacity. However, each addition or withdrawal of a request from a wavelength requires optical to electrical conversion and the use of cross-connect equipment with expensive ports of high densities. The objective is to minimize the number of required ports of the cross-connect equipment. We deal with backbone optical networks, therefore with networks with a moderate number of nodes (14 to 20) but thousands of requests. Further difficulties arise from the symmetries in wavelength assignment and traffic loading. Traditional multi-commodity network flow approaches are not suited for this problem. Instead, four alternative models relying on Dantzig-Wolfe and/or Benders' decomposition are introduced and compared. The formulations are strengthened using symmetry breaking restrictions, variable domain reduction, zero-one discretization of integer variables, and cutting planes. The resulting dual bounds are compared to the values of primal solutions obtained through hierarchical optimization and rounding procedures. For realistic size instances, our best approaches provide solutions with optimality gap of approximately 5% on average in around two hours of computing time.

7.9. Dense sphere packing

In [27], we consider the sphere packing problem in arbitrary dimension: what is the maximum fraction Δ_n of the Euclidean space $\mathbb{R}_n$ that can be covered by unit balls with pairwise disjoint interiors?

Δ_n is known for only for some small values of n , and when n grows, we only have lower bounds. A trivial lower bound states that for every n, $\Delta_n \geq 2^{-n}$. Minkowski and Hlwaka's Theorem (1905) improves this lower bound by a factor 2: $\Delta_n \geq 2 \times 2^{-n}$. Asymptotic improvements of this bound were obtained (from Rogers, 1947 up to Ball, 1992), all of them being of the form $\Delta_n \geq cn2^{-n}$ where c is a constant.

This problem has a natural reformulation in graph theoretic terms as follows: let G denote the graph whose vertices are the points of the Euclidean space and edges are pair of vertices at distance at most 2 one from the other. The independent sets of G are the sphere packings: so, finding a maximum-density sphere packing is the same as finding a maximum-density independent set in this infinite graph. By using graph theoretic arguments only, Krivelevich et al. established that $\Delta_n \geq 0.01n2^{-n}$ for sufficiently large n.

In a recent breakthrough, Venkatesh introduced the first superlinear improvement: there are infinitely many n such that $\Delta_n \geq cn \log\log n2^{-n}$, where c is a constant. Venkatesh's result is however non-constructive.

In this joint work with C. Bachoc and P. Moustrou, we give a constructive proof of Venkatesh's lower bound.

This study has been carried out with financial support from the French State, managed by the French National Research Agency (ANR) in the frame of the " Investments for the future " Programme IdEx Bordeaux - CPU (ANR-10-IDEX-03-02).

8. Bilateral Contracts and Grants with Industry

8.1. Contract with EDF on robust maintenance planning

Our project with EDF concerns the optimization of the long term energy production planning, allowing for nuclear power plants maintenance. The challenges are to handle the large-scale instance of a five year planning and to handle the stochastic aspects of the problem: the stochastic variation of the electricity demand, the production capacity and the duration of maintenance period. The key decisions to be optimized are the dates of outages (for maintenance) and the level refuelling that determines the production of the year to come. We previously developed a column generation approach based on extended formulation which enables to solve within a few minutes a deterministic instance of the problem, which is within the time frame of the operational tools currently used by EDF. We now investigate stochastic and robust versions of the problem, where the duration of maintenance operations and the power demand are uncertain. Our approaches shall be evaluated on real life instances within a rolling horizon framework.

8.2. Collaboration with ERTUS on phytosanitary treatment planning

In planning winary operations (most importantly phytosanitary treatments on the wine tree) under wheather forcast uncertainty, one searches for solutions that remain feasible and "cheap" in case of perturbation in the data. We consider the planning and scheduling of the operations that arise over a one-year horizon. More precisely, the operations to be sheduled include tasks related to soil care, or grape tree care: cutting, line building, thinning out leaves, ..., and chemical treatments. The latter are a main focus of our study since one of the principal goals of better planning is to reduce the amount of chemical treatments by selecting the appropriate products and schemes, but also by spacing out treatements while guarantying a desease free vineyard with some confidence. Each of the scheduled tasks requires its own resource, so the planning also triggers equipement and raw products selection decisions. The objective is to minimize both equipment and product costs augmented by an evaluation of the hazard of chemical product use. The planning should be "robust" to seasonal variations on the proper time frame for scheduling tasks.

8.3. Collaboration with B-Travel on a yield management problem

Through the PhD thesis of Martin Bué (in collaboration with inria team Dolphin), we are now working with society B-Travel on pricing and yield management. The goal is to find the best prices and incentives in the context of professional travel. The techniques used are based on network-flow formulations and mathematical programming.

8.4. Collaboration with Asys on an employee-scheduling problem

Through the PhD thesis of Matthieu Gérard, we have investigated a very rich version of employee-scheduling problem. We have designed an efficient algorithm for computing the best shift for each employee, based on dynamic programming. This method is used in a greedy algorithm to find solutions in a faster manner, and in a branch-and-price method to prove the optimality of the solution.

8.5. Collaboration with Renault S.A. on truck loading problem

The goal of this one year industrial contrat was to analyze the algorithmic solutions used by Renault S.A. for packing items into trucks. The outcome of the contract was a report on their approach and how hints to improve it.

8.6. Collaboration with St-Gobain Recherche on glass cutting

Through the PhD of Quentin Viaud, we study a hard glass-cutting problem. The objective is to minimize the quantity of trim loss when rectangular pieces are cut from large rectangles. This first study has shown that our methodologies are able to cope with this problem for medium-size instances. Solving the problem with large instances is a scientific challenge that we will address in the a follow-up contract.

9. Partnerships and Cooperations

9.1. Regional Initiatives

We have received support from the regional authorities (Region Aquitaine) for a research project on the planning under uncertainty. A postdoc, Agnès Leroux, has been recruited on this project. She currently develops dynamic programming approaches for scheduling problems and their application to building planning for phytosanitary treatments.

9.2. National Initiatives

9.2.1. ANR

9.2.1.1. ANR Solhar (ANR-13-MONU-0007)

This project aims at studying and designing algorithms and parallel programming models for implementing direct methods for the solution of sparse linear systems on emerging computing platforms equipped with accelerators. This project proposes an innovative approach which relies on the efficiency and portability of runtime systems, such as the StarPU tool. The focus of RealOpt in this project is on the scheduling aspect. Indeed, executing a heterogeneous workload with complex dependencies on a heterogeneous architecture is a very challenging problem that demands the development of effective scheduling algorithms. These will be confronted with possibly limited views of dependencies among tasks and multiple, and potentially conflicting objectives, such as minimizing the makespan, maximizing the locality of data or, where it applies, minimizing the memory consumption.

See also: http://solhar.gforge.inria.fr/

9.2.1.2. ANR SONGS (ANR 11 INFRA 13)

The goal of the SONGS project is to extend the applicability of the SimGrid simulation framework from Grids and Peer-to-Peer systems to Clouds and High Performance Computation systems. Any sound study of such systems through simulations relies on the following pillars of simulation methodology: Efficient simulation kernel; Sound and validated models; Simulation analysis tools; Campaign simulation management. The contribution of RealOpt in this project revolves around enabling peer-to-peer simulation, and providing use cases for Cloud Comupting simulations.
See also: http://infra-songs.gforge.inria.fr/

9.3. International Initiatives

9.3.1. Inria Associate Team

9.3.1.1. SAMBA

Title: Synergies for Ameliorations and Mastering of Branch-and-Price Algorithms

International Partner (Institution - Laboratory - Researcher):

Universidade Federal Fluminense (Brazil) & Universidad Adolfo Ibanez (Chile)

Start year: 2011-13 and 2014-16

See also: https://realopt.bordeaux.inria.fr/?page_id=573

SAMBA is a research project between the Inria project team ReAlOpt (Bordeaux, France), the ADT-Lab Pontifícia Universidade Católica do Rio de Janeiro, and the LOGIS at the Universidade Federal Fluminense. The project is supported by Inria under the “associate team” framework for an initial period of three years (2011-2013) and was renewed for another three years period (2014-2016) with additional partners at the Operations Research and Complex Systems Group School of Business, Universidad Adolfo Ibanez, Chile, and the LIRMM at the University of Montpellier.

Quantitative models are important tools for strategic, tactical, and operational decision-making. Many underlying optimization problems are discrete in nature. They are modeled as linear programs with integer variables, so called Mixed Integer Programs (MIP). Their solution is es- sentially based on enumeration techniques, which is notoriously difficult given the huge size ofthe solution set. Powerful generic commercial solvers for MIP are available, but despite continuous progress, the existing tools can be overwhelmed when problem complexity or size increases.

Decomposition approaches are primary tools to expand the capabilities of MIP solution techniques. When the application presents a decomposable constraint system, the so-called “Dantzig-Wolfe decomposition” consists in reformulating the problem as a selection of a specific solution for each individual subsystems that together satisfy the linking constraints. In practice, the individual subsystem solutions are brougth in the formulation in the course of the opti- mization if they can lead to improvement in the objective value. On the other hand, “Benders’ decomposition applies when the the application presents a decomposable system of variables, as traditional in stocahstic two-stage optimization models where main decisions are taken prior to knowing the realization ofr random data, while second stage decision are adjusments that can be done once the true value of data is revealed. In this context, one solves the first stage model and check a posteriori the feasibilility of the second stage. In case the second stage is infeasible, a constraint on the first stage variables is induced that aim to account for the cause of second stage infeasibility, and the processus reiterates.

Both of these decomposition approaches are perceived as requiring an application specific implementation for tractability in scaling-up to real-life applications. Our research aim at developing generic methods for these and algorithmic enhancements to can yield significant speed-ups in practice and have sound theoretical basis. Such research includes methodological developments (such as stabilization techniques for improved convergence, preprocessing rules, dynamic aggregation-and-disagregation), algorithms strategies (such as mutli-column/cut generation strategies, pre-evaluation of enumerated subproblem strategies – so-called strong branching), and efficient implementations (code re-engineering of our software platform BaPCod).

Beyond the methodological developments, our motivations are to set new benchmarks on standard combinatorial problems and industrial applications. In particular, we proceed to extend our techniques to the context of dynamic optimization. In a stochastic environment, the aim is to build a planning that are robust to perturbations in the sense that it can be adapted dynamically in reaction to the observed changes in the predicted data.

The project builds on the accumulated experience of both the Brazilian, the Chilean and the French teams that have done pioneering work in tackling complex applications and deriving generic solution strategies using this decomposition approach.

9.4. International Research Visitors

9.4.1. Visits of International Scientists

- Alexander Lazarev (Russia) visited us in Bordeaux in January 2015.
- Eduardo Uchoa (Brasil) visited us in Bordeaux on the second week of January 2015.
- Michael Poss visited us in Bordeaux on the first week of May 2015.
- Eduardo Moreno (Chile) visited us in Bordeaux for 10 days in November 2015.

9.4.2. Visits to International Teams

9.4.2.1. Sabbatical programme

Sadykov Ruslan

Date: Aug 2015 - Jul 2016

Institution: Universidade Federal Fluminense (Brazil)

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific events organisation

10.1.1.1. General chair, scientific chair

- François Clautiaux has organized an optimization challenge with multinational automobile manufacturer Renault (prizes of 30.000 euros).
- François Vanderbeck has been chosen by MOS as the general chair of the next triennal Symposium on Mathematical Optimization (ISMP-2018)

10.1.2. Scientific events selection

10.1.2.1. Member of the conference program committees

The team members are members of the following program committees:

- Olivier Beaumont: IPDPS'15: IEEE International Parallel & Distributed Processing Symposium.
- Olivier Beaumont: ISCIS'15, 30th International Symposium on Computer and Information Sciences.
- Olivier Beaumont: SC'15: IEEE ACM International Conference for High Performance Computing, Networking, Storage and Analysis.
- Olivier Beaumont: HCW'15 24th International Heterogeneity in Computing Workshop.
- Olivier Beaumont: HeteroPar'2015: Thirteenth International Workshop on Algorithms, Models and Tools for Parallel Computing on Heterogeneous Platforms.
- Olivier Beaumont and Lionel Eyraud-Dubois: HIPC'15: IEEE International Conference on High Performance Computing.
- François Clautiaux: Roadef 2015: French Operational Research Society Conference.
- Arnaud Pêcher: JGA 2015: Journées Graphes et Algorithmes 2015.
- Pierre Pesneau: INOC 2015: 7th International Network Optimization Conference.

10.1.3. Journal

10.1.3.1. Member of the editorial boards

- Olivier Beaumont is editor for IEEE Transactions on Parallel and Distributed Systems (TPDS)
- François Vanderbeck is Associate Editor for the EURO Journal on Computational Optimization

10.1.3.2. Reviewer - Reviewing activities

The team members are regular referees for the best journals of the field.

10.1.4. Invited talks

Arnaud Pêcher: "On dense sphere packings", International Conference on Graph Theory and its Applications, Coimbatore, India, 2015

10.1.5. Research administration

- Olivier Beaumont is the scientific deputy of Inria Bordeaux Sud-Ouest.
- François Vanderbeck is taking care of the team OptimAl ("Optimisation Mathématique Modèle Aléatoire et Statistique") at the Mathematics Institute of Bordeaux.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence : A. Pêcher, Programmation Impérative, 10h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Conception Objet, 42h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Programmation objet en Java, 44h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Algorithmique Avancée, 32h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Assembleur, 24h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Programmation Mobile, 24h, DUT, Université de Bordeaux, France

Licence : P. Pesneau, Système et Programmation en Fortran 90, 59h, L2, Université de Bordeaux, France

Licence : P. Pesneau, Modèles et Méthodes d'Optimisation, 30h, L2, Université de Bordeaux, France

Licence : P. Pesneau, Recherche Opérationnelle, 24h, DUT, Université de Bordeaux, France

Master : O. Beaumont, Big Data, 4h, M1, Institut National Polytechnique de Bordeaux, France

Master : O. Beaumont, Optimisation en Cloud Computing et Big Data, 15h, M2, Université de Bordeaux, France

Master : O. Beaumont, Distributed Computing, 4h, M2, Institut National Polytechnique de Bordeaux, France

Master : F. Clautiaux, Programmation Linéaire 1, 15h, M1, Université de Bordeaux, France

Master : F. Clautiaux, Introduction à la Programmation en Variables Entières, 15h, M1, Université de Bordeaux, France

Master : F. Clautiaux, Gestion des Opérations et Planification de la Production, 30h, M2, Université de Bordeaux, France

Master : F. Clautiaux, Problèmes combinatoires et routage, 30h, M1, Université de Bordeaux et Institut National Polytechnique de Bordeaux, France

Master : B. Detienne, Programmation Linéaire 1, 14h, M1, Université de Bordeaux, France

Master : B. Detienne, Optimisation Convexe Non Linéaire, 29h, M1, Université de Bordeaux, France

Master : B. Detienne, Recherche Opérationnelle, 16h, M1, Institut National Polytechnique de Bordeaux, France

Master : B. Detienne, Introduction à la Programmation en Variables Entières, 14h, M1, Université de Bordeaux, France

Master : B. Detienne, Gestion des Opérations et Planification de la Production, 28h, M2, Université de Bordeaux, France

Master : B. Detienne, Optimisation Stochastique, 58h, M2, Université de Bordeaux, France

Master : L. Eyraud-Dubois, Introduction à la Programmation par Contraintes, 30h, M2, Université de Bordeaux, France

Master : L. Eyraud-Dubois, Optimisation en Cloud Computing et Big Data, 15h, M2, Université de Bordeaux, France

Master : J. Guillot, Modèles de flot, 14h, M1, Université de Bordeaux, France

Master : P. Pesneau, Problèmes combinatoires et routage, 8h, M1, Université de Bordeaux, France

Master : P. Pesneau, Programmation Linéaire 1, 10h, M1, Université de Bordeaux, France

Master : P. Pesneau, Algorithmique et Programmation Objet, 60h, M1, Université de Bordeaux, France

Master : P. Pesneau, Modèles de flot, 15h, M1, Université de Bordeaux, France

Master : P. Pesneau, Programmation Linéaire 2, 14h, M1, Université de Bordeaux, France

Master : R. Sadykov, Modélisation, Optimisation, Complexité et Algorithmes, 50h, M2, CNAM Aquitaine, Bordeaux, France

Master : I. Tahiri, Outils et Logiciels pour l'Optimisation, 30h, M1, Université de Bordeaux, France

Master : F. Vanderbeck, Recherche Opérationnelle, 15h, M1, Institut National Polytechnique de Bordeaux, France

Master : F. Vanderbeck, Programmation Entière, 58h, M2, Université de Bordeaux, France

10.2.2. Supervision

PhD : Matthieu Gérard, Heuristiques basées sur la génération de colonnes pour un proble`me de planification du personnel, University of Lille, December 9th 2015, François Clautiaux (dir) and Manuel Davy (dir) and Ruslan Sadykov (co-dir).

PhD : Hugo Kramer, Software clustering problems, Universtate Federal de Flumense. Eduardo Uchoa (dir) and Francois Vanderbeck (co-dir).

PhD in progress : Jérémy Guillot, Optimisation de problèmes de partionnement, September 2014, François Clautiaux (dir) and Pierre Pesneau (dir).

PhD in progress : Quentin Viaud, Méthodes de programmation mathématiques pour des problèmes complexes de découpe, January 2015, François Clautiaux (dir), Ruslan Sadykov (dir), and François Vanderbeck (co-dir)).

PhD in progress : Martin Bué, Gestion du revenu dans le cadre du voyage professionnel, September 2012, François Clautiaux (dir), Luce Brotcorne (dir).

PhD in progress : Rodolphe Griset, Robust planning in Electricity production, November 2015, Boris Detienne (dir) and François Vanderbeck (dir).

PhD in progress : Imen Ben Mohamed, Location routing problems, October 2015, Walid Klibi (dir) and François Vanderbeck (dir).

PhD in progress : Thomas Bellitto, Infinite graphs, September 2015, Arnaud Pêcher (dir) and Christine Bachoc (dir).

PhD in progress : Philippe Moustrou, Codes, September 2014, Arnaud Pêcher (dir) and Christine Bachoc (dir).

10.2.3. Juries

- Olivier Beaumont: Evaluation (rapporteur) of the habilitation thesis (HDR) of Georges Da Costa (IRIT Toulouse).
- Olivier Beaumont: Evaluation (rapporteur) of the PhD thesis of Nathalie Herr (University of Besançon).
- François Clautiaux: Evaluation (directeur) of the PhD thesis of Matthieu Gérard (University of Lille).
- Ruslan Sadykov: Evaluation (encadrant) of the PhD thesis of Matthieu Gérard (University of Lille).
- Ruslan Sadykov: Evaluation (examinateur) of the PhD thesis of Hugo Kramer (University Federal Fluminense, Niteroi, Brazil).
- Ruslan Sadykov: Pre-evaluation of the PhD thesis of André Soares Velasco (University Federal Fluminense, Niteroi, Brazil).
- Ruslan Sadykov: Evaluation (examinateur) of the Master thesis of Daniel Dias de Olieira Neto (University Federal Fluminense, Niteroi, Brazil).

10.3. Popularization

François Clautiaux is a member of the board of AMIES, the French Agency for Interaction in Mathematics with Business and Society. AMIES is a national organization that aims to develop relations between academic research teams in mathematics and business, especially SMEs.

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