informatics mathematics

IN PARTNERSHIP WITH:
CNRS

Université de Lorraine

Activity Report 2016

Project-Team ALICE

Geometry and Lighting

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Interaction and visualization

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Project-Team ALICE

Creation of the Project-Team: 2006 January 09

Keywords:

Computer Science and Digital Science:

5.5.1. - Geometrical modeling
5.5.2. - Rendering
6.2.1. - Numerical analysis of PDE and ODE
6.2.8. - Computational geometry and meshes
7.2. - Discrete mathematics, combinatorics
7.5. - Geometry, Topology

Other Research Topics and Application Domains:

3.3.1. - Earth and subsoil
5.1. - Factory of the future
5.7. - 3D printing
9.2.2. - Cinema, Television
9.2.3. - Video games
9.4.1. - Computer science
9.4.2. - Mathematics
9.4.3. - Physics

1. Members

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2. Overall Objectives

2.1. Overall Objectives

ALICE is a project-team in Computer Graphics. The fundamental aspects of this domain concern the interaction of *light* with the *geometry* of the objects. The lighting problem consists in designing accurate and efficient *numerical simulation* methods for the light transport equation. The geometrical problem consists in developing new solutions to *transform and optimize geometric representations*. Our original approach to both issues is to restate the problems in terms of *numerical optimization*. We try to develop solutions that are *provably correct*, *numerically stable* and *scalable*.

To reach these goals, our approach consists in transforming the physical or geometric problem into a numerical optimization problem, studying the properties of the objective function and designing efficient minimization algorithms. Besides Computer Graphics, our goal is to develop cooperations with researchers and people from the industry, who test applications of our general solutions to various domains, comprising CAD, industrial design, oil exploration, plasma physics... Our solutions are distributed in both open-source software (Graphite, OpenNL, CGAL) and industrial software (Gocad, DVIZ).

Since 2010, we started to develop techniques to model not only virtual objects, but also real ones. Our "modeling and rendering" research axis evolved, and we generalized our results on by-example texture synthesis to the production of real objects, using 3D printers. As compared to virtual objects, this setting defines higher requirements for the geometry processing techniques that we develop, that need to be adapted to both numerical simulation and computer-aided fabrication. We study how to include *computational physics* into the loop, and simulation methods for various phenomena (*e.g.*, fluid dynamics).

3. Research Program

3.1. Introduction

Computer Graphics is a quickly evolving domain of research. These last few years, both acquisition techniques (*e.g.*, range laser scanners) and computer graphics hardware (the so-called GPU's, for Graphics Processing Units) have made considerable advances. However, despite these advances, fundamental problems still remain open. For instance, a scanned mesh composed of hundred millions triangles cannot be used directly in real-time visualization or complex numerical simulation. To design efficient solutions for these difficult problems, ALICE studies two fundamental issues in Computer Graphics:

- the representation of the objects, *i.e.*, their geometry and physical properties;
- the interaction between these objects and light.

Historically, these two issues have been studied by independent research communities. However, we think that they share a common theoretical basis. For instance, multi-resolution and wavelets were mathematical tools used by both communities [29]. We develop a new approach, which consists in studying the geometry and lighting from the *numerical analysis* point of view. In our approach, geometry processing and light simulation are systematically restated as a (possibly non-linear and/or constrained) functional optimization problem. This type of formulation leads to algorithms that are more efficient. Our long-term research goal is to find a formulation that permits a unified treatment of geometry and illumination over this geometry.

3.2. Geometry Processing for Engineering

Keywords: Mesh processing, parameterization, splines

Geometry processing recently emerged (in the middle of the 90's) as a promising strategy to solve the geometric modeling problems encountered when manipulating meshes composed of hundred millions of elements. Since a mesh may be considered to be a *sampling* of a surface - in other words a *signal* - the *digital signal processing* formalism was a natural theoretic background for this subdomain (see *e.g.*, [30]). Researchers of this domain then studied different aspects of this formalism applied to geometric modeling.

Although many advances have been made in the geometry processing area, important problems still remain open. Even if shape acquisition and filtering is much easier than 30 years ago, a scanned mesh composed of hundred million triangles cannot be used directly in real-time visualization or complex numerical simulation. For this reason, automatic methods to convert those large meshes into higher level representations are necessary. However, these automatic methods do not exist yet. For instance, the pioneer Henri Gouraud often mentions in his talks that the *data acquisition* problem is still open [19]. Malcolm Sabin, another pioneer of the "Computer Aided Geometric Design" and "Subdivision" approaches, mentioned during several conferences of the domain that constructing the optimum control-mesh of a subdivision surface so as to approximate a given surface is still an open problem [28]. More generally, converting a mesh model into a higher level representation, consisting of a set of equations, is a difficult problem for which no satisfying solutions have been proposed. This is one of the long-term goals of international initiatives, such as the AIMShape European network of excellence.

Motivated by gridding application for finite elements modeling for oil and gas exploration, in the frame of the Gocad project, we started studying geometry processing in the late 90's and contributed to this area at the early stages of its development. We developed the LSCM method (Least Squares Conformal Maps) in cooperation with Alias Wavefront [24]. This method has become the de-facto standard in automatic unwrapping, and was adopted by several 3D modeling packages (including Maya and Blender). We explored various applications of the method, including normal mapping, mesh completion and light simulation [21].

However, classical mesh parameterization requires to partition the considered object into a set of topological disks. For this reason, we designed a new method (Periodic Global Parameterization) that generates a continuous set of coordinates over the object [26]. We also showed the applicability of this method, by proposing the first algorithm that converts a scanned mesh into a Spline surface automatically [23].

We are still not fully satisfied with these results, since the method remains quite complicated. We think that a deeper understanding of the underlying theory is likely to lead to both efficient and simple methods. For this reason, in 2012 we studied several ways of discretizing partial differential equations on meshes, including Finite Element Modeling and Discrete Exterior Calculus. In 2013, we also explored Spectral Geometry Processing and Sampling Theory (more on this below).

3.3. Computer Graphics

Keywords: texture synthesis, shape synthesis, texture mapping, visibility

Content creation is one of the major challenges in Computer Graphics. Modeling shapes and surface appearances which are visually appealing and at the same time enforce precise design constraints is a task only accessible to highly skilled and trained designers.

In this context the team focuses on methods for by-example content creation. Given an input example and a set of constraints, we design algorithms that can automatically generate a new shape (geometry+texture). We formulate the problem of content synthesis as the joint optimization of several objectives: Preserving the local appearance of the example, enforcing global objectives (size, symmetries, mechanical properties), reaching user defined constraints (locally specified geometry, contacts). This results in a wide range of optimization problems, from statistical approaches (Markov Random fields), to combinatorial and linear optimization techniques.

As a complement to the design of techniques for automatic content creation, we also work on the representation of the content, so as to allow for its efficient manipulation. In this context we develop data structures and algorithms targeted at massively parallel architectures, such as GPUs. These are critical to reach the interactive rates expected from a content creation technique. We also propose novel ways to store and access content defined along surfaces [27] or inside volumes [18] [22].

The team also continues research in core topics of computer graphics at the heart of realistic rendering and realistic light simulation techniques; for example, mapping textures on surfaces, or devising visibility relationships between 3D objects populating space.

4. Application Domains

4.1. Geometric Tools for Simulating Physics with a Computer

Numerical simulation is the main targeted application domain for the geometry processing tools that we develop. Our mesh generation tools are tested and evaluated in the frame of our cooperation with the Gocad consortium, with applications in oil exploration and geomechanics, through co-advised Ph.D. thesis (Arnaud Botella, Julien Renaudeau). We think that the hex-dominant meshes that we generate have geometrical properties that make them suitable for some finite element analyses. We work on evaluating and measuring their impact with simple problems (heat equation, linear elasticity) and then practical applications (unfolding geological layer), with the Ph.D. thesis of Maxence Reberol.

In numerical simulation, developing discrete formulations that satisfy the conservation laws (conservation of mass, conservation of energy, conservation of momentum) is important to ensure that the numerical simulation faithfully reflects the behavior of the physics. There are interesting relations with optimal transport theory, as explained by Benamou and Brenier who developed a numerical algorithm for optimal transport that uses a fluid dynamics formulation [17]. Conversely, some dynamics can be approximated by a series of optimal transport problems, as in the Jordan-Kinderlehrer-Otto scheme [20] and in recent works by Mérigot. We started developing efficient geometric algorithms and optimisation methods that may serve as the basis for implementing these numerical methods in 3D. We started discussions / cooperation projects with Quentin Mérigot (MOKAPLAN project).

4.2. Fabrication

Our work around fabrication and additive manufacturing finds applications in different fields. Our algorithms for fast geometric computations on solids (boolean operations, morphological operations) are useful to model a variety of shapes, from mechanical engineering parts to prosthetics for medical applications.

Our by-example techniques allow for simpler modeling and processing of very intricate geometries and therefore also find applications in art and design, for unusual shapes that would be very difficult to obtain otherwise. Extensions of these techniques also find applications for reproducing naturally occurring micro-structures from a scanned sampled.

5. Highlights of the Year

5.1. Highlights of the Year

5.1.1. Geometry processing

Meshes composed of hexahedra (deformed cubes) are desirable for certain numerical simulations, they can improve both performances and precision. They are very difficult to generate. We developed in 2010 one of the first fully automatic algorithms that generates a "hex-dominant" hybrid mesh (top part of the image), with hexahedra and other elements (colored). This year, we made a quantum leap, and significantly reduced the number of non-hex elements (bottom part of the image). Our approach is based on an optimization of a direction field [11] and a global parameterization steered by the direction field [9].

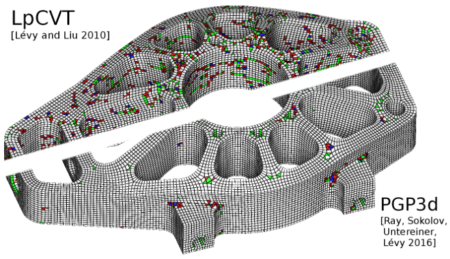

Figure 1. Improvements in hexahedral dominant remeshing.

5.1.2. Additive manufacturing

The advent of additive manufacturing enables the fabrication of shapes with unprecedented complexity, in particular embedding intricate micro-structures with details in the order of tens of microns. There is a strong interest in different fields for such structures, in medical science (prosthetics), mechanical engineering (strong but lightweight materials), art and design (aesthetics, material strength and flexibility). Unfortunately, we lack the software tools to model these structures efficiently. This year we made two significant advances in this area. We first proposed a novel methodology to create procedural micro-structures that exhibit good mechanical properties and can be fabricated [7]. As the definition of the micro-structure is procedural, they are not pre-computed. Instead their geometry is evaluated on the fly, slice after slice, during the additive manufacturing process. Yet, their elasticity can be progressively varied within the shape to align with geometric features. Our second contribution is a novel algorithm to synthesize intricate filigree patterns along a surface, from basic elements [5]. This is achieved by relaxing a strict geometric packing problem by to allow for partial overlaps between elements that preserve local geometric details. The shapes are optimized for strength during the synthesis.

6. New Software and Platforms

6.1. GEOGRAM

GEOGRAM : A functions library for geometric programming
KEYWORD: 3D modeling

GEOGRAM is a programming library with a set of basic geometric algorithms, such as search data structures (AABB tree, Kd tree), geometric predicates, triangulations (Delaunay triangulation, Regular triangulation), intersection between a simplicial mesh and a Voronoi diagram (restricted Voronoi diagram). GEOGRAM also includes a code generator for predicates (PCK: Predicate Construction Kit) and an efficient implementation of expansion arithmetics in arbitrary precision. GEOGRAM is shipped with WARPDRIVE, the first program that computes semi-discrete optimal transport in 3D.

- Participant: Bruno Lévy
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/software/geogram/doc/html/index.html

6.2. GLE

GraphiteLifeExplorer
KEYWORDS: 3D modeling - Biology
GLE is a 3D modeler, developed as a plugin of Graphite, dedicated to molecular biology. Biologists need simple spatial modeling tools to help in understanding the role of the relative position of objects in the functioning of the cell. In this context, we develop a tool for easy DNA modeling. The tool generates DNA along any user-given curve, open or closed, allows for fine-tuning of atoms position and, most importantly, exports to PDB (the Protein Daba Bank file format).

- Participant: Samuel Hornus
- Partner: Fourmentin Guilbert foundation
- Contact: Samuel Hornus
- URL: https://members.loria.fr/samuel.hornus/FFG/gle.html

6.3. Graphite

Graphite: The Numerical Geometry Workbench
KEYWORDS: 3D modeling - Numerical Geometry - Texturing - Lighting - CAD - Visualization
Graphite is an experimental 3D modeler, built on top of the Geogram programming library. It has data structures and efficient OpenGL visualization for pointsets, surfacic meshes (triangles and polygons), volumetric meshes (tetrahedra and hybrid meshes). It has state-of-the-art mesh repair, remeshing, reconstruction algorithms. It also has an interface to the Tetgen tetrahedral mesh generator (by Hang Si). This year, Graphite3 was released. It is a major rewrite, based on Geogram, with increased software quality standards (zero warnings on all platforms, systematic documentation of all classes / all functions / all parameters, dramatically improved performances). It embeds Geogram (and optionally Vorpaline) with an easy-to-use Graphic User Interface.
Graphite is a dedicated software platform in numerical geometry that enables, among other things, 3D modelling and texture baking.

- Participants: Dobrina Boltcheva, Samuel Hornus, Bruno Lévy, David Lopez, Jeanne Pellerin and Nicolas Ray
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/software/graphite

6.4. IceSL

The software allows us to modelize throught CSG's operations 3D's objects. These objects can be directly prepared to be send through a 3d printer without forming an intermediary mesh.

- Participants: Sylvain Lefebvre, Jérémie Dumas, Jean Hergel, Frederic Claux, Jonas Martinez Bayona and Samuel Hornus
- Contact: Sylvain Lefebvre
- URL: http://shapeforge.loria.fr/icesl

6.5. LibSL

LibSL: Simple Library For Graphics
LibSL is a toolbox for rapid prototyping of computer graphics algorithms, under both OpenGL, DirectX 9/10, Windows and Linux.

- Participant: Sylvain Lefebvre
- Contact: Sylvain Lefebvre
- URL: http://members.loria.fr/Sylvain.Lefebvre/libsl

6.6. OpenNL

OpenNL: Open Numerical Library
KEYWORDS: 3D modeling - Numerical algorithm
SCIENTIFIC DESCRIPTION

Open Numerical Library is a library for solving sparse linear systems, especially designed for the Computer Graphics community. The goal of OpenNL is to be as small as possible, while offering the subset of functionalities required by this application field. The Makefiles of OpenNL can generate a single .c + .h file, very easy to integrate in other projects. The distribution includes an implementation of the Least Squares Conformal Maps parameterization method.

- Participants: Bruno Lévy, Rhaleb Zayer and Nicolas Ray
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/index.php/software/4-library/23-opennl.html

6.7. VORPALINE

VORPALINE mesh generator
KEYWORDS: 3D modeling - Unstructured heterogeneous meshes
VORPALINE is a surfacic and volumetric mesh generator, for simplicial meshes (triangles and tetrahedra), for quad-dominant and hex-dominant meshes.

- Participant: Bruno Lévy
- Contact: Bruno Lévy
- URL: http://alice.loria.fr/index.php/erc-vorpaline.html

7. New Results

7.1. Hexahedral-dominant Remeshing

Participants: Dmitry Sokolov, Nicolas Ray, Bruno Lévy, Maxence Reberol

Representing the geometry of complex objects in a computer is usually achieved by a mesh: the object is decomposed in cells that have a simple geometry. Each cell is defined by a set of facets. The simplest choice is to use meshes with tetrahedral cells that are relatively easy to produce and to work with. However, some applications involving numerical simulations better work with hexahedral cells. Such hexahedral meshes are very difficult to produce, even when it is completely done by a designer. Our objective is to relax the intrinsic difficulties of full hexahedral remeshing by allowing the process to generate a few tetrahedra in the hexahedral mesh (hexahedral-dominant meshes). Our approach is to produce as many hexahedra as possible by filling most of the volume with a deformed 3D grid, and to rely on more classic meshing techniques everywhere else. We also develop tools to evaluate how our remeshing impacts results of FEM simulations.

7.1.1. Generation of Hexahedral-dominant Meshes

The traditional approach to produce hexahedral dominant meshes is by advancing front: first hexahedra are produced near the object boundary, then additionals hexahedra are attached to them. An alternative solution is to consider an hexahedral mesh as a deformed 3D grid: the hexahedral remeshing problem is then restated as finding the (geometric) deformation that will transform the hexahedral mesh into the regular grid. This approach is able to generate very good hexhaderal meshes, but it is often impossible to entirely remesh the input object.

Our objective is to produce hexahedra from the mapping approach, then complete the mesh with traditional approaches that may leave some tetrahedra. We proposed a first solution [9]: we compute a mapping, extract the vertices of the deformed 3D grid, generate a tetrahedral mesh having these vertices, then merge sets of tetrahedra into hexahedra with an extension of [25]. Using the mapping as a heuristic made this solution very competitive with other hexahedral dominant methods. We are now developing a software pipeline that makes it easy for different algorithms (frame field, mapping and classic remeshing) to work together. With a simple implementation of each step, we already observe better performances than previous works, and we foresee many opportunities to improve it.

7.1.2. Impact on FEM Performance

It is admitted by our scientific community that hexahedral meshes are better than tetrahedral meshes for some FEM simulation. We would like to demonstrate evidence of this belief, including fair comparisons with equal running time and/or result accuracy, with the best function basis for each case. For hexahedral dominant meshes, we have developed a new specific function basis devoted to properly link tetrahedral and hexahedral elements. Using a combination of tri-linear and quadratic hexahedra, we can build an approximation space made of continuous functions, even at non-conforming interfaces between hexahedra and tetrahedra. But in practice, hexahedral-dominant meshes are mainly useful to mesh complicated 3D domains. In such cases, there are no analytical solutions of partial differential equations and thus it is not straightforward to evaluate the accuracy of a new numerical method. To measure the differences between finite element solutions defined on different meshes of the same 3D model, we are developing a fast and efficient sampling method which exploits GPU hardware. These topics are addressed in the (ongoing) Ph.D. thesis of Maxence Reberol.

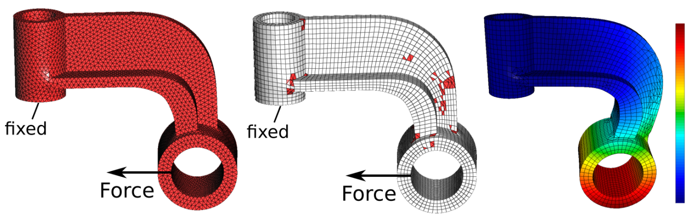

Figure 2. Mechanical problem on the Hanger 3D model. (left) Sandard tetrahedral mesh. (center) Our hex-dominant mesh, hex in gray and tet in red. (right) Solution of the problem with mixed hexahedral-tetrahedral finite elements, color is the amplitude of the displacement.

7.2. Optimal transport

Participant: Bruno Lévy

7.2.1. *Optimal transport:*

Optimal Transport is not only a fundamental problem with a rich structure, but also a new computational tool, with many possible applications. To name but a few, applications of Optimal Transport comprise image registration, reflector and refractor design, histogram interpolation, artificial intelligence. In astrophysics, it is used by Early Universe Reconstruction, a difficult inverse problem that reconstructs the time evolution of the universe from the observed current state. It can be also used in meteorology, to simulate certain phenomena (semi-geostrophic currents). It is also the main component of solvers for certain equations, based on a variational formulation that leads to a gradient flow. All these applications and future developments depend on a single component: an efficient solver for the Monge-Ampère equation. We developed a new algorithm that overcome by several order of magnitude the speed of the classical "auction algorithm" (that solves in $O(n \log(n))$ a discrete version of the problem). The *semi-discrete* version of the problem that we study can be solved by extremizing a smooth objective function, thus a significantly faster speed is obtained as compared to the previous combinatorial algorithm. This year we improved our Quasi-Newton solver and replaced it with a Full-Newton solver, that gains one additional order of magnitude in speed, and we can solve semi-discrete problems with 1 million Dirac masses in a matter of minutes. We also experimented with applications of this solver to fluid simulation. Last winter (December 2015) Wenping Wang visited Nancy, and we discussed several ideas on Optimal Transport. We proposed together this year (2016) a new method to sample a surface with a power diagram [31]. The positions of the samples are optimized by a criterion similar to centroidal Voronoi tessellations, and the associated weights are used to control the areas of the power cells with prescribed values. We give the expressions of the derivatives of the combined objective function, and propose a quasi-Newton algorithm to optimize it. We describe several applications of the algorithm.

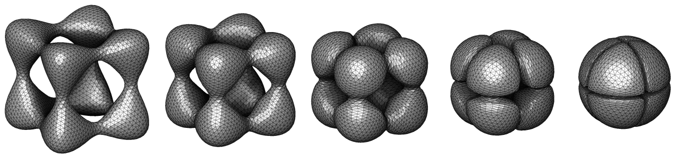

Figure 3. Semi-discrete optimal transport between a shape and a sphere, computed by our algorithm

7.3. Spectral Clustering of Plant Units From 3D Point Clouds

Participant: Dobrina Boltcheva

High-resolution terrestrial Light Detection And Ranging (tLiDAR), a 3-D remote sensing technique, has recently been applied for measuring the 3-D characteristics of vegetation from grass to forest plant species. The resulting data are known as a point cloud which shows the 3-D position of all the hits by the laser beam giving a raw sketch of the spatial distribution of plant elements in 3-D, but without explicit information on their geometry and connectivity. We have developed a new approach based on a delineation algorithm that clusters a point cloud into elementary plant units such as internodes, petioles and leaves. The algorithm creates a graph (points + edges) to recover plausible neighboring relationships between the points and embeds this graph in a spectral space in order to segment the point-cloud into meaningful elementary plant units. This work has been published in the International Journal of Remote Sensing [6].

7.4. Surface Reconstruction From 3D Point Clouds

Participants: Dobrina Boltcheva, Bruno Lévy

We have developed a practical reconstruction algorithm that generates a surface triangulation from an input pointset. In the result, the input points appear as vertices of the generated triangulation. The algorithm has several desirable properties: it is very simple to implement, it is time and memory efficient, and it is trivially parallelized. On a standard hardware (core i7, 16Gb) it takes less than 10 seconds to reconstruct a surface from 1 million points, and scales-up to 36 million points (then it takes 350 seconds). On a smartphone (ARMV7 Neon, quad core), it takes 55 seconds to reconstruct a surface from 900K points. The algorithm computes the Delaunay triangulation of the input pointset restricted to a "thickening" of the pointset (similarly to several existing methods, like alpha-shapes, crust and co-cone). By considering the problem from the Voronoi point of view (rather than Delaunay), we use a simple observation (radius of security) that makes the problem simpler. The Delaunay triangulation data structure and associated algorithms are replaced by simpler ones (KD-Tree and convex clipping) while the same set of triangles is provably obtained. The restricted Delaunay triangulation can thus be computed by an algorithm that is not longer than 200 lines of code, memory efficient and parallel. The so-computed restricted Delaunay triangulation is finally post-processed to remove the non-manifold triangles that appear in regions where the sampling was not regular/dense enough. Sensitivity to outliers and noise is not addressed here. Noisy inputs need to be pre-processed with a pointset filtering method. In the presented experimental results, we are using two iterations of projection onto the best approximating plane of the 30 nearest neighbors (more sophisticated ones may be used if the input pointset has many outliers).

This work has been published in the Research Report [13] and is currently in revision for Eurographics 2017.

7.5. Microstructures for additive manufacturing

Participants: Jonas Martinez, Sylvain Lefebvre

Nowadays, there is a big interest in the functional optimization of microstructures for additive manufacturing, as reflected by the high number of recent publications on the subject. This also comes not only from research but also industry, as controlling the macroscopic elasticity of materials has a wide range of industrial applications. For instance, to fabricate flexible prosthetic body parts, or to produce rigid but porous prosthetics for surgery. In particular, controlling material elasticity will enable the design of lightweight and resistant materials, and in turn, reduce material consumption.

Most of the existing software either optimize for periodic tilings of microstructures, or generate random microstructures without precise control of their functionality. We recently introduced a method [7] to generate stochastic structures (figure 4) while having unique computational advantages, and precisely controlling their functionality. Our optimization approach of stochastic porous structures deviates significantly from both the periodic tiling of microstructures and the optimization of macrostructures, by making a link between microstructures, and procedural solid textures with controlled statistics in Computer Graphics. We believe there are many other such structures left to be discovered, and hope our work will spark further interest in procedurally generated, stochastic microstructures.

7.6. Towards Zero-Waste Furniture Design

Participants: Bongjin Koo, Jean Hergel, Sylvain Lefebvre, Niloy J. Mitra.

This project considered the optimization of parametric models of furniture to reduce the wastage of material used to fabricate the model. Our approach uses a 2D packing algorithm to pack the different parts of the furniture in a wooden plank. Then we optimize locally the wastage by editing smoothly the parameters with only moving smoothly the parts in the packing space. We produced full size objects with laser cutter to prove the efficiency of our method. This work has been accepted in Transaction on Visualization and Computer Graphics.

7.7. Anti-aliasing for fused filament deposition

Participants: Hai-Chuan Song, Nicolas Ray, Dmitry Sokolov, Sylvain Lefebvre

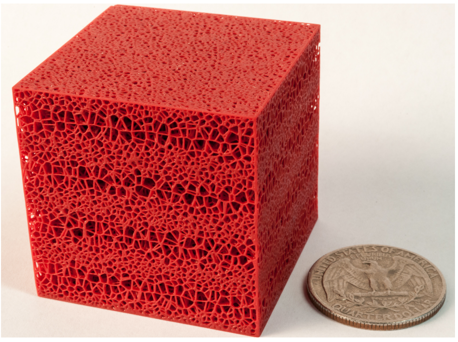

Figure 4. Anisotric microstructures

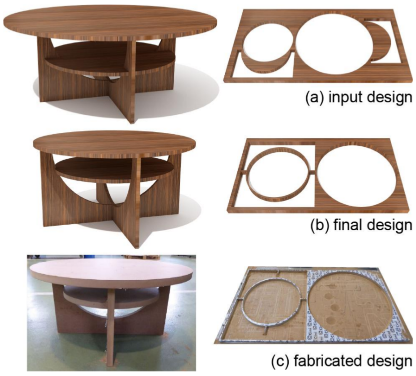

Figure 5. Our technique modifies the parameters of the input design (a) to improve the packing and waste less material (b). The produced furniture is shown in (c).

Layered manufacturing inherently suffers from staircase defects along surfaces that are gently sloped with respect to the build direction. Reducing the slice thickness improves the situation but never resolves it completely as flat layers remain a poor approximation of the true surface in these regions. In addition, reducing the slice thickness largely increases the print time. In this project we focus on a simple yet effective technique to improve the print accuracy for layered manufacturing by filament deposition. Our method [16] works with standard three-axis 3D filament printers (e.g. the typical, widely available 3D printers), using standard extrusion nozzles. It better reproduces the geometry of sloped surfaces without increasing the print time. Our key idea is to perform a local anti-aliasing, working at a sub-layer accuracy to produce slightly curved deposition paths and reduce approximation errors. We show that the necessary deviation in height compared to standard slicing is bounded by half the layer thickness. Therefore, the height changes remain small and plastic deposition remains reliable. We further split and order paths to minimize defects due to the extruder nozzle shape, avoiding any change in the existing hardware. We apply and analyze our approach on 3D printed examples, showing that our technique greatly improves surface accuracy and silhouette quality while keeping the print time nearly identical.

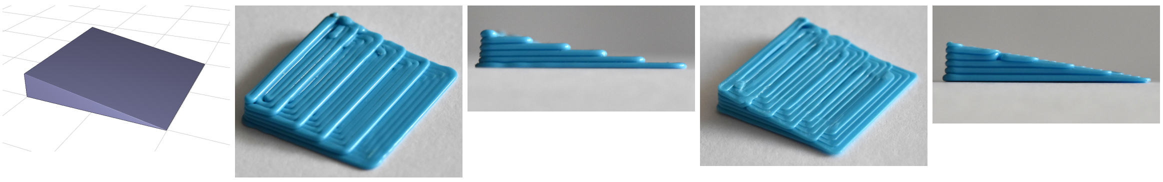

Figure 6. Printing a wedge model clearly reveals the staircase defects that plague 3D printing. (a) Input 3D model; the bottom edge length is 20mm and the angle of the incline plane is 10. (b) Global view and (c) side view of a standard, flat layer printed result. (d) Global view and (e) side view of our anti-aliased printed result, revealing the improvement in surface accuracy and silhouette smoothness

8. Partnerships and Cooperations

8.1. Regional Initiatives

CPER (2014-2020) 50 k€. Sylvain Lefebvre coordinates a work package for the CPER 2014-2020. It involves several members of ALICE as well as laboratories within the Nancy area (Institut Jean Lamour, LRGP, ERPI). Our goal is to consider the interaction between software and material in the additive manufacturing process, with a focus on filament-based printers.

PIC (2015-2017) 150 k€. The PIC project (Polymères Innovants Composites) is a collaboration between Inria, Institut Jean Lamour and Ateliers Cini, funded by Région Lorraine. The goal is to develop a new additive manufaturing process using filament of composite materials with applications in mechanical engineering and the medical domain. Our goal in the project is to provide novel ways to deposit the filament that is better suited to the considered materials and improves the quality of the final parts.

8.2. National Initiatives

8.2.1. ANR BECASIM (2013 – 2016)

890 k€. X. Antoine heads the second partner, which includes Bruno Lévy. Budget for Nancy: 170 k€ of which 100 k€ are for IECL (team CORIDA). This project is managed by Inria. Becasim is a thematic "Numerical Models" ANR project granted by the French Agence Nationale de la Recherche for years 2013-2016. The

acronym Becasim is related to Bose-Einstein Condensates: Advanced SIMulation Deterministic and Stochastic Computational Models, HPC Implementation, Simulation of Experiments. The members of the ANR Project Becasim belong to 10 different laboratories.

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

8.3.1.1. SHAPEFORGE

Title: ShapeForge: By-Example Synthesis for Fabrication

Programm: FP7

Duration: December 2012 - November 2017

Coordinator: Inria

Inria contact: Sylvain Lefebvre

Despite the advances in fabrication technologies such as 3D printing, we still lack the software allowing for anyone to easily manipulate and create useful objects. Not many people possess the required skills and time to create elegant designs that conform to precise technical specifications. 'By–example' shape synthesis methods are promising to address this problem: New shapes are automatically synthesized by assembling parts cutout of examples. The underlying assumption is that if parts are stitched along similar areas, the result will be similar in terms of its low–level representation: Any small spatial neighborhood in the output matches a neighborhood in the input. However, these approaches offer little control over the global organization of the synthesized shapes, which is randomized. The ShapeForge challenge is to automatically produce new objects visually similar to a set of examples, while ensuring that the generated objects can enforce a specific purpose, such as supporting weight distributed in space, affording for seating space or allowing for light to go through. These properties are crucial for someone designing furniture, lamps, containers, stairs and many of the common objects surrounding us. The originality of our approach is to cast a new view on the problem of 'by–example' shape synthesis, formulating it as the joint optimization of 'by–example' objectives, semantic descriptions of the content, as well as structural and fabrication objectives. Throughout the project, we will consider the full creation pipeline, from modeling to the actual fabrication of objects on a 3D printer. We will test our results on printed parts, verifying that they can be fabricated and exhibit the requested structural properties in terms of stability and resistance.

8.4. International Initiatives

8.4.1. Inria Associate Teams Not Involved in an Inria International Labs

8.4.1.1. PREPRINT3D

Title: Model Preparation for 3D Printing

International Partner (Institution - Laboratory - Researcher):

University of Hong Kong, Computer science department, with Li-Yi Wei and Wenping Wang

Start year: 2014

We seek to develop novel ways to prepare objects for 3D printing which better take into account limitations of the fabrication processes as well as real-world properties such as the mechanical strength of the printed object. This is especially important when targeting an audience which is not familiar with the intricacies of industrial design. We target complex, intricate shapes such as models of vegetation and highly detailed meshes, as well as models with thin walls such as architectural models. Our methods will modify the object geometry and topology while remaining as close as possible to its initial appearance.

8.4.2. Inria International Partners

8.4.2.1. Informal International Partners

We have on-going collaborations with Marc Alexa (TU Berlin) regarding slicing algorithms for additive manufacturing and Niloy Mitra (University College London) on minimal wastage design of furniture.

8.5. International Research Visitors

8.5.1. Visits of International Scientists

8.5.1.1. Internships

Denis Salem (CESI-EXIA), 6-months intern started in September 2016, working on point distributions along surfaces using GPU algorithms. Théo Poisson (CESI-EXIA) was an intern from February to May 2016, working on quality testing and improvements to our software IceSL. Yuexin Ma, PhD student with Wenping Wang (HKU), 1 month visit in the context of the PrePrint3D associated team.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

Sylvain Lefebvre is program co-chair for SMI 2017.

9.1.1.2. Member of the Organizing Committees

Bruno Lévy was conference co-chair of GMP 2016.

9.1.2. Scientific Events Selection

9.1.2.1. Chair of Conference Program Committees

Bruno Lévy was member of the program committee of ACM/EG Symposium on Geometry Processing, Pacific Graphic, Shape Modeling International

9.1.2.2. Member of the Conference Program Committees

Sylvain Lefebvre was member of the SIGGRAPH 2016 program committee, and is a member of the EUROGRAPHICS 2017 program committee.

9.1.2.3. Reviewer

- Jonas Martinez: Eurographics'16 Short papers.
- N. Ray: SIGGRAPH, SIGGRAPH ASIA, eurographics

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

Sylvain Lefebvre is associate editor for ACM TOG. Bruno Lévy is associate editor of ACM Transactions on Graphics, The Visual Computer and Graphical Models

9.1.3.2. Reviewer - Reviewing Activities

- D. Sokolov: Computer Graphics Forum, The Visual Computer
- N. Ray: ACM TOG, TVCG, Electronics Letters
- Dobrina Boltcheva : International Journal of Discrete & Computational Geometry
- Jonas Martinez: Computers & Graphics journal.
- Laurent Alonso: Graphs and Combinatorics, Discrete Applied Mathematics, ACM Translation of Algorithms, ESA 2016, LATIN 2016

9.1.4. Invited Talks

- Dmitry Sokolov: "Towards hexahedral meshes" Numerical geometry, grid generation and scientific computing NUMGRID 2016, Moscow
- Sylvain Lefebvre: Computer Graphics lab of Aachen, led by Pr. Leif Kobbelt
- Sylvain Lefebvre: IST Austria, visiting Pr. Bernt Bickel
- Sylvain Lefebvre: invited at CNR Imati (Genova, Italy) for a week to present his work and collaborate on a joint survey paper submitted to EUROGRAPHICS 2017.

9.1.5. Scientific Expertise

Bruno Lévy evaluated projects for the ANR, for the European Research Council.

9.1.6. Research Administration

Samuel Hornus was moderator of the CDT (Commission for Technological Development) of Inria nancy.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Licence : Enseignant, titre du cours, nombre d'heures en équivalent TD, niveau (L1, L2, L3), université, pays

Licence : Cédric Zanni, Informatique 1, 20h ETD, L3, École des Mines de Nancy, France

License: Dobrina Boltcheva, Licence Pro ISN, Computer Graphics 20h, Image Processing 20h, IUT Saint-Dié-des-Vosges

License: Dobrina Boltcheva, 2A DUT INFO, Advanced Java Programming 100h, IUT Saint-Dié-des-Vosges

License: Dobrina Boltcheva, 1A DUT INFO, Programming 60h, IUT Saint-Dié-des-Vosges

Licence : Dmitry Sokolov, Informatique graphique, 55h ETD, L1, Université de Lorraine, France

Licence : Dmitry Sokolov, Programmation Avancée, 40h ETD, L2, Université de Lorraine, France

Licence : Dmitry Sokolov, Logique, 20h ETD, L3, Université de Lorraine, France

Licence : Dmitry Sokolov, Introduction à la robotique mobile, 30h ETD, L2, Université de Lorraine, France

License : Samuel Hornus, Mathématiques Appliquées pour l'Informatique, 32 h ETD, L3, Télécom Nancy

Master : N. Ray, communication, 10h equivalent TD, M1, Universitée de Lorraine, France

Master :Cédric Zanni, Software Engineering, 19h ETD, M1, École des Mines de Nancy, France

Master :Cédric Zanni, Operating System, 14h ETD, M1, École des Mines de Nancy, France

Master :Cédric Zanni, Modèles et simulations d'instrument de musique, 9h ETD, M1, École des Mines de Nancy, France

Master :Dmitry Sokolov, Optimisation Combinatoire, 30h ETD, M1, Université de Lorraine, France

Master :Dmitry Sokolov, Infographie, 30h ETD, M1, Université de Lorraine, France

Master :Dmitry Sokolov, Algorithmique et programmation, 45h ETD, M1, Université de Lorraine, France

Master :Dmitry Sokolov, Algorithmique et Programmation Avancée, 60h ETD, M2, Université de Lorraine,France

Master, Sylvain Lefebvre, Programmation pour le jeux vidéo, 30h ETD, Ecole des Mines de Nancy, France.

Master, Sylvain Lefebvre, Introduction au parallélisme et au graphisme, 9h ETD, ENSG Nancy, France.

Master, Sylvain Lefebvre, Introduction à la fabrication additive, 9h ETD, ENSEM Nancy, France.

Master, Sylvain Lefebvre, Synthèse procédurale pour le design, 9h ETD, ENSAD, Nancy, France.

E-learning

Pedagogical resources tinyRender: Dmitry Sokolov, Computer graphics course. Short series of lectures on what is under the hood of OpenGL/DirectX. The course is already adopted in several universities throughout the world, and is now included in the distribution of Bullet physics library. Microsoft includes its C# port in the distribution of Xamarin Workbooks .

9.2.2. Supervision

HdR : Dmitry Sokolov, *Modélisation géométrique*, Université de Lorraine, 10 juin 2016

PhD thesis submitted, defense 01/02/2017 : Jean Hergel, *Modeling under Fabrication Constraints*, Oct. 2013, supervised by Sylvain Lefebvre

PhD thesis submitted, defense 03/02/2017 : Jérémie Dumas, *By-example shape synthesis for Fabrication*, Sept. 2013, supervised by Sylvain Lefebvre

PhD in progress : Maxence Reberol, *Finite elements for non-conformal mixed hexahedral-tetrahedral meshes*, Janvier 2015, co-supervised by Bruno Lévy and Sylvain Lefebvre

9.2.3. Juries

Dobrina Boltcheva was a member of the recruitment committee for an associate professor position (maître de conférence) at the University of Poitiers (MCF 27 - XLIM/IRIAF).

Sylvain Lefebvre was a member of the CR2 recruitment jury for Inria Nancy Grand-Est. He was reviewer ('rapporteur') for the PhD defense of Hugo Loi (Université de Grenoble).

9.3. Popularization

Sylvain Lefebvre participated in the 2016 science festival (stand and general public conference), and gave a presentation in front of an industrial public in an event organized by Fondation Charles Hermite.

10. Bibliography

Major publications by the team in recent years

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Articles in International Peer-Reviewed Journals

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International Conferences with Proceedings

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