



## Activity Report 2016

# Team ANJA

## ANJA

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions).

RESEARCH CENTER  
**Rennes - Bretagne-Atlantique**

THEME  
**Optimization, machine learning and  
statistical methods**



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# Team ANJA

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## Keywords:

### Computer Science and Digital Science:

- 3.4.1. - Supervised learning
- 3.4.2. - Unsupervised learning
- 3.4.4. - Optimization and learning
- 3.4.5. - Bayesian methods
- 6.1.2. - Stochastic Modeling (SPDE, SDE)
- 6.1.3. - Discrete Modeling (multi-agent, people centered)
- 8.2. - Machine learning

### Other Research Topics and Application Domains:

- 9.5.2. - Juridical science
- 9.5.3. - Economy, Finance
- 9.5.5. - Sociology
- 9.5.6. - Archeology, History

## 1. Members

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## 2. Overall Objectives

### 2.1. Overall Objectives

Applications of mathematics in social sciences (economy, sociology, law,...) are increasing at a fast pace. In a growing number of areas, decisions are being made in an *actuarial* way rather than in a *clinical* one, as has been the norm for decades or even centuries. This means that actions are based on the outputs of (mathematical) models instead on the informed judgement of experts. This occurs daily in financial economics (for instance for option pricing, portfolio management and risk assessment), and is also on the verge of becoming routine in certain domains of law and certain countries (most notably the USA, where examples include evidence-based sentencing and the setting of certain fines).

Such a paradigm shift certainly has several benefits, but it may also be harmful at least because it *rigidifies* the system. What we mean by this is that, when most actors in a given field use the same model, with possibly minor variants, massive unanticipated effects will occur:

1. if the model is either wrong, or wrongly specified or used beyond its nominal conditions, the fact that everyone makes the same erroneous decisions may lead either to system-wide crises (this has happened several times in finance in recent years) or to profound uncontrolled modifications of reality (this has also occurred in finance but also in law). This second case, leading to “silent revolutions”, is maybe more problematic;
2. even when the model is approximately correct or used in normal conditions, hard and fast, uniform answers will reduce diversity in an often damaging way.

Both situations above are related to the concept of *performativity* introduced by social scientists. More precisely, item 1 leads to what we call convergent or divergent performativity, while item 2 leads systematically to convergent performativity.

How can one eliminate or at least reduce the negative impacts of the generalizing use of actuarial approaches in social sciences ? Clearly, the trend seems unstoppable and even if it were, this would probably not be desirable: quantitative, mathematically well-founded methods are likely to permit great progress in many domains. There is for instance a strong need for good models that would help judges fix financial amounts in various areas including spousal support and fines in cases of violation of intellectual property. The aim of Anja is to propose ways to minimize the drawbacks of model-based approaches using essentially the following devices:

1. be aware of the performative effect of models: by taking into account the impact of actuarially-based actions on reality when setting-up a model, this impact will be endogenised and, thus, to some extent, controlled. In other words, whenever possible, we study the feedback between reality and the proposed model.
2. insist that the output of models be systematically probabilistic: instead of a hard answer, always propose a range of answers with associated probability distribution. This should greatly reduce the rigidification of the system by allowing to re-introduce a part of expertise, thus preserving some diversity, and in turn minimise the occurrence of crises or unwanted modifications of reality. From a theoretical point of view, this amounts to using tools introduced in the now well-developed area of *uncertainty propagation*. The basic idea is that uncertainties on the various parameters are propagated at all stages of the modelling process, so that, at each step, computations are made on probability densities rather than numbers. This approach raises various difficulties when the models are non-linear, as will typically be the case in our applications.

Although our long term aim is to provide a general frame enabling to fulfil the above objectives, our first studies will focus on a limited number of well identified situations in economy and law described in Sections 4.1 and 4.2.

Our program clearly requires strong interdisciplinary collaborations. Anja is actively involved in on-going work with financial economists and lawyers.

## 3. Research Program

### 3.1. Research Program

The aim of Anja is to develop mathematical models in selected areas of SHS, which include, at this time, economy/finance, law, and archaeology. These models are essentially probabilistic. This entails that our theoretical studies mainly lie in the fields of probability and statistics.

A major focus of Anja is on providing mathematical analyses of how performativity operates in economy/finance and law, where performativity is understood as the phenomenon by which applying models co-constructs a new reality by the very fact that the existing reality was not properly apprehended. We are chiefly concerned with performativity that results from mathematical models. Indeed, while performativity exists before and independently of such models, mathematics may, and already have, strengthened the performative power in a significant way. This has occurred so far in an uncontrolled fashion, and thus in a typically damaging way. The essence of our work is to shed light on the mechanisms mediating the increase of performativity brought by mathematical models, thus allowing one to manage their effects and hopefully orient them towards an improvement of the reality they transform.

We stress the important fact that this program allows us to go well beyond what is typically achieved in sociological studies. For instance, many such studies have evidenced the role of performativity in the context of financial theory and how it shapes today's markets [46]. This is certainly useful in order to exert political control on the tools proposed by, e.g., economists. However, such an exogenous control is not fully satisfactory because it does not provide explicit procedures to enhance the models. This is due to the fact that these studies have not permeated the technical literature to a point where it would have a significant impact on the definition and practical use of models. One explanation for this is that, though convincing, these analyses do not provide mathematical or otherwise applicable tools to modify practices: they explain general mechanisms through which, for instance, economics performs the reality of economies, but do not shed light on the precise mathematics that mediate these mechanisms. We believe that it is important that mathematicians tackle this issue. In other words, we think that it is extremely useful to reverse that statement made in [48]: "*en souscrivant au programme de la performativité, la recherche en sociologie économique ne se contente plus de partager ses objets d'études avec les sciences économiques, elle inclut ces dernières dans ses propres objets d'étude*"<sup>1</sup>. Our "reverse" statement is that economical sciences, and, more generally, mathematics applied in SHS, should include in their research objectives the sociological impact they create through performativity: mathematicians need to take into account in their models the fact that reality will be transformed by them, and thus model also this transformation.

The first goal of Anja is precisely to fill this gap, that is, to pinpoint which parts of a given mathematical model are responsible for performative effects and how this occurs. It is important to stress that, in our view, this means that the performativity of mathematical models will be itself assessed with mathematical tools. This endogenisation permits to measure quantitatively the impact of models on reality. In turn, this quantification opens the way to our second and more ambitious goal: indeed, we propose paradigms allowing one to control the performativity of mathematical models. The main mechanisms we use in that view are as follows:

- systematically take into account the fact that the models will perform reality. This means that, when defining our models, we try as much as possible to foresee how applying them will transform practices, and then adjust them in such a way that these modifications are desirable and under control;
- impose that the output of our models always be *probability distributions* rather than hard prescriptions. In other words, recognizing that modelling in SHS, in addition to being typically extremely complex with a large number of variables and with many sources of errors in the process of calibrating the parameters, always involves addressing a moving reality that will be transformed by the very application of the model, we insist that, at all stages of the analysis, *uncertainties* be propagated, as is routinely done in industrial fields such as aeronautics, so that the answer of the system will be probabilistic. We use in particular Bayesian analysis in that view, in order to incorporate information on unknown parameters using prior probability distributions that are sequentially updated after the acquisition of each new observation.

As a longer term perspective, we intend to propose a general mathematical model of performativity. The current literature has already proposed general analyses of mechanisms through which theories can become performative: most notably, [35] has identified three main such channels, namely institutional design, social

<sup>1</sup>"By subscribing to the performativity program, research in economical sociology goes beyond sharing its objects of study with economical sciences: it includes the latter in its own objects of study.

norms, and language, as well as the way in which culture and accountability affect their course of operation. Our aim will be more focused: we will concentrate on the sole performativity of mathematical models, but in this restricted frame, we wish to propose quantitative, mathematical analyses. In other words, a mathematical model of performativity should allow one to answer questions such as: when can one expect that a theory is likely to be performative, what exactly are the conditions favouring performativity, which indices should one look for in order to detect a performative influence, how can one predict whether performativity will be convergent or divergent, which aspects of reality a theory will affect and how, and finally what are the means to minimize its negative effects.

One important motivation of Anja is that we feel that, as researchers in mathematics, we are partly responsible for the way mathematics is used in social sciences. In particular, while we strongly believe that mathematical models have already and will continue to enhance our social lives as they have improved our understanding and control in natural sciences, extra caution is needed because of their performative power explained above. The core of our work is that such caution can be exercised (a) by recognizing that models impact reality and by taking into account their performative power in their very definition, and (b) by using systematically probability and statistics: in a nutshell, imposing that mathematical answers to questions in social sciences always take the form of a probability distribution should (1) remind users that no mathematical model is able to provide a definitive hard and fast answer when it comes, e.g., to computing the amount of a fine in competition law, and (2) allow one to tame the inherent complexity of human-related matters, thus providing useful guides for making informed decisions.

Of course, we do not address performativity issues in all social and human sciences. Rather, we focus on two domains where we already have an expertise: economy/finance, and law. Details on our studies in these fields are given in sections 4.1 and 4.2.

Our program cannot be realistically realised without strong collaborations from specialists in the SHS fields we deal with. In law, our expertise is brought by Jérôme Dupré, which holds a Ph.D. in law and is also a former judge. As far as archaeology is concerned, we collaborate with Philippe Lanos (senior researcher at CNRS). He is an expert in archaeomagnetism and its applications to materials dating in archaeology.

## 4. Application Domains

### 4.1. Economy and finance

#### 4.1.1. *Basel III and Solvency 2 regulations*

As amply demonstrated above, economy is a field where the performativity of mathematical models is particularly noticeable. This has become even more so in recent years in finance because international regulations have fundamentally changed since the Basel II Accords. Among other evolutions, Basel II and III explicitly impose that computations of capital requirements be model-based. The same is true of the Solvency 2 directive, a European regulation aiming in particular at evaluating the amount of capital that insurance companies must hold to reduce the risk of insolvency, much in the spirit in the Basel Accords.

This paradigm shift in risk management has been the source of strong debates among both practitioners and academics, who question whether such model-based regulations are indeed more efficient.

A common feeling in the industry is that regulations will sometimes give a false impression of security: risk managers tend to think that a financial company that would fulfil all the criteria of, say, the Basel III Accords on capital adequacy, is not necessarily on the safe side. This is so mainly because many risks, and most significantly systemic or system-wide risks, are not properly modelled, and also because it is easy to manipulate to some extent various risk measures, such as Value at Risk (VaR).

In parallel, a fast growing body of academic research provides various arguments explaining why current regulations are not well fitted to address risk management in an adequate way, and may even, in certain cases, worsen the situation. In other words, they have a divergent performativity effect.



Our first angle to tackle the performativity of these regulations is to question the Gaussian assumption that is implicitly made in designing them. More precisely, we have already shown in [11], [12] that, in some situations, and because of this assumption, prudential rules are themselves the source of a systemic risk. In [12], it was explained how a wrong model of price dynamics coupled to the regulatory VaR constraint tends to systematically increase Tail Conditional Expectation. [11] details how trying to minimize VaR under Gaussian beliefs for the dynamics of returns when actual movements are stable non-Gaussian results in fact in maximization of VaR. Along with the concept of endogenous risk put forward in [44], this body of work provides a mathematical description of how models perform financial reality: this is a perfect example of divergent performativity, since, because of a wrong model, (mandatory) actions are taken that make financial markets even less similar to the model. More technically, assume the simplest model of returns movements, that is, Brownian motion. Brownian motion is the symmetric stable motion characterized by the stability index  $\alpha = 2$  and a given scale parameter  $\sigma^2$ . Under reasonable assumptions, minimizing VaR in a Brownian market amounts to minimizing the variance. However, in a stable market where  $\alpha < 2$ , which therefore is subject to jumps, minimizing VaR requires to maximize  $\alpha$  while choosing an intermediate value of  $\sigma$ . Furthermore, actions taken under a Brownian belief will tend not only to minimize  $\sigma$  but also  $\alpha$ : therefore, implementing VaR-based regulations founded on the wrong Brownian model tends to decrease  $\alpha$ , making the market even “more” non-Brownian. This is exactly the definition of divergent performativity.

The work in [11], [12] is only one possible mechanism of performativity, although maybe the simplest one. Starting from this, one may progress in two directions: propose regulations that will avoid at least the particular kind of performativity just described, and study more complex models and their performative effects.

As for the first direction, assuming a stable non-Brownian market, we need to understand what kind of constraints would lead to actions favouring an increase rather than a decrease of  $\alpha$ . Our first idea is to explore counter-cyclical measures, as current regulations are often blamed for their pro-cyclical effect. In a nutshell, pro-cyclicality is entailed by the fact that, in market downs, actors will be forced by regulations to reduce their exposure, thus amplifying downwards movements. We plan to investigate how this translates into modifications of the  $(\alpha, \sigma)$  couple, and check whether basing regulations on the time evolution of this couple would be efficient. For instance, one might imagine measuring  $(\alpha, \sigma)$  as a function of time, and let financial companies increase or decrease their solvency capital requirements based on the coupled evolution.

As for the second direction, we remark that, since regulations tend to endogenously modify both volatility and jump intensity, it seems natural to define and study processes where the local regularity varies in time, possibly in relation with the value of the process. We have introduced such classes of processes in recent years. We plan to deepen their study in the light of their possible adequacy for the mathematical modelling of performativity. We briefly describe now the first actions we will take in this respect.

#### 4.1.2. *Multistable and self-stabilizing processes for financial modelling*

It is widely accepted that the dynamics of most financial instruments display jumps and there is a huge literature dealing with jump processes in all areas of financial engineering [32]. In order to get a better understanding of these dynamics, we have developed in recent years various instances of *multistable processes*. These processes were introduced in [4] and further studied e.g. in [8]. Their main feature is that their local intensity of jumps varies in time. In view of their application, we plan to study the following points:

- Recognizing that the local characteristics (intensity of jumps and scale) vary in time implies that evolution equations these parameters must be proposed for these parameters. We have started to develop Hull and White-like models, where auxiliary EDS are satisfied by both scale and the intensity of jumps. This will hopefully allow one to model in a satisfactory manner implicit volatility surfaces.

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<sup>2</sup>recall that a stable motion is a process with independent and identically distributed increments, where each increment follows a stable law  $S_\alpha(\sigma, \beta, \mu)$ . The parameter  $\alpha \in (0, 2]$  characterizes the jump intensity - the smallest  $\alpha$ , the largest the jump intensity, with no jumps when  $\alpha = 2$ , that is, for Brownian motion -,  $\sigma$  is the scale parameter - proportional to the variance when  $\alpha = 2$  -,  $\beta$  is the skewness parameter and  $\mu$  the location one.

- Robust statistical estimation of  $\alpha(t)$  (or of the couple  $(\alpha(t), h(t))$  in the case of the so-called linear multifractional multistable motion) is necessary. Some results are presented in [45], but other methods should be studied.
- Self-regulating processes are processes where the local regularity is a function of the amplitude. They were introduced in [1] and further studied e.g. in [3]. It seems natural to follow the same approach and define “self-stabilizing processes” as processes where the local index of stability is a function of the amplitude. Certain tools used for defining some SRP, namely the fixed point theorem, could be adapted, with the difference that the underlying space will not be the one of continuous functions, but the one of càdlàg functions. As a consequence, the Prohorov metric may have to be considered instead of the sup-norm. We have some preliminary results in this direction, which also include the definition of Markovian self-stabilizing processes. Statistical issues (that is, the estimation of the “self-stabilizing” function) need also be addressed.

#### 4.1.3. Multifractional and self-regulating processes for financial modelling

Besides multistable motions, we will also continue to investigate the use of multifractional Brownian motion in financial modelling. Previous works [29] have shown the potential of this approach, in particular for reproducing certain features of the volatility process [51], and we plan to pursue this line of study. More precisely, we will investigate the following matters:

- The instance of self-regulating processes built so far [1] are not progressive, in the sense that paths are constructed globally rather than in a chronological manner. For this reason, they do not provide adequate models for time series encountered in economy and finance. We will put some effort in trying to construct progressive self-regulating processes. Our first attempts will be based on pathwise stochastic integrals as well as on Skohorod integrals.
- Once progressive self-regulating processes have been built and their basic probabilistic properties been investigated, the second step will consist in constructing estimators for the self-regulating function (that is, the function relating amplitude and regularity). This is of course essential for applications.
- We will finally investigate precisely which economical or financial times series display self-regulation, and examine the performative effect of current regulations when such models are in force.

#### 4.1.4. Performativity of monetary policies

It seems clear that, besides prudential regulations, monetary policies such as quantitative easing used by central banks in Europe, Japan and the USA have a strong impact on economy<sup>3</sup>. There is already a huge literature studying this impact. From a broader perspective, many actions taken by financial authorities are designed in a conceptual frame where volatility is all there is to risk. We believe that incorporating at least another dimension related to jumps is essential for proper control. In this respect, we plan to analyse in a quantitative way what is the impact on the stability of markets of the various measures taken by central banks in recent years, such as Zero Interest Rates Policies, Large Scale Assets Purchases, Forward Guidance or Long Term Refinancing Operations, when one takes into account the jump dimension of risk. Such measures have led to typically very low volatility on the markets. But, as C. Borio of BIS recently stated [30], “history teaches us that low volatility and risk premia are not the signs of smaller risk, but rather than investors are ready to take large risks. The less investors fear risk, the more dangerous the situation is”. In other words, recent monetary policies seem to have lowered volatility at the expense of increasing the intensity of jumps. This view is supported by a number of studies in recent years by the BIS. For instance, [26] argues that the accommodative monetary policy have pushed volatility to low levels in various ways: directly by reducing the amplitude of interest rate movements and by removing to a large extent uncertainty about interest rate changes; and indirectly because an environment of low yields on high- quality benchmark bonds favours risk-taking. Investors then tend to have a lower perception of risk, and thus be inclined to take riskier positions.

<sup>3</sup>In a nutshell, quantitative easing is an unconventional monetary policy by which central banks create new money to buy financial assets in view of stimulating the economy.

Studying such a performative effect is typically in the focus of Anja. Our first attempts in this direction will be again to use stable or multistable processes in place of the Brownian motion as a source of randomness. The obvious approach is to rewrite current models with this modification. This will however require to define several new notions adapted to this situation. More precisely, most computations in classical models crucially depend on the fact that all the quantities involved are square integrable, a property not available when one deals with (multi-)stable processes. As a consequence, correlations, for instance, are not well-defined; this is a problem as they serve as a fundamental tool in such studies. One possible way out would be to use CGMY or other tempered stable processes instead of stable ones, since this would bring us back in the realm of  $L^2$  random variables. The price to pay is that we lose stability, meaning that aggregate behaviours are more difficult to assess. A more ambitious but potentially more fruitful approach is to start again from the modified classical models but to extend their study in a stable frame so as to be able to compute joint distributions.

Another, very different path, is to use the mathematical theory of causality to tackle these questions [49]. We will recall in the next section some facts about causality. Recent studies have tried to tackle the question of determining the causal structure among economic quantities. For instance, results in [33] suggest that per capita real balances and real per capita private gross domestic product are both causes of real per capita consumption expenditures and that real per capita consumption expenditures and real per capita private gross domestic product in turn cause real per capita gross private domestic fixed investment in a four-variables vector autoregressive model of US macro-economic data for the period January 1949 to April 2002. We plan to use both constraint-based methods and Bayesian approaches to study the causal structure in a graph where the nodes are the various quantities manipulated by quantitative easing policies. As always, one of the main problems will be to define the set of sufficient variables.

## 4.2. Law

There are now many ways in which mathematics are applied to law. They include the following approaches:

1. the classical domain of *Law and Economics*
2. the more recent statistical approaches
3. approaches using tools of mathematical logic.

Given our expertise, we are concerned with approaches 1 and 2: our first applications are based either on a mix of economic and statistical methods, or on purely statistical ones. We will also develop original probabilistic models.

From a general point of view, the benefits of using actuarial models in law is twofold:

- mathematical models should allow for a more profound understanding of law structures and rules. Indeed, as explained in [47], law can be seen as an information technology in the sense that it provides information to the community about the content of legal norms and, in its common law form, elicits information about the world from the disputes before a court. In this two-way path, tension between law's potential for certainty and its capacity for discovery reflects in part the imperfect circulation of information. The joint use of adequate mathematical models and big data tools should greatly enhance this circulation, thus improving the efficiency of the system as a whole;
- in a more complex and more informed world, legal procedures are likely to become more frequent. However, the state resources devoted to law cannot increase without bounds. Making available tools that would facilitate amicable settlement is then of strong interest. In particular, models allowing one to estimate outputs of legal decisions, at least in certain areas and in a rough way, would certainly draw people to be more inclined to negotiate rather than go to court, thus reducing the burden put on the legal system. This tendency is already quite noticeable in particular in the USA, where so-called *on-line dispute resolution systems* gain popularity.

We contribute to both these goals, paying in addition extra caution to the performative aspects. Our first studies are detailed in the next sections.

### 4.2.1. Law-Mathematics correspondences

In order to root our subsequent studies on firm bases, we intend to start by evidencing some parallel notions in law and mathematics, and to study if they are profound enough to yield useful tools. While this will inevitably be sometimes rather qualitative, it will definitely shed some light on how to model legal reasoning in a mathematical way.

An example of such a qualitative link is the fact judges, as mathematicians, when faced with a question, often have immediately a intuition of their answer. In a second phase, lawyers try to find which legal texts or jurisprudence allow them to justify this answer, while mathematicians invoke a series of computations and known theorems to do the same. In both cases, if no path is found to the initial answer (that is, no legal texts or no valid sequence of computations), the practitioner tries to defend or prove the opposite one. We have no idea yet how to formalize this parallelism, but this will be a topic of study. More quantitative ones are the following:

#### 1. Weights and linear models

Judges often say that they weigh different factors when they need to make a decision. The obvious corresponding mathematical notion is the one of linear models, where variables are linearly combined to produce an output. We will choose some simple domains, such as for instance child support, to check whether the decided amount is indeed obtained by weighting the criteria that judges are supposed to take into account.

This requires to analyse a large amount of case law and assessing the fit of various linear or generalised linear models. State-of-the-art techniques in machine learning are used in this connection.

#### 2. Causality

Finally, an obvious and probably fruitful correspondence between both domains rests on the notion of causality. Determining which events are causes of others is clearly a crucial task in courts, since evidencing responsibilities is at the core of making informed judgements.

On the other hand, statisticians have, until rather recently, avoided to consider causal questions, concentrating on correlations. This is still true today, where most researchers and practitioners would claim that statistics can only evidence dependencies between random variables but cannot assess causal links, except when controlled experiments may be performed. It is hard to think of a situation in law where one could perform such experiments.

However, a growing community has started to develop what now seems to be a somewhat coherent theory, termed causality theory, that allows one to efficiently decide if a variable  $X$  is indeed a cause of a variable  $Y$  under some conditions [49]. Apart from theoretical developments, this theory has been applied in various domains, and most notably in economy and biomedical studies. We are not aware of any applications in law.

We study this area in two ways:

- the most direct one is to choose a specific domain, analyse some decisions in it in light of the legal and jurisprudential criteria that are supposed to base them, and check whether they are indeed causes of the decision in the sense of causality theory. More generally, we try to construct the whole Bayesian network associated with a given field;
- a more ambitious goal is to question whether the way law sees and organizes causality is anything like what is performed in statistical causality theory. This task requires an abstract model of legal causality that must be constructed from scratch. This is a long term aim.

### 4.2.2. Scales and performativity

We have just won a call “Droit, justice et numérique” of the “Mission de recherche Droit et Justice”, a “groupement d’intérêt public” created by the French ministry of justice and CNRS. Our proposal is a joint project with L. Godefroy (Faculté de droit et science politique, Nice University), who has expertise in the relations between the digital world and law, and F. Lebaron (Versailles St Quentin University). F. Lebaron is

a sociologist and a specialist of performativity. We aim at studying the performative effects of scales from a general point of view by using our respective knowledges in law, sociology and statistics. More precisely, we will first choose some domains where scales have been introduced, like for instance child support or competition law. Statistical studies based on sociological insights will then be performed to measure how much these scales have performed as compared to the previous, scale-free, situation. This step will require to construct models in order to enhance the estimation step and thus the interpretation of the results. Based on the analysis of the current performative effects and our models, we will, if needed, propose modifications allowing one to reduce unwanted effects.

As a last step, we hope that a global pattern of how scales perform will emerge, maybe from a comparative analysis of the models in different areas. This could open the way to the construction a general theory.

#### **4.2.3. *Quantifying legal risk***

Our most successful application to date is in the quantification of legal risk: once one is prepared to accept that a legal decision is a random variable, one realizes that legal risk, which is a special component of the global risk companies or even citizens face, may be treated as are other risks. In particular, financial risks have been the topic of extensive studies in recent years, partly in response to the several crises we have witnessed. One lesson from this area is that, although one cannot of course predict the future state of a market, one is able to estimate its probability distribution. This allows one for instance to compute Values at Risk and thus to control one's risk.

We have designed an approach that can quantify legal risk in the same way as financial risk: given a specific domain, e.g. spousal support or dismissal without fair cause, we carefully design a set of legal criteria and analyse a large amount of cases in light of these criteria. We then use refined machine learning techniques to produce a probability distribution that reflects the decisions that would be taken by the judges in our database. This probability distribution takes into account both inter- and intra-judges variability. The mathematical result is that, when the size of the database tends to infinity, the estimated probability distribution tends, under some assumptions, to the actual one.

We have applied this theory to two fields so far : spousal support and dismissal without fair cause. Our future plans include in particular areas in labour law.

In view of to the strong interest this tool has raised among professionals (lawyers, insurance companies, but also the french ministry of justice), we are thinking of creating a start-up company that would commercialize it. As a consequence, we are not able to detail the mathematics involved in this study.

#### **4.2.4. *Intellectual property***

This project is conducted in the frame of an ISN-funded collaboration between Inria and CERDI (University Paris Sud). Its aim is to help judges make informed decisions concerning the amount of fines in cases of violation of intellectual property. Indeed, in this domain, the fundamental rule that the amount is fixed so as to make good the damage suffered is not adequate: a person who commits a fault with a view to gain can be condemned, in addition to compensatory damages, to pay punitive damages. This rule has been introduced in 2007 under the impulsion of European law. In practice, it seems that it has not been implemented with great success. Our contribution studies a Bayesian network model for understanding how judges compute such amounts. We construct two such networks, one based on law and jurisprudence from Canada and one from France. This project has started in the fall of 2015.

### **4.3. *Archaeology***

We have been working since 2011 on the construction of new Bayesian approach for chronological modeling: this is an important issue in archaeology and paleo-environmental sciences. The archaeologists base their interpretations on a wide range of sources of information. A priori knowledge about the parameters of the model is often available, and so it should be considered along with the model and the data. This motivates the Bayesian choice.

In our case the data are the measurements  $M_i$  provided by dating laboratories e.g. 14C). The prior information contains historical evidence (e.g. an event must have occurred between two calendar dates,..) or geological information (e.g. a stratigraphic information,..). All the measurements require a calibration step to be converting into calendar date.

#### *Tools for Constructing Chronologies*

The aim is to provide probabilistic estimation of a chronology; a crucial aspect is to obtain a robust approach with respect to outliers due to the sampling in the field or the measurement process in the laboratory.

The solution proposed in [7], [6] is based on the "event model". We define the Event as the date  $\theta$  of an archeological context determined from a collection of contemporaneous artifacts. The model with random effect can be written as follows

$$\begin{aligned} M_i &= g_i(t_i) + S_i \rho_i \\ t_i &= \theta + \sigma_i \lambda_i \end{aligned}$$

where  $g_i$  is the calibration function and  $(\rho_1, \dots, \rho_n, \lambda_1, \dots, \lambda_n)$  are iid standard Gaussian random variables. The random variables  $(\lambda_i)_i$  and  $(\epsilon_i)_i$  are interpreted as follows :

- $S_i \rho_i$  represents the experimental error provided by the laboratory and the calibration step.
- $\sigma_i \lambda_i$  represents the irreducible error between  $t_i$  and  $\theta$  due to sampling problems external to the laboratory

In [7], [6], we show the ability of the variance  $\sigma_i^2$  to take large values, in order to automatically penalize an outlier.

To enrich the chronological modelling, we wish to incorporate archaeological "phases". Contrary to an "event", a phase suggests duration. The objective is then to estimate the parameters that characterize the phase (beginning /end/duration), and then to develop Bayesian tests on the duration of the phase or the existence of a gap (hiatus) between two phases.

#### *Calibration*

The dating processes provide measurements, which are converted into calendar dates using calibration reference curves. We plan to explore issues related to calibration for different dating methods.

Optically stimulated luminescence (OSL) dating is a quantitative dating method to determine the time of last exposure of sand and silt to sunlight. Our aim is to complete the model constructed in [2] in order to obtain an OSL age determination.

We generally observe a overestimation of the age of a sample by OSL dating. This can be explaining by an insufficient resetting of the optically stimulated luminescence signal prior to sediment deposition. Therefore detection of so-called poor bleaching is of prime importance in OSL dating.

## 5. New Software and Platforms

### 5.1. New Software

#### 5.1.1. Law

In order for our research to become used in a non-technological domain as is law, we make special efforts to develop user-friendly software tools. We have already made available an iOS application for computing the amount of child support. Another application is currently being developed for spousal support.

Future user-friendly software we intend to propose will be concerned with the areas of personal injury and intellectual property.

### 5.1.2. Economy and finance

The software toolbox FracLab (<http://fraclab.saclay.inria.fr>) contains a number of tools related to multistable processes, including their simulation and calibration. We intend to pursue these developments in particular by proposing methods dealing with self-stabilizing processes.

We also hope to be able to collaborate with financial institutions, in view of advancing the idea that current regulations, because they are model-based, perform the economy in a way that needs to be carefully scrutinized.

### 5.1.3. Archaeology

Our software ChronoModel provides a set of Bayesian statistical tools for constructing chronologies using measurement coming from different dating methods (e.g. 14C, TL/OSL, AM, typo-chronology). Chronomodel is a free and open-source cross-platform software that can be downloaded at <http://www.chronomodel.fr>

It offers a user-friendly interface for entering both data and archaeological facts : stratigraphy, temporal order constraints, contemporaneous events.

ChronoModel is currently used by researchers in laboratories of archaeology and earth science, and also by archaeologists working in the organizations of rescue archaeology.

## 6. New Results

### 6.1. Legal aspects of systems designed to judicial risk quantification

Participants : Jérôme Dupré

Within the ANJA team systems designed to calculate judicial risk using machine learning technology (AI) have been developed. A former French magistrate is one of the team member who has participated to these researches. In the meantime, he endeavored to contribute to design a legal framework applicable to this activity.

Artificial intelligence (AI), particularly when applied to justice, is liable to encounter rules of law, which are applicable even in the absence of a specific law. As with any new field of activity (eg the Internet), the notion of “legal vacuum” must not be confused with that of “legislative void”. It is therefore necessary to identify how to protect these technologies, what is the responsibilities of each, whether designer or / and user, that could already be applicable.

The two main concerns are relating to property and liability.

1.Regarding property, one can observe that predictive/quantitative solutions based on artificial intelligence result from a combination of technical criterion, databases, algorithms and software, each subject to specific legal protections. These elements may hence be protected by the copyright (for technical criterion); the database law, copyright and unfair competition (for databases); the trade secret (for algorithms), the copyright (for software). It may be questioned whether it would be irrelevant to create, ultimately, a unified legal status specific to this complex reality. But it is probably too early to legislate.

At the heart of the solution is the algorithm, an immaterial element which, in France, is the least well protected by law (it belongs to the domain of “ideas”), justifying its secret nature.

2. Considering liability aspects, we observe that this “black box” - the secret being a consequence of complexity and investments made - may be at the origin of a prejudice, either because of a bad use, or because it is not correctly designed.

The French law offers a range of solutions to the victim, depending on the origin of the damage (see 6.2).

Trust in results is also a factor to be considered. Thus, in the absence of a technical problem peculiar to the solution, misuse by the legal professional providing legal advice may justify, for example, his/her contractual liability. From this standpoint, the reliance granted to the technology and the way it is presented are essential. It justifies a specific attention to the way contracts relating to these services are drafted.



The designer of a defective solution may be required to guarantee against the hidden defects of Article 1641 of the French Civil Code. (When there is no contract, one can also be liable on the grounds of Article 1242 paragraph 1 of the same Code).

Standardization of algorithms, which could be tested by an independent body and subject to secrecy, is also an option, but presents a risk of possible paralysis of a promising market in the field of mathematics.

More generally, it seems necessary to comply with the CNIL (the French Data Protection Authority) provisions relating to personal data (Law No 78-17 of 6 January 1978, spec. article 10, and soon Regulation EU 2016/679 of the European Parliament and of the Council of 27 April 2016 applicable in 2018) and with Privacy Law... A huge amount of data is indeed likely to reveal information not resulting from each data taken separately. But this risk is probably more present in the field of big data than in algorithms, the data used for learning being "dissolved" in the formula.

## 6.2. Liability and ethics

Participants : Jacques Lévy Véhel, Jérôme Dupré

Some legal issues are specifically related to the calculation error. The system user may not have entered the data correctly and then will be responsible for the displayed result (but it is possible to display the entered data with the results to limit this risk).

The user may also have entered the data correctly and: -he/she is aware of the error of the system, in which case the user regains a share of responsibility, -he/she is not aware of the error of the system and the error is not easily detectable, then the responsibility should move towards the authors of the system conception (with its different steps that result from each other: definition of the search criteria, creation of database and creation of the algorithm, development and integration of the software, etc.). -he/she is not aware of the error of the system and the error is easily detectable, in which case the user should regain at least some responsibility.

The use of systems designed to judicial risk quantification entails paradigm shift The probabilistic approach, where all things are equal, seems very far from the hierarchy of legal norms and legal causality. In the absence of any upheaval in the law, we can at least expect an upheaval in legal practice.

Ultimately, the use of predictive tools, which favors the discovery of correlations, may lead to less attention to the causes of events, hence the need to maintain a vigilance on this aspect. The human who "delegates everything" to the machine should not avoid responsibility.

From an ethical perspective, is it acceptable to calculate the sentence which is likely to be pronounced in penal law, e.g. for a crime? Should we accept to profile judges using all their past rulings?

More generally, a reflection as well as a study on the place left to the human at all the stages of the process of elaboration and use (the decision-making) of the predictive tools seems necessary.

## 6.3. Statistical inference methods for panel of random-coefficient AR(1) data

Participants: A. Philippe and D. Surgailis R. Leipus and V. Pilipauskaitė (Vilnius university)

We study the statistical inference methods for panel of random-coefficient AR(1) data [17]. We propose a nonparametric estimation of the distribution function of random coefficients by the empirical distribution of lag 1 sample correlations of individual AR(1) processes. Consistency and asymptotic normality of the empirical distribution function and a class of kernel density estimators is established under some regularity conditions on  $G(x)$  as  $N$  and  $n$  increase to infinity. An extension of this work consists in testing the presence of long memory. The procedure is based on the tail index of  $G$  and the théorie of extreme values. In the same direction, a new frequency-domain test statistic is introduced to test for short memory versus long memory, see [23]

## 6.4. New Bayesian approach for chronological modeling

Participants: Anne. Philippe and Marie-Anne Vibet in collaboration with IRAMAT - Université Bordeaux Montaigne



We have been working on the construction of new Bayesian approach for chronological modeling: this is an important issue in archaeology and paleo-environmental sciences. The proposed solution is based on the "event model". We define the Event as the date of an archeological context determined from a collection of contemporaneous artifacts. We obtain a robust approach with respect to outliers due to the sampling in the field or the measurement process in the laboratory.

In [25] We propose new tools to analyse the chronologies especially regarding phases. They are implemented in R package 'RChronoModel'.

In [22], [18], [5], we propose bayesian models for optically stimulated luminescence dating. It consists in estimating a central equivalent dose from a set of luminescence measurements. Then a calibration step is required to convert equivalent dose into calendar date.

## 6.5. Self-regulated processes

Participants : Jacques Lévy Véhel, Anne Philippe, Caroline Robet

We wish to construct various instances of processes  $Z$  such that, at each point  $t$ , almost surely, the pointwise Hölder exponent of  $Z$  at  $t$ , denoted  $\alpha_Z(t)$ , verifies

$$\alpha_Z(t) = g(Z(t))$$

where  $g \in \mathcal{C}^1(\mathbb{R}, [a, b])$  is a deterministic function. Then, we would estimate the function  $g$  which control the regularity.

The pointwise Hölder exponent at  $t$  of a function or a process  $f : \mathbb{R} \rightarrow \mathbb{R}$ , which is  $\mathcal{C}^1$  nowhere, is the real  $\alpha_f(t)$  such that :

$$\alpha_f(t) = \sup \left\{ \beta, \limsup_{h \rightarrow 0} \frac{|f(t+h) - f(t)|}{|h|^\beta} = 0 \right\}$$

We worked first on pathwise integrals :

**Theorem 1** Let  $g \in \mathcal{C}^1(\mathbb{R}, [a, b])$ ,  $0 < a < b < 1$ . Provided  $\|g'\|_\infty$  is small enough, there exists a unique continuous process  $Z$  verifying almost surely on  $[0, T]$

$$Z_t = \int_0^t (t-u)^{g(Z_u)-1} W_u du$$

where  $W$  is an almost surely continuous process.

A random condition ( $\|g'\|_\infty \|W(\omega)\|_\infty C(a, T) < 1$ ) appears in the application of Banach fixed point theorem (in  $(\mathcal{C}^0([0, T]; \mathbb{R}), \|\cdot\|_\infty)$ ). It implies that it is possible to have existence et uniqueness only on  $[0, t']$ ,  $t' < T$ . We simulated pathwise integrals and showed some cases without uniqueness. We studied some easier processes in order to find the regularity of  $Z$ .

**Theorem 2** Let  $h \in ]0, 1[$  and  $U$  defined on  $[0, T]$  by

$$U_t = \int_0^t (t-u)^{h-1} W_u du$$

Then  $\forall t \in [0, T]$ ,  $\alpha_U(t) \geq h$ .

**Theorem 3** Let  $g \in \mathcal{C}^1(\mathbb{R}, [a, b])$ ,  $0 < a < b < 1$ . Provided  $\|g'\|_\infty$  is small enough, there exists a unique continuous process  $Y$  verifying almost surely on  $[0, T]$

$$Y_t = \int_0^t (t-u)^{g(Y_t)-1} W_u du$$

where  $W$  is an almost surely continuous process. Furthermore,  $\forall t \in [0, T]$ ,  $\alpha_Y(t) \geq g(Y_t)$

Then, we adapted the multifractional Brownian Motion [50], [31] (which a representation is  $B_t = \int_0^t K_{H(t)}(t, u)W(du)$ ,  $W$  Brownian Motion et  $H \in \mathcal{C}^1$ ) to construct the modified multifractional Brownian Motion :  $Z_t = \int_0^t K_{H(u)}(t, u)W(du)$ . We expect obtain a self-regulated process  $Y_t = \int_0^t K_{g(Y_u)}(t, u)dW(u)$ .

**Theorem 4** Let  $g \in \mathcal{C}^1(\mathbb{R}, [a, b])$ ,  $0 < a < b < 1$ . Provided  $\|g'\|_\infty$  is small enough, there exists a unique continuous adapted process  $Y$  include in  $\mathcal{C}^0([0, T]; L^2(\Omega))$  verifying almost surely on  $[0, T]$

$$Y_t = \int_0^t K_{g(Y_u)}(t, u)dW(u)$$

where  $W$  is the Brownian motion.

## 6.6. Causal inference by independent component analysis with Application of to American macro-economic data

Participants : Jacques Lévy-Vehel, Anne Philippe, Marie-Anne Vibet

The aim of this work is to study the causal relationships existing among macro-economic variables under investigation, and trace out how economically interpreted random shocks affect the system. Structural vector of autoregressive models (SVAR) are usually applied in this kind of study and the causal structure is driven by the data. In this work, independent component analysis (ICA) is implemented in order to guaranty the identifiability of the causal structure. However, the use of ICA can only be done under the hypothesis that the residuals are non-Gaussian, an hypothesis easily verified with economic data.

The vector of autoregressive (VAR) model has the following reduced representation :

$$Y_t = A_1 Y_{t-1} + \dots + A_p Y_{t-p} + u_t, \text{ for } t = 1, \dots, T$$

where,  $Y_t$ , is the vector of contemporaneous variables of dimension  $K \times 1$ ,  $p$  is the number of autoregressive variables,  $A_j$ , for  $j = 1, \dots, p$ , are matrices of dimension  $K \times K$  estimated by the model and  $u_t$  is the vector of random disturbances of dimension  $K \times 1$  and assumed to be a zero-mean white noise process,  $u_t \sim N(0_K, \Sigma_u)$ . Given enough data, both  $\Sigma_u$  and all matrices  $A_j$  can be correctly estimated by the VAR model.

However, the VAR model is not sufficient for policy analysis. Indeed, using the Moving Average representation of a stable VAR :

$$Y_t = \sum_{j=0}^{\infty} \Phi_j u_{t-j} \quad (1)$$

where  $\Phi_0 = I_K$  and  $\Phi_j, j \geq 1$ , are the coefficients matrices representing the impulse responses of the elements of  $Y_t$  to the disturbances  $u_{t-j}$ . This representation is not unique.

The structural VAR (SVAR) is essentially a VAR equipped with a particular choice of a matrix  $P$  so that  $Y_t = \sum_{j=0}^{\infty} \Phi_j P P^{-1} u_{t-j} = \sum_{j=0}^{\infty} \Psi_j \epsilon_{t-j}$

where  $\epsilon_{t-j}$  are independent random shocks economically interpreted. To this aim, the ICA procedure is then used to find the proper matrix  $P$  using the hypothesis that the residuals,  $\epsilon_{t-j}$ , are non-Gaussian.

We used the VAR-LINGAM procedure developed by Moneta *et al* [28] and their package written for R software. We started by testing this procedure with a series of simulations study. We tackled the following questions : Are the coefficients of the matrices  $B$  and  $A$  well estimated by the VARLINGAM procedure ? Is the bootstrap function appropriate, and in particular, does it estimate properly the standard error of the coefficients of matrices  $A$  and  $B$  ? And how long should the economic data be in order to estimate correctly the coefficients of the matrices  $B$  ?

As the conclusion to all these studies were correct enough, we went on analysing our real data that consists of 6 weekly time series US macro-economic data, reported from the first week of January 1996 to April 2016 : The BofA Merrill Lynch US Broad Market Index, The Bofa Merrill Lynch US Corporate Index, Equity Indices S&P, 500, Federal Funds Rates, Treasury Bills, Other Factors Draining Reserve Balances.

The conclusions of this work is in discussion with economists and a paper will soon be written.

## 6.7. SAR image denoising using an irregularity-preserved denoising technique based on the global Hölder exponent

Participants : Jacques Lévy-Vehel, Yue Huang

This work addresses the speckle noise reduction for SAR images by using the irregularity-preserved denoising technique proposed in [34]. This irregularity preserving denoising scheme in [34] may be summarized as a three-step process in the following:

1. Apply a Discrete Wavelet Transform (DWT) on the noisy signal and represent the resulting coefficients distribution over scales. Estimate the cut-off scale and the global Hölder exponent  $\alpha_f$  using linear regression of  $\max_k (\log_2 |\langle f, \psi_{j,k} \rangle|)$  at larger scales.
2. Extrapolate the larger scale regression line to smaller scales and limit coefficient at smaller scales ( $j \geq j_{\text{cut-off}}$ ) to the boundary value obtained from the linear regression
3. Reconstruct the filtered signal from the set of modified coefficients

where  $f$  is the signal under analysis,  $\psi_{j,k}$  is the wavelet basis, and  $\langle f, \psi_{j,k} \rangle$  is the wavelet coefficient of  $f$  at scale  $j$  and location  $k$ . As it has been shown by simulations in [34], to retrieve irregular signals affected by additive noise, this technique outperforms conventional denoising techniques that apply hard or soft thresholding to the wavelet coefficients.

Considering a speckle-affected SAR image, a complex SAR signal may be represented by:

$$y(l) = s(l)u(l)$$

where  $l$  represents one of  $L$  realizations, and the noise term  $u(l)$  follows a complex circular centered Gaussian white distribution with unit variance, i.e.  $u \sim \mathcal{N}_C(0, 1)$ ,  $E(u(i)u^*(j)) = \delta_{(i-j)}$ . The texture of SAR image significantly depends on the backscattering power  $\sigma(l) = |y(l)|^2$ .

We aim to use the irregularity-preserved denoising technique to denoise SAR image and enhance its texture. We tested firstly on the simulated signals affected by multiplicative noise and then on real SAR images. This denoising scheme showed potential to reduce the speckle noise, preserve the irregularity of image texture and enhance target signature.

Although the results have been compared with other SAR speckle filtering techniques, we still need more efforts for validation. As long as the results are validated, the work will be written in a paper.

## 6.8. Underfoliage object imaging using SAR tomography and wavelet-based sparse estimation methods

Participants : Yue Huang, Jacques Lévy-Vehel

Hybrid environments refer to a scenario of deterministic objects embedded in a host natural random environment and their scattering patterns consist of a complex mixture of diverse mechanisms, like, in the case of this study, volume scattering from the canopy, double bounce reflection between the ground and under-foilage objects as well as between objects and trunks, surface scattering from the underlying ground, etc. The resulting SAR information is characterized by a strong complexity, and its analysis using 2-D images or even data acquired in InSAR configuration remains difficult. Using Multi-baseline(MB) InSAR data, SAR tomography can be applied to reconstruct in 3-D the measured scattering responses and polarimetric patterns. Natural volumes, such as forest canopies, being composed of a large number of scatterers whose responses cannot be discriminated at the resolution of analysis, their scattering patterns are generally considered as a vertical density of random or speckle-affected reflectivity. On the other hand, localized objects, such as artificial targets on the ground are associated to point-like contributions, that may be separable in the vertical direction. The global response of under-foilage objects with a deterministic scattering response embedded in surrounding distributed environments, can be described by a mixed spectrum. Conventional tomographic techniques like the Capon and Beamforming methods, estimate continuous Power Spectral Density (PSD) and hence are well adapted to the characterization of continuous volumetric media, but cannot discriminate closely-spaced scatterers, e.g. scattering responses from trucks, due to limited spatial resolution. Conventional high-resolution methods like MUSIC and subspace fitting estimators as well as sparse estimation techniques such as LASSO [52] and FOCUSS [40], are well adapted to the characterization of discrete scatterers like truck top, truck-ground interaction and calibrators over bare soils, or buildings over urban areas [53], but cannot properly handle the high dimensionality of the scattering responses of natural volumes. Usual tomographic techniques cannot simultaneously cope with both types of spectrum, and not able to deal with mixed spectral estimation problems, characteristic of underfoilage object imaging scenario.

Wavelet-based techniques present a high potential for such applications, as they permit to parameterize in a sparse way continuous functions, i.e. canopy PSDs in the present case. Wavelet-based tomographic techniques have been used for tomographic imaging of forested areas [27], and for such regular signals, large wavelet coefficients being often concentrated in the approximation space, scale thresholding may be implemented to extract the most significant wavelet coefficients for an accurate volume signal recovery [27]. In the underfoilage object scenario, discrete scatterers embedded in a continuous medium, result in a mixed vertical PSD that may be associated to an irregular signal with wavelet coefficients distributed both in the approximation and detail spaces, and a simple scale cut-off is hence not adapted to separate the wavelet coefficients of discrete scatterers from those of continuous media. Therefore, we propose a new wavelet-based method to extract underfoilage objects from their speckle-affected distributed environment and characterize them with a high resolution.

For an MB-InSAR configuration with  $M$  acquisition positions, considering an azimuth-range resolution cell containing a mixture of backscattering contributions from object ( $o$ ) and volume ( $v$ ) scatterers located at different heights  $z$ , the observed data vector at  $l$ th realization can be represented by:

$$\mathbf{y}(l) = \mathbf{A}_o(\mathbf{z}_o)\mathbf{s}_o(l) + \mathbf{A}_v(\mathbf{z}_v)\mathbf{s}_v(l) + \mathbf{n}(l) \quad (2)$$

where the steering matrix,  $\mathbf{A}_x(\mathbf{z}_x)$ , contains the interferometric phase information associated to the InSAR responses of the scatterers located at the unknown elevation positions  $\mathbf{z}_x = [z_{x_1}, \dots, z_{x_{N_x}}]$  above the reference focusing plane, and the source signal vector,  $\mathbf{s}_x = [s_{x_1} \dots s_{x_{N_x}}]^T \in \mathbb{C}^{N_x \times 1}$ , contains the unknown complex backscattering coefficients of the  $N_x$  source scatterers. The vertical reflectivity function can be represented as  $\mathbf{p}_x = E(|\mathbf{s}_x|^2)$  ( $x = o, v$ ).

Over speckle-affected environments, unknown reflectivity and elevation parameters are generally estimated from second-order statistics, i.e. from the covariance matrix  $\widehat{\mathbf{R}} \in \mathbb{C}^{M \times M}$  of the observed MB-InSAR data  $\mathbf{y} \in \mathbb{C}^{M \times 1}$ . The proposed tomographic processing technique is based on the minimization of the Least-Square (LS) fitting between the observed and modeled data covariance  $\|\mathbf{R} - \widehat{\mathbf{R}}\|_F$ . The modeled covariance matrix is composed by the covariances of object and volume contributions  $\mathbf{R} = \mathbf{R}_o + \mathbf{R}_v$ , each of them being simply related to its discretized vertical density of reflectivity  $\mathbf{p}_x$  through  $\mathbf{R}_x = \mathbf{A}(\mathbf{z}_x) \text{diag}(\mathbf{p}_x) \mathbf{A}^H(\mathbf{z}_x) \in \mathbb{C}^{M \times M}$ .

The proposed method can be represented by a  $l_1$  norm minimization in a transformed space subject to quadratic constraints between the observed and modeled data covariance:

$$\min_{\mathbf{p}} \|\mathbf{B}\mathbf{p}\|_1 \quad \text{subject to} \quad \|\mathbf{R} - \widehat{\mathbf{R}}\|_F \leq \epsilon \quad (3)$$

where

- $\mathbf{p} = [\mathbf{p}_o^T \quad \mathbf{p}_v^T]^T \in \mathbb{R}^{+N_s \times 1}$  stands for vertical backscattering power distribution for the resolution cell under analysis,
- $\mathbf{B} = \begin{bmatrix} \mathbf{I}_{(N_o \times N_o)} & \mathbf{0} \\ \mathbf{0} & \Psi_{(N_v \times N_v)} \end{bmatrix} \in \mathbb{R}^{(N_s \times N_s)}$  represents the hybrid sparsifying basis with the wavelet basis  $\Psi$

This tomographic technique is suitable for the mixed-spectrum estimation problem, because it maintains the spectral continuity for the backscattering power of forest canopies and the high-resolution for the vertical reflectivity of objects. The effectiveness of this new approach is demonstrated using L-band airborne tomographic SAR data acquired by the DLR over Dornstetten, Germany. The undeniable performance can be shown by the results in [21] and [20].

This work has been presented in European SAR conference 2016 . Some refined results have been presented in IGARSS conference 2016 as an invited talk. By extending this work in details, a journal paper [24] has been submitted to IEEE Geoscience and Remote Sensing Letters (GRSL) and is currently under reviewing.

## 6.9. Detection of objects concealed beneath forest canopies using Time-Frequency techniques

Participants : Yue Huang, Jacques Lévy-Vehel

In the scenario of hybrid environments where objects with a deterministic response are embedded in a speckle affected environment, the parameter estimation for this type of scatterers becomes a problem of mixed-spectrum estimation. To isolate and characterize these different scattering contributions, a novel method proposed by Huang et al. was used to extract isolated scatterers (IS) from their surrounding distributed environments, named IS extraction in [42]. Incorporating the Weighted Subspace Fitting (WSF) estimator, this method estimated scattering responses within one resolution cell and then distinguishes isolated scatterers from distributed ones by calculating the cross-correlation between the measured data and the estimated scattering responses. Moreover, to compare the detection performance for coherent scatterers, two statistical methods have been applied to analyse hybrid environments in [43]: GLRT (generalized likelihood ratio test)-based and SSF (weighted Signal Subspace Fitting)-based detection procedures. However, the above mentioned methods based on discrete high-resolution tomographic estimation, require to preselect the number of scattering contributions, which may induce reliability issues due to model order selection.

This paper proposes a new tomographic estimator based on Time-Frequency (TF) techniques using Multi-baseline Polarimetric and Interferometric SAR data. The coherent TF analysis of polarimetric SAR has been introduced in [38], [39] for the study of anisotropic scattering behaviors and then applied in [37], [36] for dense urban environment characterization. Time-frequency techniques can represent spectral properties around specific spatial locations or spatial features at specific spectral positions, leading to describe local variations of spectral or spatial features. Considering SLC SAR images, the spectral locations can be linked to azimuth looking angle and illumination frequency in such a way:

$$w_{az} = \frac{4\pi}{c} f_c v_{SAR} \sin \phi, \quad w_{rg} = \frac{4\pi}{c} (f - f_c)$$

with  $f_c$  central frequency and  $\phi$  azimuth looking angle. The TF technique can be used to analyze scattering behaviors at different illuminated positions and frequency components during SAR integration. Based on the correlation between different spectral positions, the TF indicator proposed in [37] can extract coherent components in complex random SAR responses. Polarimetric TF indicator has been developed in [41] for ship discrimination. In this paper, the new tomographic estimator extends 2-D TF analysis to 3-D, which provides an efficient cancellation for clutters from speckle-affected random scattering environments, and discriminates the deterministic responses from coherent scatterers in 3-D space. The effectiveness of this new tomographic approach is demonstrated by using L-band MB-PolInSAR data set acquired over the test site of Dornstetten where the underfoliage objects are set up. The fully polarimetric version of this TF tomographic estimator is also developed to improve the detection efficiency. This work has been accepted for oral presentation at the Polinsar 2017 Workshop and the final paper will be written by the end of Workshop.

## 7. Dissemination

### 7.1. Promoting Scientific Activities

#### 7.1.1. Scientific Events Organisation

##### 7.1.1.1. Member of the Organizing Committees

Bayesian workshop : Bayesian statistics applied to archaeology - May 2016 Nantes  
Anne Philippe and Marie-Anne Vibet were main organizers.

#### 7.1.2. Journal

Anne Philippe is an Associated editor of Computational Statistics and Data analysis  
Jacques Lévy Véhel is associate editor of the journal « Fractals »

#### 7.1.3. Invited Talks

Anne Philippe and Marie-Anne Vibet were invited to the Ibercrono conference, Barcelone Spain, October 2016

Anne Philippe was invited to ArcheoFoss, Cagliari Italy October 2016

Yue Huang gave a talk at the invited session of advanced SAR technologies at IEEE International Geoscience and Remote Sensing Symposium, Beijing Chine, 2016

#### 7.1.4. Research Administration

Anne Philippe is a member of commits national du CNRS

### 7.2. Teaching - Supervision - Juries

#### 7.2.1. Teaching

- Marie-Anne Vibet: Introduction to SAS software, Master 2 Ingénierie Mathématique, January and November 2016 (12hrs)
- Anne Philippe : Statistical inference, Master 1 Ingénierie Mathématique, Automne 2016 (24hrs)
- Anne Philippe : Bayesian statistics, Master 2 Ingénierie Mathématique, Automne 2016 (36hrs)

#### 7.2.2. Supervision

- PhD in progress: Vytautė Pilipauskaitė, supervised by Anne Philippe and Donatas Surgailis
- PhD in progress: Caroline Robet, supervised by Anne Philippe and Jacques Lévy-Véhel

#### 7.2.3. Juries

Anne Philippe was in the jury of PhD theses :

- President of jury for Kaniav Canary
- Reporter and member of jury for Lilliam Urrego, Le Quyen Thieu

## 8. Bibliography

### Major publications by the team in recent years

- [1] O. BARRIERE, A. ECHELARD, J. LÉVY VÉHEL. *Self-Regulating Processes*, in "Electronic Journal of Probability", December 2012 [DOI : 10.1214/EJP.v17-2010], <https://hal.inria.fr/hal-00749742>
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- [5] M. FROUIN, S. HUOT, S. KREUTZER, C. LAHAYE, M. LAMOTHE, A. PHILIPPE, N. MERCIER. *An improved radiofluorescence single-aliquot regenerative dose protocol for K-feldspars*, in "Quaternary Geochronology", 2017, vol. 38, pp. 13 - 24
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- [10] J. LÉVY VÉHEL, P.-E. LÉVY VÉHEL, V. LÉVY VÉHEL. *A model investigating incentives for illegal cultural goods sharing sites to become legal*, January 2015, working paper or preprint, <https://hal.inria.fr/hal-01103048>
- [11] J. LÉVY VÉHEL. *A simple isochore model evidencing regulation risk*, in "IME'2015, Liverpool", 2015
- [12] J. LÉVY VÉHEL, C. WALTER. *Prudential regulation: is there a danger in reducing the volatility?*, in "9-th Actuarial Research Conference, Santa Barbara", 2014



- [13] A. PHILIPPE, M.-A. VIBET. *Analysis of archaeological phases using the CRAN package RChronoModel' A tutorial*, July 2016, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-01347895>

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### Articles in International Peer-Reviewed Journals

- [14] F. BARSOTTI, A. PHILIPPE, P. ROCHET. *Hypothesis testing for Markovian models with random time observations*, in "Journal of Statistical Planning and Inference", June 2016, vol. 173, pp. 87 - 98 [DOI : 10.1016/J.JSPI.2015.12.009], <https://hal.archives-ouvertes.fr/hal-01428011>
- [15] L. J. FERMIN, J. LÉVY VÉHEL. *Variability and singularity arising from poor compliance in a pharmacokinetic model II: the multi-oral case*, in "Journal of Mathematical Biology", 2016 [DOI : 10.1007/s00285-016-1041-1], <https://hal.inria.fr/hal-00868621>
- [16] M. FROUIN, S. HUOT, S. KREUTZER, C. LAHAYE, M. LAMOTHE, A. PHILIPPE, N. MERCIER. *An improved radiofluorescence single-aliquot regenerative dose protocol for K-feldspars*, in "Quaternary Geochronology", March 2017, vol. 38, pp. 13 - 24 [DOI : 10.1016/J.QUAGEO.2016.11.004], <https://hal.archives-ouvertes.fr/hal-01427990>
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### Invited Conferences

- [20] Y. HUANG, J. LEVY VEHEL, L. FERRO-FAMIL, A. REIGBER. *3D imaging for underfoliage targets using L-band multibaseline polinsar data and sparse estimation methods*, in "IGARSS 2016 - IEEE International Geoscience and Remote Sensing Symposium", Beijing, China, July 2016, <https://hal.inria.fr/hal-01418435>

### International Conferences with Proceedings

- [21] Y. HUANG, J. LÉVY VÉHEL, L. FERRO-FAMIL, A. REIGBER, S. FORTUNATI. *3D Characterization of Underfoliage Targets Using L-band Tomographic SAR Data and A Wavelet-Based Approach*, in "EUSAR 2016 - 11th European Conference on Synthetic Aperture Radar", Hamburg, Germany, June 2016, <https://hal.inria.fr/hal-01418677>

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- [22] B. COMBÈS, A. PHILIPPE. *Bayesian analysis of individual and systematic multiplicative errors for estimating ages with stratigraphic constraints in optically stimulated luminescence dating*, November 2016, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-01346868>
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