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Activity Report 2016

Project-Team AROMATH

AlgebRa geOmetry Modelling and AlgoriTHms

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
Algorithmics, Computer Algebra and Cryptology

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Project-Team AROMATH

Creation of the Project-Team: 2016 July 01

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2. Overall Objectives

2.1. Overall Objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of “real” phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to “act” on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, ... are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects “at home” from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimisation loops where the numerical simulation results are used to improve the “performance” of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world...), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

3. Research Program

3.1. High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in CAGD, to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at the seams. Although these models have many advantages and have become the standard for designing nowadays CAD models,they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

3.2. Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally, as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods, which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One is based on linearization of algebraic problems based on "elimination of variables" or projection into a space of smaller dimension. Resultant theory provides strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progresses have been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered.This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute "only" the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems.The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exits in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

4. Application Domains

4.1. Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing, ... Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeal for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5 axis machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

4.2. Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree ≤ 2).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modelled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

5. New Software and Platforms

5.1. AXEL

KEYWORDS: CAO - Algebraic geometric modeler
SCIENTIFIC DESCRIPTION

Axel is an algebraic geometric modeler that aims at providing “algebraic modeling” tools for the manipulation and computation with curves, surfaces or volumes described by semi-algebraic representations. These include parametric and implicit representations of geometric objects. Axel also provides algorithms to compute intersection points or curves, singularities of algebraic curves or surfaces, certified topology of curves and surfaces, etc. A plugin mechanism allows to extend easily the data types and functions available in the plateform.

FUNCTIONAL DESCRIPTION

Axel is a cross platform software to visualize, manipulate and compute 3D objects. It is composed of a main application and several plugins. The main application provides atomic geometric data and processes, a viewer based on VTK, a GUI to handle objects, to select data, to apply process on them and to visualize the results. The plugins provides more data with their reader, writer, converter and interactors, more processes on the new or atomic data. It is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python. The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows.

- Participants: Nicolas Douillet, Anaïs Ducoffe, Valentin Michelet, Bernard Mourrain, Meriadeg Perrinel, Stéphane Chau and Julien Wintz
- Contact: Bernard Mourrain
- URL: http://axel.inria.fr/

Collaboration with Elisa Berrini (MyCFD, Sophia), Tor Dokken (Gotools library, Oslo, Norway), Angelos Mantzaflaris (GISMO library, Linz, Austria), Laura Saini (Post-Doc GALAAD/Missler, TopSolid), Gang Xu (Hangzhou Dianzi University, China).

6. New Results

6.1. Flat extensions in $_*$-algebras

Participant: Bernard Mourrain.

The main result of the paper [9] is a flat extension theorem for positive linear functionals on $_*$-algebras. The theorem is applied to truncated moment problems on cylinder sets, on matrices of polynomials and on enveloping algebras of Lie algebras.

This is a joint work with K. Schmüdgen.

6.2. On deflation and multiplicity structure

Participant: Bernard Mourrain.

The paper [6] presents two new constructions related to singular solutions of polynomial systems. The first is a new deflation method for an isolated singular root. This construction uses a single linear differential form defined from the Jacobian matrix of the input, and defines the deflated system by applying this differential form to the original system. The advantages of this new deflation is that it does not introduce new variables and the increase in the number of equations is linear in each iteration instead of the quadratic increase of previous methods. The second construction gives the coefficients of the so-called inverse system or dual basis, which defines the multiplicity structure at the singular root. We present a system of equations in the original variables plus a relatively small number of new variables that completely deflates the root in one step. We show that the isolated simple solutions of this new system correspond to roots of the original system with given multiplicity structure up to a given order. Both constructions are "exact " in that they permit one to treat all conjugate roots simultaneously and can be used in certification procedures for singular roots and their multiplicity structure with respect to an exact rational polynomial system.

This is a joint work with J. Hauenstein and A. Szanto.

6.3. On the construction of general cubature formula by flat extensions

Participant: Bernard Mourrain.

We describe a new method to compute general cubature formulae [1]. The problem is initially transformed into the computation of truncated Hankel operators with flat extensions. We then analyse the algebraic properties associated to flat extensions and show how to recover the cubature points and weights from the truncated Hankel operator. We next present an algorithm to test the flat extension property and to additionally compute the decomposition. To generate cubature formulae with a minimal number of points, we propose a new relaxation hierarchy of convex optimization problems minimizing the nuclear norm of the Hankel operators. For a suitably high order of convex relaxation, the minimizer of the optimization problem corresponds to a cubature formula. Furthermore cubature formulae with a minimal number of points are associated to faces of the convex sets. We illustrate our method on some examples, and for each we obtain a new minimal cubature formula.

This is a joint work with Marta Abril-Bucero and C. Bajaj (Univ. of Austin, Texas, USA).

6.4. Geometrically continuous splines for surfaces of arbitrary topology

Participant: Bernard Mourrain.

In the paper [10], we analyze the space of geometrically continuous piecewise polynomial functions or splines for quadrangular and triangular patches with arbitrary topology and general rational transition maps. To define these spaces of G^1 spline functions, we introduce the concept of topological surface with gluing data attached to the edges shared by faces. The framework does not require manifold constructions and is general enough to allow non-orientable surfaces. We describe compatibility conditions on the transition maps so that the space of differentiable functions is ample and show that these conditions are necessary and sufficient to construct ample spline spaces. We determine the dimension of the space of G^1 spline functions which are of degree k on triangular pieces and of bi-degree (k, k) on quadrangular pieces, for k big enough. A separability property on the edges is involved to obtain the dimension formula. An explicit construction of basis functions attached respectively to vertices, edges and faces is proposed and examples of bases of G^1 splines of small degree for topological surfaces with boundary and without boundary are detailed.

This is a joint work with N. Villamizar and R. Vidunas.

6.5. Border Basis for Polynomial System Solving and Optimization

Participant: Bernard Mourrain.

We describe in [15] the software package BORDERBASIX dedicated to the computation of border bases and the solutions of polynomial equations. We present the main ingredients of the border basis algorithm and the other methods implemented in this package: numerical solutions from multiplication matrices, real radical computation, polynomial optimization. The implementation parameterized by the coefficient type and the choice function provides a versatile family of tools for polynomial computation with modular arithmetic, floating point arithmetic or rational arithmetic. It relies on linear algebra solvers for dense and sparse matrices for these various types of coefficients. A connection with SDP solvers has been integrated for the combination of relaxation approaches with border basis computation. Extensive benchmarks on typical polynomial systems are reported, which show the very good performance of the tool.

This is a joint work with M. Abril Bucero and Ph. Trébuchet.

6.6. Bit complexity of bivariate systems

Participant: Ioannis Emiris.

The paper [14] studies the bit complexity of solving systems of bivariate polynomial equations. By means of adapted resultant formulations we thus improve upon the existing general bounds.

6.7. Compact formulae in sparse elimination

Participant: Ioannis Emiris.

This invited talk [12] describes three aspects of constructing compact formulae in toric (or sparse) elimination algebraic theory. We start with the most general existing formula for computing the mixed volume of a square algebraic system, then sketch older and recent progress in matrix formulae for the sparse resultant of an overconstrained system, and conclude with recent work in a matrix formula for the multivariate disccriminant of a specific class of well-constrained systems.

6.8. Computation of the Invariants of Finite Abelian Groups

Participant: Evelyne Hubert.

In [7] we investigate the computation and applications of rational invariants of the linear action of a finite abelian group in the nonmodular case. By diagonalization, such a group action can be described by integer matrices of orders and exponents. We make use of integer linear algebra to compute a minimal generating set of invariants along with the substitution needed to rewrite any invariant in terms of this generating set. In addition, we show how to construct a minimal generating set that consists only of polynomial invariants. As an application, we provide a symmetry reduction scheme for polynomial systems whose solution set is invariant by a finite abelian group action. Finally, we also provide an algorithm to find such symmetries given a polynomial system.

This is joint work with George Labahn (University of Waterloo, Canada).

6.9. Extraction of cylinders and cones from minimal point sets

Participants: Laurent Busé, André Galligo.

In [3], we propose new algebraic methods for extracting cylinders and cones from minimal point sets, including oriented points. More precisely, we are interested in computing efficiently cylinders through a set of three points, one of them being oriented, or through a set of five simple points. We are also interested in computing efficiently cones through a set of two oriented points, through a set of four points, one of them being oriented, or through a set of six points. For these different interpolation problems, we give optimal bounds on the number of solutions. Moreover, we describe algebraic methods targeted to solve these problems efficiently.

6.10. Resultant of an equivariant polynomial system with respect to the symmetric group

Participants: Laurent Busé, Anna Karasoulou.

Given a system of n homogeneous polynomials in n variables which is equivariant with respect to the canonical actions of the symmetric group of n symbols on the variables and on the polynomials, we prove in [4] that its resultant can be decomposed into a product of several smaller resultants that are given in terms of some divided differences. As an application, we obtain a decomposition formula for the discriminant of a multivariate homogeneous symmetric polynomial.

6.11. A Line/Trimmed NURBS Surface Intersection Algorithm Using Matrix Representations

Participant: Laurent Busé.

In the work [11], we contribute a reliable line/surface intersection method for trimmed NURBS surfaces, based on a novel matrix-based implicit representation and numerical methods in linear algebra such as singular value decomposition and the computation of generalized eigenvalues and eigenvectors. A careful treatment of degenerate cases makes our approach robust to intersection points with multiple pre-images. We then apply our intersection algorithm to mesh NURBS surfaces through Delaunay refinement. We demonstrate the added value of our approach in terms of accuracy and treatment of degenerate cases, by providing comparisons with other intersection approaches as well as a variety of meshing experiments.

This is a joint work in collaboration with Pierre Alliez from TITANE Inria project-team and Jingjing SHen and Neil Dodgson both from Cambridge University.

6.12. Effective criteria for bigraded birational maps

Participant: Laurent Busé.

In [2], we consider rational maps whose source is a product of two subvarieties, each one being embedded in a projective space. Our main objective is to investigate birationality criteria for such maps. First, a general criterion is given in terms of the rank of a couple of matrices that became to be known as *Jacobian dual matrices*. Then, we focus on rational maps from $\mathbb{P}^1 \times \mathbb{P}^1$ to $\mathbb{P}^2$ in very low bidegrees and provide new matrix-based birationality criteria by analyzing the syzygies of the defining equations of the map, in particular by looking at the dimension of certain bigraded parts of the syzygy module. Finally, applications of our results to the context of geometric modeling are discussed at the end of the paper.

This is a joint work with N. Botbol (University of Buenos Aires, ARgentina), M. Chardin (UMPC, France), S. H. Hassanzadeh (University of Rio, Brazil), A. Simis (University of Pernambuco, Brazil), Q. H. Tran (UMPC, France). It has been done in the framework of the SYRAM project.

6.13. Geometric model for shape deformation

Participants: Elisa Berrini, Bernard Mourrain.

In [13], we describe a new parametric modeller for an automatic shape optimization loop. The modeller enables the generation of shapes by selecting a set of design parameters that controls a twofold parameterization: geometrical – based on a skeleton approach – and architectural – based on the experience of practitioners, to impact the system performance. The resulting forms are relevant and effective, thanks to a smoothing procedure that ensures the consistency of the shapes produced.

The skeleton consists of a set of B-Spline curves composed of a generating curve and section curves. The deformation of the shape is performed by changing explicit parameters of the representation or implicit parameters such as architectural parameters. The new shape is obtained by minimizing a distance function between the current parameters and the target parameters in combination with a smoothing term to ensure shape consistency. Finally, a 3D surface is reconstructed around the skeleton with an iterative method handling multi-patches and boundary constraints.

Thanks to this approach, architects can directly use a CAD-model based on NURBS representations in the modeller tool that allows a straightforward modification of the initial design to improve performance. The methodology developed can be applied to any shape that can be described by a skeleton, e.g. hulls, foils, bulbous bows, but also wind turbines, airships, etc.

As application, we consider the optimization of the shape of a bulbous bow. The modeller is linked to the RANSE-CFD solver FINE/Marine. The aim is to reduce the total drag of the hull with variation of its bulbous bow shape.

6.14. Shape-optimization of 2D hydrofoils using an Isogeometric BEM solver

Participant: Panagiotis Kaklis.

In [8], an optimization procedure, based on an Isogeometric BEM solver for the potential flow, is developed and used for the shape optimization of hydrofoils. The formulation of the exterior potential-flow problem reduces to a Boundary-Integral Equation (BIE) for the associated velocity potential exploiting the null-pressure jump Kutta condition at the trailing edge. The numerical solution of the BIE is performed by an Isogeometric Boundary-Element Method (BEM) combining a generic B-splines parametric modeler for generating hydrofoil shapes, using a set of eight parameters, the very same basis of the geometric representation for representing the velocity potential and collocation at the Greville abscissas of the knot vector of the hydrofoil's B-splines representation. Furthermore, the optimization environment is developed based on the geometric parametric modeler for the hydrofoil, the Isogeometric BEM solver and an optimizer employing a controlled elitist genetic algorithm. Multi-objective hydrofoil shape optimization examples are demonstrated with respect to the criteria i) maximum lift coefficient and ii) minimum deviation of the hydrofoil area from a reference area.

This is a joint work with K. Kostas (Nazarbayev University), A. Ginnis (National Technical University of Athens), C. Politis (Technological Educational Institute of Athens).

6.15. Algebraic method for constructing singular steady solitary waves: A case study

Participant: André Galligo.

The article [5] describes the use of algebraic methods in a phase plane analysis of ordinary differential equations. The method is illustrated by the study of capillary-gravity steady surface waves propagating in shallow water. We consider the (fully nonlinear, weakly dispersive) Serre-Green-Naghdi equations with surface tension, because it provides a tractable model that, in the same time, is not too simple so the interest of the method can be emphasised. In particular, we analyse a special class of solutions, the solitary waves, which play an important role in many fields of Physics. In capillary-gravity regime, there are two kinds of localised infinitely smooth travelling wave solutions – solitary waves of elevation and of depression. However, if we allow the solitary waves to have an angular point, the "zoology" of solutions becomes much richer and the main goal of this study is to provide a complete classification of such singular localised solutions using the methods of the effective Algebraic Geometry.

This is a joint work with D. Clamond (Laboratoire Jean Alexandre Dieudonné, Université de Nice Sophia-Antipolis) and Denys Dutykh (Laboratoire de Mathématiques, Université de Savoie).

7. Partnerships and Cooperations

7.1. Regional Initiatives

7.1.1. Inria SAM Action Transverse

Participants: Paul Görlach, Evelyne Hubert.

Finding biomarkers of abnormalities of the white matter is one important problem in dMRI processing. As these biomarkers need to be independent of the orientation of the head, they are functions of the rotational invariants of the shapes that characterize the diffusion probabilities in the white matter. While the situation is well understood for second order tensors, these are not powerful enough to represent crossings in the white matter. Acquisitions made with the HARDI scheme allow for a richer description of probabilities. In particular, the project-team ATHENA has modelled them as (positive) ternary quartics (symmetric tensors of order 4). But invariants of these quartics are not well known. For a long period, only six were known, when there should be at least 12. Strategies were developed in the project-team ATHENA to compute more invariants, either algebraic [25] or polynomial [21]. The former suffered some instability issues in their evaluations, the latter did not form a minimal set. The goal of this "Transverse action" was to team up with expertise in algebraic computation and leverage the methods [23], [24], [22] [19], [7] developed in the project team AROMATH to gain more insight in this problem of rotational invariants of ternary quartics.

This action is done in collaboration with Théodore Papadopoulo (ATHENA team).

7.1.2. CIMI thematic project

Participant: Evelyne Hubert.

Labex CIMI Toulouse supports the project *Joint Implicit and Parametric Representation based on Skeleton* where the PI are Géraldine Morin (IRIT, Vortex team) and Evelyne Hubert. This project aims at developing a mathematical model and software for surfaces, based on a joint parametric and implicit representation, with a skeleton.

7.2. European Initiatives

7.2.1. FP7 & H2020 Projects

Program: Marie Skłodowska-Curie ITN

Project acronym: ARCADES

Project title: Algebraic Representations in Computer-Aided Design for complEx Shapes

Duration: January 2016 - December 2019

Coordinator: I.Z. Emiris (NKUA, Athens, Greece, and ATHENA Research Innovation Center)

Scientist-in-charge at Inria: L. Busé

Other partners: U. Barcelona (Spain), Inria Sophia-Antipolis (France), J. Kepler University, Linz (Austria), SINTEF Institute, Oslo (Norway), U. Strathclyde, Glascow (UK), Technische U. Wien (Austria), Evolute GmBH, Vienna (Austria).

Webpage: http://arcades-network.eu/

Abstract: ARCADES aims at disrupting the traditional paradigm in Computer-Aided Design (CAD) by exploiting cutting-edge research in mathematics and algorithm design. Geometry is now a critical tool in a large number of key applications; somewhat surprisingly, however, several approaches of the CAD industry are outdated, and 3D geometry processing is becoming increasingly the weak link. This is alarming in sectors where CAD faces new challenges arising from fast point acquisition, big data, and mobile computing, but also in robotics, simulation, animation, fabrication and manufacturing, where CAD strives to address crucial societal and market needs. The challenge taken up by ARCADES is to invert the trend of CAD industry lagging behind mathematical breakthroughs and to build the next generation of CAD software based on strong foundations from algebraic geometry, differential geometry, scientific computing, and algorithm design. Our game-changing methods lead to real-time modelers for architectural geometry and visualisation, to isogeometric and design-through-analysis software for shape optimisation, and marine design & hydrodynamics, and to tools for motion design, robot kinematics, path planning, and control of machining tools.

7.3. International Initiatives

7.3.1. Participation in Other International Programs

7.3.1.1. PICS project

Participant: Laurent Busé.

We participate to a bilateral collaboration between France and Spain which is supported as a PICS from CNRS. This project, titled *Diophantine Geometry and Computer Algebra*, aims at exploring interactions between diophantine geometry and computer algebra by stimulating collaborations between experts in both domains. The research program focuses on five particular topics : toric varieties and height, equidistribution, Diophantine geometry and complexity, factorization of multivariate polynomials by means of toric geometry and study of singularities of toric parameterizations.

The Spanish partner is the University of Barcelona, with participants J. Burgos, C. D'Andrea, Martin Sombra, and the French partners are the university of Caen, with participants F. Amoroso and M. Weimann, the University of Paris 6, with participants M. Chardin and P. Philippon and the Inria project-team AROMATH, with participant L. Busé.

7.3.1.2. SYRAM project

Participants: Laurent Busé, Bernard Mourrain, André Galligo.

Title: Geometry of SYzygies of RAtional Maps with applications to geometric modeling (SYRAM)

We coordinate a research project which is funded by the regional program Math-AmSud for two years : 2015-2016. This project is composed by research teams from Argentina, Universidad de Buenos Aires (Nicolás Botbol, Alicia Dickenstein), Brazil, Universidade Federal de Rio de Janeiro, de Pernambuco e de Sergipe (Sayed Hamid Hassanzadeh, Aron Simis) and France, Institut de Mathématiques de Jussieu (Marc Chardin) and the Inria project-team AROMATH.

The study of rational maps is of theoretical interest in algebraic geometry and commutative algebra, and of practical importance in geometric modeling. This research proposal focuses on rational maps in low dimension, typically parameterizations of curves and surfaces embedded in the projective space of dimension three, but also dominant rational maps in dimension two and three. The two main objectives amount to unravel geometric properties of these rational maps from the syzygies of their projective coordinates. The first one aims at extending and generalizing the determination of the closed image of a rational map, as well as its geometric features, whereas the second one will focus on the study of dominant rational maps, in particular on the characterization of those that are generically one-to-one.

7.4. International Research Visitors

7.4.1. Visits of International Scientists

Cordian Riener (University of Konstanz, Germany) visited from September 4-9th, 2016 to collaborate on symmetry, orthogonal polynomials and cubature with Evelyne Hubert and Bernard Mourrain.

Lan Nguyen (University of Vietnam at Hanoï) visited to collaborate on implicitization of rational maps with Laurent Busé. His visits received the financial support of LIAFV (International Laboratory for France-Vietnam collaborations in mathematics).

Aron Simis (University of Pernambuco, Brazil) visited to collaborate on syzygies of rational maps with Laurent Busé.

Nicolas Botbol (Universidad de Buenos Aires, Argentina) visited to collaborate on distance function to rational curves and surfaces with Laurent Busé.

7.4.1.1. Internships

Paul Görlach (University of Bonn) came to work on the *CRISAM - Transverse action* between the project teams AROMATH and ATHENA (August-December).

Akshit Goyal and Deepak Bhatt (IIT Dehli) worked during their internship on “Meshing Singular Isosurfaces” and “Isosurface of the distance function” (May-July).

Antoine Deharveng, student at the engineer school of the University of Nice Sophia Antipolis, came since June 15 to work on the extraction of geometric primitives in a 3D point cloud under the supervision of Laurent Busé.

7.4.2. Visits to International Teams

7.4.2.1. Sabbatical programme

Evelyne Hubert was in Ontario from September 1st 2015 to February 29th 2016, with the sabbatical programme of Inria DPEI. For the period of January and February 2016 she was hosted and supported by University of Waterloo, visiting the Symbolic Computation Lab, and more particularly Pr. George Labahn.

Bernard Mourrain was invited at Univ. of Texas, Austin, for a collaboration with Pr. Chandajit Bajaj (7th-19th May).

8. Dissemination

8.1. Promoting Scientific Activities

8.1.1. Scientific Events Organisation

8.1.1.1. Member of the Organizing Committees

Laurent Busé was the main organizor of the BIRS-CMO conference "*Computational Algebra and Geometric Modeling*" that took place at Oaxaca, Mexico, August 7-12 2016. He also co-organized with A. Dimca (Univ. Nice) a mini-workshop "*commutative algebra and applications*" that took place at the laboratory of mathematics of the university of Nice, September 22-23. He also co-organized a *week of studies maths-industry* (SEME) that took place at the CRI-SAM January 25-29.

Evelyne Hubert was part of the organizing committee of the collaborative research workshop *Women in Shape: Modeling Boundaries of Objects in 2- and 3-Dimensions* that took place June 6-12 at the Nesin Mathematics Village in Turkey.

8.1.2. Scientific Events Selection

8.1.2.1. Reviewer

Laurent Busé, Evelyne Hubert and Bernard Mourrain reviewed submissions for the conference ISSAC'16.

8.1.3. Journal

8.1.3.1. Member of the Editorial Boards

Bernard Mourrain is associate editor of the Journal of Symbolic Computation (since 2007) and of the SIAM Journal on Applied Algebra and Geometry (since 2016).

Ioannis Emiris is associate editor of the Journal of Symbolic Computation (since 2003) and of Mathematics in Computer Science (since 2016).

Evelyne Hubert is associate editor of the Journal of Symbolic Computation (since 2007) and became a reviewer for Mathematical Reviews (MathSciNet) this year.

8.1.3.2. Reviewer - Reviewing Activities

Laurent Busé wrote reviews for the following international journals: Journal of Symbolic Computation, Journal of Algebra, Computer Aided Geometric Design, Mathematical and Computational Applications, Graphical Models, Linear Algebra and its Applications, SIAM Journal on Applied Algebra and Geometry, Transactions on Graphics, the Quarterly Journal of Mathematics and Math. Zeitschrift. He also wrote reviews for the ISSAC 2016 and the Eurographics 2017 international conferences.

Evelyne Hubert reviewed for the journal *Mathematics of Computation*, the *Journal of Symbolic Computation*, the *Journal of Pure & Applied Algebra*, the journal *Mathematics in Computer Science*, and Springer book series *Texts and Monographs in Symbolic Computation*,

Bernard Mourrain reviewed for the journal *Advances in Computational Mathematics*, the *Journal of Algebra and Applications*, the journal *Collectanea Mathematica*, the journal *Computer Aided Design*, the journal *Computer Aided Geometric Design*, the journal *Foundations of Computational Mathematics*, the *Journal of Pure and Applied Algebra*, the journal *SIAM Journal on Optimization*, the *Transactions on Mathematical Softwares*.

8.1.4. Invited Talks

Laurent Busé was invited to give a talk at the Inria project-team ARIC seminar, May 26, at the conference "Computational Algebra, Algebraic Geometry and Applications", in honor of Alicia Dickenstein, that took place at Buenos Aires, Argentina, August 1-3 2016, at the *H2020 day* organized at the CRI-SAM to give a testimony on the writing of the successful MCA-ITN proposal ARCADES.

Ioannis Emiris gave an invited talk at ACM International Symposium on Symbolic & Algebraic Computation, Waterloo, Canada, July 2016.

Evelyne Hubert was invited to give a talk at the *Computational Mathematics Colloquium* at University of Waterloo, Canada (January 2016); at the workshop on *Théorie Effective des Invariants*, at the Insitut de Mathématiques de Marseille (June 2016); at the workshop on *Symmetry, Invariants, Reduction* in RWTH Aachen University (September 2016); at the BIRS-CMO conference *Sparse Interpolation, Rational Approximation and Exponential Analysis* in Oaxaca, Mexico (November 2016). She was also invited (and supported) to partitipate to the American Institute of Mathematics workshop *Algebraic Vision* in San Jose, California (May 2016); and the BIRS-CMO conference *Computational Algebra and Geometric Modeling* in Oaxaca, Mexico (August 2016); and to the CIRM conference *Multivariate Approximation and Interpolation with Applications* where she presented a poster (September 2016).

Bernard Mourrain was invited to give a talk at the MFO workshop *Mathematical Foundations of Isogeometric Analysis* Oberwolfach, Germany (February 2016), at the conference "Computational Algebra, Algebraic Geometry and Applications", in honor of Alicia Dickenstein, that took place at Buenos Aires, Argentina, August 1-3 2016, at the BIRS-CMO conference *Computational Algebra and Geometric Modeling* in Oaxaca, Mexico (August 2016), at the CIRM conference *Multivariate Approximation and Interpolation with Applications* Marseille, France (September 2016), at the BIRS-CMO conference *Sparse Interpolation, Rational Approximation and Exponential Analysis* in Oaxaca, Mexico (November 2016).

8.1.5. Leadership within the Scientific Community

Evelyne Hubert, in collaboration with Géraldine Morin, lead a collaborative research group at the workshop *Women in Shape: Modeling Boundaries of Objects in 2- and 3-Dimensions*.

8.1.6. Scientific Expertise

Bernard Mourrain was member of the committee of the HCERES for the evaluation of IRMAR, University of Rennes.

Evelyne Hubert was a member of the admissibility jury for the *Chargé de Recherche* position in CRI-Rennes Bretagne Atlantique.

Laurent Busé was a member of the CRI-SAM committee "Actions Marquantes", March 25. He is also a board member of the (national) labex AMIES (CRI-SAM representative) and a member of the steering committee of the MSI, *Maison de la Modélisation, de la Simulation et des Interactions* of the University Côte d'Azur.

8.1.7. Research Administration

Evelyne Hubert is an elected member of the Inria national *Commission d'Evaluation*.

Laurent Busé is an elected member of the CPRH (Commission Permanente de Ressources Humaines) of the math laboratory of the university of Nice. He was also appointed Inria representative at the "Academic Council" and the "Research Commission" of the university of Nice.

8.2. Teaching - Supervision - Juries

8.2.1. Teaching

Licence : Ioannis Emiris, Discrete Math, 53, L1, NKU Athens, Greece.

Licence : Ioannis Emiris, Soft development for algorithmic problems, 53, L3, NKU Athens, Greece

Master : Laurent Busé, Curves and Surfaces, 66h ETD, M1, EPU of the university of Nice-Sophia Antipolis.

Master : Laurent Busé, Geometric Modeling, 27h ETD, M2, EPU of the university of Nice-Sophia Antipolis.

8.2.2. Supervision

PhD in progress: Elisa Berrini, Parametric modeling for ship hull deformation and optimization. CIFRE with MyCFD, started in January 2014, supervised by Bernard Mourrain.

PhD in progress: Ahmed Blidia, New geometric models for the design and computation of complex shapes. ARCADES Marie Skłodowska-Curie ITN, started in September 2016, supervised by Bernard Mourrain.

PhD in progress: Jouhayna Harmouch, Low rank structured matrix decomposition and completion. Cotutelle Univ. Liban, started in November 2015, cosupervised by Houssam Khalil and Bernard Mourrain.

PhD in progress: Anna Karasoulou, Exploiting structure in polynomial systems. Excellence awards (Greece), started in November 2011, supervised by Ioannis Emiris.

PhD in progress: Ioannis Psarros, Geometric approximation algorithms. Thales network (Greece), started in May 2015, supervised by Ioannis Emiris.

PhD in progress: Evangelos Bartzos, Modeling motion. ARCADES Marie Skłodowska-Curie ITN, started in May 2016, supervised by Ioannis Emiris.

PhD in progress: Evangelos Anagnostopoulos, Geometric algorithms for massive datasets. Started in May 2016, supervised by Ioannis Emiris.

PhD in progress: Clement Laroche, Change of representation in CAGD. ARCADES Marie Skłodowska-Curie ITN, started in Nov. 2016, supervised by Ioannis Emiris.

PhD in progress: Alvaro-Javier Fuentes-Suarez, Skeleton-based modeling of smooth shapes. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Evelyne Hubert.

Master in Computer Science: Paul Görlach, University of Bonn. Rotational invariants of ternary quartics. CRI-SAM tranverse action AROMATH-ATHENA. August-December 2016, supervised by Evelyne Hubert.

PhD in progress: Fatmanur Yildirim, Distances between points, rational Bézier curves and surfaces by means of matrix-based implicit representations. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Laurent Busé.

8.2.3. Juries

Evelyne Hubert was a referee for the PhD of Louis Dumont entitled *Algorithmes rapides pour le calcul symbolique de certaines intégrales de contour à paramètre*, Université Paris-Saclay, École Polytechnique, Inria Saclay Île-de-France.

Bernard Mourrain was a referee for the PhD of Emil Horobet, entitled *Tensors of low rank*, Univ. of Technology, Eindhoven, Netherland.

Laurent Busé was a referee for the PhD of Thibaut Verron entitled *Régularisation du calcul de bases de Gröbner pour des systèmes avec poids et déterminantiels, et application en imagerie médicale*, Université Pierre et Marie Curie, Paris, France, September 26.

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[2] N. BOTBOL, L. BUSÉ, M. CHARDIN, S. H. HASSANZADEH, A. SIMIS, Q. H. TRAN. *Effective criteria for bigraded birational maps*, in "Journal of Symbolic Computation", 2016 [*DOI :* 10.1016/J.JSC.2016.12.001], https://hal.inria.fr/hal-01278405

[3] L. BUSÉ, A. GALLIGO, J. ZHANG. *Extraction of cylinders and cones from minimal point sets*, in "Graphical Models", 2016, vol. 86, pp. 1-12, https://hal.inria.fr/hal-01288325

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Invited Conferences

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International Conferences with Proceedings

[13] E. BERRINI, B. MOURRAIN, Y. ROUX, G. FONTAINE, E. JEAN. *Parametric shape modeler for hulls and appendages*, in "COMPIT'16", Lecce, Italy, V. BERTRAM (editor), 15th International Conference on

Computer and IT Applications in the Maritime Industries COMPIT'16, May 2016, vol. 15, pp. 255-263, https://hal.archives-ouvertes.fr/hal-01373867

[14] I. Z. EMIRIS, A. MANTZAFLARIS, E. TSIGARIDAS. *On the Bit Complexity of Solving Bilinear Polynomial Systems*, in "ISSAC '16 - 41st International Symposium on Symbolic and Algebraic Computation", Waterloo, Canada, ACM, July 2016, pp. 215-222 [*DOI :* 10.1145/2930889.2930919], https://hal.inria.fr/hal-01401134

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