IN PARTNERSHIP WITH:
CNRS

Ecole Polytechnique

Ecole nationale supérieure des techniques avancées

Activity Report 2016

Project-Team COMMANDS

Control, Optimization, Models, Methods and Applications for Nonlinear Dynamical Systems

IN COLLABORATION WITH: Centre de Mathématiques Appliquées (CMAP), Unité de Mathématiques Appliquées (UMA - ENSTA)

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team COMMANDS

Creation of the Project-Team: 2009 January 01

Keywords:

Computer Science and Digital Science:

6.2.1. - Numerical analysis of PDE and ODE
6.2.6. - Optimization
6.2.7. - High performance computing
6.3.2. - Data assimilation
6.4.1. - Deterministic control
6.4.2. - Stochastic control

Other Research Topics and Application Domains:

4.4.1. - Smart grids
7.1.2. - Road traffic
7.1.3. - Air traffic
7.2.1. - Smart vehicles

1. Members

Research Scientists

J. Frédéric Bonnans [Team leader, Inria, Senior Researcher, HDR]
Axel Kröner [Inria, Starting Research position]
Pierre Martinon [Inria, Researcher]

Engineer

Jinyan Liu [Inria, from Feb 2016]

PhD Students

Benjamin Heymann [Ecole Polytechnique, until Sep 2016]
Arthur Le Rhun [Ifpen, from Apr 2016]
Cédric Rommel [Safety Line, granted by CIFRE]
Justina Gianatti [U. Rosario (Argentina), intern, until Jul 2016]

Administrative Assistant

Jessica Gameiro [Inria]

Other

Luis Alberto Croquevielle [Inria, intern, until Mar 2016]

2. Overall Objectives

2.1. Scientific directions

Commands is a team devoted to dynamic optimization, both for deterministic and stochastic systems. This includes the following approaches: trajectory optimization, deterministic and stochastic optimal control, stochastic programming, dynamic programming and Hamilton-Jacobi-Bellman equation.

Our aim is to derive new and powerful algorithms for solving numerically these problems, with applications in several industrial fields. While the numerical aspects are the core of our approach it happens that the study of convergence of these algorithms and the verification of their well-posedness and accuracy raises interesting and difficult theoretical questions, such as, for trajectory optimization: qualification conditions and second-order optimality condition, well-posedness of the shooting algorithm, estimates for discretization errors; for the Hamilton-Jacobi-Bellman approach: accuracy estimates, strong uniqueness principles when state constraints are present, for stochastic programming problems: sensitivity analysis.

2.2. Industrial impact

For many years the team members have been deeply involved in various industrial applications, often in the framework of PhD theses. The Commands team itself has dealt since its foundation in 2009 with several types of applications:

- Space vehicle trajectories, in collaboration with CNES, the French space agency.
- Aeronautics, in collaboration with the startup Safety Line.
- Production, management, storage and trading of energy resources, in collaboration with EDF, GDF and TOTAL.
- Energy management for hybrid vehicles, in collaboration with Renault and IFPEN.

We give more details in the Bilateral contracts section.

3. Research Program

3.1. Historical aspects

The roots of deterministic optimal control are the "classical" theory of the calculus of variations, illustrated by the work of Newton, Bernoulli, Euler, and Lagrange (whose famous multipliers were introduced in [45]), with improvements due to the "Chicago school", Bliss [32] during the first part of the 20th century, and by the notion of relaxed problem and generalized solution (Young [51]).

Trajectory optimization really started with the spectacular achievement done by Pontryagin's group [50] during the fifties, by stating, for general optimal control problems, nonlocal optimality conditions generalizing those of Weierstrass. This motivated the application to many industrial problems (see the classical books by Bryson and Ho [38], Leitmann [47], Lee and Markus [46], Ioffe and Tihomirov [43]).

Dynamic programming was introduced and systematically studied by R. Bellman during the fifties. The HJB equation, whose solution is the value function of the (parameterized) optimal control problem, is a variant of the classical Hamilton-Jacobi equation of mechanics for the case of dynamics parameterized by a control variable. It may be viewed as a differential form of the dynamic programming principle. This nonlinear first-order PDE appears to be well-posed in the framework of *viscosity solutions* introduced by Crandall and Lions [39]. The theoretical contributions in this direction did not cease growing, see the books by Barles [30] and Bardi and Capuzzo-Dolcetta [29].

3.2. Trajectory optimization

The so-called *direct methods* consist in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure. So the two main problems are the choice of the discretization and the nonlinear programming algorithm. A third problem is the possibility of refinement of the discretization once after solving on a coarser grid.

In the *full discretization approach*, general Runge-Kutta schemes with different values of control for each inner step are used. This allows to obtain and control high orders of precision, see Hager [42], Bonnans [35]. In an interior-point algorithm context, controls can be eliminated and the resulting system of equation is easily solved due to its band structure. Discretization errors due to constraints are discussed in Dontchev et al. [40]. See also Malanowski et al. [48].

In the *indirect* approach, the control is eliminated thanks to Pontryagin's maximum principle. One has then to solve the two-points boundary value problem (with differential variables state and costate) by a single or multiple shooting method. The questions are here the choice of a discretization scheme for the integration of the boundary value problem, of a (possibly globalized) Newton type algorithm for solving the resulting finite dimensional problem in IR^n (n is the number of state variables), and a methodology for finding an initial point.

For state constrained problems or singular arcs, the formulation of the shooting function may be quite elaborate [33], [34], [28]. As initiated in [41], we focus more specifically on the handling of discontinuities, with ongoing work on the geometric integration aspects (Hamiltonian conservation).

3.3. Hamilton-Jacobi-Bellman approach

This approach consists in calculating the value function associated with the optimal control problem, and then synthesizing the feedback control and the optimal trajectory using Pontryagin's principle. The method has the great particular advantage of reaching directly the global optimum, which can be very interesting when the problem is not convex.

Characterization of the value function >From the dynamic programming principle, we derive a characterization of the value function as being a solution (in viscosity sense) of an Hamilton-Jacobi-Bellman equation, which is a nonlinear PDE of dimension equal to the number n of state variables. Since the pioneer works of Crandall and Lions [39], many theoretical contributions were carried out, allowing an understanding of the properties of the value function as well as of the set of admissible trajectories. However, there remains an important effort to provide for the development of effective and adapted numerical tools, mainly because of numerical complexity (complexity is exponential with respect to n).

Optimal stochastic control problems occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky and non-risky assets, super-replication with uncertain volatility, management of power resources (dams, gas). Air traffic control is another example of such problems.

Nonsmoothness of the value function. Sometimes the value function is smooth and the associated HJB equation can be solved explicitly. Still, the value function is not smooth enough to satisfy the HJB equation in the classical sense. As for the deterministic case, the notion of viscosity solution provides a convenient framework for dealing with the lack of smoothness, see Pham [49], that happens also to be well adapted to the study of discretization errors for numerical discretization schemes [44], [31].

For solving stochastic control problems, we studied the so-called Generalized Finite Differences (GFD), that allow to choose at any node, the stencil approximating the diffusion matrix up to a certain threshold [37]. Determining the stencil and the associated coefficients boils down to a quadratic program to be solved at each point of the grid, and for each control. This is definitely expensive, with the exception of special structures where the coefficients can be computed at low cost. For two dimensional systems, we designed a (very) fast algorithm for computing the coefficients of the GFD scheme, based on the Stern-Brocot tree [36].

4. Application Domains

4.1. Fuel saving by optimizing airplanes trajectories

We have a collaboration with the startup Safety Line on the optimization of trajectories for civil aircrafts. Key points include the reliable identification of the plane parameters (aerodynamic and thrust models) using data from the flight recorders, and the robust trajectory optimization of the climbing and cruise phases. We use both local (quasi-Newton interior-point algorithms) and global optimization tools (dynamic programming).

4.2. Hybrid vehicles

We started a collaboration with IFPEN on the energy management for hybrid vehicles. A significant direction is the analysis and classification of traffic data. We have preliminary results on the choice of the routing which amounts to some type of constrained shortest path.

5. Highlights of the Year

5.1. Highlights of the Year

We started at the beginning of 2016 an Innovation Lab (Ilab) 'OSCAR', jointly with the startup Safety Line. The subject of the Ilab is the design of algorithmic tools for the (i) identification of aircraft dynamics, based on flight data recorders, and (ii) the computation of energy efficient flight trajectories.

6. New Software and Platforms

6.1. BOCOP

Boite à Outils pour le Contrôle OPtimal
KEYWORDS: Energy management - Numerical optimization - Biology - Identification - Dynamic Optimization - Transportation
FUNCTIONAL DESCRIPTION

Bocop is an open-source toolbox for solving optimal control problems, with collaborations with industrial and academic partners. Optimal control (optimization of dynamical systems governed by differential equations) has numerous applications in transportation, energy, process optimization, energy and biology. Bocop includes a module for parameter identification and a graphical interface, and runs under Linux / Windows / Mac.

- Participants: Joseph Frédéric Bonnans, Pierre Martinon, Benjamin Heymann and Jinyan Liu
- Contact: Pierre Martinon
- URL: http://bocop.org

6.2. Bocop Avion

KEYWORDS: Optimization - Aeronautics
FUNCTIONAL DESCRIPTION

Optimize the climb speeds and associated fuel consumption for the flight planning of civil airplanes.

- Participants: Joseph Frédéric Bonnans, Pierre Martinon, Stéphan Maindrault, Cindie Andrieu, Pierre Jouniaux and Karim Tekkal
- Contact: Pierre Martinon
- URL: http://www.safety-line.fr

6.3. Bocop HJB

- Participants: Joseph Frédéric Bonnans, Pierre Martinon, Benjamin Heymann and Jinyan Liu
- Contact: Joseph Frédéric Bonnans
- URL: http://bocop.org

7. New Results

7.1. Optimal control of ordinary and partial differential equations

7.1.1. On the Design of Optimal Health Insurance Contracts under Ex Post Moral Hazard

Participant: Pierre Martinon.

With Pierre Picard and Anasuya Raj, Ecole Polytechnique.
We analyze in [27] the design of optimal medical insurance under ex post moral hazard, i.e., when illness severity cannot be observed by insurers and policyholders decide on their health expenditures. We characterize the trade-o§ between ex ante risk sharing and ex post incentive compatibility, in an optimal revelation mechanism under hidden information and risk aversion. We establish that the optimal contract provides partial insurance at the margin, with a deductible when insurersí rates are a§ected by a positive loading, and that it may also include an upper limit on coverage. We show that the potential to audit the health state leads to an upper limit on out-of-pocket expenses.

7.1.2. Optimal control of infinite dimensional bilinear systems: application to the heat and wave equations

Participants: J. Frédéric Bonnans, Axel Kröner.

With Soledad Aronna, FGV, Rio de Janeiro. In this paper [13] we consider second order optimality conditions for a bilinear optimal control problem governed by a strongly continuous semigroup operator, the control entering linearly in the cost function. We derive first and second order optimality conditions, taking advantage of the Goh transform. We then apply the results to the heat and wave equations.

7.1.3. Optimal control of PDEs in a complex space setting; application to the Schrödinger equation

Participants: J. Frédéric Bonnans, Axel Kröner.

With Soledad Aronna, FGV, Rio de Janeiro. This paper [22] presents some optimality conditions for abstract optimization problems over complex spaces. We then apply these results to optimal control problems with a semigroup structure. As an application we detail the case when the state equation is the Schrödinger one, with pointwise constraints on the "bilinear'" control. We derive first and second order optimality conditions and address in particular the case that the control enters the state equation and cost function linearly.

7.1.4. Approximation and reduction of optimal control problems in infinite dimension

Participant: Axel Kröner.

With Michael D. Chekroun, UCLA) and H. Liu, Virginia Tech. Nonlinear optimal control problems in infinite dimensions are considered for which we establish approximation theorems and reduction procedures. Approximation theorems and reduction procedures are available in the literature. The originality of our approach relies on a combination of Galerkin approximation techniques with reduction techniques based on finite-horizon parameterizing manifolds. The numerical approximation of the control in a feedback form based on Hamilton-Jacobi-Equation become also affordable within this approach. The approach is applied to optimal control problems of delay differential equations and nonlinear parabolic equations.

7.2. Stochastic control, electricity production and planning

7.2.1. MIDAS: A Mixed Integer Dynamic Approximation Scheme

Participant: J. Frédéric Bonnans.

With Andy Philpott and Faisal Wahid, U. Auckland. Mixed Integer Dynamic Approximation Scheme (MIDAS) [23] is a new sampling-based algorithm for solving finite-horizon stochastic dynamic programs with monotonic Bellman functions. MIDAS approximates these value functions using step functions, leading to stage problems that are mixed integer programs. We provide a general description of MIDAS, and prove its almost-sure convergence to an epsilon-optimal policy when the Bellman functions are known to be continuous, and the sampling process satisfies standard assumptions.

7.2.2. Long term aging : an adaptative weights dynamic programming algorithm

Participants: J. Frédéric Bonnans, Benjamin Heymann, Pierre Martinon.

We introduce [26] a class of optimal control problems with periodic data. A state variable that we call the age of the system represents the negative impact of the operations on the system qualities over time: other things being equal, older systems have higher operating costs. Many industrial problems relate to this class. If we envision to perform an optimization over a large number of periods, there is a tradeoff between minimizing repeatedly the one-period criterion in a short sighted way and taking into account the impact of the decision on the aging speed (which modifies the minimal one period criterion). In general, because the aging process is slow, short term optimization strategies-such as one period sliding horizon strategies-either neglect it or use rule-of-thumb penalization terms in the criterion, which leads to suboptimal solutions. On the other hand, for most applications it is unrealistic to envision a brute-force numerical resolution by dynamic programming of the long term problem because of the computation burden. We introduce a two-scale method to reduce this computation burden. The method relies on Lagrangian duality and some monotony properties. We expose the theoretical foundations of the method and discuss some practical aspects: approximation errors, asymptotic estimation, computation burden, possible extensions, etc. Since our initial motivation was the difficulty to take long term battery aging in Energy Management Systems into account, we implement the method on a toy long term microgrid energy management problem.

7.2.3. *Continuous Optimal Control Approaches to Microgrid Energy Management*

Participants: J. Frédéric Bonnans, Benjamin Heymann, Pierre Martinon.

With Francisco Silva XLIM, U. Limoges, Fernando Lanas and Guillermo Jimenez, U. Chile.

We propose in [18] a novel method for the microgrid energy management problem by introducing a continuous-time, rolling horizon formulation. The energy management problem is formulated as a deterministic optimal control problem (OCP). We solve (OCP) with two classical approaches: the direct method [1], and Bellman's Dynamic Programming Principle (DPP) [2]. In both cases we use the optimal control toolbox BOCOP [3] for the numerical simulations. For the DPP approach we implement a semi-Lagrangian scheme [4] adapted to handle the optimization of switching times for the on/off modes of the diesel generator. The DPP approach allows for an accurate modeling and is computationally cheap. It finds the global optimum in less than 3 seconds, a CPU time similar to the Mixed Integer Linear Programming (MILP) approach used in [5]. We achieve this performance by introducing a trick based on the Pontryagin Maximum Principle (PMP). The trick increases the computation speed by several orders and also improves the precision of the solution. For validation purposes, simulation are performed using datasets from an actual isolated microgrid located in northern Chile. Results show that DPP method is very well suited for this type of problem when compared with the MILP approach.

7.2.4. *A Stochastic Continuous Time Model for Microgrid Energy Management*

Participants: J. Frédéric Bonnans, Benjamin Heymann.

With Francisco Silva XLIM U. Limoges, Guillermo Jimenez, U. Chile.

We propose in [20] a novel stochastic control formulation for the microgrid energy management problem and extend previous works on continuous time rolling horizon strategy to uncertain demand. We modelize the demand dynamics with a stochastic differential equation. We decompose this dynamics into three terms: an average drift, a time-dependent mean-reversion term and a Brownian noise. We use BOCOPHJB for the numerical simulations. This optimal control toolbox implements a semi-Lagrangian scheme and handle the optimization of switching times required for the discrete on/off modes of the diesel generator. The scheme allows for an accurate modelling and is computationally cheap as long as the state dimension is small. As described in previous works, we use a trick to reduce the search of the optimal control values to six points. This increases the computation speed by several orders. We compare this new formulation with the deterministic control approach using data from an isolated microgrid located in northern Chile.

7.2.5. *Mechanism Design and Auctions for Electricity Network*

Participant: Benjamin Heymann.

With Alejandro Jofré, CMM - Center for Mathematical Modeling, U. Chile, Santiago. We present in [25] some key aspects of wholesale electricity markets modeling and more specifically focus our attention on auctions and mechanism design. Some of the results arising from those models are the computation of an optimal allocation for the Independent System Operator, the study of the equilibria (existence and unicity in particular) and the design of mechanisms to increase the social surplus. From a more general perspective, this field of research provides clues to discuss how wholesale electricity market should be regulated. We start with a general introduction and then present some results the authors obtained recently. We also briefly expose some undergoing related work. As an illustrative example, a section is devoted to the computation of the Independent System Operator response function for a symmetric binodal setting with piece-wise linear production cost functions.

7.2.6. Mechanism design and allocation algorithms for network markets with piece-wise linear costs and externalities

Participant: Benjamin Heymann.

With Alejandro Jofré, CMM - Center for Mathematical Modeling, U. Chile, Santiago. In [24], motivated by market power in electricity market, we introduce a mechanism design for simplified markets of two agents with linear production cost functions. In standard procurement auctions, the market power resulting from the quadratic transmission losses allow the producers to bid above their true value (i.e. production cost). The mechanism proposed in the previous paper reduces the producers margin to the society benefit. We extend those results to a more general market made of a finite number of agents with piecewise linear cost functions, which make the problem more difficult, but at the same time more realistic. We show that the methodology works for a large class of externalities. We also provide two algorithms to solve the principal allocation problem.

7.2.7. Variational analysis for options with stochastic volatility and multiple factors

Participants: J. Frédéric Bonnans, Axel Kröner.

In this ongoing work we discuss the variational analysis for stochastic volatility models with correlation and their applications for the pricing equations for European options is discussed. The considered framework is based on weigthed Sobolev spaces. Furthermore, to verify continuity of the rate term in the pricing equation an approach based on commutator analysis is developed.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

8.1.1. Ifpen

In the framework of the PhD thesis of Arthur Le Rhun, we study the energy management of hybrid (parallel) vehicles, and more specifically the optimal use of the thermal engine. Before the PhD, a 4-month internship was focused on the eco-routing problem for hybrid vehicles, ie computing the optimal path. We proposed a method based on graphs: the road network is defined by a graph, and to take into account the hybrid aspect of the vehicle, we dicretized the State of Charge on each node. Then a simple shortest path algorithm (A*) applied to this extended graph is able to solve the routing problem. Numerical simulations indicate that the solution of our discrete eco-routing problem converges to the correct solution when a sufficiently fine discretization of SoC is used. We illustrate the method on the Ille-et-Vilaine department, see Fig. 1 and Table 1. The main disadvantage of the method is the increasingly large computation time when the size of the extended graph grows.

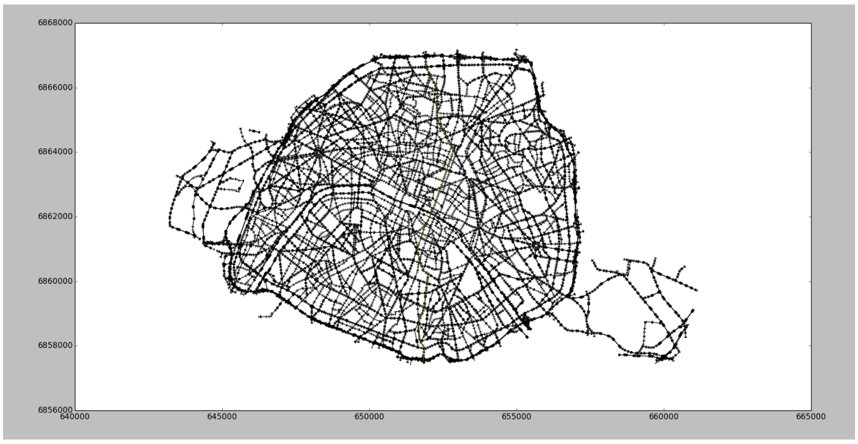

Figure 1.

Table 1. Results on the Ille-et-Vilaine department over 100 simulations

SoC disc.	improved cases	Fuel savings	CPU time (s)
3	19%	0.9753	6.03
5	65%	0.8531	14.64
10	88%	0.5831	52.80
20	88%	0.4222	283.43

8.1.2. Safety Line

In the framework of an Ilab with Safety Line (a startup in aeronautics), we design tools for the optimization of fuel consumption for civil planes. A first part is devoted to the identification of the aerodynamic and thrust characteristics of the plane, using recorded data from hundreds of flights. Fig. 2 shows the drag and lift coefficients for a Boeing 737, as functions of Mach and angle of attack. A second part is optimizing the fuel consumption during the climb and cruise phases. Fig. 3 shows a simulated climb phase, along with recorded data from the actual flight. This collaboration relies significantly on the toolboxes BOCOP and BOCOPHJB developed by Commands since 2010.

9. Partnerships and Cooperations

9.1. Regional Initiatives

- Gaspard Monge Program for Optimization and Operational Research (Fondation Jacques Hadamard)

Title	:	Optimal control of partial differential equations using parameterizing manifolds, model reduction, and dynamic programming,
Funding	:	9,000 Euro (for 2015-16), 10,000 Euro (for 2016-17)
PI	:	Axel Kröner
Period	:	2015 – 2017
Further members	:	Frédéric Bonnans (Inria Saclay and CMAP, École Polytechnique), Mickaël Chekroun (UCLA, Los Angeles), Martin Gubisch (University of Konstanz), Karl Kunisch (University of Graz), Hasnaa Zidani (ENSTA ParisTech).

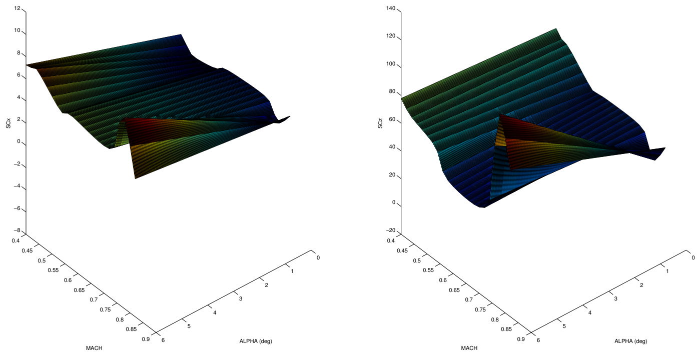

Figure 2.

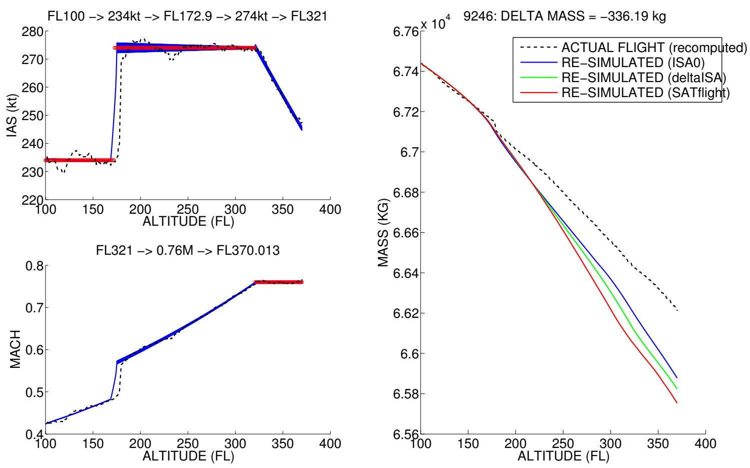

Figure 3.

9.2. International Initiatives

9.2.1. Inria International Partners

9.2.1.1. Informal International Partners

- Michael D. Chekroun, U.C.L.A, collaboration on the approximation and reduction of optimal control problems in infinite dimension.
- Alejandro Jofré, CMM, U. Chile, Santiago de Chile. Cosupervision of B. Heymann's PhD thesis.
- Pablo Lotito, U. Tandil, Argentina, supervision of Justina Gianatti's PhD.

9.3. International Research Visitors

9.3.1. Visits of International Scientists

- M. Chekroun (University of California, Los Angeles), 12.-14.12.2016.
- Johannes Pfefffer (Technische Universität München), 12.-14.12.2016.

9.3.1.1. Internships

- Luis Alberto Croquevielle Rendic: Classification of probability measures based on Optimal Transportation theory. January-March 2016. U. Catolica, Santiago, Chile.
- Justina Gianatti, Discretization of stochastic control problems, U. Rosario (Argentina), May-July 2016.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific events selection

10.1.1.1. Member of the conference program committees

- F. Bonnans, 14th EUROPT Workshop on Advances in Continuous Optimization, Poznań, July 3-6, 2016.

10.1.2. Journal

10.1.2.1. Member of the editorial boards

- F. Bonnans: Corresponding Editor of "ESAIM:COCV" (Control, Optimization and Calculus of Variations), and Associate Editor of "Applied Mathematics and Optimization", "Optimization, Methods and Software", and "Series on Mathematics and its Applications, Annals of The Academy of Romanian Scientists".

10.1.2.2. Reviewer - Reviewing activities

Reviews for major journals in the field such as Applied Mathematics and Optimization, Automatica, J. Diff. Equations, J. of Optimization Theory and Applications the SIAM J. Optimization, SIAM J. Control and Optimization, Inverse problems, Journal of Numerical Mathematics, Operations Research, Optimization, Process Control, Math. Reviews.

10.1.3. Invited talks

- A. Kröner: Seminars in U. Konstanz, U. Hamburg, 2016.
- Minisymposium Numerical methods for time-dependent transportation and optimal control problems. Computational Methods in Applied Mathematics (CMAM), Jyväskyla, Finland, July 31-Aug. 6,2016;
- Minisymposium 'Optimal Control - Theory and Applications', Emerging Trends in Applied Mathematics and Mechanics, Perpignan, May 30-June 3, 2016;
- Minisymposium 'Numerical aspects of controllability of PDEs and inverse problems', CANUM (Congrès d'Analyse Numérique), Obernai, May 9-13, 2016;
- A. Kröner: Workshop on *Numerical methods for Hamilton-Jacobi equations in optimal control and related fields*, Linz, Austria, Nov., 2016;

10.1.4. Leadership within the scientific community

- F. Bonnans: French representative to the IFIP-TC5 committee (International Federation of Information Processing; TC7 devoted to System Modeling and Optimization).
- F. Bonnans: member of the PGMO board and Steering Committee (Gaspard Monge Program for Optimization and Operations Research, EDF-FMJH).
- F. Bonnans: member of the Broyden Prize committee (from the Journal Optimization Methods and Software).

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Master :
F. Bonnans: *Numerical analysis of partial differential equations arising in finance and stochastic control*, 36h, M2, Ecole Polytechnique and U. Paris 6, France.
F. Bonnans: *Optimal control*, 15h, M2, Optimization master (U. Paris-Saclay) and Ensta, France.
F. Bonnans: *Stochastic optimization*, 15h, M2, Optimization master (U. Paris-Saclay), France.
A. Kröner : Optimal control of partial differential equations, 20h, M2, Optimization master (U. Paris-Saclay), France.

E-learning F. Bonnans, several lecture notes on the page
http://www.cmap.polytechnique.fr/∼bonnans/notes.html

10.2.2. Supervision

- PhD : Benjamin Heymann, Dynamic optimization with uncertainty; application to energy production. Polytechnique fellowship, defense October 2016, F. Bonnans and A. Jofre.
- PhD in progress : Cédric Rommel, Data exploration for the optimization of aircraft trajectories. Started November 2015, F. Bonnans and P. Martinon. CIFRE fellowship (Safety Line).
- PhD in progress : Arthur Le Rhun, Optimal and robust control of hybrid vehicles (IFPEN fellowship), started Sept. 2016, F. Bonnans and P. Martinon.

10.2.3. Juries

- HDR Juries: F. Silva (Limoges), A. Rondepierre (Toulouse, rapporteur).

10.3. Popularization

- J.F. Bonnans: *Comment optimiser la gestion d'un micro-réseau électrique intelligent ?* Cahier de l'Institut Louis Bachelier N. 23 (2016), p. 12-13.

11. Bibliography

Major publications by the team in recent years

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