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Activity Report 2016

Project-Team GECO

Geometric Control Design

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team GECO

Creation of the Team: 2011 May 01, updated into Project-Team: 2013 January 01

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1. Members

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2. Overall Objectives

2.1. Overall Objectives

Motion planning is not only a crucial issue in control theory, but also a widespread task of all sort of human activities. The aim of the project-team is to study the various aspects preceding and framing *motion planning*: accessibility analysis (determining which configurations are attainable), criteria to make choice among possible trajectories, trajectory tracking (fixing a possibly unfeasible trajectory and following it as closely as required), performance analysis (determining the cost of a tracking strategy), design of implementable algorithms, robustness of a control strategy with respect to computationally motivated discretizations, etc. The viewpoint that we adopt comes from geometric control: our main interest is in qualitative and intrinsic properties and our focus is on trajectories (either individual ones or families of them).

The main application domain of GECO is *quantum control*. The importance of designing efficient transfers between different atomic or molecular levels in atomic and molecular physics is due to its applications to photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing.

A second application area concerns the control interpretation of phenomena appearing in *neurophysiology*. It studies the modeling of the mechanisms supervising some biomechanics actions or sensorial reactions such as image reconstruction by the primary visual cortex, eyes movement and body motion. All these problems can be seen as motion planning tasks accomplished by the brain.

As a third applicative domain we propose a system dynamics approach to *switched systems*. Switched systems are characterized by the interaction of continuous dynamics (physical system) and discrete/logical components. They provide a popular modeling framework for heterogeneous aspects issuing from automotive and transportation industry, energy management and factory automation.

3. Research Program

3.1. Geometric control theory

The main research topic of the project-team is **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [22], [56] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;

- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([76], [63], [69]), those exploiting the possible flatness of the system ([50]) and those based on the continuation method ([88]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([75]), geometric measure theory ([51], [26]) and hypoelliptic operator theory ([38]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [43]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [23] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [81], [86]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [42]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [24],
- bounds on the controllability time [20],
- STIRAP processes [91],
- simultaneous control [64],
- optimal control ([60], [33], [44]),
- numerical simulations [70].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [25], [61], [82], [39]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [29], [40], [57], [30]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [90]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [83], [73]. These negative results have been more recently completed by positive ones. In [31], [32] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [46]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [72], [77], [58]. While [72] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [77], [58] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [49], [36].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [55]. Optimal control approaches have also been considered [28], [41]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [79]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

  The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word "geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

  A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([80]) and then modified by Citti and Sarti ([45]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [54] who received the Nobel prize in 1981.

  In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

  From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in robotics and neurophysiology. It could help to design better control strategies for robots and artificial limbs, yielding smoother and more progressive movements. Another underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson's disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain's choices. For instance, for the problem of human locomotion (see [27]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [89] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [59] or for Markov processes [78]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([66]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [87], [67]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [68]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [74] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [34], [35], [47]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [48] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [65], [21].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [53].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [62]. It is known that, in this context, stability cannot be tested on periodic signals alone [37].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([71], [84], [85]) and some recent papers on optimal control ([52], [92]).

5. Highlights of the Year

5.1. Highlights of the Year

The European Research Council (ERC) has awarded Ugo Boscain with a “Proof of concept grant" for his project *An Artificial Visual Cortex for Image Processing*.

6. New Software and Platforms

6.1. ARTIV1 INPAINTING

ARTIV1 INPAINTING

FUNCTIONAL DESCRIPTION

ARTIV1 INPAINTING is a software for reconstruction of corrupted and damaged images. One of the main features of the algorithm on which the software is based is that it does not require any information about the location and character of the corrupted places. Another important advantage is that this method is massively parallelizable, this allows to work with sufficiently large images. Theoretical background of the presented method is based on the model of geometry of vision due to Petitot, Citti and Sarti. The main step is numerical solution of the equation of 3D hypoelliptic diffusion. A new version of the software has just be submitted for protection at APP (Agence pour la protection des programmes).

- Contact: Ugo Boscain

7. New Results

7.1. New results: geometric control

Let us list some new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- In [2] we compare different notions of curvature on contact sub-Riemannian manifolds. In particular we introduce canonical curvatures as the coefficients of the sub-Riemannian Jacobi equation. The main result is that all these coefficients are encoded in the asymptotic expansion of the horizontal derivatives of the sub-Riemannian distance. We explicitly compute their expressions in terms of the standard tensors of contact geometry. As an application of these results, we obtain a sub-Riemannian version of the Bonnet-Myers theorem that applies to any contact manifold.
- In [3] we provide the small-time heat kernel asymptotics at the cut locus in three relevant cases: generic low-dimensional Riemannian manifolds, generic 3D contact sub-Riemannian manifolds (close to the starting point) and generic 4D quasi-contact sub-Riemannian manifolds (close to a generic starting point). As a byproduct, we show that, for generic low-dimensional Riemannian manifolds, the only singularities of the exponential map, as a Lagragian map, that can arise along a minimizing geodesic are A_3 and A_5 (in Arnol'd's classification). We show that in the non-generic case, a cornucopia of asymptotics can occur, even for Riemannian surfaces.

- In [5] we study the evolution of the heat and of a free quantum particle (described by the Schrödinger equation) on two-dimensional manifolds endowed with the degenerate Riemannian metric $ds^2 = dx^2 + |x|^{-2\alpha} d\theta^2$, where $x \in \mathbb{R}$, $\theta \in S^1$ and the parameter $\alpha \in \mathbb{R}$. For $\alpha \leq -1$ this metric describes cone-like manifolds (for $\alpha = -1$ it is a flat cone). For $\alpha = 0$ it is a cylinder. For $\alpha \geq 1$ it is a Grushin-like metric. We show that the Laplace-Beltrami operator Δ is essentially self-adjoint if and only if $\alpha \notin (-3, 1)$. In this case the only self-adjoint extension is the Friedrichs extension Δ_F, that does not allow communication through the singular set $\{x = 0\}$ both for the heat and for a quantum particle. For $\alpha \in (-3, -1]$ we show that for the Schrödinger equation only the average on θ of the wave function can cross the singular set, while the solutions of the only Markovian extension of the heat equation (which indeed is Δ_F) cannot. For $\alpha \in (-1, 1)$ we prove that there exists a canonical self-adjoint extension Δ_N, called bridging extension, which is Markovian and allows the complete communication through the singularity (both of the heat and of a quantum particle). Also, we study the stochastic completeness (i.e., conservation of the L^1 norm for the heat equation) of the Markovian extensions Δ_F and Δ_B, proving that Δ_F is stochastically complete at the singularity if and only if $\alpha \leq -1$, while Δ_B is always stochastically complete at the singularity.
- In [6] we study spectral properties of the Laplace–Beltrami operator on two relevant almost-Riemannian manifolds, namely the Grushin structures on the cylinder and on the sphere. As for general almost-Riemannian structures (under certain technical hypothesis), the singular set acts as a barrier for the evolution of the heat and of a quantum particle, although geodesics can cross it. This is a consequence of the self-adjointness of the Laplace–Beltrami operator on each connected component of the manifolds without the singular set. We get explicit descriptions of the spectrum, of the eigenfunctions and their properties. In particular in both cases we get a Weyl law with dominant term $E \log E$. We then study the effect of an Aharonov-Bohm non-apophantic magnetic potential that has a drastic effect on the spectral properties. Other generalized Riemannian structures including conic and anti-conic type manifolds are also studied. In this case, the Aharonov-Bohm magnetic potential may affect the self-adjointness of the Laplace-Beltrami operator.
- Generic singularities of line fields have been studied for lines of principal curvature of embedded surfaces. In [7] we propose an approach to classify generic singularities of general line fields on 2D manifolds. The idea is to identify line fields as bisectors of pairs of vector fields on the manifold, with respect to a given conformal structure. The singularities correspond to the zeros of the vector fields and the genericity is considered with respect to a natural topology in the space of pairs of vector fields. Line fields at generic singularities turn out to be topologically equivalent to the Lemon, Star and Monstar singularities that one finds at umbilical points.
- In [10] we prove that any corank 1 Carnot group of dimension $k + 1$ equipped with a left-invariant measure satisfies the measure contraction property $\mathrm{MCP}(K, N)$ if and only if $K \leq 0$ and $N \geq k + 3$. This generalizes the well known result by Juillet for the Heisenberg group H^{k+1} to a larger class of structures, which admit non-trivial abnormal minimizing curves. The number $k + 3$ coincides with the geodesic dimension of the Carnot group, which we define here for a general metric space. We discuss some of its properties, and its relation with the curvature exponent (the least N such that the $\mathrm{MCP}(0, N)$ is satisfied). We prove that, on a metric measure space, the curvature exponent is always larger than the geodesic dimension which, in turn, is larger than the Hausdorff one. When applied to Carnot groups, our results improve a previous lower bound due to Rifford. As a byproduct, we prove that a Carnot group is ideal if and only if it is fat.
- In [14] we relate some basic constructions of stochastic analysis to differential geometry, via random walk approximations. We consider walks on both Riemannian and sub-Riemannian manifolds in which the steps consist of travel along either geodesics or integral curves associated to orthonormal frames, and we give particular attention to walks where the choice of step is influenced by a volume on the manifold. A primary motivation is to explore how one can pass, in the parabolic scaling limit, from geodesics, orthonormal frames, and/or volumes to diffusions, and hence their infinitesimal generators, on sub-Riemannian manifolds, which is interesting in light of the fact that there is no completely canonical notion of sub-Laplacian on a general sub-Riemannian manifold. However,

even in the Riemannian case, this random walk approach illuminates the geometric significance of Ito and Stratonovich stochastic differential equations as well as the role played by the volume.

- By adapting a technique of Molchanov, we obtain in [15] the heat kernel asymptotics at the sub-Riemannian cut locus, when the cut points are reached by a r-dimensional parametric family of optimal geodesics. We apply these results to the bi-Heisenberg group, that is, a nilpotent left-invariant sub-Riemannian structure on $\mathbb{R}^5$ depending on two real parameters α_1 and α_2. We develop some results about its geodesics and heat kernel associated to its sub-Laplacian and we point out some interesting geometric and analytic features appearing when one compares the isotropic $(\alpha_1 = \alpha_2)$ and the non-isotropic cases $(\alpha_1 \neq \alpha_2)$. In particular, we give the exact structure of the cut locus, and we get the complete small-time asymptotics for its heat kernel.
- The Whitney extension theorem is a classical result in analysis giving a necessary and sufficient condition for a function defined on a closed set to be extendable to the whole space with a given class of regularity. It has been adapted to several settings, among which the one of Carnot groups. However, the target space has generally been assumed to be equal to $\mathbb{R}^d$ for some $d \geq 1$. We focus in [17] on the extendability problem for general ordered pairs (G_1, G_2) (with G_2 non-Abelian). We analyze in particular the case $G_1 = \mathbb{R}$ and characterize the groups G_2 for which the Whitney extension property holds, in terms of a newly introduced notion that we call pliability. Pliability happens to be related to rigidity as defined by Bryant an Hsu. We exploit this relation in order to provide examples of non-pliable Carnot groups, that is, Carnot groups so that the Whitney extension property does not hold. We use geometric control theory results on the accessibility of control affine systems in order to test the pliability of a Carnot group.
- In [19] we study the cut locus of the free, step two Carnot groups G^k with k generators, equipped with their left-invariant Carnot–Carathéodory metric. In particular, we disprove the conjectures on the shape of the cut loci proposed in the literature, by exhibiting sets of cut points $C \subset G^k$ which, for $k \geq 4$, are strictly larger than conjectured ones. Furthermore, we study the relation of the cut locus with the so-called abnormal set. For each $k \geq 4$, we show that, contrarily to the case $k = 2, 3$, the cut locus always intersects the abnormal set, and there are plenty of abnormal geodesics with finite cut time. Finally, and as a straightforward consequence of our results, we derive an explicit lower bound for the small time heat kernel asymptotics at the points of C. The question whether C coincides with the cut locus for $k \geq 4$ remains open.

We also edited the two volumes [13] and [12], containing some of the lecture notes of the courses given during the IHP triemster on “Geometry, Analysis and Dynamics on sub-Riemannian Manifolds” which we organized in Fall 2014. The second volume also contains a chapter [11] co-authored by members of the team.

7.2. New results: quantum control

- In recent years, several sufficient conditions for the controllability of the Schrödinger equation have been proposed. In [16], we discuss the genericity of these conditions with respect to the variation of the controlled or the uncontrolled potential. In the case where the Schrödinger equation is set on a domain of dimension one, we improve the results in the literature, removing from the previously known genericity results some unnecessary technical assumptions on the regularity of the potentials.

7.3. New results: neurophysiology

In [4] we propose a supervised object recognition method using new global features and inspired by the model of the human primary visual cortex V1 as the semidiscrete roto-translation group $\mathrm{SE}(2, N) = \mathbb{Z}_N \rtimes \mathbb{R}^2$. The proposed technique is based on generalized Fourier descriptors on the latter group, which are invariant to natural geometric transformations (rotations, translations). These descriptors are then used to feed an SVM classifier. We have tested our method against the COIL-100 image database and the ORL face database, and compared it with other techniques based on traditional descriptors, global and local. The obtained results have shown that our approach looks extremely efficient and stable to noise, in presence of which it outperforms the other techniques it has been compared with.

7.4. New results: switched systems

- In [8] we address the exponential stability of a system of transport equations with intermittent damping on a network of $N \geq 2$ circles intersecting at a single point O. The N equations are coupled through a linear mixing of their values at O, described by a matrix M. The activity of the intermittent damping is determined by persistently exciting signals, all belonging to a fixed class. The main result is that, under suitable hypotheses on M and on the rationality of the ratios between the lengths of the circles, such a system is exponentially stable, uniformly with respect to the persistently exciting signals. The proof relies on a representation formula for the solutions of this system, which allows one to track down the effects of the intermittent damping. A similar representation formula is used in [18] to study the relative controllability of linear difference equations with multiple delays in the state. Thanks to such formula, we characterize relative controllability in time T in terms of an algebraic property of the matrix-valued coefficients, which reduces to the usual Kalman controllability criterion in the case of a single delay. Relative controllability is studied for solutions in the set of all functions and in the function spaces L^p and C^k. We also compare the relative controllability of the system for different delays in terms of their rational dependence structure, proving that relative controllability for some delays implies relative controllability for all delays that are “less rationally dependent" than the original ones. Finally, we provide an upper bound on the minimal controllability time for a system depending only on its dimension and on its largest delay.
- In [9] we address the stability of transport systems and wave propagation on general networks with time-varying parameters. We do so by reformulating these systems as non-autonomous difference equations and by providing a suitable representation of their solutions in terms of their initial conditions and some time-dependent matrix coefficients. This enables us to characterize the asymptotic behavior of solutions in terms of such coefficients. In the case of difference equations with arbitrary switching, we obtain a delay-independent generalization of the well-known criterion for autonomous systems due to Hale and Silkowski. As a consequence, we show that exponential stability of transport systems and wave propagation on networks is robust with respect to variations of the lengths of the edges of the network preserving their rational dependence structure. This leads to our main result: the wave equation on a network with arbitrarily switching damping at external vertices is exponentially stable if and only if the network is a tree and the damping is bounded away from zero at all external vertices but at most one.

8. Partnerships and Cooperations

8.1. Regional Initiatives

- Project *Stabilité des systèmes à excitation persistante*, Program MathIng, Labex LMH, 2013-2016. This project is about different stability properties for systems whose damping is intermittently activated. The coordinator is Mario Sigalotti. The other members are Yacine Chitour and Guilherme Mazanti.
- iCODE is the Institute for Control and Decision of the Idex Paris Saclay. It was launched in March 2014 for two years until June 2016. We have been involved in three actions funded by iCODE:
  - one action on control of quantum systems, in collaboration with Nicoals Boulant of Neurospin. The action was coordinated by Ugo Boscain;
  - one action on control of wave propagation on networks. The action was coordinated by Mario Sigalotti;
  - one action on switched system. The action was coordinated by Marianne Akian (and handled by MAXPLUS).

  Starting from November 2016, iCODE has been renewed for three years as a IRS (*Institut de Recherche Strategique*) by the Idex Paris Saclay. The funded actions have still not been identified.

- Starting from the end of 2015, we obtained a grant by PGMO (Gaspard Monge Program for Optimisation and operational research) on Geometric Optimal Control. The grant duration is one year, has been renewed in 2016 and is still renewable for a third year. The grant is coordinated by Mario Sigalotti (up to August, it was co-coordinated by Luca Rizzi as well).

8.2. National Initiatives

8.2.1. ANR

The ANR SRGI starts at the end of 2015, for a duration of four years. GECO is one of one of the partners of the ANR. The national coordinator is Emmanuel Trélat (UPMC) and the local one Ugo Boscain.

SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.

8.2.2. Other initiatives

Ugo Boscain and Mario Sigalotti are members of the project DISQUO of the program Inphyniti of the CNRS (duration: one year renewable). Coordinator: Thomas Chambrion (Nancy).

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

Program: ERC Starting Grant

Project acronym: GeCoMethods

Project title: Geometric Control Methods for the Heat and Schroedinger Equations

Duration: Initially accepted from 1/5/2010 to 1/5/2015, the project has been extended for one additional year, up to 1/5/2016.

Coordinator: Ugo Boscain

Abstract: The aim of this project is to study certain PDEs for which geometric control techniques open new horizons. More precisely we plan to exploit the relation between the sub-Riemannian distance and the properties of the kernel of the corresponding hypoelliptic heat equation and to study controllability properties of the Schroedinger equation.

All subjects studied in this project are applications-driven: the problem of controllability of the Schroedinger equation has direct applications in Laser spectroscopy and in Nuclear Magnetic Resonance; the problem of nonisotropic diffusion has applications in cognitive neuroscience (in particular for models of human vision).

Participants. Main collaborator: Mario Sigalotti. Other members of the team: Andrei Agrachev, Riccardo Adami, Thomas Chambrion, Grégoire Charlot, Yacine Chitour, Jean-Paul Gauthier, Frédéric Jean.

8.4. International Initiatives

8.4.1. Inria International Partners

8.4.1.1. Informal International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

8.4.2. Participation in Other International Programs

- Laboratoire Euro Maghrébin de Mathématiques et de leurs Interactions (LEM2I)
  http://lem2i.math.cnrs.fr/
- GDRE Control of Partial Differential Equations (CONEDP)
  http://www.ceremade.dauphine.fr/~glass/GDRE/

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Andrei Agrachev (SISSA, Italy) is visiting the GECO team for one year, starting in September 2016.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. Member of the Organizing Committees

- Mario Sigalotti was member of the organizing committee of the *Workshop on switching dynamics & verification*, IHP, Paris, January 28-29, 2016.
- Ugo Boscain and Mario Sigalotti were member of the organizing committee of the *Workshop on quantum dynamics & control*, IHP, Paris, May 23-24, 2016.

9.1.2. Journal

9.1.2.1. Member of the Editorial Boards

- Ugo Boscain is Associate Editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing Editor of Journal of Dynamical and Control Systems
- Mario Sigalotti is Associate Editor of Journal of Dynamical and Control Systems
- Ugo Boscain is Associate Editor of ESAIM Control, Optimisation and Calculus of Variations
- Ugo Boscain is Associate Editor of Mathematical Control and Related Fields
- Ugo Boscain is Associate editor of Analysis and Geometry in Metric Spaces

9.1.3. Invited Talks

- Mario Sigalotti gave an invited talk at the “ExQM Miniworkshop: Mathematics of Quantum Control”, Munich, Germany, February 2016.
- Ugo Boscain gave an invited talk at the conference “Geometric Analysis in Control and Vision Theory", Voss, Norway, May 2016.
- Ugo Boscain gave an invited talk at the seminar of the *Departement de Mathématiques d’Orsay*, May 2016.
- Ugo Boscain gave an invited talk at the conference “Recent Trends in Differential equations”, Aveiro, Portugal, June 2016.
- Mario Sigalotti gave an invited talk at the *Séminaire de géométrie sous-riemannienne*, IHP, Paris, June 2016.
- Ugo Boscain gave the opening talk at the conference “Geometry, PDE’s and Lie Groups in Image Analysis", Eindhoven, The Netherlands, August 2016.
- Mario Sigalotti gave an invited talk at the seminar of the *Dipartimento di Matematica - Università degli Studi di Trento*, Italy, September 2016.

- Ugo Boscain gave an invited talk at the conference “Nouvelles directions en analyse semiclassique", Chalès, France, December 2016.

9.1.4. Research Administration

- Mario Sigalotti is member of the IFAC technical committee “Distributed Parameter Systems”.
- Mario Sigalotti is member of the steering committee of the *Institut pour le Contrôle et la Décision* of the Idex Paris-Saclay.

9.2. Teaching - Supervision - Juries

9.2.1. Supervision

- PhD (concluded): Guiherme Mazanti, “Stabilité et taux de convergence pour les systèmes à excitation persistante" [1], supervisors: Yacine Chitour, Mario Sigalotti. Discussed on September 2016.
- PhD in progress: Ludovic Sacchelli, “Sub-Riemannian geometry, hypoelliptic operators, geometry of vision", started in September 2015, supervisors: Ugo Boscain, Mario Sigalotti.
- PhD in progress: Nicolas Augier, “Contrôle adiabatique des systèmes quantiques", started in September 2016, supervisors: Ugo Boscain, Mario Sigalotti.
- PhD in progress: Mathieu Kohli, “Volume and curvature in sub-Riemannian geometry", started in September 2016, supervisors: Davide Barilari, Ugo Boscain.
- PhD in progress: Jakub Orłowski, “Modeling and steering brain oscillations based on in vivo optogenetics data", started in September 2016, supervisors: Antoine Chaillet, Alain Destexhe, and Mario Sigalotti.

9.2.2. Juries

- Ugo Boscain was member of the commission for the PhD defense of Valentina Franceschi, Padue, March 2016.
- Mario Sigalotti was member of the commission for the PhD defense of Francesco Boarotto, SISSA, Trieste, September 2016.
- Ugo Boscain was reviewer and member of the commission for the PhD defense of Jérémy Rouot, Nice, November 2016.

9.3. Popularization

Ugo Boscain gave a concert-seminar at the event “Musique & Mathématiques 2016”, Besanoņ, November 2016.

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

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Articles in International Peer-Reviewed Journals

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[4] A. BOHI, D. PRANDI, V. GUIS, F. BOUCHARA, J.-P. GAUTHIER. *Fourier Descriptors Based on the Structure of the Human Primary Visual Cortex with Applications to Object Recognition*, in "Journal of Mathematical Imaging and Vision", July 2016, pp. 1-17 [*DOI :* 10.1007/S10851-016-0669-1], https://hal.archives-ouvertes.fr/hal-01383846

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Scientific Books (or Scientific Book chapters)

[11] A. AGRACHEV, D. BARILARI, U. BOSCAIN. *Introduction to geodesics in sub-Riemannian geometry*, in "Geometry, Analysis and Dynamics on Sub-Riemannian Manifolds - Volume II", EMS Series of Lectures in Mathematics, 2016, https://hal.inria.fr/hal-01392516

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