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Project-Team MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Institut Mathématique de Bourgogne, Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
**Optimization and control of dynamic
systems**

Table of contents

1. Members	1
2. Overall Objectives	2
3. Research Program	2
3.1. Control Systems	2
3.2. Structure of nonlinear control systems	3
3.3. Optimal control and feedback control, stabilization	3
3.3.1. Optimal control.	3
3.3.2. Feedback, control Lyapunov functions, stabilization.	4
3.4. Optimal Transport	4
3.5. Small controls and conservative systems, averaging	5
4. Application Domains	5
4.1. Space engineering, satellites, low thrust control	5
4.1.1. Low thrust	5
4.1.2. Typical problems	6
4.1.3. Properties of the control system.	6
4.2. Quantum Control	6
4.3. Swimming at low-Reynolds number	7
4.4. Applications of optimal transport	7
4.5. Applications to some domains of mathematics.	8
5. New Software and Platforms	8
6. New Results	8
6.1. Advances in optimal control	8
6.1.1. Algebraic and geometric techniques in medical resonance imaging	8
6.1.2. Local minima, second order conditions	9
6.1.3. Solving chance-constrained optimal control problems in aerospace engineering via Kernel Density Estimation	9
6.2. Averaging and filtering for optimal control in Space mechanics	9
6.2.1. Convergence properties of the Maximum principle	9
6.2.2. Approximation by filtering in optimal control and applications	10
6.2.3. Averaging with reconstruction of the fast variable	10
6.3. Fully controlled slender microswimmers	10
6.3.1. The N -link micro-swimmer	10
6.3.2. Optimal periodic strokes for the Copepod and Purcell micro-swimmers	10
6.4. Modelization and Controllability of “Magneto-elastic” Micro-swimmers	11
6.5. Sub-Riemannian Geometry and Optimal Transport	11
6.6. Geometric Control and Dynamics	11
7. Bilateral Contracts and Grants with Industry	12
8. Partnerships and Cooperations	12
8.1. Regional Initiatives	12
8.2. National Initiatives	12
8.2.1. ANR	12
8.2.2. Others	12
8.3. European Initiatives	13
8.3.1.1. Bilateral program with Portugal	13
8.3.1.2. Bilateral program with Germany	13
9. Dissemination	13
9.1. Promoting Scientific Activities	13
9.1.1. Scientific Events Organisation	13
9.1.1.1. General Chair, Scientific Chair	13

9.1.1.2. Member of the Organizing Committees	13
9.1.2. Books	14
9.1.2.1. Springer briefs	14
9.1.2.2. Springer Maths and Industry	14
9.1.2.3. Radon Series on Comput. and Applied Math.	14
9.1.3. Journals	14
9.1.3.1. Member of the Editorial Boards	14
9.1.3.2. Reviewer - Reviewing Activities	14
9.1.4. Invited Talks	14
9.1.5. Leadership within the Scientific Community	15
9.1.6. Scientific Expertise	15
9.1.7. Research Administration	15
9.2. Teaching - Supervision - Juries	15
9.2.1. Teaching	15
9.2.2. Supervision	16
9.2.3. Juries	16
9.3. Popularization	16
10. Bibliography	16

Project-Team MCTAO

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- 5.10.3. - Planning
- 5.10.4. - Robot control
- 6.1. - Mathematical Modeling
 - 6.1.1. - Continuous Modeling (PDE, ODE)
 - 6.1.4. - Multiscale modeling
- 6.2.1. - Numerical analysis of PDE and ODE
- 6.2.6. - Optimization
- 6.4. - Automatic control
 - 6.4.1. - Deterministic control
 - 6.4.3. - Observability and Controlability
 - 6.4.4. - Stability and Stabilization

Other Research Topics and Application Domains:

- 2.6. - Biological and medical imaging
- 5.2.3. - Aviation
- 5.2.4. - Aerospace
- 6.6. - Embedded systems

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2. Overall Objectives

2.1. Overall Objectives

The core endeavor of this team is to develop methods in control theory for finite-dimensional nonlinear systems, as well as in optimal transport, and to be involved in real applications of these techniques. Some mathematical fields like dynamical systems and optimal transport may benefit from control theory techniques. Our primary domain of industrial applications will be space engineering, namely designing trajectories in space mechanics using optimal control and stabilization techniques: transfer of a satellite between two Keplerian orbits, rendez-vous problem, transfer of a satellite from the Earth to the Moon or more complicated space missions. A second field of applications is quantum control with applications to Nuclear Magnetic Resonance and medical image processing. A third and more recent one is the control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number.

3. Research Program

3.1. Control Systems

Our effort is directed toward efficient methods for the *control* of real (physical) systems, based on a *model* of the system to be controlled. *System* refers to the physical plant or device, whereas *model* refers to a mathematical representation of it.

We mostly investigate nonlinear systems whose nonlinearities admit a strong structure derived from physics; the equations governing their behavior are then well known, and the modeling part consists in choosing what phenomena are to be kept in the model used for control design, the other phenomena being treated as perturbations; a more complete model may be used for simulations, for instance. We focus on systems that admit a reliable finite-dimensional model, in continuous time; this means that models are controlled ordinary differential equations, often nonlinear.

Choosing accurate models yet simple enough to allow control design is in itself a key issue; however, modeling or identification as a theory is not per se in the scope of our project.

The extreme generality and versatility of linear control do not contradict the often heard sentence “most real life systems are nonlinear”. Indeed, for many control problems, a linear model is sufficient to capture the important features for control. The reason is that most control objectives are local, first order variations around an operating point or a trajectory are governed by a linear control model, and except in degenerate situations (non-controllability of this linear model), the local behavior of a nonlinear dynamic phenomenon is dictated by the behavior of first order variations. Linear control is the hard core of control theory and practice; it has been pushed to a high degree of achievement –see for instance some classics: [64], [55]– that lead to big successes in industrial applications (PID, Kalman filtering, frequency domain design, H^∞ robust control, etc...), it is taught to future engineers, and it is still a topic of ongoing research.

Linear control by itself however reaches its limits in some important situations:

1. **Non local control objectives.** Steering the system from a region to a reasonably remote other one, as in path planning and optimal control, is outside the scope of information given by a local linear approximation. It is why these are by essence nonlinear.
Stabilisation with a basin of attraction larger than the region where the linear approximation is dominant also needs more information than one linear approximation.
2. **Local control at degenerate equilibria.** Linear control yields local stabilization of an equilibrium point based on the tangent linear approximation if the latter is controllable. It is *not* the case at interesting operating points of some physical systems; linear control is irrelevant and specific nonlinear techniques have to be designed. This is an extreme case of the second part of the above item: the region where the linear approximation is dominant vanishes.
3. **Small controls.** In some situations, actuators only allow a very small magnitude of the effect of control compared to the effect of other phenomena. Then the behavior of the system without control plays a major role and we are again outside the scope of linear control methods.

3.2. Structure of nonlinear control systems

In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution. In control systems, it is often crucial to analyze the structure of the model, deduced from physical principles, of the plant to be controlled; this may lead to putting it via some transformations in a simpler form, or a form that is most suitable for control design. For instance, equivalence to a linear system may allow to use linear control; also, the so-called “flatness” property drastically simplifies path planning [59], [70].

A better understanding of the “set of nonlinear models”, partly classifying them, has another motivation than facilitating control design for a given system and its model: it may also be a necessary step towards a theory of “nonlinear identification” and modeling. Linear identification is a mature area of control science; its success is mostly due to a very fine knowledge of the structure of the class of linear models: similarly, any progress in the understanding of the structure of the class of nonlinear models would be a contribution to a possible theory of nonlinear identification.

These topics are central in control theory, but raise very difficult mathematical questions: static feedback classification is a geometric problem which is feasible in principle, although describing invariants explicitly is technically very difficult; and conditions for dynamic feedback equivalence and linearization raise unsolved mathematical problems, that make one wonder about decidability ¹.

3.3. Optimal control and feedback control, stabilization

3.3.1. Optimal control.

Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [39], [68], [46], [76]. Relying on Hamiltonian dynamics is now prevalent, instead of the standard Lagrangian formalism of the calculus of variations. Also, coming from control engineering, constraints on the control (for instance the control is a force or a torque, which are naturally bounded) or the state (for example in the shuttle atmospheric re-entry problem there is a constraint on the thermal flux) are imposed; the ones on the state are usual but these on the state yield more complicated necessary optimality conditions and an increased intrinsic complexity of the optimal solutions. Also, in the modern treatment, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions.

¹Consider the simple system with state $(x, y, z) \in \mathbb{R}^3$ and two controls that reads $\dot{z} = (\dot{y} - z\dot{x})^2 \dot{x}$ after elimination of the controls; it is not known whether it is equivalent to a linear system, or flat; this is because the property amounts to existence of a formula giving the general solution as a function of two arbitrary functions of time and their derivatives up to a certain order, but no bound on this order is known a priori, even for this very particular example.

What makes optimal control an applied field is the necessity of computing these optimal trajectories, or rather the controls that produce these trajectories (or, of course, close-by trajectories). Computing a given optimal trajectory and its control as a function of time is a demanding task, with non trivial numerical difficulties: roughly speaking, the Pontryagin Maximum Principle gives candidate optimal trajectories as solutions of a two point boundary value problem (for an ODE) which can be analyzed using mathematical tools from geometric control theory or solved numerically using shooting methods. Obtaining the *optimal synthesis* –the optimal control as a function of the state– is of course a more intricate problem [46], [51].

These questions are not only academic for minimizing a cost is *very* relevant in many control engineering problems. However, modern engineering textbooks in nonlinear control systems like the “best-seller” [61] hardly mention optimal control, and rather put the emphasis on designing a feedback control, as regular and explicit as possible, satisfying some qualitative (and extremely important!) objectives: disturbance attenuation, decoupling, output regulation or stabilization. Optimal control is sometimes viewed as disconnected from automatic control... we shall come back to this unfortunate point.

3.3.2. Feedback, control Lyapunov functions, stabilization.

A control Lyapunov function (CLF) is a function that can be made a Lyapunov function (roughly speaking, a function that decreases along all trajectories, some call this an “artificial potential”) for the closed-loop system corresponding to *some* feedback law. This can be translated into a partial differential relation sometimes called “Artstein’s (in)equation” [42]. There is a definite parallel between a CLF for stabilization, solution of this differential inequation on the one hand, and the value function of an optimal control problem for the system, solution of a HJB equation on the other hand. Now, optimal control is a quantitative objective while stabilization is a qualitative objective; it is not surprising that Artstein (in)equation is very under-determined and has many more solutions than HJB equation, and that it may (although not always) even have smooth ones.

We have, in the team, a longstanding research record on the topic of construction of CLFs and stabilizing feedback controls.

3.4. Optimal Transport

We believe that matching optimal transport with geometric control theory is one originality of our team. We expect interactions in both ways.

The study of optimal mass transport problems in the Euclidean or Riemannian setting has a long history which goes from the pioneer works of Monge [72] and Kantorovitch [65] to the recent revival initiated by fundamental contributions due to Brenier [52] and McCann [71].

The same transportation problems in the presence of differential constraints on the set of paths —like being an admissible trajectory for a control system— is quite new. The first contributors were Ambrosio and Rigot [40] who proved the existence and uniqueness of an optimal transport map for the Monge problem associated with the squared canonical sub-Riemannian distance on the Heisenberg groups. This result was extended later by Agrachev and Lee [37], then by Figalli and Rifford [56] who showed that the Ambrosio-Rigot theorem holds indeed true on many sub-Riemannian manifolds satisfying reasonable assumptions. The problem of existence and uniqueness of an optimal transport map for the squared sub-Riemannian distance on a general complete sub-Riemannian manifold remains open; it is strictly related to the regularity of the sub-Riemannian distance in the product space, and remains a formidable challenge. Generalized notions of Ricci curvatures (bounded from below) in metric spaces have been developed recently by Lott and Villani [69] and Sturm [80]. A pioneer work by Juillet [62] captured the right notion of curvature for subriemannian metric in the Heisenberg group; Agrachev and Lee [38] have elaborated on this work to define new notions of curvatures in three dimensional sub-Riemannian structures. The optimal transport approach happened to be very fruitful in this context. Many things remain to be done in a more general context.

3.5. Small controls and conservative systems, averaging

Using averaging techniques to study small perturbations of integrable Hamiltonian systems dates back to H. Poincaré or earlier; it gives an approximation of the (slow) evolution of quantities that are preserved in the non-perturbed system. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit; see for instance [41], [79].

When the “perturbation” is a control, these techniques may be used after deciding how the control will depend on time and state and other quantities, for instance it may be used after applying the Pontryagin Maximum Principle as in [44], [45], [53], [60]. Without deciding the control a priori, an “average control system” may be defined as in [43].

The focus is then on studying into details this simpler “averaged” problem, that can often be described by a Riemannian metric for quadratic costs or by a Finsler metric for costs like minimum time.

This line of research stemmed out of applications to space engineering, see section 4.1.

4. Application Domains

4.1. Space engineering, satellites, low thrust control

Space engineering is very demanding in terms of safe and high-performance control laws (for instance optimal in terms of fuel consumption, because only a finite amount of fuel is onboard a satellite for all its “life”). It is therefore prone to real industrial collaborations.

We are especially interested in trajectory control of space vehicles using their own propulsion devices, outside the atmosphere. Here we discuss “non-local” control problems (in the sense of section 3.1 point 1): orbit transfer rather than station keeping; also we do not discuss attitude control.

In the geocentric case, a space vehicle is subject to

- gravitational forces, from one or more central bodies (the corresponding acceleration is denoted by F_{grav} below),
- a thrust, the control, produced by a propelling device; it is the $G u$ term below; assume for simplicity that control in all directions is allowed, *i.e.* G is an invertible matrix
- other “perturbating” forces (the corresponding acceleration is denoted by F_2 below; in simplified models, it is not present). In position-velocity coordinates, its dynamics can be written as

$$\ddot{x} = F_{\text{grav}}(x, t) + F_2(x, \dot{x}, t) + G(x, \dot{x}) u, \quad \|u\| \leq u_{\text{max}}. \quad (1)$$

In the case of a single attracting central body (the earth) and in a geocentric frame, F_{grav} does not depend on time, or consists of a main term that does not depend on time and smaller terms reflecting the action of the moon or the sun, that depend on time. The second term is often neglected in the design of the control at first sight; it contains terms like atmospheric drag or solar pressure. G could also bear an explicit dependence on time (here we omit the variation of the mass, that decreases proportionally to $\|u\|$).

4.1.1. Low thrust

Low thrust means that u_{max} is small, or more precisely that the maximum magnitude of $G u$ is small with respect to the one of F_{grav} . (but in general not compared to F_2). Hence the influence of the control is very weak instantaneously, and trajectories can only be significantly modified by accumulating the effect of this low thrust on a long time. Obviously this is possible only because the free system is somehow conservative. This was “abstracted” in section 3.5.

Why low thrust ? The common principle to all propulsion devices is to eject particles, with some relative speed with respect to the vehicle; conservation of momentum then induces, from the point of view of the vehicle alone, an external force, the “thrust” (and a mass decrease). Ejecting the same mass of particles with a higher relative speed results in a proportionally higher thrust; this relative speed (specific impulse, I_{sp}) is a characteristic of the engine; the higher the I_{sp} , the smaller the mass of particles needed for the same change in the vehicle momentum. Engines with a higher I_{sp} are highly desirable because, for the same maneuvers, they reduce the mass of “fuel” to be taken on-board the satellite, hence leaving more room (mass) for the payload. “Classical” chemical engines use combustion to eject particles, at a somehow limited speed even with very efficient fuel; the more recent electric engines use a magnetic field to accelerate particles and eject them at a considerably higher speed; however electrical power is limited (solar cells), and only a small amount of particles can be accelerated per unit of time, inducing the limitation on thrust magnitude.

Electric engines theoretically allow many more maneuvers with the same amount of particles, with the drawback that the instant force is very small; sophisticated control design is necessary to circumvent this drawback. High thrust engines allow simpler control procedures because they almost allow instant maneuvers (strategies consist in a few burns at precise instants).

4.1.2. Typical problems

Let us mention two.

- *Orbit transfer or rendez-vous.* It is the classical problem of bringing a satellite to its operating position from the orbit where it is delivered by the launcher; for instance from a GTO orbit to the geostationary orbit at a prescribed longitude (one says rendez-vous when the longitude, or the position on the orbit, is prescribed, and transfer if it is free). In equation (1) for the dynamics, F_{grav} is the Newtonian gravitation force of the earth (it then does not depend on time); F_2 contains all the terms coming either from the perturbations to the Newtonian potential or from external forces like radiation pressure, and the control is usually allowed in all directions, or with some restrictions to be made precise.
- *Three body problem.* This is about missions in the solar system leaving the region where the attraction of the earth, or another single body, is preponderant. We are then no longer in the situation of a single central body, F_{grav} contains the attraction of different planets and the sun. In regions where two central bodies have an influence, say the earth and the moon, or the sun and a planet, the term F_{grav} in (1) is the one of the restricted three body problem and dependence on time reflects the movement of the two “big” attracting bodies.

An issue for future experimental missions in the solar system is interplanetary flight planning with gravitational assistance. Tackling this global problem, that even contains some combinatorial problems (itinerary), goes beyond the methodology developed here, but the above considerations are a brick in this puzzle.

4.1.3. Properties of the control system.

If there are no restrictions on the thrust direction, i.e., in equation (1), if the control u has dimension 3 with an invertible matrix G , then the control system is “static feedback linearizable”, and a fortiori flat, see section 3.2. However, implementing the static feedback transformation would consist in using the control to “cancel” the gravitation; this is obviously impossible since the available thrust is very small. As mentioned in section 3.1, point 3, the problem remains fully nonlinear in spite of this “linearizable” structure ².

4.2. Quantum Control

These applications started by a collaboration between B. Bonnard and D. Sugny (a physicist from ICB) in the ANR project Comoc (now ended). The problem was the control of the orientation of a molecule

²However, the linear approximation around any feasible trajectory is controllable (a periodic time-varying linear system); optimal control problems will have no singular or abnormal trajectories.

using a laser field, with a model that does take into account the dissipation due to the interaction with the environment, molecular collisions for instance. The model is a dissipative generalization of the finite dimensional Schrödinger equation, known as Lindblad equation. It is a 3-dimensional system depending upon 3 parameters, yielding a very complicated optimal control problem that we have solved for prescribed boundary conditions. In particular we have computed the minimum time control and the minimum energy control for the orientation of a two-level system, using geometric optimal control and appropriate numerical methods (shooting and numerical continuation) [49], [48].

More recently, based on this project, we have reoriented our control activity towards Nuclear Magnetic Resonance (MNR). In MNR medical imaging, the contrast problem is the one of designing a variation of the magnetic field with respect to time that maximizes the difference, on the resulting image, between two different chemical species; this is the “contrast”. This research is conducted with Prof. S. Glaser (TU-München), whose group is performing both *in vivo* and *in vitro* experiments; experiments using our techniques have successfully measured the improvement in contrast between materials chemical species that have an importance in medicine, like oxygenated and de-oxygenated blood, see [47]; this is however still to be investigated and improved. The model is the Bloch equation for spin $\frac{1}{2}$ particles, that can be interpreted as a sub-case of Lindblad equation for a two-level system; the control problem to solve amounts to driving in minimum time the magnetization vector of the spin to zero (for parameters of the system corresponding to one of the species), and generalizations where such spin $\frac{1}{2}$ particles are coupled: double spin inversion for instance.

A reference book by B. Bonnard and D. Sugny has been published on the topic [50].

4.3. Swimming at low-Reynolds number

The study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. This feature, known as the scallop theorem in that context needs to be circumvented when one wants to swim with strokes that produce a net motion of the swimmer. In this regime, it turns out that the dynamic of a micro-swimmer could be expressed as an ordinary differential equation. First of all, by stating that the swimmer controls its own shape, we focus on finding the best strategy to swim (by minimizing a time or an energy). Moreover, we work on the control and optimal control of magnetic micro-swimmers. The latter micro-device is charged in order to be deformed by an external magnetic field. In this case, the control functions are the external magnetic field. And we wonder whether it is possible to control the position of the swimmer by acting on this external magnetic field. We are also interested in the associated optimal control problem (acting on the magnetic field in such a way that the swimmer reaches a desired position as soon as possible).

4.4. Applications of optimal transport

Optimal Transportation in general has many applications. Image processing, biology, fluid mechanics, mathematical physics, game theory, traffic planning, financial mathematics, economics are among the most popular fields of application of the general theory of optimal transport. Many developments have been made in all these fields recently. Three more specific examples:

- In image processing, since a grey-scale image may be viewed as a measure, optimal transportation has been used because it gives a distance between measures corresponding to the optimal cost of moving densities from one to the other, see e.g. the work of J.-M. Morel and co-workers [73].
- In representation and approximation of geometric shapes, say by point-cloud sampling, it is also interesting to associate a measure, rather than just a geometric locus, to a distribution of points (this gives a small importance to exceptional “outlier” mistaken points); this was developed in Q. Mérigot’s PhD [74] in the GEOMETRICA project-team. The relevant distance between measures is again the one coming from optimal transportation.

- The specific to the type of costs that we have considered in some mathematical work, i.e. these coming from optimal control, are concerned with evolutions of densities under state or velocity constraints. A fluid motion or a crowd movement can be seen as the evolution of a density in a given space. If constraints are given on the directions in which these densities can evolve, we are in the framework of non-holonomic transport problems.

4.5. Applications to some domains of mathematics.

Control theory (in particular thinking in terms of inputs and reachable set) has brought novel ideas and progresses to mathematics. For instance, some problems from classical calculus of variations have been revisited in terms of optimal control and Pontryagin's Maximum Principle [63]; also, closed geodesics for perturbed Riemannian metrics were constructed in [66], [67] using control techniques.

Inside McTAO, a work like [58], [57] is definitely in this line, applying techniques from control to construct some perturbations under constraints of Hamiltonian systems to solve longstanding open questions in the field of dynamical systems.

5. New Software and Platforms

5.1. Hampath

KEYWORDS: Geometric control - Second order conditions - Differential homotopy - Ordinary differential equations

FUNCTIONAL DESCRIPTION

Hampath is an open-source software developed to solve optimal control problems by coupling shooting and homotopy methods. More generally, it can be used to solve general Hamiltonian boundary value problems. It also implements an efficient computation of Jacobi fields (allowing in particular second order optimality condition checks) based on the repeated use of automatic differentiation.

- Participants: Jean-Baptiste Caillaud, Olivier Cots and Joseph Gergaud
- Contact: Oliver Cots
- URL: <http://www.hampath.org>

6. New Results

6.1. Advances in optimal control

6.1.1. Algebraic and geometric techniques in medical resonance imaging

Participants: Bernard Bonnard, Jean-Charles Faugère [EPI PolSys], Alain Jacquemard [Univ. de Bourgogne], Mohab Safey El Din [EPI PolSys], Thibaut Verron [EPI PolSys].

In the framework of the ANR-DFG project Explosys (see Section 8.3) we use computer algebra methods to analyze the controlled Bloch equations, modeling the contrast problem in MRI. The problem boils down to analyzing the so called singular extremals associated to the problem. Thanks to the linear dependence of the problem with respect to the state variables and the relaxation parameters the problem is algebraic and is equivalent to determining equilibrium points and eigenvalues of the linearized system at such points together with the algebraic classification of the surface associated to the switches between bang and singular arcs. Preliminary results are described in ISSAC paper [12] using Grobner basis and stratifications of singularities of determinantal varieties. This work was a part of T. Verron's PhD and is continuing in particular with him (Post doc APO-ENSEEIH).

6.1.2. *Local minima, second order conditions*

Participants: Jean-Baptiste Caillau, Zheng Chen, Yacine Chitour [Univ. Paris-Sud], Ariadna Farrés [Univ. Barcelona].

It is well known that the PMP gives necessary conditions for optimality, but curves satisfying this condition may be local minima or critical saddle points. Roughly speaking, the PMP is a first order condition. Higher order conditions give finer necessary conditions (and sufficient in some special cases), but they require differentiability that is not always satisfied when commutations occur. Furthermore, these local conditions cannot distinguish local from global minima. In [4] and [19], we make contributions respectively to extending higher order conditions to non-smooth cases and to exploring local and global minima on an example of interest.

Second order systems whose drift is defined by the gradient of a given potential are considered, and minimization of the L^1 -norm of the control is addressed in [4]. An analysis of the extremal flow emphasizes the role of singular trajectories of order two [78], [81]; the case of the two-body potential is treated in detail. In L^1 -minimization, regular extremals are associated with bang-bang controls (saturated constraint on the norm); in order to assess their optimality properties, sufficient conditions are given for broken extremals and related to the no-fold conditions of [75]. Two examples of numerical verification of these conditions are proposed on a problem coming from space mechanics.

In another direction, we have been studying the structure of local minima for time minimization in the controlled three-body problem. In [19], several homotopies are systematically used to unfold the structure of these local minimizers, and the resulting singularity of the path associated with the value function is analyzed numerically.

6.1.3. *Solving chance-constrained optimal control problems in aerospace engineering via Kernel Density Estimation*

Participants: Jean-Baptiste Caillau, Max Cerf [Airbus Industries], Achille Sassi, Emmanuel Trélat [Univ. P. & M. Curie], Hasna Zidani [ENSTA ParisTech].

The goal of [30] is to show how non-parametric statistics can be used to solve chance-constrained optimization and optimal control problems by reformulating them into deterministic ones, focusing on the details of the algorithmic approach. We use the Kernel Density Estimation method to approximate the probability density function of a random variable with unknown distribution, from a relatively small sample. In the paper it is shown how this technique can be applied to a class of chance-constrained optimization problem, focusing on the implementation of the method. In particular, in our examples we analyze a chance-constrained version of the well known problem in aerospace optimal control: the Goddard problem.

6.2. *Averaging and filtering for optimal control in Space mechanics*

Participants: Jean-Baptiste Caillau, Thierry Dargent, Florentina Nicolau, Jean-Baptiste Pomet, Jérémy Rouot.

Investigating averaging in optimal control for space mechanics with low thrust, or more generally with conservative systems with “small” controls is an ongoing subject in the team. It is also central in the research contract with CNES mentioned in Section 7.1.

6.2.1. *Convergence properties of the Maximum principle*

Part of Jérémy Rouot’s PhD [2] was devoted to convergence properties in the Hamiltonian system resulting from Pontryagin’s Maximum principle when the small parameter representing the ratio between slow and fast velocities tends to zero. The difference with previous work is that we give a clear method to sort fast and slow variables in the adjoint variables, and we provide convergence of these under some conditions. A more complete publication is under preparation.

6.2.2. Approximation by filtering in optimal control and applications

Minimum time control of slow-fast systems is considered. In the case of only one fast angle, averaging techniques are available for such systems. The approach introduced in [54] and [43] is recalled, then extended to time dependent systems by means of a suitable filtering operator. The process relies upon approximating the dynamics by means of sliding windows. The size of these windows is an additional parameter that provides intermediate approximations between averaging over the whole fast angle period and the original dynamics. The method was applied to problems coming from space mechanics, and is exposed in [31].

6.2.3. Averaging with reconstruction of the fast variable

We have been studying a way to modify the initial condition of the average equation in order to approach better (but in the mean) the slow variable while reconstructing asymptotically the fast variable. This follows an idea that was shown to work numerically in [54].

In [32], we give a construction for Cauchy problems. It is lighter than second order averaging, in that oscillating signals and ODEs are not used, and still provides a second order error *in the mean*, together with convergence of the fast variable. This remains to be developed for two-point boundary value problems like in optimal control.

6.3. Fully controlled slender microswimmers

6.3.1. The N -link micro-swimmer

Participants: François Alouges [École Polytechnique], Antonio Desimone [SISSA Trieste], Laetitia Giraldi, Marta Zopello [Univ. di Padova].

We discussed a reduced model to compute the motion of slender swimmers which propel themselves by changing the curvature of their body. Our approach is based on the use of Resistive Force Theory for the evaluation of the viscous forces and torques exerted by the surrounding fluid, and on discretizing the kinematics of the swimmer by representing its body through an articulated chain of N rigid links capable of planar deformations. The resulting system of ODEs, governing the motion of the swimmer, is easy to assemble and to solve, making our reduced model a valuable tool in the design and optimization of bio-inspired artificial microdevices. We prove that the swimmer composed by almost 3 segments is controllable in the whole plane. As a direct result, there exists an optimal swimming strategy to reach a desired configuration in minimum time. Numerical experiments for in the case of the Purcell swimmer suggest that the optimal strategy is periodic, namely a sequence of identical strokes. Our results indicate that this candidate for an optimal stroke, indeed gives a better displacement speed than the classical Purcell stroke.

This is presented in [36] (accepted as a Book chapter in Multi-scale Models in Mechano and Tumor Biology: Modeling, Homogenization and Applications, Lecture Notes in Computational Science and Engineering, June 2016).

6.3.2. Optimal periodic strokes for the Copepod and Purcell micro-swimmers

Participants: Piernicola Bettioli [Uni. Bretagne Ouest], Bernard Bonnard, Alice Nolot, Jérémy Rouot.

We have analyzed the problem of optimizing the efficiency of the displacement of two micro swimmers with slender links, namely the following two models: the symmetric micro swimmer introduced by Takagi (see [29]); this is a model to describe the locomotion of the micro crustaceans named copepod, and the historical three link Purcell swimmer. The problems are studied in the framework of optimal control theory and SR geometry vs the standard curvature control point of view. Our contributions are to determine the optimal solutions combining geometric analysis and adapted numerical scheme. In particular the nilpotent models introduced in SR geometry allow to make a neat analysis of the problem of determining optimal strokes with small amplitudes and numerical continuation methods are then applied to compute more general stroke. This approach is completely original in optimal control. Also necessary and sufficient optimality conditions are applied to select the topology of optimal strokes (simple loops) and to determine the optimal solution in both cases. For the references see [17] and [27]. Also note that in collaboration with D. Takagi and M. Chyba

this approach is currently at the experimental level at the university of Hawaii using a robot micro swimmer mimicking a copepod, see above. More theoretical issues in relation with SR geometry are investigated in the framework of A. Nolot's starting PhD (started August, 2016). Other publication relating these advances are [25], [26], [11].

6.4. Modelization and Controllability of “Magneto-elastic” Micro-swimmers

Participants: François Alouges [École Polytechnique], Antonio Desimone [SISSA Trieste], Laetitia Giraldi, Pierre Lissy [Univ. Paris Dauphine], Clément Moreau [ENS Cachan and York University], Jean-Baptiste Pomet, Marta Zopello [Univ. di Padova].

It is not realistic for *artificial* micro-swimmers built as micro-robots, to have an actuator at each joint. A possibility is as follows: each link of the swimmer bears is magnetized and the movement is controlled via an exterior magnetic field. These models also bear an internal elastic force, that can be modelled as a torsional spring at each joint and tends to asymptotically restore the straight shape in the absence of other forces.

Control strategies for these models have been proved successful numerically. It can also be proved mathematically via an asymptotic analysis that it is possible to steer the swimmer along a chosen direction with some well chosen oscillating magnetic field, provided some obstruction, like symmetries, are avoided. This is exposed in [23] for a Purcell magnetic swimmer (3 links).

For the smallest magneto-elastic micro-swimmer (2 links), we have been able to prove a strong local controllability result (weaker than STLC) around the straight position of the swimmer, again except for values of the parameters that correspond to symmetries preventing controllability. This is exposed in [8], and a note is under preparation, that shows that STLC is indeed *not* satisfied. This analysis is difficult because the straight position corresponds to the equilibria but is very degenerate from the control point of view.

To avoid this degeneracy, a possibility is to “twist” one of the torsional springs so that the equilibria no longer occur for a straight shape. This is exposed in [34] for a 3-link magnetic microswimmer (local controllability has not been proved for this system without the twist). A local partial controllability result around the equilibrium is proved in that case and a constructive method to find the magnetic field that allows the swimmer to move along a prescribed trajectory is described.

6.5. Sub-Riemannian Geometry and Optimal Transport

Participants: Zeinab Badreddine, André Belotto Da Silva [University of Toronto], Ludovic Rifford.

We have studied the Sard Conjecture and its link with the problem of existence and uniqueness of an optimal transport map for a cost given by the square of a sub-Riemannian distance. Given a totally non-holonomic distribution on a smooth manifold, the Sard Conjecture is concerned with the size of the set of points that can be reached by singular horizontal paths starting from a same point. In the setting of rank-two distributions in dimension three, the Sard conjecture states that that set should be a subset of the so-called Martinet surface of 2-dimensional Hausdorff measure zero. In [24], A. Belotto da Silva and L. Rifford proved that the conjecture holds in the case where the Martinet surface is smooth. Moreover, they address the case of singular real-analytic Martinet surfaces and show that the result holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. The methods rely on the control of the divergence of vector fields generating the trace of the distribution on the Martinet surface and some techniques of resolution of singularities. In a work in progress, the control on the divergence of this "generating" vector field is the key ingredient used by Z. Badreddine to obtain results of existence and uniqueness of optimal transport map for rank-two distribution in dimension four.

6.6. Geometric Control and Dynamics

Participants: Ayadi Lazrag, Ludovic Rifford, Rafael Ruggiero [PUC-RIO].

Following [77], [57] and [58], we apply techniques from geometric control to the study of perturbations of Hamiltonian flows. In [9], we prove a uniform Franks' lemma at second order for geodesic flows and apply the result in persistence theory.

7. Bilateral Contracts and Grants with Industry

7.1. CNES - Inria - UB Contract

Contract number: 130777/00. Call Number: R-S13/BS-005-012

"Perturbations and averaging for low thrust" (Poussée faible et moyennation).

Research contract between CNES and McTAO (both the Inria and the Université de Bourgogne parts). It runs for the period 2014-2017. It concerns averaging techniques in orbit transfers around the earth while taking into account many perturbations of the main force (gravity for the earth considered as circular). The objective is to validate numerically and theoretically the approximations made by using averaging, and to propose methods that refine the approximation. It has co-funded the PhD thesis of Jeremy Rouot [2] (also co-funded by Région PACA) and fully funded the postdoc of Florentina Nicolau [32], [31].

8. Partnerships and Cooperations

8.1. Regional Initiatives

The PhD thesis of Jeremy Rouot [2] has been co-funded by Région PACA.

8.2. National Initiatives

8.2.1. ANR

Weak KAM beyond Hamilton-Jacobi (WKBHJ). Started 2013 (decision ANR-12-BS01-0020 of December 19, 2012), duration: 4 years. L. Rifford is in the scientific committee.

Sub-Riemannian Geometry and Interactions (SRGI). Started 2015 (decision ANR-15-CE40-0018), duration: 4 years. L. Rifford is a member.

Intérazions Systèmes Dynamiques Équations d'Évolution et Contrôle (ISDEEC). Started 2016 (decision ANR-16-CE40-0013), duration: 4 years. L. Rifford is a member.

8.2.2. Others

The McTAO team participates in the **GdR MOA**, a CNRS network on Mathematics of Optimization and Applications.

PEPS project of **AMIES** Labex, "Dealing with exclusion constraints in orbital transfer" with Thalès Alenia Space (PI J.-B. Caillau). This project funded two master internships during summer 2016 (M. Brunengo and Y. El Alaoui Faris, co-supervised with T. Dargent from Thalès).

PGMO grant (2016-2017) on "Metric approximation of minimizing trajectories and applications" (PI J.-B. Caillau). This project involves colleagues from Université Paris Dauphine and has funding for one year, including one internship (M2 level).

J.-B. Caillau is associate researcher of the team **Optimization & Control** at ENSTA-Paristech and of the CNRS team **Parallel Algorithms & Optimization team** at ENSEEIHT, Univ. Toulouse.

8.3. European Initiatives

8.3.1. Collaborations in European Programs, other than FP7 & H2020

8.3.1.1. Bilateral program with Portugal

Program: FCT (Fundação para a Ciência e a Tecnologia)

Grant no. : PTDC/MAT-CAL/4334/2014

Project title: "Extremal spectral quantities and related problems"

Duration: 05/2016-05/2019

Coordinator: P. Freitas (Univ. Lisbon)

Team member involved: J.-B. Caillau

Other partners: Univ. Lisbon, Univ. Luxembourg, Czech Nuclear Physics Institute, Univ. Bern

Link: <https://team.inria.fr/mctao/fct-project-extremal-spectral-quantities-and-related-problems-2016-2019>

8.3.1.2. Bilateral program with Germany

Program: Projets de recherche collaborative-internationale ANR-DFG (Germany)

Grant no. : ANR-14-CE35-0013-01; DFG-GI 203/9-1

Project title: "Exploring the physical limits of spin systems (Explosys)."

Duration: 11/2014-10/2018

Coordinator: D. Sugny (Univ. de Bourgogne) for France, Glaser (TU München) for Germany.

Team member involved: Bernard Bonnard is in the (scientific committee).

Other partners: TU München, Univ. de Bourgogne (IMB and UCB).

This project involves specialists in physics and control theory in order to make important progresses in the use of spin dynamics, in particular for Magnetic Resonance Medical Imaging.

Link: <http://www.agence-nationale-recherche.fr/fileadmin/aap/2014/selection/pa-2014-selection-franco-allemand-dfg.pdf>

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

The cut locus: A bridge over differential geometry, optimal control and transport, Bangkok, August 2016 (B. Bonnard, J.-B. Caillau, K. Kondo, L. Rifford, M. Tanaka). The conference was organized with the support of the Thai KMITL University and gathered 30 people mostly from Japan, Thailand and France.

9.1.1.2. Member of the Organizing Committees

Séminaire de géométrie hamiltonienne, Paris 6 (J.-B. Caillau). Bi-mensual seminar.

Journée McTAO, Inria Sophia, January 2016. One day event organized by the team with four invited speakers.

10th International Young Researcher Workshop on Geometry, Mechanics and Control, Paris (IHP), January 2016 (J.-B. Caillau). The workshop gathered 50 people for conferences and mini-courses.

Journées SMAI-MODE, Toulouse, March 2016 (J.-B. Caillau). The conference gathered 140 researchers, and was coupled with a series of two mini-courses co-organized with **GdR MOA**.

Journée MokaTAO, Inria Paris, October 2016. Two-day event co-organized by Mokaplan (Inria Paris) and McTAO teams, with talks by members of these teams. The two teams share common interests in optimization and optimal transportation.

Groupe de travail "Optimisation et applications", Inria Sophia Antipolis, November 2016 (J.-B. Caillau). One day event with four invited speakers.

9.1.2. Books

9.1.2.1. Springer briefs

B. Bonnard and J. Rouot, together with M. Chyba, have written the series of notes [28], submitted as Springer briefs Publications. They were the basis of courses at the Phd level given at the University of Burgundy and at the institute of Mathematics for industry at Fukuoka (Japan)

9.1.2.2. Springer Maths and Industry

B. Bonnard, together with M. Chyba, served as an editor of the volume [18], which gather contributions on the subject by specialists of both academics and space agencies.

9.1.2.3. Radon Series on Comput. and Applied Math.

J.-B. Caillau, together with M. Bergounioux, G. Peyré, C. Schnörr and T. Haberkorn, served as an editor for the volume [21]. With a focus on the interplay between mathematics and applications of imaging, the first part covers topics from optimization, inverse problems and shape spaces to computer vision and computational anatomy. The second part is geared towards geometric control and related topics, including Riemannian geometry, celestial mechanics and quantum control.

9.1.3. Journals

9.1.3.1. Member of the Editorial Boards

L. Rifford has been a member of the Editorial Board of the journal "Discrete and Continuous Dynamical Systems - A" since 2014.

9.1.3.2. Reviewer - Reviewing Activities

The team members have reviewed articles in 2016 for the following journals: SIAM J. Control & Optim., ESAIM Control Optim. and Calc. Var., Automatica, J. Dyn. Control Syst., J. Optim. Theory Appl., J. Math. Pures Appl., Inventiones Mathematicae, Mathematische Zeitschrift, Advances in Differential Equations, Memoirs of the American Mathematical Society, Annales Scientifiques de l'ENS, Nonlinear Analysis, Communications in Mathematical Physics

9.1.4. Invited Talks

J.-B. Caillau

03/2016: *Séminaire Géométrie et dynamique*, Nice

04/2016: *Séminaire de Géométrie hamiltonienne*, Paris

05/2016: *Emerging Trends in Applied Mathematics and Mechanics*, Perpignan

06/2016: *Alicante-Limoges-Elche Meeting on Optimization*, Cartagena

08/2016: *The cut locus: A bridge over differential geometry, optimal control and transport*, Bangkok

09/2016: *Séminaire Astrogéo*, Observatoire de la côte d'azur, Sophia

L. Giraldi

11/2016: *Controllability and hysteresis*, Trento, Italie

L. Rifford

02/2016: *Rencontre d'Analyse Mathématique et ses Applications*, Ouargla (Algeria)

02/2016: *Séminaire de Calculs des Variations et EDP*, Aix-Marseille University

03/2016: *CIMPA Research School "Géométrie et Analyse"*, Abidjan (Ivory Coast)

03/2016: *Séminaire Bourbaki*, Institut Henri Poincaré, Paris

04/2016: *Colloquium de l'Institut de Mathématiques*, University of Neuchâtel (Switzerland)
 04/2016: *International Conference of the GE2MI*, Hammamet (Tunisia)
 05/2016: *Colloquium du Département de Mathématiques*, University of Orsay
 05/2016: *Geometric control and sub-Riemannian geometry*, Course given at the University of Isfahan (Iran)
 06/2016: *Analysis, Geometry, and Optimal Transport*, KIAS, Seoul (South Korea)
 08/2016: *The cut locus*, KIMTL, Bangkok (Thailand)
 11/2016: *Dynamical Systems Seminar*, ETH Zurich
 11/2016: *Dynamics and Geometry Seminar*, University Nice Sophia Antipolis
 12/2016: *Seminar of Hamiltonian Geometry*, Paris VI
 12/2016: *2016 Analysis School in Benin*, IMPS, Benin

9.1.5. Leadership within the Scientific Community

J.-B. Caillau has been head of the **SMAI-MODE** group (2014-2016), the group on optimization of the French Society for Applied and Industrial Mathematics.

L. Rifford has been executive director of CIMPA since September 2016.

9.1.6. Scientific Expertise

J.-B. Caillau is member of the scientific committees of the “**Institut de Mécanique Céleste de Calcul des Éphémérides**” and of the **GdR Calcul**, and corresponding member in Dijon for the Labex **AMIES**.

J.-B. Pomet is a member of the steering committee of “C4PO”, a structuring project of the IDEX UCA^{JEDI}.

9.1.7. Research Administration

J.-B. Caillau is the joint head of the CNRS team **Statistique, Probabilités, Optimisation & Contrôle** at the Math. Institute of Univ. Bourgogne & Franche-Comté.

J.-B. Pomet has been an elected member of the Inria Evaluation Committee (commission d'évaluation) since 2014.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

J.-B. Caillau has managed L1 math and the master of applied mathematics (M2 **MIGS**) of Univ. Bourgogne Franche-Comté (2014-2016). He has been a designated member of the Conseil pédagogique du Département de Mathématiques at UBFC (2014-2016), and of the Conseil de l'UFR Mathématiques & Informatique at Univ. Paris I (2014-2016). His teaching duties in 2016 include:

Licence : cours d'analyse (analysis), 100 H équivalent TD, niveau L1, Univ. Bourgogne Franche-Comté, France

Master : approximation géométrique (geometric approximation), 50 H équivalent TD, niveau M1, Univ. Bourgogne Franche-Comté, France

Master : optimisation, 50 H équivalent TD, niveau M2, Univ. Bourgogne Franche-Comté, France

Master : contrôle optimal (optimal control), 20 H équivalent TD, niveau M2, ENSTA-Paristech, France

L. Giraldi is responsible of the following courses:

Licence : “*colles de mathématiques*”, *MPSI and MP*, 4 H équivalent TD/week, L1 L2, Lycée International de Valbonne,

Licence : numerical analysis, 20 H équivalent TD, L3, Polytech Nice Sophia,

Master : numerical analysis project, 30 H équivalent TD, M1, Polytech Nice Sophia.

9.2.2. Supervision

PhD: Zheng Chen, L^1 -minimization in space mechanics, Univ. Paris Saclay, defended September, 2016, J.-B. Caillau (co-supervised with Y. Chitour) [1]

PhD: Jérémy Rouot, , Univ. Nice Sophia Antipolis, Geometric and numerical methods in optimal control and application to orbit transfer and swimming at low Reynolds number. Defended November, 2016, co-supervised by B. Bonnard and J.-B. Pomet. [2]

PhD in progress: Achille Sassi, to be defended 01/2017, Numerical methods for hybrid control and chance-constrained optimization problems, Univ. Paris Saclay, J.-B. Caillau (co-supervised with M. Cerf, E. Trélat and H. Zidani)

PhD in progress: Michaël Orioux, started 10/2015, Dynamical systems and optimal control, Univ. Paris Dauphine, J.-B. Caillau (co-supervised with J. Féjóz)

PhD in progress: Zeinab Badreddine, started 09/2014, Sub-Riemannian Geometry and Optimal Transport, co-supervised by L. Rifford and B. Bonnard.

PhD in progress: Sébastien Fueyo, started 09/2016, Testing stability of nonlinear amplifier by frequency-domain methods. J.-B. Pomet (co-supervised with L. Baratchart).

PhD in progress: Alice Nolot, started 09/2016, Sub-Riemannian geometry and optimal swimming at low Reynolds number. B. Bonnard.

9.2.3. Juries

In 2016, J.-B. Caillau referee for Jiamin Zhu (Univ. Paris 6) and Maxime Chupin (Univ. Paris 6) PhD theses, for the HDR of Aude Rondepierre (Univ. Toulouse), and jury member for the PhD thesis of Clément Royer (Univ. Toulouse).

9.3. Popularization

Conference for high school students, "Ne votez pas, jugez !" (J.-B. Caillau), **semaine des maths 2016**, Lycée Charles de Gaulle (Dijon) and **MASTIC** initiative (Inria Sophia Antipolis).

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

- [1] Z. CHEN. *L^1 -Minimization for Space Mechanics*, Université Paris-Saclay, September 2016, <https://tel.archives-ouvertes.fr/tel-01397008>
- [2] J. ROUOT. *Geometric and numerical methods in optimal control and application to orbit transfer and swimming at low Reynolds number*, Université Nice Sophia Antipolis, November 2016, <https://hal.inria.fr/tel-01402516>

Articles in International Peer-Reviewed Journals

- [3] L. BARBET, M. DAMBRINE, A. DANIILIDIS, L. RIFFORD. *Sard theorems for Lipschitz functions and applications in optimization*, in "Israel Journal of Mathematics", 2016, vol. 212, n^o 2, pp. 757-790 [DOI : 10.1007/s11856-016-1308-7], <https://hal.archives-ouvertes.fr/hal-01336328>
- [4] Z. CHEN, J.-B. CAILLAU, Y. CHITOUR. *L^1 -minimization for mechanical systems*, in "SIAM Journal on Control and Optimization", May 2016, vol. 54, n^o 3, pp. 1245-1265 [DOI : 10.1137/15M1013274], <https://hal.archives-ouvertes.fr/hal-01136676>

- [5] Z. CHEN. L^1 -optimality conditions for the circular restricted three-body problem, in "Celestial Mechanics and Dynamical Astronomy", June 2016, vol. 126, n^o 4, pp. 461-481 [DOI : 10.1007/s10569-016-9703-2], <https://hal.archives-ouvertes.fr/hal-01402647>
- [6] Z. CHEN. *Neighboring Optimal Control for Fixed-Time Multi-Burn Orbital Transfers*, in "Aerospace Science and Technology", December 2016 [DOI : 10.1016/J.AST.2016.11.021], <https://hal.archives-ouvertes.fr/hal-01402650>
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- [10] R. MCCANN, L. RIFFORD. *The intrinsic dynamics of optimal transport*, in "Journal de l'École polytechnique — Mathématiques", 2016, vol. 3, pp. 67-98 [DOI : 10.5802/JEP.29], <https://hal.archives-ouvertes.fr/hal-01336327>

International Conferences with Proceedings

- [11] B. BONNARD, M. CHYBA, J. ROUOT, D. TAKAGI. *A Numerical Approach to the Optimal Control and Efficiency of the Copepod Swimmer*, in "55th IEEE Conference on Decision and Control - CDC", Las Vegas, United States, December 2016, <https://hal.inria.fr/hal-01286602>
- [12] B. BONNARD, J.-C. FAUGÈRE, A. JACQUEMARD, M. SAFEY EL DIN, T. VERRON. *Determinantal sets, singularities and application to optimal control in medical imagery*, in "International symposium on symbolic and algebraic computations", Waterloo, Canada, ACM, July 2016, pp. 103-110 [DOI : 10.1145/2930889.2930916], <https://hal.inria.fr/hal-01307073>
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- [14] F. NICOLAU, W. RESPONDEK. *Flatness of two-input control-affine systems linearizable via a two-fold prolongation*, in "55th IEEE Conference on Decision and Control", Las Vegas, NV, United States, December 2016, <https://hal.archives-ouvertes.fr/hal-01351615>

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- [15] J.-B. CAILLAU, M. CERF, A. SASSI, E. TRÉLAT, H. ZIDANI. *Chance constrained optimization of a three-stage launcher*, in "Journées SMAI-MODE 2016", Toulouse, France, March 2016, <https://hal.archives-ouvertes.fr/hal-01408885>

- [16] J.-B. CAILLAU, M. CERF, A. SASSI, E. TRÉLAT, H. ZIDANI. *Kernel Density Estimation applied to the chance-constrained Goddard problem*, in "PGMO Days 2016", Palaiseau, France, November 2016, <https://hal.archives-ouvertes.fr/hal-01408897>

Scientific Books (or Scientific Book chapters)

- [17] P. BETTIOL, B. BONNARD, L. GIRALDI, P. MARTINON, J. ROUOT. *The Purcell Three-link swimmer: some geometric and numerical aspects related to periodic optimal controls*, in "Variational methods in imaging and geometric control", M. BERGOUNIOUX, G. PEYRE, C. SCHNÖRR, J.-B. CAILLAU, T. HABERKORN (editors), Radon Series on Computational and Applied Mathematics, 2016, vol. 18, pp. 328-357, <https://hal.inria.fr/hal-01143763>
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- [21] M. BERGOUNIOUX, J.-B. CAILLAU, T. HABERKORN, G. PEYRE, C. SCHNÖRR (editors). *Variational methods in imaging and geometric control*, Radon Series on Comput. and Applied Math., de Gruyter, December 2016, n° 18, <https://hal.archives-ouvertes.fr/hal-01315508>

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- [22] C. ALDANA, J.-B. CAILLAU, P. FREITAS. *Maximal determinants of Schrödinger operators*, November 2016, working paper or preprint, <https://hal.inria.fr/hal-01406270>
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