IN PARTNERSHIP WITH:
CNRS
Université de Lorraine

Activity Report 2016

Project-Team VEGAS

Effective Geometric Algorithms for Surfaces and Visibility

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Algorithmics, Computer Algebra and Cryptology

Table of contents

Project-Team VEGAS

Creation of the Project-Team: 2005 August 01, end of the Project-Team: 2016 December 31

Keywords:

Computer Science and Digital Science:

5.10.1. - Design
6.2.3. - Probabilistic methods
7.2. - Discrete mathematics, combinatorics
7.5. - Geometry, Topology
7.6. - Computer Algebra

Other Research Topics and Application Domains:

5. - Industry of the future

1. Members

Research Scientists

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Olivier Devillers [Inria, Senior Researcher, HDR]
Guillaume Moroz [Inria, Researcher]
Marc Pouget [Inria, Researcher]
Monique Teillaud [Inria, Senior Researcher, HDR]

Faculty Member

Laurent Dupont [Univ. Lorraine, Associate Professor]

Engineer

Eric Biagioli [Inria, since Nov. 2016]

PhD Students

Sény Diatta [Univ. Lorraine Mobility program since Nov. 2016]
Charles Duménil [Univ. Lorraine, since Oct. 2016]
Iordan Iordanov [Univ. Lorraine]

Post-Doctoral Fellow

Rémi Imbach [Inria, until Oct. 2016]

Visiting Scientist

Gert Vegter [Univ. Lorraine, May 2016]

Administrative Assistants

Laurence Benini [Inria, until Aug. 2016]
Laurence Félicité [Univ. Lorraine, until Mar. 2016]
Hélène Zganic [Inria, since Sept. 2016]
Virginie Priester [CNRS, since Apr. 2016]

Other

Louis Noizet [ENS Paris, Internship, from June 2016 until Aug. 2016]

2. Overall Objectives

2.1. Overall Objectives

The main scientific objective of the VEGAS research team is to *contribute to the development of an effective geometric computing* dedicated to *non-trivial geometric objects*. Included among its main tasks are the study and development of new algorithms for the manipulation of geometric objects, the experimentation of algorithms, the production of high-quality software, and the application of such algorithms and implementations to research domains that deal with a large amount of geometric data, notably solid modeling and computer graphics.

Computational geometry has traditionally treated linear objects like line segments and polygons in the plane, and point sets and polytopes in three-dimensional space, occasionally (and more recently) venturing into the world of non-linear curves such as circles and ellipses. The methodological experience and the know-how accumulated over the last thirty years have been enormous.

For many applications, particularly in the fields of computer graphics and solid modeling, it is necessary to manipulate more general objects such as curves and surfaces given in either implicit or parametric form. Typically such objects are handled by approximating them by simple objects such as triangles. This approach is extremely important and it has been used in almost all of the usable software existing in industry today. It does, however, have some disadvantages. Using a tessellated form in place of its exact geometry may introduce spurious numerical errors (the famous gap between the wing and the body of the aircraft), not to mention that thousands if not hundreds of thousands of triangles could be needed to adequately represent the object. Moreover, the curved objects that we consider are not necessarily everyday three-dimensional objects, but also abstract mathematical objects that are not linear, that may live in high-dimensional space, and whose geometry we do not control. For example, the set of lines in 3D (at the core of visibility issues) that are tangent to three polyhedra span a piecewise ruled quadratic surface, and the lines tangent to a sphere correspond, in projective five-dimensional space, to the intersection of two quadratic hypersurfaces.

Effectiveness is a key word of our research project. By requiring our algorithms to be effective, we imply that the algorithms should be *robust, efficient,* and *versatile*. By robust we mean algorithms that do not crash on degenerate inputs and always output topologically consistent data. By efficient we mean algorithms that run reasonably quickly on realistic data where performance is ascertained both experimentally and theoretically. Finally, by versatile we mean algorithms that work for classes of objects that are general enough to cover realistic situations and that account for the *exact geometry* of the objects, in particular when they are curved.

3. Application Domains

3.1. Computer Graphics

We are interested in the application of our work to virtual prototyping, which refers to the many steps required for the creation of a realistic virtual representation from a CAD/CAM model.

When designing an automobile, detailed physical mockups of the interior are built to study the design and evaluate human factors and ergonomic issues. These hand-made prototypes are costly, time consuming, and difficult to modify. To shorten the design cycle and improve interactivity and reliability, realistic rendering and immersive virtual reality provide an effective alternative. A virtual prototype can replace a physical mockup for the analysis of such design aspects as visibility of instruments and mirrors, reachability and accessibility, and aesthetics and appeal.

Virtual prototyping encompasses most of our work on effective geometric computing. In particular, our work on 3D visibility should have fruitful applications in this domain. As already explained, meshing objects of the scene along the main discontinuities of the visibility function can have a dramatic impact on the realism of the simulations.

3.2. Solid Modeling

Solid modeling, i.e., the computer representation and manipulation of 3D shapes, has historically developed somewhat in parallel to computational geometry. Both communities are concerned with geometric algorithms and deal with many of the same issues. But while the computational geometry community has been mathematically inclined and essentially concerned with linear objects, solid modeling has traditionally had closer ties to industry and has been more concerned with curved surfaces.

Clearly, there is considerable potential for interaction between the two fields. Standing somewhere in the middle, our project has a lot to offer. Among the geometric questions related to solid modeling that are of interest to us, let us mention: the description of geometric shapes, the representation of solids, the conversion between different representations, data structures for graphical rendering of models and robustness of geometric computations.

4. Highlights of the Year

4.1. Highlights of the Year

Inria signed a contract for the integration of ISOTOP within Maple.

The project-team VEGAS will terminate at the end of 2016. A new project-team GAMBLE (Geometric Algorithms and Models Beyond the Linear and Euclidean realm) is currently submitted. It intends to extend computational geometry to non-linear objects, non-Euclidean spaces and probabilistic complexities.

5. New Software and Platforms

5.1. ISOTOP

Topology and geometry of planar algebraic curves
KEYWORDS: Topology - Curve plotting - Geometric computing
Isotop is a Maple software for computing the topology of an algebraic plane curve, that is, for computing an arrangement of polylines isotopic to the input curve. This problem is a necessary key step for computing arrangements of algebraic curves and has also applications for curve plotting.

This software, registered at the APP in June 2011, has been developed since 2007 in collaboration with F. Rouillier from Inria Paris. The distributed version is based on the method described in [3], which presents several improvements over previous methods. In particular, our approach does not require generic position. This version is competitive with other implementations (such as ALCIX and INSULATE developed at MPII Saarbrücken, Germany and TOP developed at Santander Univ., Spain). It performs similarly for small-degree curves and performs significantly better for higher degrees, in particular when the curves are not in generic position.

We are currently working on an improved version integrating a new bivariate polynomial solver based on several of our recent results published in [11]. This version is not yet distributed.

Via the Inria ADT FastTrack funding, Eric Biagioli has joined the project in November 2016 for 6 months. He is porting the maple code to C code and enhancing the visualization. This work will prepare for a better diffusion of the software via a webserver and a transfert to Maplesoft with which Inria has signed a contract in April 2016.

- Contact: Sylvain Lazard & Marc Pouget
- URL: http://vegas.loria.fr/isotop/

5.2. SubdivisionSolver

KEYWORDS: Numerical solver - Polynomial or analytical systems

The software SubdivisionSolver solves square systems of analytic equations on a compact subset of a real space of any finite dimension. SubdivisionSolver is a numerical solver and as such it requires that the solutions in the subset are isolated and regular for the input system (i.e. the Jacobian must not vanish). SubdivisionSolver is a subdivision solver using interval arithmetic and multiprecision arithmetic to achieve certified results. If the arithmetic precision required to isolate solutions is known, it can be given as an input parameter of the process, otherwise the precision is increased on the fly. In particular, SubdivisionSolver can be interfaced with the Fast_Polynomial library (https://bil.inria.fr/en/software/view/2423/tab) to solve polynomial systems that are large in terms of degree, number of monomials and bit-size of coefficients.

The software is based on a classic branch and bound algorithm using interval arithmetic: an initial box is subdivided until its sub-boxes are certified to contain either no solution or a unique solution of the input system. Evaluation is performed with a centered evaluation at order two, and existence and uniqueness of solutions is verified thanks to the Krawczyk operator.

SubdivisionSolver uses two implementations of interval arithmetic: the C++ boost library that provides a fast arithmetic when double precision is enough, and otherwise the C mpfi library that allows to work in arbitrary precision. Considering the subdivision process as a breadth first search in a tree, the boost interval arithmetic is used as deeply as possible before a new subdivision process using higher precision arithmetic is performed on the remaining forest.

The software has been improved and a technical report published [28].

- Contact: Rémi Imbach
- URL: https://bil.inria.fr/fr/software/view/2605/tab

6. New Results

6.1. Non-linear Computational Geometry

Participants: Laurent Dupont, Rémi Imbach, Sylvain Lazard, Guillaume Moroz, Marc Pouget.

6.1.1. Numeric and Certified Algorithm for the Topology of the Projection of a Smooth Space Curve

Let a smooth real analytic curve embedded in $\mathbb{R}^3$ be defined as the solution of real analytic equations of the form $P(x, y, z) = Q(x, y, z) = 0$ or $P(x, y, z) = \frac{\partial P}{\partial z} = 0$. Our main objective is to describe its projection $\mathcal{C}$ onto the (x, y)-plane. In general, the curve $\mathcal{C}$ is not a regular submanifold of $\mathbb{R}^2$ and describing it requires to isolate the points of its singularity locus Σ.

In previous work, we have shown how to describe the set of singularities Σ of $\mathcal{C}$ as regular solutions of a so-called ball system suitable for a numerical subdivision solver. In our current work, the space curve is first enclosed in a set of boxes with a certified path-tracker to restrict the domain where the ball system is solved. Boxes around singular points are then computed such that the correct topology of the curve inside these boxes can be deduced from the intersections of the curve with their boundaries. The tracking of the space curve is then used to connect the smooth branches to the singular points. The subdivision of the plane induced by the curve is encoded as an extended planar combinatorial map allowing point location. This work is not already published but has been presented by R. Imbach at the Summer Workshop on Interval Methods (https://swim2016.sciencesconf.org/).

The technical report [28] describes the software SubdivisionSolver (see Section 5.2) used within this project.

6.1.2. A Fast Algorithm for Computing the Truncated Resultant

Let P and Q be two polynomials in $\mathbb{K}[x,y]$ with degree at most d, where $\mathbb{K}$ is a field. Denoting by $R \in \mathbb{K}[x]$ the resultant of P and Q with respect to y, we present an algorithm to compute $R \mod x^k$ in $\widetilde{O}(kd)$ arithmetic operations in $\mathbb{K}$, where the $\widetilde{O}$ notation indicates that we omit polylogarithmic factors. This is an improvement over state-of-the-art algorithms that require to compute R in $\widetilde{O}(d^3)$ operations before computing its first k coefficients [24].

This work was done in collaboration with Éric Schost (Waterloo University, Canda).

6.1.3. Quadric Arrangement in Classifying Rigid Motions of a 3D Digital Image

Rigid motions are fundamental operations in image processing. While bijective and isometric in $\mathbb{R}^3$, they lose these properties when digitized in $\mathbb{Z}^3$. To understand how the digitization of 3D rigid motions affects the topology and geometry of a chosen image patch, we classify the rigid motions according to their effect on the image patch. This classification can be described by an arrangement of hypersurfaces in the parameter space of 3D rigid motions of dimension six. However, its high dimensionality and the existence of degenerate cases make a direct application of classical techniques, such as cylindrical algebraic decomposition or critical point method, difficult. We show that this problem can be first reduced to computing sample points in an arrangement of quadrics in the 3D parameter space of rotations. Then we recover information about the three remaining parameters of translation. We implemented an ad-hoc variant of state-of-the-art algorithms and applied it to an image patch of cardinality 7. This leads to an arrangement of 81 quadrics and we recovered the classification in less than one hour on a machine equipped with 40 cores [25].

This work was done in collaboration with Kacper Pluta (LIGM - Laboratoire d'Informatique Gaspard-Monge), Yukiko Kenmochi (LIGM - Laboratoire d'Informatique Gaspard-Monge), Pascal Romon (LAMA - Laboratoire d'Analyse et de Mathématiques Appliquées).

6.1.4. Influence of the Trajectory Planning on the Accuracy of the Orthoglide 5-axis manipulator

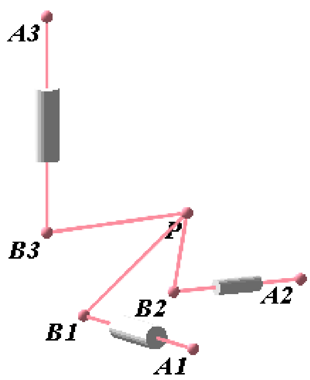

Figure 1. A configuration of the orthoglide manipulator has three orthogonal prismatic joints.

Usually, the accuracy of parallel manipulators depends on the architecture of the robot, the design parameters, the trajectory planning and the location of the path in the workspace. This paper reports the influence of static and dynamic parameters in computing the error in the pose associated with the trajectory planning made and analyzed with the Orthoglide 5-axis (Figure 1). An error model is proposed based on the joint parameters (velocity and acceleration) and experimental data coming from the Orthoglide 5-axis. Newton and Gröbner based elimination methods are used to project the joint error in the workspace to check the accuracy/error in the Cartesian space. For the analysis, five similar trajectories with different locations inside the workspace are defined using fifth order polynomial equation for the trajectory planning. It is shown that the accuracy of the robot depends on the location of the path as well as the starting and the ending posture of the manipulator due to the acceleration parameters [23].

This work was done in collaboration with Ranjan Jha (IRCCyN - Institut de Recherche en Communications et en Cybernétique de Nantes), Damien Chablat (IRCCyN - Institut de Recherche en Communications et en Cybernétique de Nantes), Fabrice Rouillier (Inria).

6.1.5. Solving Bivariate Systems and Topology of Plane Algebraic Curves

In the context of our algorithm Isotop for computing the topology of plane algebraic curves (see Section 5.1), we work on the problem of solving a system of two bivariate polynomials. We are interested in certified numerical approximations or, more precisely, isolating boxes of the solutions. But we are also interested in computing, as intermediate symbolic objects, a Rational Univariate Representation (RUR) that is, roughly speaking, a univariate polynomial and two rational functions that map the roots of the univariate polynomial to the two coordinates of the solutions of the system. RURs are relevant symbolic objects because they allow to the transformation of many queries on the system into queries on univariate polynomials. However, such representations require the computation of a separating form for the system, that is a linear combination of the variables that takes different values when evaluated at the distinct solutions of the system.

We published this year [11] results showing that, given two polynomials of degree at most d with integer coefficients of bitsize at most τ, (i) a separating form, (ii) the associated RUR, and (iii) isolating boxes of the solutions can be computed in, respectively, $\widetilde{O}_B(d^5+d^5\tau)$, bit operations in the worst case, where $\widetilde{O}$ refers to the complexity where polylogarithmic factors are omitted and O_B refers to the bit complexity. Furthermore, we also presented probabilistic Las Vegas variants of problems (i) and (ii), which have expected bit complexity $\widetilde{O}_B(d^5+d^4\tau)$. We also showed that these complexities are "morally" optimal in the sense of that improving them would essentially require to improve bounds on several other fundamental problems (on resultants and roots isolation of univariate polynomials) that have hold for decades. These progresses are substantial since, when we started woriking on these problems, their best know complexities were in $\widetilde{O}_B(d^{12}+d^{10}\tau^2)$ (2009).

This work was done in collaboration with Yacine Bouzidi (Inria Lille), Michael Sagraloff (MPII Sarrebruken, Germany) and Fabrice Rouillier (Inria Rocquencourt).

6.1.6. Reflection through Quadric Mirror Surfaces

We addressed the problem of finding the reflection point on quadric mirror surfaces, especially ellipsoid, paraboloid or hyperboloid of two sheets, of a light ray emanating from a 3D point source P_1 and going through another 3D point P_2, the camera center of projection. We previously proposed a new algorithm for this problem, using a characterization of the reflection point as the tangential intersection point between the mirror and an ellipsoid with foci P_1 and P_2. The computation of this tangential intersection point is based on our algorithm for the computation of the intersection of quadrics [5], [32]. Unfortunately, our new algorithm is not yet efficient in practice. This year, we made several improvements on this algorithm. First, we decreased from 11 to 4 the degree of a critical polynomial that we need to solve and whose solutions induce the coefficients in some other polynomials appearing later in the computations. Second, we implemented Descartes' algorithm for isolating the real roots of univariate polynomials in the case where the coefficients belong to extensions of $\mathbb{Q}$ generated by at most two square roots. Furthermore, we are currently implementing the generalization of that algorithm when the coefficients belong to extensions of $\mathbb{Q}$ generated by one root of an arbitrary polynomial. We are also interested by extensions decomposable in extensions of degree 2. These undergoing improvements should allow us to compute more directly the wanted reflection point, thus avoiding problematic approximations and making the overall algorithm tractable.

6.2. Non-Euclidean Computational Geometry

Participants: Iordan Iordanov, Monique Teillaud, Gert Vegter.

6.2.1. Closed Flat Orbifolds

The work on Delaunay triangulations of flat d-dimensional orbifolds, started several years ago in the Geometrica project team in Sophia Antipolis, was finalized this year [13].

We give a definition of the Delaunay triangulation of a point set in a closed Euclidean d-manifold, i.e. a compact quotient space of the Euclidean space for a discrete group of isometries (a so-called Bieberbach group or crystallographic group). We describe a geometric criterion to check whether a partition of the manifold actually forms a triangulation (which subsumes that it is a simplicial complex). We provide an incremental algorithm to compute the Delaunay triangulation of the manifold defined by a given set of input points, if it exists. Otherwise, the algorithm returns the Delaunay triangulation of a finite-sheeted covering space of the manifold. The algorithm has optimal randomized worst-case time and space complexity. It extends to closed Euclidean orbifolds. To the best of our knowledge, this is the first general result on this topic.

6.2.2. Closed Orientable Hyperbolic Surfaces

Motivated by applications in various fields, some packages to compute periodic Delaunay triangulations in the 2D and 3D Euclidean spaces have been introduced in the CGAL library and have attracted a number of users. To the best of our knowledge, no software is available to compute periodic triangulations in a hyperbolic space, though they are also used in diverse fields, such as physics, solid modeling, cosmological models, neuromathematics.

This would be a natural extension: 2D Euclidean periodic triangulations can be seen as triangulations of the two-dimensional (flat) torus of genus one; similarly, periodic triangulations in the hyperbolic plane can be seen as triangulations of hyperbolic surfaces. A closed orientable hyperbolic surface is the quotient of the hyperbolic plane under the action of a Fuchsian group only containing hyperbolic translations. Intuition is challenged there, in particular because such groups are non-Abelian in general.

We have obtained some theoretical results on Delaunay triangulations of general closed orientable hyperbolic surfaces, and we have investigated algorithms in the specific case of the Bolza surface, a hyperbolic surface with the simplest possible topology, as it is homeomorphic to a genus-two torus [20]. We are now studying more practical aspects and we propose a first implementation of an incremental construction of Delaunay triangulations of the Bolza surface [30].

6.3. Probabilistic Analysis of Geometric Data Structures and Algorithms

Participants: Olivier Devillers, Louis Noizet.

6.3.1. Stretch Factor of Long Paths in a Planar Poisson-Delaunay Triangulation

Let $X := X_n \cup \{(0,0),(1,0)\}$, where X_n is a planar Poisson point process of intensity n. We provide a first non-trivial lower bound for the distance between the expected length of the shortest path between $(0,0)$ and $(1,0)$ in the Delaunay triangulation associated with X when the intensity of X_n goes to infinity. Experimental values indicate that the correct value is about 1.04. We also prove that the expected number of Delaunay edges crossed by the line segment $[(0,0),(1,0)]$ is equivalent to $2.16\sqrt{n}$ and that the expected length of a particular path converges to 1.18 giving an upper bound on the stretch factor [26].

This work was done in collaboration with Nicolas Chenavier (Université Littoral Côte d'Opale).

6.3.2. Walking in a Planar Poisson-Delaunay Triangulation: Shortcuts in the Voronoi Path

Let X_n be a planar Poisson point process of intensity n. We give a new proof that the expected length of the Voronoi path between $(0,0)$ and $(1,0)$ in the Delaunay triangulation associated with X_n is $\frac{4}{\pi} \simeq 1.27$ when n goes to infinity; and we also prove that the variance of this length is $O(1/\sqrt{n})$. We investigate the length of possible shortcuts in this path, and defined a shortened Voronoi path whose expected length can be expressed as an integral that is numerically evaluated to $\simeq 1.16$. The shortened Voronoi path has the property to be *locally defined*; and is shorter than the previously known locally defined path in Delaunay triangulation such as the upper path whose expected length is $35/3\pi^2 \simeq 1.18$ [27].

6.3.3. Expected Length of the Voronoi Path in a High Dimensional Poisson-Delaunay Triangulation

Let X_n be a d dimensional Poisson point process of intensity n. We prove that the expected length of the Voronoi path between two points at distance 1 in the Delaunay triangulation associated with X_n is $\sqrt{\frac{2d}{\pi}} + O(d^{-\frac{1}{2}})$ for all $n \in \mathbb{N}$ and $d \to \infty$. In any dimension, we provide a precise interval containing the exact value, in 3D the expected length is between 1.4977 and 1.50007 [31].

This work was done in collaboration with Pedro Machado Manhães De Castro (Centro de Informática da Universidade Federal de Pernambuco).

6.4. Classical Computational Geometry and Graph Drawing

Participants: Olivier Devillers, Sylvain Lazard.

6.4.1. Monotone Simultaneous Path Embeddings in $\mathbb{R}^d$

We study the following problem: Given k paths that share the same vertex set, is there a simultaneous geometric embedding of these paths such that each individual drawing is monotone in some direction? We prove that for any dimension $d \geq 2$, there is a set of $d+1$ paths that does not admit a monotone simultaneous geometric embedding [21].

This work was done in collaboration with David Bremner (U. New Brunswick), Marc Glisse (Inria Datashape), Giuseppe Liotta (U. Perugia), Tamara Mchedlidze (Karlsruhe Institute for Technology), Sue Whitesides (U. Victoria), and Stephen Wismath (U. Lethbridge).

6.4.2. Analysis of Farthest Point Sampling for Approximating Geodesics in a Graph

A standard way to approximate the distance between two vertices p and q in a graph is to compute a shortest path from p to q that goes through one of k sources, which are well-chosen vertices. Precomputing the distance between each of the k sources to all vertices yields an efficient computation of approximate distances between any two vertices. One standard method for choosing k sources is the so-called *Farthest Point Sampling* (FPS), which starts with a random vertex as the first source, and iteratively selects the farthest vertex from the already selected sources.

We analyzed the stretch factor $\mathcal{F}_{\text{FPS}}$ of approximate geodesics computed using FPS, which is the maximum, over all pairs of distinct vertices, of their approximated distance over their geodesic distance in the graph. We showed that $\mathcal{F}_{\text{FPS}}$ can be bounded in terms of the minimal value $\mathcal{F}^*$ of the stretch factor obtained using an optimal placement of k sources as $\mathcal{F}_{\text{FPS}} \leq 2r_e^2\mathcal{F}^* + 2r_e^2 + 8r_e + 1$, where r_e is the length ratio of longest edge over the shortest edge in the graph. We further showed that the factor r_e is not an artefact of the analysis by providing a class of graphs for which $\mathcal{F}_{\text{FPS}} \geq \frac{1}{2}r_e\mathcal{F}^*$ [18].

This work was done in collaboration with Pegah Kamousi (Université Libre de Bruxelles), Anil Maheshwari (Carleton University), and Stefanie Wuhrer (Inria Grenoble Rhône-Alpes).

6.4.3. Recognizing Shrinkable Complexes is NP-complete

We say that a simplicial complex is shrinkable if there exists a sequence of admissible edge contractions that reduces the complex to a single vertex. We prove that it is NP-complete to decide whether a (three-dimensional) simplicial complex is shrinkable. Along the way, we describe examples of contractible complexes which are not shrinkable [10].

This work was done in collaboration with Dominique Attali (CNRS, Grenoble) and Marc Glisse (Inria Datashape).

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

A two years licence and cooperation agreement was signed on April 1st, 2016 between WATERLOO MAPLE INC., Ontario, Canada (represented by Laurent Bernardin, its Executive Vice President Products and Solutions) and Inria. On the Inria side, this contract involves the teams VEGAS and OURAGAN (Paris), and it is coordinated by Fabrice Rouillier (OURAGAN).

F. Rouillier and VEGAS are the developers of the ISOTOP software for the computation of topology of curves. One objective of the contract is to transfer a version of ISOTOP to WATERLOO MAPLE INC.

8. Partnerships and Cooperations

8.1. Regional Initiatives

We organized, with IECL, a «journée Charles Hermite» about geometry and probability. A regular working group on the topic was started in november.

8.2. National Initiatives

8.2.1. ANR PRESAGE

The white ANR grant PRESAGE brings together computational geometers (from the VEGAS and GEOMETRICA projects of Inria) and probabilistic geometers (from Universities of Rouen, Orléans and Poitiers) to tackle new probabilistic geometry problems arising from the design and analysis of geometric algorithms and data structures. We focus on properties of discrete structures induced by random continuous geometric objects.

The project, with a total budget of 400kE, started on Dec. 31st, 2011 and ended in March 2016. It is coordinated by Xavier Goaoc who moved from the Vegas team to Marne-la-Vallée university in 2013.

Project website: https://members.loria.fr/GMoroz/ANR-Presage/.

8.2.2. ANR SingCAST

The objective of the young-researcher ANR grant SingCAST is to intertwine further symbolic/numeric approaches to compute efficiently solution sets of polynomial systems with topological and geometrical guarantees in singular cases. We focus on two applications: the visualization of algebraic curves and surfaces and the mechanical design of robots.

After identifying classes of problems with restricted types of singularities, we plan to develop dedicated symbolic-numerical methods that take advantage of the structure of the associated polynomial systems that cannot be handled by purely symbolic or numerical methods. Thus we plan to extend the class of manipulators that can be analyzed, and the class of algebraic curves and surfaces that can be visualized with certification.

This is a 3.5 years project, with a total budget of 100kE, that started on March 1st 2014, coordinated by Guillaume Moroz.

The project funded the postdoc position of Rémi Imbach from November 2014 until October 2016. We organized two workshops in 2016 with the OPTI team in Nantes, on certified surface continuation.

Project website: https://project.inria.fr/singcast/.

8.3. International Initiatives

8.3.1. Participation in Other International Programs

8.3.1.1. Nancy Emerging Associate Team Astonishing

The objectives of the *ASsociate Team On Non-ISH euclIdeaN Geometry* is to study various structures and algorithms in non-Euclidean spaces, from a computational geometry viewpoint. Proposing algorithms operating in such spaces requires a prior deep study of the mathematical properties of the objects considered, which raises new fundamental and difficult questions that we want to tackle.

A key characteristic of the project is its interdisciplinarity: it gathers approaches, knowledge, and tools in mathematics and computer science. A mathematical study of the considered objects will be performed, together with the design of algorithms when applicable. Algorithms will be analyzed both in theory and in practice after prototype implementations. In the long term, implementations should be improved whenever it makes sense to target longer-term integrations into CGAL, in order to disseminate our results to end-users.

The partners are the Johann Bernouilli Institute of Mathematics and Computer Science of University of Groningen, the Mathematics Research Unit of University of Luxembourg, and the Talgo team of École Normale Supérieure. The project is coordinated by Monique Teillaud and supported by Inria Nancy - Grand Est.

Project website: https://members.loria.fr/Monique.Teillaud/collab/Astonishing/.

8.4. International Research Visitors

8.4.1. Visits of International Scientists

8.4.1.1. Invited Professor

Gert Vegter, Professor at Univerity of Groningen, was awarded an invited professor position by University of Lorraine and spent one month in the group in May. He is coordinating the NEAT Astonishing on the Dutch side.

8.4.1.2. PhD Visitor

Sény Diatta, Senegalese PhD student co-advised by Guillaume Moroz, Daouda Niang Diatta (Ziguinchor) and Marie-Françoise Roy (Rennes), obtained a bourse Eiffel from Campus France, which includes a salary for 10 months to visit LORIA.

8.4.2. Visits to International Teams

8.4.2.1. Research Stays Abroad

Iordan Iordanov spent one month at University of Luxembourg in June. The visit was partially supported by by University of Luxembourg and by the NEAT Astonishing.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. Member of Organizing Committees

Sylvain Lazard organized with S. Whitesides (Victoria University) the 15th Workshop on Computational Geometry at the Bellairs Research Institute of McGill University in Feb. (1 week workshop on invitation).

Monique Teillaud co-organized the workshop *20 years of CGAL*, with Efi Fogel, Michael Hoffmann, and Emo Welzl, Zurich, Switzerland, September 10-11, and she gave a talk.

9.1.2. Scientific Events Selection

9.1.2.1. Member of Conference Program Committees

Monique Teillaud was a member of the program committee of EuroCG, *European Workshop on Computational Geometry*.

9.1.2.2. Reviewer

All members of the team are regular reviewers for the conferences of our field, namely the *Symposium on Computational Geometry* (SoCG) and the *International Symposium on Symbolic and Algebraic Computation* (ISSAC) and also SODA, CCCG, EuroCG.

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

Monique Teillaud is a managing editor of JoCG, *Journal of Computational Geometry*. She is also a member of the Editorial Board of IJCGA, *International Journal of Computational Geometry and Applications*. She resigned from the Editorial Board of CGTA, *Computational Geometry: Theory and Applications*, after unsuccessfully trying to convince the Editorial Board to leave Elsevier and move to a free (libre and gratis) open-access model.

Marc Pouget and Monique Teillaud are members of the CGAL editorial board.

Olivier Devillers resigned from the Editorial Board of Graphical Models (Elsevier) after discussion to move to a free open-access model.

9.1.3.2. Reviewer - Reviewing Activities

All members of the team are regular reviewers for the journals of our field, namely *Discrete and Computational Geometry* (DCG), *Computational Geometry. Theory and Applications* (CGTA), *Journal of Computational Geometry* (JoCG), *International Journal on Computational Geometry and Applications* (IJCGA), *Journal on Symbolic Computations* (JSC), *SIAM Journal on Computing* (SICOMP), *Mathematics in Computer Science* (MCS), etc.

9.1.4. Invited Talks

Olivier Devillers was invited to give a talk at the geometry week organized by GipsaLab in Grenoble.

Guillaume Moroz was invited to give talks at the LIGM seminar in Marne-la-Vallée university, at the SpecFun team seminary in Inria Saclay and at the MSDOS workshop in CIRM.

Monique Teillaud was invited to give a talk talk at the seminar *Computer Science meets Mathematics* of the University of Luxembourg, February 8: “CGAL, geometry made practical”. She was invited to give a talk at the *Mittagsseminar* of Institute of Theoretical Computer Science of ETH Zürich on September 8: “Delaunay triangulations on orientable surfaces of low genus”.

9.1.5. Seminar Organization

We invited:

Kacper Pluta (LIGM - Laboratoire d’Informatique Gaspard-Monge),

Mickaël Buchet (Tohoku University).

Andrew Yarmola (University of Luxembourg).

9.1.6. Leadership within the Scientific Community

9.1.6.1. Steering Committees

M. Teillaud has been elected Chair of the Steering Committee of the Symposium on Computational Geometry (SoCG). She is a member of the Steering Committee of the European Symposium on Algorithms (ESA).

9.1.7. Research Administration

9.1.7.1. Hiring committees

Sylvain Lazard was president of the hiring committee for a Professor position (UL/École des Mines/LORIA).

Monique Teillaud was the representative of LORIA in the hiring committee for an Associate Professor (MCF) position (École des Mines/LORIA) and composed the committee with the president. She was also a member of the Inria CR2 Nancy - Grand Est interview committee and of the hiring committee for a Professor position (FST/LORIA).

9.1.7.2. National committees

L. Dupont is a member of "Commission Pédagogique Nationale" (CPN) Information-Communication / Métiers du Multimédia et de l'Internet.

M. Teillaud is a member of the Scientific Board of the *Société Informatique de France* (SIF).

M. Teillaud is a member of the working group for the BIL, *Base d'Information des Logiciels* of Inria.

9.1.7.3. Local Committees and Responsabilities

S. Lazard: Head of the PhD and Post-doc hiring committee for Inria Nancy-Grand Est (since 2009). Member of the *Bureau de la mention informatique* of the *École Doctorale IAE+M* (since 2009). Head of the *Mission Jeunes Chercheurs* for Inria Nancy-Grand Est (since 2011). Head of the Department Algo at LORIA (since 2014). Member of the *Conseil Scientifique* of LORIA (since 2014).

G. Moroz is member of the Mathematics Olympiades committee of the Nancy-Metz academy. G. Moroz is member of the *Comité des utilisateurs des moyens informatiques*

M. Pouget is elected at the *Comité de centre*, and member of the board of the Charles Hermite federation of labs. M. Pouget is secretary of the board of *AGOS-Nancy*.

M. Teillaud is a member of the BCP, *Bureau du Comité des Projets* and of the CDT, *Commission de développement technologique* of Inria Nancy - Grand Est.

9.1.7.4. Websites

M. Teillaud is maintaining the Computational Geometry Web Pages http://www.computational-geometry.org/, hosted by Inria Nancy - Grand Est since December. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual/international Symposium on Computational Geometry.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master : O. Devillers, *Synthèse, image et géométrie* , 12h (academic year 2015-16) and 12h (academic year 2016-2017), IPAC-R, Université de Lorraine. https://members.loria.fr/ODevillers/master/

Master: Marc Pouget, *Introduction to computational geometry*, 10.5h, M2, École Nationale Supérieure de Géologie, France.

Licence: Sylvain Lazard, *Algorithms and Complexity*, 25h, L3, Université de Lorraine, France.

Licence: Laurent Dupont, *Algorithmique*, 78h, L1, Université de Lorraine, France.

Licence: Laurent Dupont, *Web development*, 75h, L2, Université de Lorraine, France.

Licence: Laurent Dupont, *Traitement Numérique du Signal*, 10h, L2, Université de Lorraine, France.

Licence: Laurent Dupont, *Data structures*, 40h, L1, Université de Lorraine, France.

9.2.2. Supervision

PhD : Ranjan Jha, Étude de l'espace de travail des mécanismes à boucles fermées, defended in Jul. 2016, supervised by Damien Chablat, Fabrice Rouillier and Guillaume Moroz.

PhD in progress : Sény Diatta, Complexité du calcul de la topologie d'une courbe dans l'espace et d'une surface, started in Nov. 2014, supervised by Daouda Niang Diatta, Marie-Françoise Roy and Guillaume Moroz.

PhD in progress : Charles Duménil, Probabilistic analysis of geometric structures, started in Oct. 2016, supervised by Olivier Devillers.

PhD in progress : Iordan Iordanov, Triangulations of Hyperbolic Manifolds, started in Jan. 2016, supervised by Monique Teillaud.

Postdoc: Rémy Imbach, Topology and geometry of singular surfaces with numerical algorithms, supervised by Guillaume Moroz and Marc Pouget.

9.2.3. Juries

O. Devillers was president of the PhD defense committee of Vincent Despré (Univ. Grenoble-Alpes).

G. Moroz was in the PhD defense committee of Ranjan Jha (IRCCyN).

9.2.4. Teaching Responsabilities

Licence: Laurent Dupont, creation and opening of L3 (Licence Professionnelle) « Animation des Communautés et Réseaux Socionumériques », Université de Lorraine, France.

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