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Project-Team AROMATH

Algèbre géométrie Modélisation et
AlgoriTHmes

RESEARCH CENTER
Sophia Antipolis - Méditerranée

THEME
**Algorithmics, Computer Algebra and
Cryptology**

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1. Personnel

Research Scientists

Bernard Mourrain [Team leader, Inria, Senior Researcher, HDR]

Laurent Busé [Inria, Researcher, HDR]

Evelyne Hubert [Inria, Researcher, HDR]

Faculty Members

André Galligo [Univ de Nice - Sophia Antipolis, Professor]

Ioannis Emiris [Univ. Athens, Professor, HDR]

Panagiotis Kaklis [Univ. Athens, Professor]

Post-Doctoral Fellows

Alessandro Oneto [Inria, Post-Doctoral Fellow]

Christos Konaxis [Univ. Athens, Scientific collaborator]

PhD Students

Evangelos Anagnostopoulos [Univ. Athens]

Evangelos Bartzos [Univ. Athens]

Elisa Berrini [MyCFD, granted by CIFRE]

Ahmed Blidia [Inria]

Alvaro Fuentes Suarez [Inria]

Jouhayna Harmouch [Univ. Lebanon, cotutelle]

Anna Karasoulou [Univ. Athens, until June 2017]

Clément Laroche [Univ. Athens]

Ioannis Psarros [Univ. Athens]

Fatma Yildirim [Inria]

Erick David Rodriguez Bazan [Inria, from Nov 2017]

Interns

Adrien Boudin [Inria, from Jun 2017 until Sep 2017]

Pofinet Keda [Inria, from Jun 2017 until Sep 2017]

Antoine Deharveng [Univ de Nice - Sophia Antipolis, until Mar 2017]

Thomas Laporte [Inria, from Jun 2017 until Sep 2017]

Emmanouil Christoforou [Inria, from Sep 2017]

Kim Perriguy [Inria, from Dec 2017]

Administrative Assistant

Sophie Honnorat [Inria]

Visiting Scientists

Sotirios Chouliaras [University of Strachlyde, from Aug 2017 until Nov 2017]

Yairon Cid Ruiz [University of Barcelona, from Sep 2017]

Roser Homs Pons [University of Barcelona, from Jul 2017]

Ibrahim Nonkane [University of Ouagadougou, Burkina Faso, from Sep 2017 until Oct 2017]

Vlada Pototskaia [University of Göttingen, from Aug 2017 until Sep 2017]

Simon Telen [KU Leuven , Belgium, from Aug 2017 until Sep 2017]

Meng Wu [Hefei University of Technology, China, from Sep 2017 until Oct 2017]

Gang Xu [Hangzhou Dianzi University, China, Sep 2017]

2. Overall Objectives

2.1. Overall Objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of “real” phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to “act” on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, ... are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects “at home” from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimisation loops where the numerical simulation results are used to improve the “performance” of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world...), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

3. Research Program

3.1. High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in CAGD, to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at the seams. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

3.2. Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally, as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods, which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One is based on linearization of algebraic problems based on “elimination of variables” or projection into a space of smaller dimension. Resultant theory provides strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progresses have been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute “only” the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completeness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

4. Application Domains

4.1. Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axis machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

4.2. Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree ≤ 2).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are

investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

5. New Software and Platforms

5.1. Platforms

5.1.1. *Axel*

KEYWORDS: Algorithm , CAD , Numerical algorithm , Geometric algorithms

SCIENTIFIC DESCRIPTION

Axel is an algebraic geometric modeler that aims at providing “algebraic modeling” tools for the manipulation and computation with curves, surfaces or volumes described by semi-algebraic representations. These include parametric and implicit representations of geometric objects. Axel also provides algorithms to compute intersection points or curves, singularities of algebraic curves or surfaces, certified topology of curves and surfaces, etc. A plugin mechanism allows to extend easily the data types and functions available in the platform.

FUNCTIONAL DESCRIPTION

Axel is a cross platform software to visualize, manipulate and compute 3D objects. It is composed of a main application and several plugins. The main application provides atomic geometric data and processes, a viewer based on VTK, a GUI to handle objects, to select data, to apply process on them and to visualize the results. The plugins provides more data with their reader, writer, converter and interactors, more processes on the new or atomic data. It is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python. The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows.

- Participants: Nicolas Douillet, Anaïs Ducoffe, Valentin Michelet, Bernard Mourrain, Meriadeg Perrinel, Stéphane Chau and Julien Wintz
- Contact: Bernard Mourrain
- URL: <http://axel.inria.fr/>

Collaboration with Elisa Berrini (MyCFD, Sophia), Tor Dokken (Gotools library, Oslo, Norway), Angelos Mantzaflaris (GISMO library, Linz, Austria), Laura Saini (Post-Doc GALAAD/Missler, TopSolid), Gang Xu (Hangzhou Dianzi University, China), Meng Wu (Hefei University of Technology, China).

5.1.2. *Dtk-Nurbs-Probing*

KEYWORDS: CAO - Algebraic geometric modeler

SCIENTIFIC DESCRIPTION

This library offers tools for computing intersection between linear primitives and the constitutive elements of CAD objects (curves and surfaces). It is thus possible to compute intersections between a linear primitive with a trimmed NURBS surface, as well as untrimmed, moreover with a Bezier surface. It is also possible, in the xy plane, to compute the intersections between linear primitives and NURBS curves as well as Bezier curves.

FUNCTIONAL DESCRIPTION

Polynomial/rational defined primitives intersection with linear primitives under the form of a dtk plugin.

- Authors: Come Le Breton, Laurent Busé, Pierre Alliez, Julien Wintz, Thibaud Kloczko.
- Contact: Laurent Busé
- URL: <http://nurbsprobing.inria.fr/>

Collaboration with Pierre Alliez (Titane) and the industrial partner GeometryFactory (Sophia).

6. New Results

6.1. Waring-like decompositions of polynomials

Participant: Alessandro Oneto.

In [9], we consider particular types of *additive decompositions* of homogeneous polynomials. The classical decomposition is the *Waring decomposition*, where we decompose polynomials as sums of powers of linear forms. Another well studied decomposition is the sometimes-called *Chow decomposition*, where we decompose polynomials as sums of products of linear forms. These are the extremal cases of the additive decompositions considered in this work. For a fixed partition $(d_1, \dots, d_s) \vdash d$ of the degree of the polynomial, we consider decompositions as sums of degree forms of the form $\ell_1^{d_1} \cdots \ell_s^{d_s}$, where the ℓ 's are linear forms. The homogeneous polynomials of the form $\ell_1^{d_1} \cdots \ell_s^{d_s}$ are parametrized by particular linear projections of certain Segre-Veronese varieties. The main results of this work concerns the dimension of the secant varieties to these projections of Segre-Veronese varieties. In particular, we compute their dimensions in the binary case (forms in two variables) and the case of secant lines varieties for any partition and any number of variables. From these results, we deduce the dimension of higher secant varieties in some particular cases.

This is a joint work with M. V. Catalisano, Luca Chiantini, and A. V. Geramita.

6.2. Waring loci and the Strassen conjecture

Participant: Alessandro Oneto.

In [8], we introduce the notion of the *Waring locus* of a homogeneous polynomial. A *Waring decomposition* is an expression of a polynomial as sum of powers of linear forms. The smallest length of such a decomposition is called the *Waring rank* of the polynomial. A very difficult challenge is to compute the rank and a minimal decomposition of a given form. The Waring locus of a polynomial is the locus of linear forms that appear in a minimal decomposition of it. The idea behind this construction is to find an iterative approach to construct Waring decompositions *step-by-step*, by adding one power at the time. Moreover, we give a version of the famous *Strassen conjecture* on the additivity of rank for sums of polynomials in independent sets of variables. We compute the Waring loci in several cases as binary forms, quadrics, monomials and plane cubics and for some other particular families of polynomials.

This is a joint work with E. Carlini, and M. V. Catalisano.

6.3. Minkowski sums and Hadamard products of algebraic varieties

Participant: Alessandro Oneto.

In [26], we study two particular geometric constructions. Given two affine algebraic varieties, we define their *Minkowski sum* as the (Zariski) closure of the set of coefficient-wise sums of pairs of points in the two varieties. Given two projective varieties, we define their *Hadamard product* as the (Zariski) closure of the set of coefficient-wise multiplications of pairs of points in the two varieties. In particular, we focus on computing their dimensions and degrees in terms of the ones of the original varieties. Hadamard products are of particular interests as they can be used to parametrize particular families of tensors which rise naturally by studying Restricted Boltzmann Machines, which are particular structures used in Statistics and Machine Learning.

This is a joint work with N. Friedenber, and R. Williams.

6.4. Polynomial-exponential decomposition from moments

Participant: Bernard Mourrain.

In [12], we analyze the decomposition problem of multivariate polynomial-exponential functions from truncated series and present new algorithms to compute their decomposition. Using the duality between polynomials and formal power series, we first show how the elements in the dual of an Artinian algebra correspond to polynomial-exponential functions. They are also the solutions of systems of partial differential equations with constant coefficients. We relate their representation to the inverse system of the roots of the characteristic variety. Using the properties of Hankel operators, we establish a correspondence between polynomial exponential series and Artinian Gorenstein algebras. We generalize Kronecker theorem to the multivariate case, by showing that the symbol of a Hankel operator of finite rank is a polynomial-exponential series and by connecting the rank of the Hankel operator with the decomposition of the symbol. A generalization of Prony's approach to multivariate decomposition problems is presented, exploiting eigenvector methods for solving polynomial equations. We show how to compute the frequencies and weights of a minimal polynomial-exponential decomposition, using the first coefficients of the series. A key ingredient of the approach is the flat extension criteria, which leads to a multivariate generalization of a rank condition for a Carathéodory-Fejér decomposition of multivariate Hankel matrices. A new algorithm is given to compute a basis of the Artinian Gorenstein algebra, based on a Gram-Schmidt orthogonalization process and to decompose polynomial-exponential series. A general framework for the applications of this approach is described and illustrated in different problems. We provide Kronecker-type theorems for convolution operators, showing that a convolution operator (or a cross-correlation operator) is of finite rank, if and only if, its symbol is a polynomial-exponential function, and we relate its rank to the decomposition of its symbol. We also present Kronecker-type theorems for the reconstruction of measures as weighted sums of Dirac measures from moments and for the decomposition of polynomial-exponential functions from values. Finally, we describe an application of this method for the sparse interpolation of polylog functions from values.

6.5. Fast algorithm for border bases of Artinian Gorenstein algebras

Participant: Bernard Mourrain.

Given a multi-index sequence σ , we present in [23] a new efficient algorithm to compute generators of the linear recurrence relations between the terms of σ . We transform this problem into an algebraic one, by identifying multi-index sequences, multivariate formal power series and linear functionals on the ring of multivariate polynomials. In this setting, the recurrence relations are the elements of the kernel I_σ of the Hankel operator H_σ associated to σ . We describe the correspondence between multi-index sequences with a Hankel operator of finite rank and Artinian Gorenstein Algebras. We show how the algebraic structure of the Artinian Gorenstein algebra A_σ associated to the sequence σ yields the structure of the terms σ_α for all $\alpha \in \mathbb{N}^n$. This structure is explicitly given by a border basis of A_σ , which is presented as a quotient of the polynomial ring $K[x_1, \dots, x_n]$ by the kernel I_σ of the Hankel operator H_σ . The algorithm provides generators of I_σ constituting a border basis, pairwise orthogonal bases of A_σ and the tables of multiplication by the variables in these bases. It is an extension of Berlekamp-Massey-Sakata (BMS) algorithm, with improved complexity bounds. We present applications of the method to different problems such as the decomposition of functions into weighted sums of exponential functions, sparse interpolation, fast decoding of algebraic codes, computing the vanishing ideal of points, and tensor decomposition. Some benchmarks illustrate the practical behavior of the algorithm.

6.6. Structured low rank decomposition of multivariate Hankel matrices

Participants: Jouhayna Harmouch, Bernard Mourrain.

In [11], we study the decomposition of a multivariate Hankel matrix H_σ as a sum of Hankel matrices of small rank in correlation with the decomposition of its symbol σ as a sum of polynomial-exponential series. We present a new algorithm to compute the low rank decomposition of the Hankel operator and the decomposition of its symbol exploiting the properties of the associated Artinian Gorenstein quotient algebra A_σ . A basis of A_σ is computed from the Singular Value Decomposition of a sub-matrix of the Hankel matrix H_σ . The frequencies and the weights are deduced from the generalized eigenvectors of pencils of shifted sub-matrices of H_σ . Explicit formula for the weights in terms of the eigenvectors avoid us to solve a Vandermonde system. This new

method is a multivariate generalization of the so-called Pencil method for solving Prony-type decomposition problems. We analyze its numerical behavior in the presence of noisy input moments, and describe a rescaling technique which improves the numerical quality of the reconstruction for frequencies of high amplitudes. We also present a new Newton iteration, which converges locally to the closest multivariate Hankel matrix of low rank and show its impact for correcting errors on input moments.

This is a joint work with Houssam Khalil.

6.7. Decomposition of low rank multi-symmetric tensor

Participants: Jouhayna Harmouch, Bernard Mourrain.

In [22], we study the decomposition of a multi-symmetric tensor T as a sum of powers of product of linear forms in correlation with the decomposition of its dual T^* as a weighted sum of evaluations. We use the properties of the associated Artinian Gorenstein Algebra A_τ to compute the decomposition of its dual T^* which is defined via a formal power series τ . We use the low rank decomposition of the Hankel operator H_τ associated to the symbol τ into a sum of indecomposable operators of low rank. A basis of A_τ is chosen such that the multiplication by some variables is possible. We compute the sub-coordinates of the evaluation points and their weights using the eigen-structure of multiplication matrices. The new algorithm that we propose works for small rank. We give a theoretical generalized approach of the method in n dimensional space. We show a numerical example of the decomposition of a multi-linear tensor of rank 3 in 3 dimensional space.

This is a joint work with Houssam Khalil.

6.8. Tensor decomposition and homotopy continuation

Participant: Bernard Mourrain.

A computationally challenging classical elimination theory problem is to compute polynomials which vanish on the set of tensors of a given rank. By moving away from computing polynomials via elimination theory to computing pseudowitness sets via numerical elimination theory, we develop in [3] computational methods for computing ranks and border ranks of tensors along with decompositions. More generally, we present our approach using joins of any collection of irreducible and nondegenerate projective varieties $X_1, \dots, X_k \subset \mathbb{P}^N$ defined over \mathbb{C} . After computing ranks over \mathbb{C} , we also explore computing real ranks. Various examples are included to demonstrate this numerical algebraic geometric approach.

This is a joint work with Alessandra Bernardi, Noah S. Daleo, Jonathan D. Hauenstein.

6.9. Effective criteria for bigraded birational maps

Participant: Laurent Busé.

In [6], we consider rational maps whose source is a product of two subvarieties, each one being embedded in a projective space. Our main objective is to investigate birationality criteria for such maps. First, a general criterion is given in terms of the rank of a couple of matrices that became to be known as Jacobian dual matrices. Then, we focus on rational maps from the product of two projective lines to the projective plane in very low bidegrees and provide new matrix-based birationality criteria by analyzing the syzygies of the defining equations of the map, in particular by looking at the dimension of certain bigraded parts of the syzygy module. Finally, applications of our results to the context of geometric modeling are discussed at the end of the paper.

This is a joint work with Nicolás Botbol (University of Buenos Aires), Marc Chardin (University of Paris VI), Hamid Seyed Hassanzadeh (University of Rio de Janeiro), Aron Simis (University of Pernambuco) and Quang Hoa Tran (University of Paris VI).

6.10. Discriminants of complete intersection space curves

Participant: Laurent Busé.

In [19], we develop a new approach to the discriminant of a complete intersection curve in the 3-dimensional projective space. By relying on the resultant theory, we first prove a new formula that allows us to define this discriminant without ambiguity and over any commutative ring, in particular in any characteristic. This formula also provides a new method for evaluating and computing this discriminant efficiently, without the need to introduce new variables as with the well-known Cayley trick. Then, we obtain new properties and computational rules such as the covariance and the invariance formulas. Finally, we show that our definition of the discriminant satisfies to the expected geometric property and hence yields an effective smoothness criterion for complete intersection space curves. Actually, we show that in the generic setting, it is the defining equation of the discriminant scheme if the ground ring is assumed to be a unique factorization domain.

This is a joint work with Ibrahim Nonkané (University of Ouaga II).

6.11. Matrix Representations by Means of Interpolation

Participants: Ioannis Emiris, Christos Konaxis, Clément Laroche.

In [20] we examine implicit representations of parametric or point cloud models, based on interpolation matrices, which are not sensitive to base points. We show how interpolation matrices can be used for ray shooting of a parametric ray with a surface patch, including the case of high-multiplicity intersections. Most matrix operations are executed during pre-processing since they solely depend on the surface. For a given ray, the bottleneck is equation solving. Our Maple code handles bicubic patches within 1 second, though numerical issues might arise. Our second contribution is to extend the method to parametric space curves and, generally, to codimension > 1 , by computing the equations of (hyper)surfaces intersecting precisely at the given object. By means of Chow forms, we propose a new, practical, randomized algorithm that always produces correct output but possibly with a non-minimal number of surfaces. For space curves, we obtain 3 surfaces whose polynomials are of near-optimal degree; in this case, computation reduces to a Sylvester resultant. We illustrate our algorithm through a series of examples and compare our Maple prototype with other methods implemented in Maple, i.e., Gröbner basis and implicit matrix representations. Our Maple prototype is not faster but yields fewer equations and seems more robust than Maple's `implicitize`; it is also comparable with the other methods for degrees up to 6.

Joint work with I.S. Kotsireas.

6.12. Efficient certification of numeric solutions to eigenproblems

Participant: Bernard Mourrain.

In [24], we present an efficient algorithm for the certification of numeric solutions to eigenproblems. The algorithm relies on a mixture of ball arithmetic, a suitable Newton iteration, and clustering of eigenvalues that are close.

This is a joint work with Joris Van Der Hoeven.

6.13. Approximating multidimensional subset sum and minkowski decomposition of polygons

Participants: Ioannis Emiris, Anna Karasoulou.

In [10] we consider the approximation of two NP-hard problems: Minkowski Decomposition (MinkDecomp) of lattice polygons in the plane and the closely related problem of Multidimensional Subset Sum (kD-SS) in arbitrary dimension. In kD-SS we are given an input set S of k -dimensional vectors, a target vector t and we ask if there exists a subset of S that sums up to t . We prove, through a gap-preserving reduction, that, for general dimension k , kD-SS is not in APX although the classic 1D-SS is in PTAS. On the positive side, we present an $O(n^3/\epsilon^2)$ approximation grid based algorithm for 2D-SS, where n is the cardinality of the set and $\epsilon > 0$ bounds the difference of some measure of the input polygon and the sum of the output polygons. We also describe two approximation algorithms with a better experimental ratio. Applying one of these algorithms, and

a transformation from MinkDecomp to 2D-SS, we can approximate Mink-Decomp. For an input polygon Q and parameter ϵ , we return two summands A and B such that $A + B = Q'$ with Q' being bounded in relation to Q in terms of volume, perimeter, or number of internal lattice points, an additive error linear in and up to quadratic in the diameter of Q . A similar function bounds the Hausdorff distance between Q and Q' . We offer experimental results based on our implementation.

Joint with C. Tzovas.

6.14. High-dimensional approximate r -nets

Participants: Ioannis Emiris, Ioannis Psarros.

The construction of r -nets offers a powerful tool in computational and metric geometry. In [17], we focus on high-dimensional spaces and present a new randomized algorithm which efficiently computes approximate r -nets with respect to Euclidean distance. For any fixed $\epsilon > 0$, the approximation factor is $1 + \epsilon$ and the complexity is polynomial in the dimension and subquadratic in the number of points. The algorithm succeeds with high probability. More specifically, the best previously known LSH-based construction is improved in terms of complexity by reducing the dependence on ϵ , provided that ϵ is sufficiently small. Our method does not require LSH but, instead, follows Valiant's (2015) approach in designing a sequence of reductions of our problem to other problems in different spaces, under Euclidean distance or inner product, for which r -nets are computed efficiently and the error can be controlled. Our result immediately implies efficient solutions to a number of geometric problems in high dimension, such as finding the $(1 + \epsilon)$ -approximate k th nearest neighbor distance in time subquadratic in the size of the input.

Joint with G. Avarikioti, L. Kavouras.

6.15. Extraction of tori from minimal point sets

Participants: Laurent Busé, André Galligo.

In [7], a new algebraic method for extracting tori from a minimal point set, made of two oriented points and a simple point, is proposed. We prove a degree bound on the number of such tori; this bound is reached on examples, even when we restrict to smooth tori. Our method is based on pre-computed closed formulae well suited for numerical computations with approximate input data.

6.16. Scaffolding skeletons using spherical Voronoi diagrams

Participants: Alvaro Fuentes Suarez, Evelyne Hubert.

Given a skeleton made of line segments we describe how to obtain a coarse mesh (or scaffold) of a surface surrounding it. We emphasize in [21] the key result that allows us to complete a previous approach that could not treat skeletons with cycles.

6.17. G^1 -smooth splines on quad meshes with 4-split macro-patch elements

Participants: Ahmed Blidia, Bernard Mourrain.

We analyze the space of differentiable functions on a quad-mesh \mathcal{M} , which are composed of 4-split spline macro-patch elements on each quadrangular face. We describe explicit transition maps across shared edges, that satisfy conditions which ensure that the space of differentiable functions is ample on a quad-mesh of arbitrary topology. These transition maps define a finite dimensional vector space of G^1 spline functions of bi-degree $\leq (k, k)$ on each quadrangular face of \mathcal{M} . We determine the dimension of this space of G^1 spline functions for k big enough and provide explicit constructions of basis functions attached respectively to vertices, edges and faces. This construction requires the analysis of the module of syzygies of univariate b-spline functions with b-spline function coefficients. New results on their generators and dimensions are provided. Examples of bases of G^1 splines of small degree for simple topological surfaces are detailed and illustrated by parametric surface constructions.

This is a joint work with Nelly Villamizar

6.18. Hermite type spline spaces over rectangular meshes with complex topological structures

Participants: André Galligo, Bernard Mourrain.

Motivated by the Magneto HydroDynamic (MHD) simulation for Tokamaks with Isogeometric analysis, we present in [14] a new type of splines defined over a rectangular mesh with arbitrary topology, which are piecewise polynomial functions of bidegree (d, d) and C^r parameter continuity. In particular, we compute their dimension and exhibit basis functions called Hermite bases for bicubic spline spaces. We investigate their potential applications for solving partial differential equations (PDEs) over a complex physical domain in the framework of Isogeometric analysis. In particular, we analyze the property of approximation of these spline spaces for the L^2 -norm. Despite the fact that the basis functions are singular at extraordinary vertices, we show that the optimal approximation order and numerical convergence rates are reached by setting a proper parameterization.

This is a joint work with Meng Wu, Bernard Mourrain, André Galligo, Boniface Nkonga

6.19. H^1 -parameterizations of complex planar physical domains in isogeometric analysis

Participants: André Galligo, Bernard Mourrain.

Isogeometric analysis (IGA) is a method for solving geometric partial differential equations (PDEs). Generating parameterizations of a PDE's physical domain is the basic and important issues within IGA framework. In [13], we present a global H^1 -parameterization method for a planar physical domain with complex topology.

This is a joint work with Meng Wu, Boniface Nkonga.

6.20. Convergence rates with singular parameterizations for solving elliptic boundary value problems in isogeometric analysis

Participant: Bernard Mourrain.

In [15], we present convergence rates for solving elliptic boundary value problems with singular parameterizations in isogeometric analysis. First, the approximation errors with the $L^2(\Omega)$ -norm and the $H^1(\Omega)$ -seminorm are estimated locally. The impact of singularities is considered in this framework. Second, the convergence rates for solving PDEs with singular parameterizations are discussed. These results are based on a weak solution space that contains all of the weak solutions of elliptic boundary value problems with smooth coefficients. For the smooth weak solutions obtained by isogeometric analysis with singular parameterizations and the finite element method, both are shown to have the optimal convergence rates. For non-smooth weak solutions, the optimal convergence rates are reached by setting proper singularities of a controllable parameterization, even though convergence rates are not optimal by finite element method, and the convergence rates by isogeometric analysis with singular parameterizations are better than the ones by the finite element method.

This a joint work with Meng Wu, Yicao Wang, Boniface Nkonga, Changzheng Cheng.

6.21. Geometric modeling and deformation for shape optimization of ship hulls and appendages

Participants: Elisa Berrini, Bernard Mourrain.

The precise control of geometric models plays an important role in many domains such as computer-aided geometric design and numerical simulation. For shape optimization in computational fluid dynamics (CFD), the choice of control parameters and the way to deform a shape are critical. In [4], we describe a skeleton-based representation of shapes adapted for CFD simulation and automatic shape optimization. Instead of using the control points of a classic B-spline representation, we control the geometry in terms of architectural parameters. We assure valid shapes with a strong shape consistency control. Deformations of the geometry are performed by solving optimization problems on the skeleton. Finally, a surface reconstruction method is proposed to evaluate the shape's performances with CFD solvers. We illustrate the approach on two problems: the foil of an AC45 racing sail boat and the bulbous bow of a fishing trawler. For each case, we obtain a set of shape deformations and then we evaluate and analyzed the performances of the different shapes with CFD computations.

This is a joint work with Yann Roux, Matthieu Durand, Guillaume Fontaine.

6.22. Geometric model for automated multi-objective optimization of foils

Participants: Elisa Berrini, Bernard Mourrain.

The work in [18] describes a new generic parametric modeler integrated into an automated optimization loop for shape optimization. The modeler enables the generation of shapes by selecting a set of design parameters that controls a twofold parameterization: geometrical - based on a skeleton approach - and architectural - based on the experience of practitioners - to impact the system performance. The resulting forms are relevant and effective, thanks to a smoothing procedure that ensures the consistency of the shapes produced. As an application, we propose to perform a multi-objective shape optimization of an AC45 foil. The modeler is linked to the fluid solver AVANTI, coupled with Xfoil, and to the optimization toolbox FAMOSA.

This is a joint work with Régis Duvigneau, Matthieu Sacher, Yann Roux.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Grants with Industry

MISSLER Software provided a grant to the team AROMATH, related to the collaboration on geometric modeling methods for toolpath generation and machining.

8. Partnerships and Cooperations

8.1. Regional Initiatives

Our team AROMATH participates to the VADER project for VIRTUAL MODELING of RESPIRATION, UCA Jedi, axis "Modélisation, Physique et Mathématique du vivant". <http://benjamin.mauroy.free.fr/VADER>.

8.2. European Initiatives

8.2.1. FP7 & H2020 Projects

Program: Marie Skłodowska-Curie ITN

Project acronym: ARCADES

Project title: Algebraic Representations in Computer-Aided Design for complex Shapes

Duration: January 2016 - December 2019

Coordinator: I.Z. Emiris (NKUA, Athens, Greece, and ATHENA Research Innovation Center)

Scientist-in-charge at Inria: L. Busé

Other partners: U. Barcelona (Spain), Inria Sophia-Antipolis (France), J. Kepler University, Linz (Austria), SINTEF Institute, Oslo (Norway), U. Strathclyde, Glasgow (UK), Technische U. Wien (Austria), Evolute GmbH, Vienna (Austria).

Webpage: <http://arcades-network.eu/>

Abstract: ARCADES aims at disrupting the traditional paradigm in Computer-Aided Design (CAD) by exploiting cutting-edge research in mathematics and algorithm design. Geometry is now a critical tool in a large number of key applications; somewhat surprisingly, however, several approaches of the CAD industry are outdated, and 3D geometry processing is becoming increasingly the weak link. This is alarming in sectors where CAD faces new challenges arising from fast point acquisition, big data, and mobile computing, but also in robotics, simulation, animation, fabrication and manufacturing, where CAD strives to address crucial societal and market needs. The challenge taken up by ARCADES is to invert the trend of CAD industry lagging behind mathematical breakthroughs and to build the next generation of CAD software based on strong foundations from algebraic geometry, differential geometry, scientific computing, and algorithm design. Our game-changing methods lead to real-time modelers for architectural geometry and visualisation, to isogeometric and design-through-analysis software for shape optimisation, and marine design & hydrodynamics, and to tools for motion design, robot kinematics, path planning, and control of machining tools.

8.2.2. Collaborations in European Programs, Except FP7 & H2020

Program: Partnership Agreement for the Development Framework

Project acronym: RANWALK

Project title: Random walks for the computation of potential and capacitance of electronic circuits

Duration: December 2017 - May 2020

Coordinator: C. Bakolias (Helic S.A.)

Scientist-in-charge at Inria: I.Z. Emiris (NKUA, Athens, Greece, and ATHENA Research Innovation Center)

Other partners: ATHENA Research Innovation Center, Maroussi (Greece), School of Electrical Engineering, U. Patras (Greece).

Abstract: The Project aims at reducing the fabrication cost of new generation circuits and achieve significant progress in Electronic Design Automation (EDA) of Integrated Circuits with the development of innovative technology, which will radically upgrade Helic's existing products by giving them a unique lead in the global market. A key element of the modeling engine and the general approach is the method of random walks between a set of conductors, based on the solution of the Laplace equation and the calculation of the Green function in cubic-shaped areas. We target the geometric modeling of the physical design of the conductors, as well as the efficient and robust calculation of the above electrostatic parameters, with the ultimate goal of a rapid simulation of the circuit's accuracy. We focus on calculating the maximum cube gap between rectangular elements and, for this, we develop large-scale geometric software.

8.3. International Research Visitors

8.3.1. Visits of International Scientists

Vlada Pototskaia, University of Göttingen (Germany), visited from August 28th to September 15th. The collaboration with E. Hubert and B. Mourrain concerned AAK theory and its applications to approximate low rank sums of exponentials.

Ibrahim Nonkané, University of Ouagadougou, visited from September 25th to October 9th to work with L. Busé on the discriminant of complete intersections in a projective space.

Sotirios Choularias, University of Strachlyde (Scotland), visited us from August 5th to November 5th in the context of his secondment in the ITN network ARCADES, to work on boundary element methods and isogeometric analysis.

Yairon Cid Ruiz, University of Barcelona (Spain), visits us since October 1st, to work with L. Busé on the birationality of bi-graded rational maps in small dimensions.

Roser Homs Pons, University of Barcelona (Spain), visited us from October 9th to December 15th, to work with B. Mourrain on effective methods for the construction of Gorenstein algebra of low colength.

Simon Telen, University of Leuven (Belgium), visited us from August 24th to September 24th, to work with B. Mourrain on algebraic solvers and numerical linear algebra.

Meng Wu, University of Hefei (China), visited us from September 4th to September 29th, to work with B. Mourrain on isogeometric analysis and its applications.

Gang Xu, Hangzhou Dianzi University (China) visited us from September 7th to September 15th, to work with B. Mourrain on parameterization of computation domains for isogeometric analysis.

8.3.1.1. Internships

Antoine Deharveng, student at the engineer school of the University of Nice Sophia Antipolis, did his PFE (Projet de fin d'étude) with L. Busé until March 2017. He developed the interpolation of cylinders and cones passing through minimal point sets in the C++ library ASurfExt (<https://gitlab.inria.fr/lbuse/ASurfExt/wikis/home>).

Andrien Boudin did his internship with L. Busé from June 15th to September 15th. He developed and implemented a new method for the interpolation of torus through a minimal point set in the C++ library ASurfExt (<https://gitlab.inria.fr/lbuse/ASurfExt/wikis/home>).

Thomas Laporte, student at the engineer school of the University of Nice Sophia Antipolis, did his internship with A. Galligo from June 15th to September 15th. He studied "Hand modeling" and implemented in Axel a method inspired by the paper by P AULY .M, T AGLIASACCHI .A, T KACH .A. Sphere-Meshes for Real-Time Hand Modeling and Tracking. ACM Transactions on Graphics 2016. (Proc. of SIGGRAPH Asia).

Emmanouil Christoforou, Master student from NKUA, works from September 4th to December 31th on software development for the algebraic-geometric modeler Axel.

8.3.1.2. Research Stays Abroad

F. Yildirim was on secondment at Barcelona university (Spain), with Carlos D'Andrea, for 3 months (September 15-December 15).

A. Fuentes Suarez was on secondment at Athens university (Greece), with Ioannis Emiris, for 4 months (September-December).

A. Blidia was on secondment at Evolute, Vienna (Austria), with A. Schiftner (Evolute) and H. Pottmann (TUW), for 3 months (November-January).

E. Hubert received a grant from the London Mathematics Society to visit the University of Kent in Canterbury (UK) from February 21st to March 1st.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Member of the Organizing Committees

Bernard Mourrain (chair), Evelyne Hubert, André Galligo, Laurent Busé were members of the organizing committee of the conference MEGA (Effective Methods in Algebraic Geometry), held at the University of Nice – Sophia Antipolis, June 12 – 16, 2017.

Laurent Busé organized the first "Learning Week" of the ARCADES Network : « Opportunity Recognition » at Inria Sophia Antipolis, April 3-7, 2017.

Laurent Busé co-organized the workshop "Commutative Algebra, Syzygies and Singularities" at the University of Nice, December 4-6, 2017.

Ioannis Emiris and Christos Konaxis organized the 1st software workshop and the Midterm Review meeting of the ARCADES Network in Athens, November 27 to December 1, 2017.

Evelyne Hubert co-organized the mini-symposium *Symmetry and Structure in Algebraic Computation* in the conference SIAM Algebraic Geometry, July 31st to August 4th Atlanta (USA) as well as the first joint meeting of the London Mathematical Society and the Institute of Mathematics and its Applications *Symmetry and Computation*, October 12th London, UK.

Bernard Mourrain co-organized the mini-symposium *Sparse representations from moments* in the conference SIAM Algebraic Geometry, Atlanta July 31st to August 4th.

9.1.2. Scientific Events Selection

9.1.2.1. Member of the Conference Program Committees

Laurent Busé was a PC member of MACIS 2017.

Ioannis Emiris was PC member of ACM ISSAC 2017, and is a member of the Advisory Board of MEGA.

Bernard Mourrain was a member of Executive Committee of the conference MEGA 2017.

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

Ioannis Emiris is associate editor of the Journal of Symbolic Computation (since 2003) and of Mathematics in Computer Science (since 2016).

Bernard Mourrain is associate editor of the Journal of Symbolic Computation (since 2007) and of the SIAM Journal on Applied Algebra and Geometry (since 2016).

Evelyne Hubert is associate editor of the Journal of Symbolic Computation (since 2007) and the journal Foundation of Computational Mathematics (since May). She is a reviewer for Mathematical Reviews (since 2016).

9.1.3.2. Reviewer - Reviewing Activities

Laurent Busé reviewed for the journal *Math. Zeitschrift*, the journal *Computer Aided Geometric Design*, the *Journal of Computational and Applied Mathematics*, the journal *ACM Transactions on Graphics*, the *Journal on Applied Algebra and Geometry*, the *Journal of Software for Algebra and Geometry*, the *Journal of Algebra*, the journal *Annales de l'Institut Fourier*, the journal *Algebra & Number Theory*, and the conferences MEGA, ISSAC and MACIS 2017. He also reviewed an application for CIFRE PhD thesis grants for the ANRT.

Ioannis Emiris reviewed for the journals *ACM Transactions on Graphics*, *Discrete Applied Mathematics*, *Discrete & Computational Geometry*, *Graphical Models*, and the conferences MEGA, and ACM Solid & Physical Modeling 2017. He also reviewed applications for H2020 Projects.

Bernard Mourrain reviewed for the journals *Advances in Computational Mathematics*, *Computer Aided Design*, *Computer Aided Geometric Design*, *Computational Methods and Function Theory*, *Computer Methods in Applied Mechanics and Engineering*, *Discrete Applied Mathematics*, *Foundations of Computational Mathematics*, *Journal of Pure and Applied Algebra*, *Linear Algebra and Applications*, *Mathematical Programming*, and for the conferences MEGA, ISSAC. He is also guest editor of the Special Issue of the *Journal Of Symbolic Computation* after MEGA 2017.

Alessandro Oneto reviewed for the journal *Linear and Multilinear Algebra* and for *Mathematical Reviews* (MathSciNet).

Evelyne Hubert reviewed for the journals *Foundation of Computational Mathematics*, *Journal of Symbolic Computation*, *Journal of Algebra*, as well as for the Springer book series *Texts and Monographs in Symbolic Computation* and the conferences MEGA and CASC.

9.1.4. Invited Talks

Laurent Busé was invited to give a talk at the workshop on *Syzygies*, Trento, September 4-9, 2017.

Ioannis Emiris was invited to give a talk at ETH Zurich, Switzerland, in February 2017; and at EU Joint Research Center, Ispra, Italy, in October 2017.

A. Galligo was invited to give a talk at the workshop *Stillman's Conjecture and other Progress on Free Resolutions: (in honor of the sixtieth birthdays of Dave Bayer and Mike Stillman)*, July 17-21, 2017 at the University of California, Berkeley, USA.

Evelyne Hubert gave plenary lectures at the *International Symposium on Orthogonal Polynomials, Special Functions and Applications* (July 3-7 Canterbury, UK) and at the *Journées Nationales du GdR Informatique-Mathématiques* (March 14-16, Montpellier).

Evelyne Hubert was invited to give talks at the first joint meeting of the London Mathematical Society and the Institute of Mathematics and its Applications, *Symmetry and Computation*, October 12th London, UK; the international conference *Integrable systems, symmetries, and orthogonal polynomials*, September 18-22 Madrid, Spain; the workshop *Resultants, Subresultants and Applications* in the SIAM Conference on Algebraic Geometry, July 31st to August 4th Atlanta, USA; the *Symbolic Analysis* workshop in the conference *Foundations of Computational Mathematics*, July 10-19 Barcelona, Spain; the Mathematics Colloquium at University of Kent in Canterbury, February 28th, UK; *Inaugural meeting of the LMS-EMS Applied Algebra and Geometry Research Network*, February 21st Nottingham, UK.

Bernard Mourrain was invited to give a talk at USTC and HFUT, Hefei, China, April 11-12.

Alessandro Oneto was invited to give talks at the Seminario de Geometría Algebraica y Singularidades, BCAM, Bilbao (Spain) on Hadamard decompositions of tensors, May 16, 2017; at the SIAM Conference in Applied and Algebraic Geometry, GeorgiaTech, Atlanta (USA) on Hadamard decompositions of matrices (and tensors), August 02, 2017; at the Algebra and Geometry Seminar, KTH, Stockholm (Sweden) on Combinatorial tools for new questions on planar polynomial interpolation, October 11, 2017; at the workshop Commutative Algebra, Syzygies and Singularities, Nice (France) on A new question on planar polynomial interpolation and line arrangements, December 4, 2017;

9.1.5. Scientific Expertise

Bernard Mourrain was member of the committee of the HCERES for the evaluation of XLIM, Limoges.

Ioannis Emiris was elected member of the Scientific Council of the Hellenic Foundation of Research and Innovation (<http://www.elidek.gr>), responsible for Informatics and Mathematics.

9.1.6. Research Administration

Evelyne Hubert is a member of the *Conseil Académique de l'Université Côte d'Azur* (since October) and of the *Commission d'Évaluation* (since 2015), and participated to the hiring jury of junior researchers in Inria NGE and RBA.

Laurent Busé is a board member of the (national) labex AMIES (CRI-SAM representative) and a member of the steering committee of the MSI, *Maison de la Modélisation, de la Simulation et des Interactions* of the University Côte d'Azur. He is also an elected member of the CPRH (Commission Permanente de Ressources Humaines) of the math laboratory of the university of Nice, and is the Inria representative at the "Academic Council" and the "Research Commission" of the University of Nice Sophia Antipolis.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Licence : Ioannis Emiris, Discrete Math, 52 h, L1, U. Athens, Greece.
- Licence : Ioannis Emiris, Software development for algorithmic problems, 52 h, L3, U. Athens, Greece.
- Master : Ioannis Emiris, Computational Geometry, 52h, M1, U. Athens, Greece.
- Master : Ioannis Emiris, Structural Bioinformatics, 52h, M2, U. Athens, Greece.
- Master : Laurent Busé, Geometric Modeling, 27h ETD, M2, EPU of the University of Nice-Sophia Antipolis.
- Master 2: Bernard Mourrain, Computer Algebra, 15h, MDFI, Univ. Aix-Marseille, Luminy.
- Master 2: Bernard Mourrain, Symbolic-Numeric Computation, 6h, Master ACSYON, Limoges.

9.2.2. Supervision

PhD: Elisa Berrini, “Geometric modeling and deformation for automatic shape optimisation” [1], defended in June 2017. CIFRE collaboration with MyCFD. Supervised by Bernard Mourrain and Yann Roux (My-CFD, K-Epsilon).

PhD in progress: Erick Rodriguez-Bazan, Computational Invariant Theory and Applications, CORDI Inria SAM, started in November 2017, supervised by Evelyne Hubert.

PhD in progress: Evangelos Anagnostopoulos, Geometric algorithms for massive datasets. Started in May 2016, supervised by Ioannis Emiris.

PhD in progress: Evangelos Bartzos, Modeling motion. ARCADES Marie Skłodowska-Curie ITN, started in May 2016, supervised by Ioannis Emiris.

PhD in progress: Ahmed Blidia, New geometric models for the design and computation of complex shapes. ARCADES Marie Skłodowska-Curie ITN, started in September 2016, supervised by Bernard Mourrain.

PhD in progress: Alvaro-Javier Fuentes-Suarez, Skeleton-based modeling of smooth shapes. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Evelyne Hubert.

PhD in progress: Jouhayna Harmouch, Low rank structured matrix decomposition and completion. Cotutelle Univ. Liban, started in November 2015, cosupervised by Houssam Khalil and Bernard Mourrain.

PhD in progress: Clément Laroche, Change of representation in CAGD. ARCADES Marie Skłodowska-Curie ITN, started in Nov. 2016, supervised by Ioannis Emiris.

PhD in progress: Ioannis Psarros, Geometric approximation algorithms. Thales network (Greece), started in May 2015, supervised by Ioannis Emiris.

PhD in progress: Fatmanur Yildirim, Distances between points, rational Bézier curves and surfaces by means of matrix-based implicit representations. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Laurent Busé.

9.2.3. Juries

Anna Karasoulou (U. Athens) defended successfully her PhD in June 2017. She was supervised by I. Emiris, while B. Mourrain was in the three-person supervising committee of the thesis. Both were members of the seven-person exam committee.

L. Busé was a member of the committee of the PhD of Hao Quang Tran entitled *Images et fibres des applications rationnelles et algèbres d'éclatement*, University Pierre and Marie Curie, Paris, France, November 17th.

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Publications of the year

Doctoral Dissertations and Habilitation Theses

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- [2] A. KARASOULOU. *Algebraic combinatorics and resultant methods for polynomial system solving*, National and Kapodistrian University of Athens, Greece, May 2017, <https://hal.inria.fr/tel-01671507>

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- [3] A. BERNARDI, N. S. DALEO, J. D. HAUENSTEIN, B. MOURRAIN. *Tensor decomposition and homotopy continuation*, in "Differential Geometry and its Applications", August 2017, vol. 55, pp. 78-105 [DOI : 10.1016/J.DIFGEO.2017.07.009], <https://hal.inria.fr/hal-01250398>
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