IN PARTNERSHIP WITH:
Université de Lorraine

Activity Report 2017

Project-Team GAMBLE

Geometric Algorithms & Models Beyond the Linear & Euclidean realm

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
Algorithmics, Computer Algebra and Cryptology

Table of contents

1. Personnel **1**
2. Overall Objectives **2**
3. Research Program **3**
3.1. Non-linear computational geometry 3
3.2. Non-Euclidean computational geometry 4
3.3. Probability in computational geometry 5
4. Application Domains **5**
5. Highlights of the Year **6**
6. New Software and Platforms **6**
6.1. ISOTOP 6
6.2. CGAL Package : 3D periodic regular triangulations 7
6.3. CGAL Package : 2D hyperbolic triangulations 7
6.4. CGAL Package : 2D periodic hyperbolic triangulations 7
7. New Results **7**
7.1. Non-Linear Computational Geometry 7
7.1.1. Reliable location with respect to the projection of a smooth space curve 8
7.1.2. Computing effectively stabilizing controllers for a class of nD systems 8
7.2. Non-Euclidean Computational Geometry 8
7.3. Probabilistic Analysis of Geometric Data Structures and Algorithms 8
7.3.1. Delaunay triangulation of a random sample of a good sample has linear size 9
7.3.2. Delaunay triangulation of a random sampling of a generic surface 9
7.4. Classical Computational Geometry and Graph Drawing 9
7.4.1. Celestial Walk: A Terminating Oblivious Walk for Convex Subdivisions 9
7.4.2. Snap rounding polyhedral subdivisions 9
7.4.3. Explicit array-based compact data structures for triangulations 9
8. Bilateral Contracts and Grants with Industry **10**
9. Partnerships and Cooperations **10**
9.1. Regional Initiatives 10
9.2. National Initiatives 10
9.3. International Initiatives 10
9.4. International Research Visitors 11
9.4.1. Visits of International Scientists 11
9.4.2. Visits to International Teams 11
10. Dissemination **11**
10.1. Promoting Scientific Activities 11
10.1.1. Scientific Events Organisation 11
10.1.2. Scientific Events Selection 11
10.1.2.1. Member of the Conference Program Committees 11
10.1.2.2. Reviewer 11
10.1.3. Journal 12
10.1.3.1. Member of the Editorial Boards 12
10.1.3.2. Reviewer - Reviewing Activities 12
10.1.4. Invited Talks 12
10.1.5. Leadership within the Scientific Community 12
10.1.5.1. Steering Committees 12
10.1.5.2. Learned societies 12
10.1.6. Scientific Expertise 12
10.1.7. Research Administration 12
10.1.7.1. Hiring committees 12

10.1.7.2. National committees 12
10.1.7.3. Local Committees and Responsabilities 12
10.1.7.4. Websites 13
10.2. Teaching - Supervision - Juries 13
10.2.1. Teaching 13
10.2.2. Supervision 13
10.2.3. Internships 14
10.3. Popularization 14
11. Bibliography **14**

Project-Team GAMBLE

Creation of the Team: 2017 January 01, updated into Project-Team: 2017 July 01

Keywords:

Computer Science and Digital Science:

A5.5.1. - Geometrical modeling
A7.1. - Algorithms
A8.1. - Discrete mathematics, combinatorics
A8.3. - Geometry, Topology
A8.4. - Computer Algebra

Other Research Topics and Application Domains:

B1.1.1. - Structural biology
B1.2. - Neuroscience and cognitive science
B2.6. - Biological and medical imaging
B3. - Environment and planet
B5.2. - Design and manufacturing
B5.5. - Materials
B5.7. - 3D printing
B6.2.2. - Radio technology

1. Personnel

Research Scientists

Olivier Devillers [Team leader, Inria, Senior Researcher, HDR]
Sylvain Lazard [Inria, Senior Researcher, HDR]
Guillaume Moroz [Inria, Researcher]
Marc Pouget [Inria, Researcher]
Monique Teillaud [Inria, Senior Researcher, HDR]

Faculty Member

Laurent Dupont [Univ de Lorraine, Associate Professor]

Post-Doctoral Fellow

Vincent Despré [Inria, from Oct 2017]

PhD Students

Sény Diatta [Univ de Lorraine, Mobility program until Sept 2017, then ATER from Oct 2017]
Charles Duménil [Univ de Lorraine]
Iordan Iordanov [Univ de Lorraine]
George Krait [Inria, CORDI-S, from Nov 2017]

Technical staff

Eric Biagioli [Inria, until Mar 2017]

Interns

Jian Qian [Ecole Normale Supérieure Paris, from Jul 2017 until Aug 2017]
Guillermo Alfonso Reyes Guzman [Univ de Lorraine, from Mar 2017 until Jul 2017]

Administrative Assistants

Sophie Drouot [Inria]

Virginie Priester [CNRS]

Visiting Scientist

Gert Vegter [Univ of Groningen , from Sep 2017 until Oct 2017]

2. Overall Objectives

2.1. Overall Objectives

Starting in the eighties, the emerging computational geometry community has put a lot of effort to design and analyze algorithms for geometric problems. The most commonly used framework was to study the worst-case theoretical complexity of geometric problems involving linear objects (points, lines, polyhedra...) in Euclidean spaces. This so-called *classical computational geometry* has some known limitations:

- Objects: dealing with objects only defined by linear equations.
- Ambient space: considering only Euclidean spaces.
- Complexity: worst-case complexities often do not capture realistic behaviour.
- Dimension: complexities are often exponential in the dimension.
- Robustness: ignoring degeneracies and rounding errors.

Even if these limitations have already got some attention from the community [26], a quick look at the flagship conference SoCG [1] proceedings shows that these topics still need a big effort.

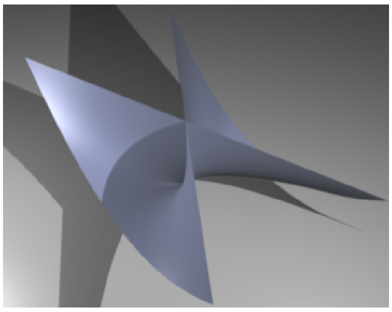
It should be stressed that, in this document, the notion of certified algorithms is to be understood with respect to robustness issues. In other words, certification does not refer to programs that are proven correct with the help of mechnical proof assistants such as Coq, but to algorithms that are proven correct on paper even in the presence of degeneracies and computer-induced numerical rounding errors.

We address several of the above limitations:

• **Non-linear computational geometry.** Curved objects are ubiquitous in the world we live in. However, despite this ubiquity and decades of research in several communities, curved objects are far from being robustly and efficiently manipulated by geometric algorithms. Our work on, for instance, quadric intersections and certified drawing of plane curves has proven that dramatic improvements can be accomplished when the right mathematics and computer science are put into motion. In this direction, many problems are fundamental and solutions have potential industrial impact in Computer Aided Design and Robotics for instance. Intersecting NURBS (Non-uniform rational basis spline) and meshing singular surfaces in a certified manner are important examples of such problems.

• **Non-Euclidean computational geometry.** Triangulations are central geometric data structures in many areas of science and engineering. Traditionally, their study has been limited to the Euclidean setting. Needs for triangulations in non-Euclidean settings have emerged in many areas dealing with objects whose sizes range from the nuclear to the astrophysical scale, and both in academia and in industry. It has become timely to extend the traditional focus on $\mathbb{R}^d$ of computational geometry and encompass non-Euclidean spaces.

• **Probability in computational geometry.** The design of efficient algorithms is driven by the analysis of their complexity. Traditionally, worst-case input and sometimes uniform distributions are considered and many results in these settings have had a great influence on the domain. Nowadays, it is necessary to be more subtle and to prove new results in between these two extreme settings. For instance, smoothed analysis, which was introduced for the simplex algorithm and which we applied successfully to convex hulls, proves that such promising alternatives exist.



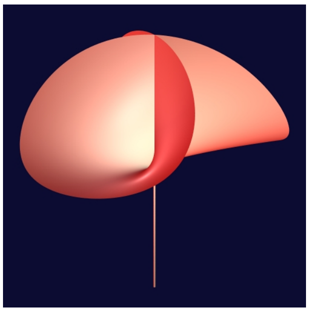


Figure 1. Two views of the Whitney umbrella (on the left, the "stick" of the umbrella, i.e., the negative z-axis, is missing). Right picture from [Wikipedia], left picture from [Lachaud et al.].

3. Research Program

3.1. Non-linear computational geometry

As mentioned above, curved objects are ubiquitous in real world problems modelings and in computer science and, despite this fact, there are very few problems on curved objects that admit robust and efficient algorithmic solutions without first discretizing the curved objects into meshes. Meshing curved objects induces some loss of accuracy which is sometimes not an issue but which can also be most problematic depending on the application. In addition, discretizing induces a combinatorial explosion which could cause a loss in efficiency compared to a direct solution on the curved objects (as our work on quadrics has demonstrated with flying colors [32], [33], [34], [36], [40]). But it is also crucial to know that even the process of computing meshes that approximate curved objects is far from being resolved. As a matter of fact there is no algorithm capable of computing in practice meshes with certified topology of even rather simple singular 3D surfaces, due to the high constants in the theoretical complexity and the difficulty of handling degenerate cases. Even in 2D, meshing an algebraic curve with the correct topology, that is in other words producing a correct drawing of the curve (without knowing where the domain of interest is), is a very difficult problem on which we have recently made important contributions [19], [20], [41].

It is thus to be understood that producing practical robust and efficient algorithmic solutions to geometric problems on curved objects is a challenge on all and even the most basic problems. The basicness and fundamentality of two problems we mentioned above on the intersection of 3D quadrics and on the drawing in a topologically certified way of plane algebraic curves show rather well that the domain is still at its infancy. And it should be stressed that these two sets of results were not anecdotical but flagship results produced during the lifetime of VEGAS team.

There are many problems in this theme that are expected to have high long-term impacts. Intersecting NURBS (Non-uniform rational basis spline) in a certified way is an important problem in computer-aided design and manufacturing. As hinted above, meshing objects in a certified way is important when topology matters. The 2D case, that is essentially drawing plane curves with the correct topology, is a fundamental problem with far-reaching applications in research or R&D. Notice that on such elementary problems it is often difficult to predict the reach of the applications; as an example, we were astonished by the scope of the applications of our software on 3D quadric intersection [2] which was used by researchers in, for instance, photochemistry, computer vision, statistics and mathematics.

[1] Symposium on Computational Geometry. http://www.computational-geometry.org/.
[2] QI: http://vegas.loria.fr/qi/.

3.2. Non-Euclidean computational geometry

Figure 2. Left: 3D mesh of a gyroid (triply periodic surface) [43]. Right: Simulation of a periodic Delaunay triangulation of the hyperbolic plane [15].

Triangulations, in particular Delaunay triangulations, in the *Euclidean space* $\mathbb{R}^d$ have been extensively studied throughout the 20th century and they are still a very active research topic. Their mathematical properties are now well understood, many algorithms to construct them have been proposed and analyzed (see the book of Aurenhammer *et al.* [14]). Some members of GAMBLE have been contributing to these algorithmic advances (see, e.g. [18], [51], [29], [17]); they have also contributed robust and efficient triangulation packages through the state-of-the-art Computational Geometry Algorithms Library CGAL, [3] whose impact extends far beyond computational geometry. Application fields include particle physics, fluid dynamics, shape matching, image processing, geometry processing, computer graphics, computer vision, shape reconstruction, mesh generation, virtual worlds, geophysics, and medical imaging. [4]

It is fair to say that little has been done on non-Euclidean spaces, in spite of the large number of questions raised by application domains. Needs for simulations or modeling in a variety of domains [5] ranging from the infinitely small (nuclear matter, nano-structures, biological data) to the infinitely large (astrophysics) have led us to consider 3D periodic Delaunay triangulations, which can be seen as Delaunay triangulations in the 3D *flat torus*, quotient of $\mathbb{R}^3$ under the action of some group of translations [24]. This work has already yielded a fruitful collaboration with astrophysicists [37], [52] and new collaborations with physicists are emerging. To the best of our knowledge, our CGAL package [23] is the only publicly available software that computes Delaunay triangulations of a 3D flat torus, in the special case where the domain is cubic. This case, although restrictive is already useful. [6] We have also generalized this algorithm to the case of general d-dimensional compact flat manifolds [25]. As far as non-compact manifolds are concerned, past approaches, limited to the two-dimensional case, have stayed theoretical [42].

Interestingly, even for the simple case of triangulations on the *sphere*, the software packages that are currently available are far from offering satisfactory solutions in terms of robustness and efficiency [22].

Moreover, while our solution for computing triangulations in hyperbolic spaces can be considered as ultimate [15], the case of *hyperbolic manifolds* has hardly been explored. Hyperbolic manifolds are quotients of a hyperbolic space by some group of hyperbolic isometries. Their triangulations can be seen as hyperbolic periodic

[3] http://www.cgal.org/

[4] See http://www.cgal.org/projects.html for details.

[5] See http://www.loria.fr/~teillaud/PeriodicSpacesWorkshop/, http://www.lorentzcenter.nl/lc/web/2009/357/info.php3?wsid=357 and http://neg15.loria.fr/.

[6] See examples at http://www.cgal.org/projects.html

triangulations. Periodic hyperbolic triangulations and meshes appear for instance in geometric modeling [44], neuromathematics [27], or physics [47]. Even the simplest possible case (a surface homeomorphic to the torus with two handles) shows strong mathematical difficulties [16], [49].

3.3. Probability in computational geometry

In most computational geometry papers, algorithms are analyzed in the worst-case setting. It often yields too pessimistic complexities that arise only in pathological situations that are unlikely to occur in practice. On the other hand, probabilistic geometry gives analyses of great precisions [45], [46], [21], but using hypotheses with much more randomness than in most realistic situations. We are developing new algorithmic designs improving state-of-the-art performances in random settings that are not overly simplified and that can thus reflect many realistic situations.

Twelve years ago, smooth analysis was introduced by Spielman and Teng analyzing the simplex algorithm by averaging on some noise on the data [50] (and they won the Gödel prize). In essence, this analysis smoothes the complexity around worst-case situations, thus avoiding pathological scenarios but without considering unrealistic randomness. In that sense, this method makes a bridge between full randomness and worst case situations by tuning the noise intensity. The analysis of computational geometry algorithms within this framework is still embryonic. To illustrate the difficulty of the problem, we started working in 2009 on the smooth analysis of the size of the convex hull of a point set, arguably the simplest computational geometry data structure; then, only one very rough result from 2004 existed [28] and we only obtained in 2015 breakthrough results, but still not definitive [31], [30], [35].

Another example of problem of different flavor concerns Delaunay triangulations, which are rather ubiquitous in computational geometry. When Delaunay triangulations are computed for reconstructing meshes from point clouds coming from 3D scanners, the worst-case scenario is, again, too pessimistic and the full randomness hypothesis is clearly not adapted. Some results exist for "good samplings of generic surfaces" [13] but the big result that everybody wishes for is an analysis for random samples (without the extra assumptions hidden in the "good" sampling) of possibly non-generic surfaces.

Trade-off between full randomness and worst case may also appear in other forms such as dependent distributions, or random distribution conditioned to be in some special configurations. Simulating these kinds of geometric distributions is currently out of reach for more than few hundred points [38] although it has practical applications in physics or networks.

4. Application Domains

4.1. Applications of computational geometry

Many domains of science can benefit from the results developed by GAMBLE. Curves and surfaces are ubiquitous in all sciences to understand and interpret raw data as well as experimental results. Still, the non-linear problems we address are rather basic and fundamental, and it is often difficult to predict the impact of solutions in that area. The short-term industrial impact is likely to be small because, on basic problems, industries have used ad hoc solutions for decades and have thus got used to it. The example of our work on quadric intersection is typical: even though we were fully convinced that intersecting 3D quadrics is such an elementary/fundamental problem that it ought to be useful, we were the first to be astonished by the scope of the applications of our software [7] (which was the first and still is the only one –to our knowledge– to compute robustly and efficiently the intersection of 3D quadrics) which has been used by researchers in, for instance, photochemistry, computer vision, statistics, and mathematics. Our work on certified drawing of plane (algebraic) curves falls in the same category. It seems obvious that it is widely useful to be able to draw curves correctly (recall also that part of the problem is to determine where to look in the plane) but it is quite hard to come up with specific examples of fields where this is relevant. A contrario, we know that certified

[7] QI: http://vegas.loria.fr/qi/.

meshing is critical in mechanical-design applications in robotics, which is a non-obvious application field. There, the singularities of a manipulator often have degrees higher than 10 and meshing the singular locus in a certified way is currently out of reach. As a result, researchers in robotics can only build physical prototypes for validating, or not, the approximate solutions given by non-certified numerical algorithms.

The fact that several of our pieces of software for computing non-Euclidean triangulations have already been requested by users long before they become public is a good sign for their wide future impact once in CGAL. This will not come as a surprise, since most of the questions that we have been studying followed from discussions with researchers outside computer science and pure mathematics. Such researchers are either users of our algorithms and software, or we meet them in workshops. Let us only mention a few names here. We have already referred above to our collaboration with Rien van de Weijgaert [37], [52] (astrophysicist, Groningen, NL). Michael Schindler [48] (theoretical physicist, ENSPCI, CNRS, France) is using our prototype software for 3D periodic weighted triangulations. Stephen Hyde and Vanessa Robins (applied mathematics and physics at Australian National University) have recently signed a software license agreement with INRIA that allows their group to use our prototype for 3D periodic meshing. Olivier Faugeras (neuromathematics, Inria Sophia Antipolis) had come to us and mentioned his needs for good meshes of the Bolza surface [27] before we started to study them. Such contacts are very important both to get feedback about our research and to help us choose problems that are relevant for applications. These problems are at the same time challenging from the mathematical and algorithmic points of view. Note that our research and our software are generic, i.e., we are studying fundamental geometric questions, which do not depend on any specific application. This recipe has made the sucess of the CGAL library.

Probabilistic models for geometric data are widely used to model various situations ranging from cell phone distribution to quantum mechanics. The impact of our work on probabilistic distributions is twofold. On the one hand, our studies of properties of geometric objects built on such distributions will yield a better understanding of the above phenomena and has potential impact in many scientific domains. On the other hand, our work on simulations of probabilistic distributions will be used by other teams, more maths oriented, to study these distributions.

5. Highlights of the Year

5.1. Highlights of the Year

The project-team VEGAS terminated at the end of 2016. Our main highlight is actually the creation of the new project-team GAMBLE (Geometric Algorithms and Models Beyond the Linear and Euclidean realm) on July 1st.

Another highlight of this year is that after two failures, both ANR projects we are coordinating finally won at the ANR lottery with two projects that will start in 2018: ASPAG (ANR-17-CE40-0017) and SoS (ANR-17-CE40-0033).

6. New Software and Platforms

6.1. ISOTOP

Topology and geometry of planar algebraic curves
KEYWORDS: Topology - Curve plotting - Geometric computing
FUNCTIONAL DESCRIPTION: Isotop is a Maple software for computing the topology of an algebraic plane curve, that is, for computing an arrangement of polylines isotopic to the input curve. This problem is a necessary key step for computing arrangements of algebraic curves and has also applications for curve plotting. This software has been developed since 2007 in collaboration with F. Rouillier from Inria Paris - Rocquencourt. It is based on the method described in [Cheng, J., Lazard, S., Pe

NEWS OF THE YEAR: In 2017, an ADT FastTrack funded a 6 months engineer contract to port the Maple code to C code. In addition, another local engineer from Inria Nancy (Benjamin Dexheimer) implemented a web server to improve the diffusion of our software.

- Participants: Elias Tsigaridas, Jinsan Cheng, Luis Penaranda, Marc Pouget and Sylvain Lazard
- Contact: Sylvain Lazard
- URL: http://vegas.loria.fr/isotop/

6.2. CGAL Package : 3D periodic regular triangulations

KEYWORDS: Flat torus - CGAL - Geometry - Geometric computing - Voronoi diagram - Delaunay triangulation - Triangulation
FUNCTIONAL DESCRIPTION: This class of CGAL (Computational Geometry Algorithms Library http://www.cgal.org) allows to build and handle periodic regular triangulations whose fundamental domain is a cube in 3D. Triangulations are built incrementally and can be modified by insertion of weighted points or removal of vertices. They offer location facilities for weighted points. The class offers nearest neighbor queries for the additively weighted distance and primitives to build the dual weighted Voronoi diagrams.

- Participants: Aymeric Pellé, Mael Rouxel-Labbe and Monique Teillaud
- Contact: Monique Teillaud
- URL: https://doc.cgal.org/latest/Manual/packages.html#PkgPeriodic3Triangulation3Summary

6.3. CGAL Package : 2D hyperbolic triangulations

KEYWORDS: Geometry - Delaunay triangulation - Hyperbolic space
FUNCTIONAL DESCRIPTION: This package implements the construction of Delaunay triangulations in the Poincaré disk model.

- Authors: Mikhail Bogdanov, Olivier Devillers and Monique Teillaud
- Contact: Monique Teillaud
- Publication: Hyperbolic Delaunay Complexes and Voronoi Diagrams Made Practical
- URL: https://github.com/CGAL/cgal-public-dev/tree/Hyperbolic_triangulation_2-MBogdanov

6.4. CGAL Package : 2D periodic hyperbolic triangulations

KEYWORDS: Geometry - Delaunay triangulation - Hyperbolic space
FUNCTIONAL DESCRIPTION: This module implements the computation of Delaunay triangulations of the Bolza surface.

- Authors: Iordan Iordanov and Monique Teillaud
- Contact: Monique Teillaud
- Publication: Implementing Delaunay Triangulations of the Bolza Surface
- URL: https://github.com/CGAL/cgal-public-dev/tree/Periodic_4_hyperbolic_triangulation_2-IIordanov

7. New Results

7.1. Non-Linear Computational Geometry

Participants: Sény Diatta, Laurent Dupont, George Krait, Sylvain Lazard, Guillaume Moroz, Marc Pouget.

7.1.1. Reliable location with respect to the projection of a smooth space curve

Consider a plane curve $\mathcal{B}$ defined as the projection of the intersection of two analytic surfaces in $\mathbb{R}^3$ or as the apparent contour of a surface. In general, $\mathcal{B}$ has node or cusp singular points and thus is a singular curve. Our main contribution [9] is the computation of a data structure for answering point location queries with respect to the subdivision of the plane induced by $\mathcal{B}$. This data structure is composed of an approximation of the space curve together with a topological representation of its projection $\mathcal{B}$. Since $\mathcal{B}$ is a singular curve, it is challenging to design a method only based on reliable numerical algorithms.

In a previous work [39], we have shown how to describe the set of singularities of $\mathcal{B}$ as regular solutions of a so-called ball system suitable for a numerical subdivision solver. Here, the space curve is first enclosed in a set of boxes with a certified path-tracker to restrict the domain where the ball system is solved. Boxes around singular points are then computed such that the correct topology of the curve inside these boxes can be deduced from the intersections of the curve with their boundaries. The tracking of the space curve is then used to connect the smooth branches to the singular points. The subdivision of the plane induced by $\mathcal{B}$ is encoded as an extended planar combinatorial map allowing point location. We experimented our method and show that our reliable numerical approach can handle classes of examples that are not reachable by symbolic methods.

7.1.2. Computing effectively stabilizing controllers for a class of nD systems

In this paper [1], we study the internal stabilizability and internal stabilization problems for multidimensional (nD) systems. Within the fractional representation approach, a multidimensional system can be studied by means of matrices with entries in the integral domain of structurally stable rational fractions, namely the ring of rational functions which have no poles in the closed unit polydisc $\overline{\mathbb{U}}^n = \{z = (z_1, ..., z_n) \in \mathbb{C}^n \mid |z_1| \leq 1, ..., |z_n| \leq 1\}$.

It is known that the internal stabilizability of a multidimensional system can be investigated by studying a certain polynomial ideal $I = \langle p_1, ..., p_r \rangle$ that can be explicitly described in terms of the transfer matrix of the plant. More precisely the system is stabilizable if and only if $V(I) = \{z \in \mathbb{C}^n \mid p_1(z) = \cdots = p_r(z) = 0\} \cap \overline{\mathbb{U}}^n = \varnothing$. In the present article, we consider the specific class of linear nD systems (which includes the class of 2D systems) for which the ideal I is zero-dimensional, i.e., the p_i's have only a finite number of common complex zeros. We propose effective symbolic-numeric algorithms for testing if $V(I) \cap \overline{\mathbb{U}}^n = \varnothing$, as well as for computing, if it exists, a stable polynomial $p \in I$ which allows the effective computation of a stabilizing controller. We illustrate our algorithms through an example and finally provide running times of prototype implementations for 2D and 3D systems.

7.2. Non-Euclidean Computational Geometry

Participants: Vincent Despré, Iordan Iordanov, Monique Teillaud.

7.2.1. Implementing Delaunay Triangulations of the Bolza Surface

The CGAL library offers software packages to compute Delaunay triangulations of the (flat) torus of genus one in two and three dimensions. To the best of our knowledge, there is no available software for the simplest possible extension, i.e., the Bolza surface, a hyperbolic manifold homeomorphic to a torus of genus two. We present an implementation based on the theoretical results and the incremental algorithm proposed recently. We describe the representation of the triangulation, we detail the different steps of the algorithm, we study predicates, and report experimental results [5]. The implementation is publicly available in the development branch of CGAL on `github` [8] and will soon be submitted for integration in the library.

7.3. Probabilistic Analysis of Geometric Data Structures and Algorithms

Participants: Olivier Devillers, Charles Duménil.

[8] https://members.loria.fr/Monique.Teillaud/DT_Bolza_SoCG17/

7.3.1. Delaunay triangulation of a random sample of a good sample has linear size

A good sample is a point set such that any ball of radius ϵ contains a constant number of points. The Delaunay triangulation of a good sample is proved to have linear size, unfortunately this is not enough to ensure a good time complexity of the randomized incremental construction of the Delaunay triangulation. In this paper we prove that a random Bernoulli sample of a good sample has a triangulation of linear size. This result allows to prove that the randomized incremental construction needs an expected linear size and an expected $O(n \log n)$ time [8].

This work was done in collaboration with Marc Glisse (Project-team DATASHAPE).

7.3.2. Delaunay triangulation of a random sampling of a generic surface

The complexity of the Delaunay triangulation of n points distributed on a surface ranges from linear to quadratic. We prove that when the points are evenly distributed on a smooth compact generic surface the expected size of the Delaunay triangulation is $O(n)$. This result has to be compared with a bound of $O(n \log n)$ when the points are a deterministic good sample of the surface under the same hypotheses on the surface [13].

7.4. Classical Computational Geometry and Graph Drawing

Participants: Olivier Devillers, Sylvain Lazard.

7.4.1. Celestial Walk: A Terminating Oblivious Walk for Convex Subdivisions

We present a new oblivious walking strategy for convex subdivisions. Our walk is faster than the straight walk and more generally applicable than the visiblity walk. To prove termination of our walk we use a novel monotonically decreasing distance measure [10].

This work was done in collaboration with Wouter Kuijper and Victor Ermolaev (Nedap Security Management).

7.4.2. Snap rounding polyhedral subdivisions

Let $\mathcal{P}$ be a set of n polygons in $\mathbb{R}^3$, each of constant complexity and with pairwise disjoint interiors. We propose a rounding algorithm that maps $\mathcal{P}$ to a simplicial complex $\mathcal{Q}$ whose vertices have integer coordinates. Every face of $\mathcal{P}$ is mapped to a set of faces (or edges or vertices) of $\mathcal{Q}$ and the mapping from $\mathcal{P}$ to $\mathcal{Q}$ can be build through a continuous motion of the faces such that (i) the L_∞ Hausdorff distance between a face and its image during the motion is at most 3/2 and (ii) if two points become equal during the motion they remain equal through the rest of the motion. In the worse, the size of $\mathcal{Q}$ is $O(n^{15})$, but, under reasonable hypotheses, this complexities decreases to $O(n^5)$.

This work was done in collaboration with William J. Lenhart (Williams College, USA).

7.4.3. Explicit array-based compact data structures for triangulations

We consider the problem of designing space efficient solutions for representing triangle meshes. Our main result is a new explicit data structure for compactly representing planar triangulations: if one is allowed to permute input vertices, then a triangulation with n vertices requires at most $4n$ references ($5n$ references if vertex permutations are not allowed). Our solution combines existing techniques from mesh encoding with a novel use of maximal Schnyder woods. Our approach extends to higher genus triangulations and could be applied to other families of meshes (such as quadrangular or polygonal meshes). As far as we know, our solution provides the most parsimonious data structures for triangulations, allowing constant time navigation. Our data structures require linear construction time, and are fast decodable from a standard compressed format without using additional memory allocation. All bounds, concerning storage requirements and navigation performances, hold in the worst case. We have implemented and tested our results, and experiments confirm the practical interest of compact data structures.

This work was done in collaboration with Luca Castelli Aleardi (LIX).

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

A two-years licence and cooperation agreement was signed on April 1st, 2016 between WATERLOO MAPLE INC., Ontario, Canada (represented by Laurent Bernardin, its Executive Vice President Products and Solutions) and Inria. On the Inria side, this contract involves the teams VEGAS and OURAGAN (Paris), and it is coordinated by Fabrice Rouillier (OURAGAN).

F. Rouillier and VEGAS are the developers of the ISOTOP software for the computation of topology of curves. One objective of the contract is to transfer a version of ISOTOP to WATERLOO MAPLE INC.

9. Partnerships and Cooperations

9.1. Regional Initiatives

We organized, with colleagues of the mathematics department (Institut Elie Cartan Nancy) a regular working group about geometry and probability.

9.2. National Initiatives

9.2.1. ANR SingCAST

The objective of the young-researcher ANR grant SingCAST is to intertwine further symbolic/numeric approaches to compute efficiently solution sets of polynomial systems with topological and geometrical guarantees in singular cases. We focus on two applications: the visualization of algebraic curves and surfaces and the mechanical design of robots.

After identifying classes of problems with restricted types of singularities, we plan to develop dedicated symbolic-numerical methods that take advantage of the structure of the associated polynomial systems that cannot be handled by purely symbolic or numerical methods. Thus we plan to extend the class of manipulators that can be analyzed, and the class of algebraic curves and surfaces that can be visualized with certification.

The project has a total budget of 100kE. It started on March 1st 2014 and will finished in August 2018. It is coordinated by Guillaume Moroz, with a participation of 60%, and Marc Pouget with a participation of 40%.

Project website: https://project.inria.fr/singcast/.

9.3. International Initiatives

9.3.1. Inria Associate Teams Not Involved in an Inria International Lab

9.3.1.1. Astonishing

Title: ASsociate Team On Non-ISH euclIdeaN Geometry

International Partners (Institution - Laboratory - Researcher):

University of Groningen (Netherlands) - Johann Bernouilli Institute of Mathematics and Computer Science - Gert Vegter

University of Luxembourg - Mathematics Research Unit - Jean-Marc Schlenker

Université Paris Est Marne-la-Vallée - Laboratoire d'Informatique Gaspard Monge - Éric Colin de Verdière

Start year: 2017

See also: https://members.loria.fr/Monique.Teillaud/collab/Astonishing/

Some research directions in computational geometry have hardly been explored. The spaces in which most algorithms have been designed are the Euclidean spaces R^d. To extend further the scope of applicability of computational geometry, other spaces must be considered, as shown by the concrete needs expressed by our contacts in various fields as well as in the literature. Delaunay triangulations in non-Euclidean spaces are required, e.g., in geometric modeling, neuromathematics, or physics. Topological problems for curves and graphs on surfaces arise in various applications in computer graphics and road map design. Providing robust implementations of these results is a key towards their reusability in more applied fields. We aim at studying various structures and algorithms in other spaces than R^d, from a computational geometry viewpoint. Proposing algorithms operating in such spaces requires a prior deep study of the mathematical properties of the objects considered, which raises new fundamental and difficult questions that we want to tackle.

9.4. International Research Visitors

9.4.1. Visits of International Scientists

Gert Vegter spent three weeks in GAMBLE in the framework of the Astonishing associate team.

9.4.2. Visits to International Teams

Olivier Devillers spent one month at Computational Geometry Lab of Carleton University http://cglab.ca/about.html.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

10.1.1.1. Member of the Organizing Committees

Sylvain Lazard organized with S. Whitesides (Victoria University) the 16th Workshop on Computational Geometry at the Bellairs Research Institute of McGill University in Feb. (1 week workshop on invitation).

Monique Teillaud co-organized with Claire Mathieu Celebrating Claude Puech's birthday, Paris, June 12.

Monique Teillaud co-organized the workshop Geometric Aspects of Materials Science with Vanessa Robins and Ileana Streinu, Brisbane, Australia, July 4–5.

Monique Teillaud co-organized with the Astonishing partners the Astonishing workshop at Loria/Inria nancy, September 25–26.

10.1.2. Scientific Events Selection

10.1.2.1. Member of the Conference Program Committees

Sylvain Lazard was a member of the program committee of SoCG, *Symposium on Computational Geometry*.

Monique Teillaud was a member of the program committee of WADS, *Algorithms and Data Structures Symposium*.

10.1.2.2. Reviewer

All members of the team are regular reviewers for the conferences of our field, namely the *Symposium on Computational Geometry* (SoCG) and the *International Symposium on Symbolic and Algebraic Computation* (ISSAC) and also SODA, CCCG, EuroCG.

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

Monique Teillaud is a managing editor of JoCG, *Journal of Computational Geometry* and a member of the editorial board of IJCGA, *International Journal of Computational Geometry and Applications*.

Marc Pouget and Monique Teillaud are members of the CGAL editorial board.

10.1.3.2. Reviewer - Reviewing Activities

All members of the team are regular reviewers for the journals of our field, namely *Discrete and Computational Geometry* (DCG), *Computational Geometry. Theory and Applications* (CGTA), *Journal of Computational Geometry* (JoCG), *International Journal on Computational Geometry and Applications* (IJCGA), *Journal on Symbolic Computations* (JSC), *SIAM Journal on Computing* (SICOMP), *Mathematics in Computer Science* (MCS), etc.

10.1.4. Invited Talks

Monique Teillaud was an invited speaker of CATS, *Computational & Algorithmic Topology*, Sydney, Australia, June 27 – July 1st.

Guillaume Moroz was invited to give a talk at the Effective Geometry and Algebra seminar at IRMAR.

10.1.5. Leadership within the Scientific Community

10.1.5.1. Steering Committees

Monique Teillaud is chairing the Steering Committee of the Symposium on Computational Geometry (SoCG). She was a member of the Steering Committee of the European Symposium on Algorithms (ESA) until September.

10.1.5.2. Learned societies

Monique Teillaud is a member of the Scientific Board of the *Société Informatique de France* (SIF).

10.1.6. Scientific Expertise

Monique Teillaud acted as a reviewer for the DFG, *Deutsche Forschungsgemeinschaft* (German Research Foundation).

10.1.7. Research Administration

10.1.7.1. Hiring committees

Olivier Devillers was the representative of LORIA in the hiring committee for an Associate Professor (MCF) position (IUT St Dié/LORIA) and composed the committee with the president.

10.1.7.2. National committees

L. Dupont is the secretary of *Commission Pédagogique Nationale Carrières Sociales / Information-Communication / Métiers du Multimédia et de l'Internet* (since May).

M. Teillaud is a member of the working group for the BIL, *Base d'Information des Logiciels* of Inria.

10.1.7.3. Local Committees and Responsabilities

O. Devillers: Elected member to *Pole AM2I* the council that gathers labs in mathematics, computer science, and control theory at *Université de Lorraine*.

L. Dupont Instigator (June 2016) and head of the Bachelor diploma *Licence Professionnele Animation des Communautés et Réseaux Socionumériques*, Université de Lorraine.

S. Lazard: Head of the PhD and Post-doc hiring committee for Inria Nancy-Grand Est (since 2009). Member of the *Bureau de la mention informatique* of the *École Doctorale IAE+M* (since 2009). Head of the *Mission Jeunes Chercheurs* for Inria Nancy-Grand Est (since 2011). Head of the Department Algo at LORIA (since 2014). Member of the *Conseil Scientifique* of LORIA (since 2014).

G. Moroz is member of the Mathematics Olympiades committee of the Nancy-Metz academy. G. Moroz is member of the *Comité des utilisateurs des moyens informatiques*

M. Pouget is elected at the *Comité de centre*, and member of the board of the Charles Hermite federation of labs. M. Pouget is secretary of the board of *AGOS-Nancy*.

M. Teillaud is a member of the BCP, *Bureau du Comité des Projets* and of the CDT, *Commission de développement technologique* of Inria Nancy - Grand Est.

10.1.7.4. Websites

M. Teillaud is maintaining the Computational Geometry Web Pages http://www.computational-geometry.org/, hosted by Inria Nancy - Grand Est since December. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual/international Symposium on Computational Geometry.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Master: Olivier Devillers, *Synthèse, image et géométrie*, 12h (academic year 2017-18), IPAC-R, Université de Lorraine. https://members.loria.fr/Olivier.Devillers/master/

Master: Olivier Devillers and Monique Teillaud, *Computational Geometry*, 24h (academic year 2017-18), Master2 Informatique, ENS Lyon https://members.loria.fr/Monique.Teillaud/Master2-ENS-Lyon/.

Licence: Sény Diatta, *Algorithme et Programmation*, 54h, L1, Université de Lorraine, France.

Licence: Sény Diatta, *Outils Informatiques et Internet*, 42h, L1, Université de Lorraine, France.

Licence: Charles Duménil, *Mathématiques*, 42h, L2, Université de Lorraine, France.

Licence: Charles Duménil, *Logiciel*, 20h, L2, Université de Lorraine, France.

Licence: Charles Duménil, *Algorithmique et programmation avancée*, 34h, M2, Université de Lorraine, France.

Licence: Laurent Dupont, *Algorithmique*, 78h, L1, Université de Lorraine, France.

Licence: Laurent Dupont, *Web development*, 75h, L2, Université de Lorraine, France.

Licence: Laurent Dupont, *Traitement Numérique du Signal*, 10h, L2, Université de Lorraine, France.

Licence: Laurent Dupont *Databases* 30h L3, Université de Lorraine, France,

Licence: Laurent Dupont *Web devloppment and Social networks* 80h L3, Université de Lorraine, France.

Licence: Iordan Iordanov, *Algorithmique et Programmation*, 64h, L1, Université de Lorraine, France.

Licence: Iordan Iordanov, *Systèmes de gestion de bases de données*, 20h, L2, Université de Lorraine, France.

Licence: Iordan Iordanov, *Algorithmique et développement web*, 28h, L2, Université de Lorraine, France.

Licence: Iordan Iordanov, *Programmation objet et événementielle*, 16h, L3, Université de Lorraine, France.

Licence: Sylvain Lazard, *Algorithms and Complexity*, 25h, L3, Université de Lorraine, France.

Master: Marc Pouget, *Introduction to computational geometry*, 10.5h, M2, École Nationale Supérieure de Géologie, France.

10.2.2. Supervision

PhD in progress: Sény Diatta, Complexité du calcul de la topologie d'une courbe dans l'espace et d'une surface, started in Nov. 2014, supervised by Daouda Niang Diatta, Marie-Françoise Roy and Guillaume Moroz.

PhD in progress: Charles Duménil, Probabilistic analysis of geometric structures, started in Oct. 2016, supervised by Olivier Devillers.

PhD in progress: Iordan Iordanov, Triangulations of Hyperbolic Manifolds, started in Jan. 2016, supervised by Monique Teillaud.

PhD in progress: George Krait, Topology of singular curves and surfaces, applications to visualization and robotics, started in Nov. 2017, supervised by Sylvain Lazard, Guillaume Moroz and Marc Pouget.

Postdoc: Vincent Despré, Triangulating surfaces with complex projective structures, started in Nov. 2017, supervised by Monique Teillaud.

10.2.3. Internships

Jian Qian, from Ècole Normale Supérieure Paris, did a L3 internship from Jul 2017 until Aug 2017 co-advised by Guillaume Moroz and Marc Pouget on a topic of ANR SingCAST.

Guillermo Alfonso Reyes Guzman, from Université de Lorraine, did a Master internship from March 2017 until July 2017 advised by O. Devillers on deletion in 3D Delaunay triangulation.

Camille Truong-Allie (Master 1, "research path", École des Mines de Nancy), Lloyd algorithm in the flat torus, started in October, supervised by Monique Teillaud.

10.3. Popularization

L. Dupont participated to several days of popularization of computerscience: Open Bidouille Camp March, 26th 2017, popularization of programming, general audience ; ISN day March, 30th 2017, popularization of computerscience for high-school teachers ; Fête de la Science 14th October 2017 Inria event, general audience, and Google Day in Nancy 21st October 2017, general audience.

11. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

[1] Y. Bouzidi, T. Cluzeau, G. Moroz, A. Quadrat. *Computing effectively stabilizing controllers for a class of nD systems*, in "IFAC-PapersOnLine", July 2017, vol. 50, n^o 1, pp. 1847 - 1852 [*DOI :* 10.1016/j.ifacol.2017.08.200], https://hal.archives-ouvertes.fr/hal-01667161

[2] O. Devillers, M. Karavelas, M. Teillaud. *Qualitative Symbolic Perturbation: Two Applications of a New Geometry-based Perturbation Framework*, in "Journal of Computational Geometry", 2017, vol. 8, n^o 1, pp. 282–315 [*DOI :* 10.20382/jocg.v8i1a11], https://hal.inria.fr/hal-01586511

[3] S. Lazard, M. Pouget, F. Rouillier. *Bivariate triangular decompositions in the presence of asymptotes*, in "Journal of Symbolic Computation", 2017, vol. 82, pp. 123 - 133 [*DOI :* 10.1016/j.jsc.2017.01.004], https://hal.inria.fr/hal-01468796

[4] P. Machado Manhães De Castro, O. Devillers. *Expected Length of the Voronoi Path in a High Dimensional Poisson-Delaunay Triangulation*, in "Discrete and Computational Geometry", 2017, pp. 1–20 [*DOI :* 10.1007/s00454-017-9866-y], https://hal.inria.fr/hal-01477030

International Conferences with Proceedings

[5] I. IORDANOV, M. TEILLAUD. *Implementing Delaunay Triangulations of the Bolza Surface*, in "33rd International Symposium on Computational Geometry (SoCG 2017)", Brisbane, Australia, July 2017, pp. 44:1 – 44:15 [*DOI :* 10.4230/LIPICS.SOCG.2017.44], https://hal.inria.fr/hal-01568002

[6] S. LAZARD, W. LENHART, G. LIOTTA. *On the Edge-length Ratio of Outerplanar Graphs*, in "International Symposium on Graph Drawing and Network Visualization", Boston, United States, 2017, https://hal.inria.fr/hal-01591699

Research Reports

[7] L. CASTELLI ALEARDI, O. DEVILLERS. *Explicit array-based compact data structures for triangulations: practical solutions with theoretical guarantees*, Inria, 2017, n^{o} RR-7736, 39 p. , https://hal.inria.fr/inria-00623762

[8] O. DEVILLERS, M. GLISSE. *Delaunay triangulation of a random sample of a good sample has linear size*, Inria Saclay Ile de France ; Inria Nancy - Grand Est, July 2017, n^{o} RR-9082, 6 p. , https://hal.inria.fr/hal-01568030

[9] R. IMBACH, G. MOROZ, M. POUGET. *Reliable location with respect to the projection of a smooth space curve*, Inria, November 2017, https://hal.archives-ouvertes.fr/hal-01632344

[10] W. KUIJPER, V. ERMOLAEV, O. DEVILLERS. *Celestial Walk: A Terminating Oblivious Walk for Convex Subdivisions*, Inria Nancy, 2017, n^{o} RR-9099, https://arxiv.org/abs/1710.01620 , https://hal.inria.fr/hal-01610205

Other Publications

[11] L. C. ALEARDI, O. DEVILLERS, E. FUSY. *Canonical ordering for graphs on the cylinder, with applications to periodic straight-line drawings on the flat cylinder and torus*, 2017, https://arxiv.org/abs/1206.1919 - 37 pages, https://hal.inria.fr/hal-01646724

[12] D. BREMNER, O. DEVILLERS, M. GLISSE, S. LAZARD, G. LIOTTA, T. MCHEDLIDZE, G. MOROZ, S. WHITESIDES, S. WISMATH. *Monotone Simultaneous Paths Embeddings in* $\mathbb{R}^d$, May 2017, working paper or preprint, https://hal.inria.fr/hal-01529154

References in notes

[13] D. ATTALI, J.-D. BOISSONNAT, A. LIEUTIER. *Complexity of the Delaunay triangulation of points on surfaces: the smooth case*, in "Proceedings of the 19th Annual Symposium on Computational Geometry", 2003, pp. 201–210 [*DOI :* 10.1145/777792.777823], http://dl.acm.org/citation.cfm?id=777823

[14] F. AURENHAMMER, R. KLEIN, D. LEE. *Voronoi diagrams and Delaunay triangulations*, World Scientific, 2013, http://www.worldscientific.com/worldscibooks/10.1142/8685

[15] M. BOGDANOV, O. DEVILLERS, M. TEILLAUD. *Hyperbolic Delaunay complexes and Voronoi diagrams made practical*, in "Journal of Computational Geometry", 2014, vol. 5, pp. 56–85

[16] M. BOGDANOV, M. TEILLAUD. *Delaunay triangulations and cycles on closed hyperbolic surfaces*, Inria, December 2013, n^o RR-8434, https://hal.inria.fr/hal-00921157

[17] J.-D. BOISSONNAT, O. DEVILLERS, S. HORNUS. *Incremental construction of the Delaunay graph in medium dimension*, in "Proceedings of the 25th Annual Symposium on Computational Geometry", 2009, pp. 208–216, http://hal.inria.fr/inria-00412437/

[18] J.-D. BOISSONNAT, O. DEVILLERS, R. SCHOTT, M. TEILLAUD, M. YVINEC. *Applications of random sampling to on-line algorithms in computational geometry*, in "Discrete and Computational Geometry", 1992, vol. 8, pp. 51–71, http://hal.inria.fr/inria-00090675

[19] Y. BOUZIDI, S. LAZARD, G. MOROZ, M. POUGET, F. ROUILLIER, M. SAGRALOFF. *Improved algorithms for solving bivariate systems via Rational Univariate Representations*, Inria, February 2015, 50 p. , https://hal.inria.fr/hal-01114767

[20] Y. BOUZIDI, S. LAZARD, M. POUGET, F. ROUILLIER. *Separating linear forms and Rational Univariate Representations of bivariate systems*, in "Journal of Symbolic Computation", May 2015, vol. 68, pp. 84-119 [*DOI :* 10.1016/J.JSC.2014.08.009], https://hal.inria.fr/hal-00977671

[21] P. CALKA. *Tessellations, convex hulls and Boolean model: some properties and connections*, Université René Descartes - Paris V, 2009, Habilitation à diriger des recherches, https://tel.archives-ouvertes.fr/tel-00448249

[22] M. CAROLI, P. M. M. DE CASTRO, S. LORIOT, O. ROUILLER, M. TEILLAUD, C. WORMSER. *Robust and Efficient Delaunay Triangulations of Points on or Close to a Sphere*, in "Proceedings of the 9th International Symposium on Experimental Algorithms", Lecture Notes in Computer Science, 2010, vol. 6049, pp. 462–473, http://hal.inria.fr/inria-00405478/

[23] M. CAROLI, M. TEILLAUD. *3D Periodic Triangulations*, in "CGAL User and Reference Manual", CGAL Editorial Board, 2009 [*DOI :* 10.1007/978-3-642-04128-0_6], http://doc.cgal.org/latest/Manual/packages.html#PkgPeriodic3Triangulation3Summary

[24] M. CAROLI, M. TEILLAUD. *Computing 3D Periodic Triangulations*, in "Proceedings of the 17th European Symposium on Algorithms", Lecture Notes in Computer Science, 2009, vol. 5757, pp. 59–70

[25] M. CAROLI, M. TEILLAUD. *Delaunay Triangulations of Point Sets in Closed Euclidean d-Manifolds*, in "Proceedings of the 27th Annual Symposium on Computational Geometry", 2011, pp. 274–282 [*DOI :* 10.1145/1998196.1998236], https://hal.inria.fr/hal-01101094

[26] B. CHAZELLE. *Application challenges to computational geometry: CG impact task force report*, in "Advances in Discrete and Computational Geometry", Providence, B. CHAZELLE, J. E. GOODMAN, R. POLLACK (editors), Contemporary Mathematics, American Mathematical Society, 1999, vol. 223, pp. 407–463

[27] P. CHOSSAT, G. FAYE, O. FAUGERAS. *Bifurcation of hyperbolic planforms*, in "Journal of Nonlinear Science", 2011, vol. 21, pp. 465–498, http://link.springer.com/article/10.1007/s00332-010-9089-3

[28] V. DAMEROW, C. SOHLER. *Extreme points under random noise*, in "Proceedings of the 12th European Symposium on Algorithms", 2004, pp. 264–274, http://dx.doi.org/10.1007/978-3-540-30140-0_25

[29] O. DEVILLERS. *The Delaunay hierarchy*, in "International Journal of Foundations of Computer Science", 2002, vol. 13, pp. 163-180, https://hal.inria.fr/inria-00166711

[30] O. DEVILLERS, M. GLISSE, X. GOAOC. *Complexity analysis of random geometric structures made simpler*, in "Proceedings of the 29th Annual Symposium on Computational Geometry", June 2013, pp. 167-175 [*DOI :* 10.1145/2462356.2462362], https://hal.inria.fr/hal-00833774

[31] O. DEVILLERS, M. GLISSE, X. GOAOC, R. THOMASSE. *On the smoothed complexity of convex hulls*, in "Proceedings of the 31st International Symposium on Computational Geometry", Lipics, 2015 [*DOI :* 10.4230/LIPICS.SOCG.2015.224], https://hal.inria.fr/hal-01144473

[32] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-optimal parameterization of the intersection of quadrics: I. The generic algorithm*, in "Journal of Symbolic Computation", 2008, vol. 43, n^{o} 3, pp. 168–191 [*DOI :* 10.1016/J.JSC.2007.10.006], http://hal.inria.fr/inria-00186089/en

[33] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-optimal parameterization of the intersection of quadrics: II. A classification of pencils*, in "Journal of Symbolic Computation", 2008, vol. 43, n^{o} 3, pp. 192–215 [*DOI :* 10.1016/J.JSC.2007.10.012], http://hal.inria.fr/inria-00186090/en

[34] L. DUPONT, D. LAZARD, S. LAZARD, S. PETITJEAN. *Near-Optimal Parameterization of the Intersection of Quadrics: III. Parameterizing Singular Intersections*, in "Journal of Symbolic Computation", 2008, vol. 43, n^{o} 3, pp. 216–232 [*DOI :* 10.1016/J.JSC.2007.10.007], http://hal.inria.fr/inria-00186091/en

[35] M. GLISSE, S. LAZARD, J. MICHEL, M. POUGET. *Silhouette of a random polytope*, in "Journal of Computational Geometry", 2016, vol. 7, n^{o} 1, 14 p. , https://hal.inria.fr/hal-01289699

[36] M. HEMMER, L. DUPONT, S. PETITJEAN, E. SCHÖMER. *A complete, exact and efficient implementation for computing the edge-adjacency graph of an arrangement of quadrics*, in "Journal of Symbolic Computation", 2011, vol. 46, n^{o} 4, pp. 467-494 [*DOI :* 10.1016/J.JSC.2010.11.002], https://hal.inria.fr/inria-00537592

[37] J. HIDDING, R. VAN DE WEYGAERT, G. VEGTER, B. J. JONES, M. TEILLAUD. *Video: the sticky geometry of the cosmic web*, in "Proceedings of the 28th Annual Symposium on Computational Geometry", 2012, pp. 421–422

[38] J. B. HOUGH, M. KRISHNAPUR, Y. PERES, B. VIRÁG. *Determinantal processes and independence*, in "Probab. Surv.", 2006, vol. 3, pp. 206-229

[39] R. IMBACH, G. MOROZ, M. POUGET. *Numeric and Certified Isolation of the Singularities of the Projection of a Smooth Space Curve*, in "Proceedings of the 6th International Conferences on Mathematical Aspects of Computer and Information Sciences", Springer LNCS, 2015, https://hal.inria.fr/hal-01239447

[40] S. LAZARD, L. M. PEÑARANDA, S. PETITJEAN. *Intersecting quadrics: an efficient and exact implementation*, in "Computational Geometry: Theory and Applications", 2006, vol. 35, n^{o} 1-2, pp. 74–99

[41] S. LAZARD, M. POUGET, F. ROUILLIER. *Bivariate triangular decompositions in the presence of ssymptotes*, Inria, September 2015, https://hal.inria.fr/hal-01200802

[42] M. MAZÓN, T. RECIO. *Voronoi diagrams on orbifolds*, in "Computational Geometry: Therory and Applications", 1997, vol. 8, pp. 219–230

[43] A. PELLÉ, M. TEILLAUD. *Periodic meshes for the CGAL library*, 2014, International Meshing Roundtable, Research Note, https://hal.inria.fr/hal-01089967

[44] G. RONG, M. JIN, X. GUO. *Hyperbolic centroidal Voronoi tessellation*, in "Proceedings of the ACM Symposium on Solid and Physical Modeling", 2010, pp. 117–126, http://dx.doi.org/10.1145/1839778.1839795

[45] A. RÉNYI, R. SULANKE. *Über die konvexe Hülle von n zufällig gerwähten Punkten I*, in "Z. Wahrsch. Verw. Gebiete", 1963, vol. 2, pp. 75–84 [*DOI :* 10.1007/BF00535300], http://www.springerlink.com/content/t5005k86665u24g0/

[46] A. RÉNYI, R. SULANKE. *Über die konvexe Hülle von n zufällig gerwähten Punkten II*, in "Z. Wahrsch. Verw. Gebiete", 1964, vol. 3, pp. 138–147 [*DOI :* 10.1007/BF00535973], http://www.springerlink.com/content/n3003x44745pp689/

[47] F. SAUSSET, G. TARJUS, P. VIOT. *Tuning the fragility of a glassforming liquid by curving space*, in "Physical Review Letters", 2008, vol. 101, pp. 155701(1)–155701(4), http://dx.doi.org/10.1103/PhysRevLett.101.155701

[48] M. SCHINDLER, A. C. MAGGS. *Cavity averages for hard spheres in the presence of polydispersity and incomplete data*, in "The European Physical Journal E", 2015, pp. 38–97, http://dx.doi.org/10.1103/PhysRevE.88.022315

[49] M. SCHMITT, M. TEILLAUD. *Meshing the hyperbolic octagon*, Inria, 2012, n^{o} 8179, http://hal.inria.fr/hal-00764965

[50] D. A. SPIELMAN, S.-H. TENG. *Smoothed analysis: why the simplex algorithm usually takes polynomial time*, in "Journal of the ACM", 2004, vol. 51, pp. 385 - 463, http://dx.doi.org/10.1145/990308.990310

[51] M. TEILLAUD. *Towards dynamic randomized algorithms in computational geometry*, Lecture Notes Comput. Sci., Springer-Verlag, 1993, vol. 758 [*DOI :* 10.1007/3-540-57503-0], http://www.springer.com/gp/book/9783540575030

[52] R. VAN DE WEYGAERT, G. VEGTER, H. EDELSBRUNNER, B. J. JONES, P. PRANAV, C. PARK, W. A. HELLWING, B. ELDERING, N. KRUITHOF, E. BOS, J. HIDDING, J. FELDBRUGGE, E. TEN HAVE, M. VAN ENGELEN, M. CAROLI, M. TEILLAUD. *Alpha, Betti and the megaparsec universe: on the homology and topology of the cosmic web*, in "Transactions on Computational Science XIV", Lecture Notes in Computer Science, Springer-Verlag, 2011, vol. 6970, pp. 60–101 [*DOI :* 10.1007/978-3-642-25249-5_3], http://www.springerlink.com/content/334357373166n902/