IN PARTNERSHIP WITH:
**Université de Technologie de Troyes**

# Activity Report 2017

# Project-Team GAMMA3

## Automatic mesh generation and advanced methods

RESEARCH CENTER
**Saclay - Île-de-France**

THEME
**Numerical schemes and simulations**

# Table of contents

# Project-Team GAMMA3

*Creation of the Project-Team: 2010 January 01*

**Keywords:**

**Computer Science and Digital Science:**

A2.5. - Software engineering
A5.2. - Data visualization
A6.1. - Mathematical Modeling
A6.2. - Scientific Computing, Numerical Analysis & Optimization
A7.1. - Algorithms
A8.3. - Geometry, Topology

**Other Research Topics and Application Domains:**

B5.2.3. - Aviation
B5.2.4. - Aerospace

# 1. Personnel

**Research Scientists**

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Adrien Loseille [Inria, Researcher]
David Marcum [Inria, International Chair, Advanced Research Position]

**Faculty Member**

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Julien Vanharen [Inria, from Jun 2017]

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Rémi Feuillet [École Nationale Supérieure de Techniques Avancées]
Loïc Frazza [Ecole polytechnique]
Julien Vanharen [Inria, May 2017]

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**Administrative Assistants**

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**External Collaborators**

Eléonore Gauci [Univ Pierre et Marie Curie, from Sep 2017]
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Loic Marechal [Inria]

# 2. Overall Objectives

## 2.1. Introduction

Une branche importante des sciences de l'ingénieur s'intéresse aux calculs des solutions d'équations aux dérivées partielles très variées (en mécanique du solide, en mécanique des fluides, en modélisation de problèmes thermiques, ...) par la méthode des éléments ou des volumes finis. Ces méthodes utilisent comme support spatial des calculs un maillage du domaine sur lequel les équations sont formulées. Par suite, les algorithmes (de construction) de maillages occupent un rôle important dans toute simulation par la méthode des éléments ou des volumes finis d'un problème modélisé en équations aux dérivées partielles. En particulier, la précision, voire la validité, des solutions calculées est liée aux propriétés du maillage utilisé [12].

L'équipe-projet GAMMA3 a été créé en 2010 à la suite du projet GAMMA. L'équipe est bilocalisée avec une partie à l'UTT (Troyes) et l'autre à Rocquencourt puis Saclay. Les thèmes du projet regroupent un ensemble d'activités concernant les points indiqués ci-dessus, en particulier, l'aspect génération automatique de maillages afin de construire les supports utilisés par les méthodes d'éléments ou de volumes finis. Sont également étudiés les aspects de modélisation géométrique, de post-traitement et de visualisation des résultats issus de tels calculs [13].

L'évolution de la demande en termes de génération automatique de maillages implique une évolution des méthodes classiques de création de maillages vers des méthodes permettant de construire des maillages contrôlés. Les maillages doivent donc être soit isotropes, le contrôle portant sur des tailles souhaitées, soit anisotropes, le contrôle portant à la fois sur des directions et des tailles selon ces dernières.

Le développement d'algorithmes de maillages gouvernés sert de support naturel à la conception de boucles de maillages adaptatifs qui, via un estimateur d'erreurs *a posteriori*, permettent de contrôler la qualité des solutions. Les estimateurs d'erreurs sont issus d'applications en mécanique des fluides (Inria) et du solide (UTT). Leurs validations reposent sur le développement de solveurs avancés, en particulier, dans ces disciplines. Ces deux points (estimateurs et solveurs) constituent au moins la moitié de nos recherches.

Ces préoccupations amènent à considérer le problème du maillage des domaines de calculs en eux-mêmes tout comme celui du maillage ou du remaillage des courbes et surfaces, frontières de ces domaines.

La taille, en termes de nombre de nœuds, des maillages nécessaires pour certaines simulations, amène à travailler sur la parallélisation des processus de calculs. Cette problématique conduit également à s'intéresser à l'aspect multi-cœurs au niveau des algorithmes de maillages proprement dits.

Simultanément, le volume des résultats obtenus dans de telles simulations, nécessite d'envisager le post-traitement de ces résultats en parallèle ou par des méthodes appropriées.

Par ailleurs, de nombreux problèmes partent de saisies *scanner* (ou autre système discret) des géométries à traiter et demandent d'en déduire des maillages de surfaces aptes à être, par la suite, traités par les méthodes classiques (de remaillage, d'optimisation, de calculs).

Enfin, la maturité de certaines méthodes (victimes de leur succès) conduit les utilisateurs à demander plus et à considérer des problèmes de maillage ou des conditions d'utilisations extrêmes induisant des algorithmes *a priori* inattendus.

Les objectifs du projet GAMMA3 consistent à étudier l'ensemble des points mentionnés ci-dessus afin de rendre automatique le calcul de la solution d'un problème donné avec une précision imposée au départ. Par ailleurs, certaines des techniques utilisées dans les problématiques de maillage sont utilisables dans d'autres disciplines (compression d'images pour ne citer qu'un seul exemple).

# 3. New Software and Platforms

## 3.1. ABL4FLO

KEYWORDS: Boundary layers - Hybrid meshes

FUNCTIONAL DESCRIPTION: ABL4FLO is designed to generate 3D adapted boundary layer meshes by using a cavity-based operator.

- Participant: Adrien Loseille
- Contact: Adrien Loseille

## 3.2. AMA4FLO

*Anisotropic Mesh Adaptation 4 FLOw*
KEYWORDS: 3D - Mesh adaptation
FUNCTIONAL DESCRIPTION: 3D, surface, 2D anisotropic mesh generation

- Participant: Adrien Loseille
- Contact: Adrien Loseille
- URL: http://pages.saclay.inria.fr/adrien.loseille/index.php?page=softwares

## 3.3. BL2D

KEYWORDS: Abstraction - Meshing - Isotropic - Anisotropic - Delaunay
FUNCTIONAL DESCRIPTION: This software package stems from a former one called BL2D-V1. The meshing method is of controled Delaunay type, isotropic or anisotropic. The internal point generation follows a frontal logic, and their connection is realised as in a classical Delaunay approach. Quadrilaterals are obtained by a pairing process. The direct construction of degree 2 element has been made possible via the control of the domain boundary mesh, in order to ensure the desired compatibility. The boundary middle nodes are located according to the curvilinear abscissa. The internal middle nodes are, by default, at the middle of the corresponding edges.

RELEASE FUNCTIONAL DESCRIPTION: Par rapport à la version V1, il offre de nombreuses possibilités nouvelles : méthode frontale, triangles quadratiques courbes, quadrilatères de degré 1ou 2, frontières déformables, allocation dynamique de mémoire, etc

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: http://pages.saclay.inria.fr/patrick.laug/logiciels/bl2d-v2/INDEX.html

## 3.4. BL2D-ABAQ

KEYWORDS: Anisotropic - Delaunay - Automatic mesher
FUNCTIONAL DESCRIPTION: The meshing method is the same as BL2D in an adaptive process. An a posteriori error estimation of a solution at the nodes of the current mesh results in a size map. A new mesh staisfying these size specifications (made continuous is built, and the solution is interpolated on the new mesh.

- Participants: Abel Cherouat, Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug

## 3.5. BLGEOL

KEYWORDS: Automatic mesher - Geologic structure

FUNCTIONAL DESCRIPTION: BLGEOL-V1 software can generate hex-dominant meshes of geologic structures complying with different geometric constraints: surface topography (valleys, reliefs, rivers), geologic layers and underground workings. First, a reference 2D domain is obtained by projecting all the line constraints into a horizontal plane. Different size specifications are given for rivers, outcrop lines and workings. Using an adaptive methodology, the size variation is bounded by a specified threshold in order to obtain a high quality quad-dominant mesh. Secondly, a hex-dominant mesh of the geological medium is generated by a vertical extrusion, taking into account the surfaces found (interfaces between two layers, top or bottom faces of underground workings). The generation of volume elements follows a global order established on the whole set of surfaces to ensure the conformity of the resulting mesh.

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: https://team.inria.fr/gamma3/project-presentation/gamma-software/

## 3.6. BLMOL

KEYWORDS: Mesher - Molecular surface
SCIENTIFIC DESCRIPTION: An increasingly important part of quantum chemistry is devoted to molecular surfaces. To model such a surface, each constituting atom is idealized by a simple sphere. Surface mesh generation techniques are then used either for visualization or for simulation, where mesh quality has a strong influence on solution accuracy. First, a boundary representation (B-rep) of the surface is obtained, i.e. a set of patches and the topological relations between them. Second, an appropriate parameterization and a metric map are computed for each patch. Third, meshes of the parametric domains are generated with respect to an induced metric map, using a combined advancing-front generalized-Delaunay approach. Finally these meshes are mapped onto the entire surface. Several application examples illustrate various capabilities of our method.
FUNCTIONAL DESCRIPTION: BLMOL is a molecular surface mesher.

- Participants: Houman Borouchaki and Patrick Laug
- Contact: Patrick Laug
- URL: http://pages.saclay.inria.fr/patrick.laug/logiciels/blmol/INDEX.html

## 3.7. BLSURF

KEYWORDS: Automatic mesher - Molecular surface
FUNCTIONAL DESCRIPTION: An indirect method for meshing parametric surfaces conforming to a user-specifiable size map is used. First, from this size specification, a Riemannian metric is defined so that the desired mesh is one with unit length edges with respect to the related Riemannian space (the so-called

- Participants: Houman Borouchaki and Patrick Laug
- Partner: Université de Technologie de Troyes
- Contact: Patrick Laug
- URL: https://team.inria.fr/gamma3/project-presentation/gamma-software/

## 3.8. FEFLOA-REMESH

KEYWORDS: Scientific calculation - Anisotropic - Mesh adaptation
FUNCTIONAL DESCRIPTION: FEFLOA-REMESH is intended to generate adapted 2D, surface and volume meshes by using a unique cavity-based operator. The metric-aligned or metric-orthogonal approach is used to generate high quality surface and volume meshes independently of the anisotropy involved.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Adrien Loseille
- URL: http://pages.saclay.inria.fr/adrien.loseille/index.php?page=softwares

## 3.9. GAMANIC 3D

KEYWORDS: Tetrahedral mesh - Delaunay - Anisotropic size and direction control - Automatic mesher
FUNCTIONAL DESCRIPTION: GAMANIC3D is a volume mesher governed by a (anisotropic) size and directional specification metric field.

- Participants: Adrien Loseille, Éric Saltel, Frédéric Alauzet, Frederic Hecht, Houman Borouchaki and Paul Louis George
- Contact: Paul Louis Georges
- URL: http://www.meshgems.com/volume-meshing.html

## 3.10. GAMHIC 3D

KEYWORDS: Tetrahedral mesh - Delaunay - Isotropic - Automatic mesher
FUNCTIONAL DESCRIPTION: GAMHIC3D is a volume mesher governed by a (isotropic) size specification metric field.

- Participants: Adrien Loseille, Éric Saltel, Frédéric Alauzet, Frederic Hecht, Houman Borouchaki and Paul Louis George
- Contact: Paul Louis George
- URL: http://www.meshgems.com/volume-meshing.html

## 3.11. GHS3D

KEYWORDS: Tetrahedral mesh - Delaunay - Automatic mesher
FUNCTIONAL DESCRIPTION: GHS3D is an automatic volume mesher

- Participants: Adrien Loseille, Éric Saltel, Frédéric Alauzet, Frederic Hecht, Houman Borouchaki and Paul Louis George
- Contact: Paul Louis George
- URL: http://www.meshgems.com/volume-meshing.html

## 3.12. HEXOTIC

KEYWORDS: 3D - Mesh generation - Meshing - Unstructured meshes - Octree/Quadtree - Multi-threading - GPGPU - GPU
FUNCTIONAL DESCRIPTION: Input: a triangulated surface mesh and an optional size map to control the size of inner elements.

Output: a fully hexahedral mesh (no hybrid elements), valid (no negative jacobian) and conformal (no dangling nodes) whose surface matches the input geometry.

The software is a simple command line that requires no knowledge on meshing. Its arguments are an input mesh and some optional parameters to control elements sizing, curvature and subdomains as well as some features like boundary layers generation.

- Participant: Loïc Maréchal
- Partner: Distene
- Contact: Loïc Maréchal
- URL: https://team.inria.fr/gamma3/project-presentation/gamma-software/hexotic/

## 3.13. Nimbus 3D

KEYWORDS: Surface reconstruction - Point cloud

FUNCTIONAL DESCRIPTION: Nimbus3D is a surface reconstruction method piece of software

- Participants: Houman Borouchaki and Paul Louis George
- Contact: Paul Louis George
- URL: http://www.meshgems.com/volume-meshing.html

## 3.14. VIZIR

*Maillages Clés en Main pour la Simulation Numérique*
KEYWORD: Mesh
FUNCTIONAL DESCRIPTION: VIZIR is intended to visualize and modify interactively simplicial, hybrid and high order curved meshes.

- Participants: Adrien Loseille, Alexis Loyer and Julien Castelneau
- Contact: Adrien Loseille
- URL: http://pages.saclay.inria.fr/adrien.loseille/index.php?page=softwares

## 3.15. Wolf

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Numerical solver for the Euler and compressible Navier-Stokes equations with turbulence modelling. ALE formulation for moving domains. Modules of interpolation, mesh optimisation and moving meshes. Wolf is written in C++, and may be later released as an opensource library. FELiScE was registered in July 2014 at the Agence pour la Protection des Programmes under the Inter Deposit Digital Number IDDN.FR.001.340034.000.S.P.2014.000.10000.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.16. Wolf-Bloom

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-Bloom is a structured boundary layer mesh generator using a pushing approach. It start from an existing volume mesh and insert a structured boundary layer by pushing the volume mesh. The volume mesh deformation is solved with an elasticity analogy. Mesh-connectivity optimizations are performed to control volume mesh element quality.

- Participants: Adrien Loseille, David Marcum and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.17. Wolf-Elast

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-Elast is a linear elasticity solver using the P1 Finite-Element method. The Young and Poisson coefficient can be parametrized. The linear system is solved using the Conjugate Gradient method with the LUSGS preconditioner.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.18. Wolf-Interpol

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-Interpol is a tool to transfer scalar, vector and tensor fields from one mesh to another one. Polynomial interpolation (from order 2 to 4) or conservative interpolation operators can be used. Wolf-Interpol also extract solutions along lines or surfaces.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.19. Wolf-MovMsh

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-MovMsh is a moving mesh algorithm coupled with mesh-connectivity optimization. Mesh deformation is computed by means of a linear elasticity solver or a RBF interpolation. Smoothing and swapping mesh optimization are performed to maintain good mesh quality. It handles rigid bodies or deformable bodies, and also rigid or deformable regions of the domain.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Paul Louis George
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.20. Wolf-Nsc

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-Nsc is numerical flow solver solving steady or unsteady turbulent compressible Euler and Navier-Stokes equations. The available turbulent models are the Spalart-Almaras and the Menter SST k-omega. A mixed finite volume - finite element numerical method is used for the discretization. Second order spatial accuracy is reached thanks to MUSCL type methods. Explicit or implicit time integration are available. It also resolved dual (adjoint) problem and compute error estimate for mesh adaptation.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

## 3.21. Wolf-Spyder

KEYWORD: Scientific calculation
FUNCTIONAL DESCRIPTION: Wolf-Spyder is a metric-based mesh quality optimizer using vertex smoothing and edge/face swapping.

- Participants: Adrien Loseille and Frédéric Alauzet
- Contact: Frédéric Alauzet
- URL: https://www.rocq.inria.fr/gamma/Frederic.Alauzet/code_eng.html

# 4. New Results

## 4.1. Element metric, element quality and interpolation error metric

**Participants:** Paul Louis George [correspondant], Houman Borouchaki.

The metric of a simplex of $\mathbb{R}^d$ is a metric tensor (symmetric positive definite matrix) in which the element is unity (regular with unit edge lengths). This notion is related to the problem of interpolation error of a given field over a mesh. Let $K$ be a simplex and let us denote by $v_{ij}$ the vector joining vertex $i$ and vertex $j$ of $K$. The metric of $K$ can be written as:

$$\mathcal{M} = \frac{d+1}{2} \Big(\sum_{i<j} v_{ij}\,{}^t v_{ij}\Big)^{-1},$$

where $v_{ij}\,{}^t v_{ij}$ is a $d \times d$ rank 1 matrix related to edge $ij$.

The metric of a simplex also characterizes the element shape. In particular, if it is the identity, the element is unity. Hence, to define the shape quality of an element, one can determine the gap of the element metric $\mathcal{M}$ and the identity using different measures based on the eigenvalues $\lambda_i = \frac{1}{h_I^2}$ of $\mathcal{M}$ or those of $\mathcal{M}^{-1}$, e.g. $h_i^2$. Notice that metric $\mathcal{M}^{-1}$ is directly related to the geometry of the element (edge length, facet area, element volume). The first algebraic shape quality measure ranging from 0 to 1 is defined as the ratio of the geometric average of the eigenvalues of $\mathcal{M}^{-1}$ and their arithmetic average:

$$q(K) = \frac{\left(\prod_i h_i^2\right)^{\frac{1}{d}}}{\frac{1}{d}\sum_{i=1}^{d} h_i^2} = d\,\frac{\left(det(\mathcal{M}^{-1})\right)^{\frac{1}{d}}}{tr(\mathcal{M}^{-1})}.$$

As the geometric average is smaller than the arithmetic average, this measure is well defined. In addition, it is the algebraic reading of the well-known quality measure defined by:

$$q^{\frac{d}{2}}(K) = (d!) d^{\frac{d}{2}} (d+1)^{\frac{d-1}{2}} \frac{|K|}{\left(\sum_{i<j} l_{ij}^2\right)^{\frac{d}{2}}},$$

where the volume and the square of the edge lengths are involved. The algebraic meaning justifies the above geometric measure. The second algebraic shape quality measure is defined as the ratio of the harmonic average of the eigenvalues of $\mathcal{M}^{-1}$ and their arithmetic average (ranging also from 0 to 1):

$$q(K) = \frac{\left\{\frac{1}{d}\sum_{i=1}^{d} \frac{1}{h_i^2}\right\}^{-1}}{\frac{1}{d}\sum_{i=1}^{d} h_i^2} = \frac{d^2}{tr(\mathcal{M})tr(\mathcal{M}^{-1})}.$$

As above, this measure is well defined, the harmonic average being smaller the arithmetic one. From this measure, one can derive another well-known measure involving the roundness and the size of an element (measure which is widely used for convergene issues in finite element methods).

Note that these measures use the invariants of $\mathcal{M}^{-1}$ or $\mathcal{M}$ and thus can be evaluated from the coefficients of the characteristical polynomial of those matrices (avoiding the effective calculation of their eigenvalues). Another advantage of the above algebraic shape measures is their easy extensions in an arbitrary Euclidean space. Indeed, if $\mathcal{E}$ is the metric of such a space, the algebraic shape measures read:

$$q_{\mathcal{E}}(K) = d\,\frac{\left(det(\mathcal{M}^{-1}\mathcal{E})\right)^{\frac{1}{d}}}{tr(\mathcal{M}^{-1}\mathcal{E})} \quad , \quad q_{\mathcal{E}}(K) = \frac{d^2}{tr(\mathcal{E}^{-1}\mathcal{M})tr(\mathcal{M}^{-1}\mathcal{E})}.$$

Following this notion of a element metric, a natural work was done regarding how to define the element metric so as to achieve a given accuracy for the interpolation error of a function using a finite element approximation by means of simplices of arbitrary degree.

This is a new approach for the majoration of the interpolation error of a polynomial function of arbitrary degree $n$ interpolated by a polynomial function of degree $n-1$. From that results a metric, the so-called interpolation metric, which allows for a control of the error. The method is based on the geometric and algebraic properties of the metric of a given element, metric in which the element is regular and unit. The interpolation metric plays an important role in advanced computations based on mesh adaptation. The method relies in a Bezier reading of the functions combined with Taylor expansions. In this way, the error in a given element is fully controled at the time the edges of the element are controled.

It is shown that the error in bounded as

$$|e(X)| \leq C \sum_{i<j} f^{(n)}(.)(v_{ij}, v_{ij}, ..., v_{ij}),$$

where $C$ is a constant depending on $d$ and $n$, $v_{ij}$ is the edge from the vertices of $K$ of index $i$ and $j$, $f^{(n)}(.)$ is the derivative of order $n$ of $f$ applied to a $n$-uple uniquely composed of $v_{ij}$. If we consider the case $d=2$ and $u=(x,y)$ is a vector in $\mathbb{R}^2$, we have

$$f^{(n)}(.)(u,u,...,u) = \sum_{i=0}^{n-2} x^{n-2-i} y^i \ {}^t u \left( C_i^{n-2} \mathcal{H}_{(n-2,n-2-i,i)} \right) u \ ,$$

where the quadratic forms $\mathcal{H}_{(n-2,n-2-i,i)}$ are defined by the matrices of order 2 (with constant entries):

$$\mathcal{H}_{(n-2,n-2-i,i)} = \begin{pmatrix} \frac{\partial^{(n)} f}{\partial x_1^{n-i} \partial x_2^{i}} & \frac{\partial^{(n)} f}{\partial x_1^{n-1-i} \partial x_2^{i+1}} \\ \\ \frac{\partial^{(n)} f}{\partial x_1^{n-1-i} \partial x_2^{i+1}} & \frac{\partial^{(n)} f}{\partial x_1^{n-2-i} \partial x_2^{i+2}} \end{pmatrix},$$

those matrices being the hessians of the derivatives of $f$ of order $n-2$.

This work resulted in a paper submitted in a journal and currently under revision.

## 4.2. Realistic modeling of fractured geologic media

**Participants:** Patrick Laug [correspondant], Géraldine Pichot.

This study, in collaboration with project-team Serena, aims to model, in a realistic and efficient manner, natural fractured media. These media are characterized by their diversity of structures and organizations. Numerous studies in the past decades have evidenced the existence of characteristic structures at multiple scales. At fracture scale, the aperture distribution is widely correlated and heterogeneous. At network scale, the topology is complex resulting from mutual mechanical interactions as well as from major stresses. Geometric modeling of fractured networks combines in a non-standard way a large number of 2D fractures interconnected in the 3D space. Intricate local configurations of fracture intersections require original methods of geometric modeling and mesh generation. We have developed in 2016 a software package that automatically builds geometric models and surface meshes of random fracture networks. The results are highly promising and we now want to continue this research to further improve the element quality in complex configurations, take into account multiple size scales in large fracture networks (up to thousands of fractures), and compare several modeling strategies (mixed hybrid finite elements, projected grids, mortar elements).

## 4.3. Parallel meshing of surfaces defined by collections of connected regions

**Participant:** Patrick Laug [correspondant].

In CAD (computer aided design) environments, a surface is commonly modeled as a collection of connected regions represented by parametric mappings. For meshing such a composite surface, a parallelized indirect approach with dynamic load balancing can be used on a shared memory system. However, this methodology can be inefficient in practice because most existing CAD systems use memory caches that are only appropriate to a sequential process. As part of the sabbatical year of P. Laug at Polytechnique Montréal in 2014/2015, two solutions have been proposed, referred to as the Pirate approach and the Discrete approach. In the first approach, the Pirate library can be efficiently called in parallel since no caching is used for the storage or evaluation of geometric primitives. In the second approach, the CAD environment is replaced by internal procedures interpolating a discrete geometric support. In 2016, the dynamic load balancing has been analyzed and improved. Significant modifications to the Pirate library have been made, and new numerical tests on three different computers (4, 8 and 64 cores) have been carried out, now showing an almost linear scaling of the method in all cases.

## 4.4. Discrete CAD model for visualization and meshing

**Participants:** Patrick Laug [correspondant], Houman Borouchaki.

During the design of an object using a CAD (computer aided design) platform, the user can visualize the ongoing model at every moment. Visualization is based on a discrete representation of the model that coexists with the exact analytical representation of the object. Most CAD systems have this discrete representation available, and each of them applies its own construction methodology. We have developed in 2016 a method to build a discrete model for CAD surfaces (the model is quadtree-based and subdivided into quadrilaterals and triangles). The method presents two major particularities: most elements are aligned with iso-parametric curves and the accuracy of the surface approximation is controlled. In addition, we have proposed a new technique of surface mesh generation that is based on this discrete model. This approach has been implemented as a part of a surface mesher called ALIEN, and several examples have demonstrate the robustness and computational efficiency of the program, as well as the quality of the geometric support.

## 4.5. Visualization and modification of high-order curved meshes

**Participants:** Alexis Loyer, Dave Marcum, Adrien Loseille [correspondant].

During the partnership between Inria and Distene, a new visualization software has been designed. It address the typical operations that are required to quickly assess the newly algorithm developed in the team. In particular, interactive modifications of high-order curved mesh and hybrid meshes has been addressed.

## 4.6. Adaptation de maillages pour des écoulements visqueux en turbomachine

**Participants:** Frédéric Alauzet, Loïc Frazza, Adrien Loseille [correspondant].

### *4.6.1. Calcul.*

Les prémices d'une adaptation pour les écoulements Navier-Stokes turbulents ont été testés sur des calculs de turbomachine. Pour ce faire nous avons tout d'abord traité les particularités liées aux calculs en turbomachine: - Les aubes présentent en général une périodicité par rotation et on ne simule donc qu'une période afin d'alléger les calculs. Il faut donc traiter cette périodicité de façon appropriée dans le code CFD et l'adaptation de maillage. - Afin de prendre en compte la rotation des pales sans employer de maillages mobiles et simulations instationnaires on peut se placer dans le référentiel tournant de l'aube en corrigeant les équations. - Les écoulements en turbomachine sont des écoulements clos, les conditions limites d'entrée et de sortie ont donc une influence très forte et peuvent de plus se trouver très près de la turbine afin de simuler la présence d'autres étages en amont ou aval. Des conditions limites bien précises ont donc été développées afin de traiter correctement ces effets.

### *4.6.2. Adaptation.*

Pour l'adaptation de maillages deux particularités doivent être traitées ici, la périodicité du maillage et la couche limite turbulente.

En 2D, la couche limite turbulente est automatiquement adaptée avec la méthode metric orthogonal et la périodicité du maillage est garantie par un traitement spécial des frontières. Les estimateurs d'erreurs Navier-Stokes et RANS n'étant pas encore au point nous avons utilisé la Hessienne du Mach de l'écoulement comme senseur ce qui donne déjà des résultats satisfaisants.

En 3D la méthode metric orthogonal est beaucoup plus complexe à mettre en oeuvre et n'est pas encore au point. La couche limite a donc été exclue de l'adaptation, le maillage est adapté uniquement dans le volume en utilisant la Hessienne du Mach de l'écoulement comme senseur. La périodicité n'étant pas traitée non plus, les frontières périodiques restent inchangées ce qui garantie leur périodicité.

### *4.6.3. Norm-Oriented.*

Dans le cadre de la théorie Norm-Oriented, afin de contrôler l'erreur implicite des schémas numériques, un correcteur a été développé et testé. Etant donné un maillage et la solution numérique obtenue avec, le résidu de cette solution projeté sur un maillage deux fois plus fin est accumulé sur le maillage initial. Ce défaut de résidu est utilisé comme terme source dans une seconde simulation plus courte. La nouvelle solution toujours sur le meme maillage est plus proche de la solution exacte et donne une bonne estimation de l'erreur.

## 4.7. Parallel mesh adaptation

**Participants:** Frédéric Alauzet, Adrien Loseille [correspondant].

We devise a strategy in order to generate large-size adapted anisotropic meshes $O(10^8 - 10^9)$ as required in many fields of application in scientific computing. We target moderate scale parallel computational resources as typically found in R&D units where the number of cores ranges in $O(10^2 - 10^3)$. Both distributed and shared memory architectures are handled. Our strategy is based on hierarchical domain splitting algorithm to remesh the partitions in parallel. Both the volume and the surface mesh are adapted simultaneously and the efficiency of the method is independent of the complexity of the geometry. The originality of the method relies on (i) a metric-based static load-balancing, (ii) dedicated hierarchical mesh partitioning techniques to (re)split the (complex) interfaces meshes, (iii) anisotropic Delaunay cavity to define the interface meshes, (iv) a fast, robust and generic sequential cavity-based mesh modification kernel, and (v) out-of-core storing of completing parts to reduce the memory footprint. We are able to generate (uniform, isotropic and anisotropic) meshes with more than 1 billion tetrahedra in less than 20 minutes on 120 cores.

## 4.8. Unsteady adjoint computation on dynamic meshes

**Participants:** Eléonore Gauci, Frédéric Alauzet [correspondant].

Adjoint formulations for unsteady problems are less common due to the extra complexity inherent in the numerical solution and storage but these methods are a great option in engineering because it takes more into account the cost function we want to minimize. Moreover the engineering applications involve moving bodies and this motion must be taken into account by the governing flow equations. We develop a model of unsteady adjoint solver on moving mesh problems. The derivation of the adjoint formulation based on the ALE form of the equations requires consideration of the dynamic meshes. Our model takes into account the DGCL.

## 4.9. Line solver for efficient stiff parse system resolution

**Participants:** Loïc Frazza, Frédéric Alauzet [correspondant].

Afin d'accélérer la résolution des problèmes raides, un line-solver á été développé. Cette méthode extrait tout d'abord des lignes dans le maillage du problème selon des critères géométriques ou physiques. Le problème peut alors être résolu exactement le long des ces lignes à moindre coût. Cette méthode est particulièrement bien adaptée aux cas où l'information se propage selon une direction privilégiée tels que les chocs, les couches limites ou les sillages. Ces cas sont généralement associés à des maillages très étirés ce qui conduit à des problèmes raides mais quasi-unidimensionnels. Ils peuvent donc être résolu efficacement par un line-solver, réduisant ainsi les temps de calculs tout en gagnant en robustesse.

## 4.10. Error estimate for high-order solution field

**Participants:** Olivier Coulaud, Adrien Loseille [correspondant].

Afin de produire des solveurs d'ordre élevé, et ainsi répondre aux exigences inhérentes à la résolution de problèmes physiques complexes, nous développons une méthode d'adaptation de maillage d'ordre élevé. Celle-ci est basée sur le contrôle par une métrique de l'erreur d'interpolation induite par le maillage du domaine. Plus précisément, pour une solution donnée, l'erreur d'interpolation d'ordre $k$ est paramétrée par la forme différentielle $(k+1)^{\text{ième}}$ de cette solution, et le problème se réduit à trouver la plus grande ellipse incluse dans une ligne de niveau de cette différentielle. La méthode que nous avons mise au point théoriquement et numériquement est appelée "log-simplexe", et permet de produire des maillages adaptés d'ordre élevé dans un temps raisonnable, et ce en dimension 2 et 3. À l'occasion de l'International Meshing Roundtable 2016, ce travail a été présenté et publié. D'autres applications de cette méthode sont en cours d'exploitation, comme par exemple la génération de maillages adaptés courbes de surface, ou le couplage avec un solveur d'ordre élevé.

## 4.11. Méthode d'immersion de frontières pour la mécanique des fluides

**Participants:** Frédéric Alauzet [correspondant], Rémi Feuillet, Adrien Loseille.

Dans les méthodes de résolution classiques des problèmes d'interaction fluide-structure, il est usuel de représenter l'objet de manière exacte dans le maillage, c'est-à-dire avec des éléments conformes à l'objet : le maillage possède des triangles dont une arête correspond avec le bord de la géométrie immergée. Cette méthode quoique plus précise est très coûteuse en préprocessing. C'est dans ce cadre qu'est introduite la notion d'immersion de frontière (embedded geometry en anglais). Cette méthode consiste à représenter la géométrie de manière fictive. Le maillage de calcul n'est de fait plus nécessairement conforme à la géométrie de l'objet. Il s'agit donc de s'intéresser aux modifications nécessaires sur les méthodes classiques pour faire un calcul dans le cadre de l'immersion de frontières. Cela concerne les conditions aux limites et l'avancée en temps. On s'intéresse également à l'adaptation de maillage pour le cas de l'immersion. La finalité de tout ce travail est d'effectuer des calculs de coefficients aérodynamiques (portance, traînée) et de trouver des résultats du même ordre de précision que ceux en géométrie inscrite.

## 4.12. Boundary layer mesh generation

**Participants:** Frédéric Alauzet [correspondant], Adrien Loseille, Dave Marcum.

A closed advancing-layer method for generating high-aspect-ratio elements in the boundary layer (BL) region has been developed. This approach provides an answer to the mesh generation robustness issue as it starts from an existing valid mesh and always guarantuees a valid mesh in output. And, it handles very efficiently and naturally BL front collisions and it produces a natural smooth anisotropic blending between colliding layers. In addition, it provides a robust strategy to couple unstructured anisotropic mesh adaptation and high-aspect-ratio element pseudo-structured BL meshes. To this end, the mesh deformation is performed using the metric field associated with the given anisotropic meshes to maintain the adaptivity while inflating the BL. This approach utilizes a recently developed connectivity optimization based moving mesh strategy for deforming the volume mesh as the BL is inflated. In regards to the BL mesh generation, it features state-of-art capabilities, including, optimal normal evaluation, normal smoothing, blended BL termination, mixed-elements BL, varying growth rate, and BL imprinting on curved surfaces. Results for typical aerospace configurations are presented to assess the proposed strategy on both simple and complex geometries.

# 5. Bilateral Contracts and Grants with Industry

## 5.1. Bilateral Contracts with Industry

- The Boeing Company,

- Safran-Tech,
- Projet Rapid (DGA) avec Lemma.

# 6. Partnerships and Cooperations

## 6.1. European Initiatives

### *6.1.1. FP7 & H2020 Projects*

- UMRIDA https://sites.google.com/a/numeca.be/umrida/

## 6.2. International Initiatives

### *6.2.1. Inria Associate Teams Not Involved in an Inria International Labs*

*6.2.1.1. AM2NS*

Title: Advanced Meshing Methods for Numerical Simulations

International Partner (Institution - Laboratory - Researcher):

Mississippi State University (United States) - Center for Advanced Vehicular Systems - Computational Fluid Dynamics Dept. (CAVS-CFD) - Marcum David

Start year: 2017

See also: http://pages.saclay.inria.fr/frederic.alauzet/AssociateTeam_AM2NS/AT_am2ns.html

The purpose of the AM2NS Associate Team is to mutualize the knowledge of all teams in order to develop the next generation of meshing methods and their parallelization to address the new challenges in numerical simulations for industrial problems. The Associate Team is composed of four partners: Inria, Mississippi State University, The Boeing Company and Massachusetts Institute of Technology.

*6.2.1.2. MODIS*

Title: High-order discrete geometric modeling

International Partner (Institution - Laboratory - Researcher):

Polytechnique Montréal (Canada) - Computer Science - François Guibault

Start year: 2017

In the area of geometric modeling, major challenges are linked to the efficient visualization of CAD surfaces and to the generation of meshes adapted to numerical simulation. In this context, the conception of a discrete geometric model provides a simple and universal representation model, without the need for CAD. A first study has been carried out for the conception of a model of order 1 (one) defined by a “triangulation” composed of quadrilaterals and triangles. The advantage of this model of order 1 lies in its geometric simplicity. However, in the case of complex surfaces, it may require a very large number of elements, and besides it is not sufficiently rich to give certain essential characteristics like geometric curvatures. The main goal of this project is to extend this discrete model of order 1 to higher orders.

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