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CNRS

Ecole Polytechnique

Activity Report 2017

Project-Team GECO

Geometric Control Design

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team GECO

Creation of the Team: 2011 May 01, updated into Project-Team: 2013 January 01, end of the Project-Team: 2017 December 31

Keywords:

Computer Science and Digital Science:

A1.5. - Complex systems
A5.3. - Image processing and analysis
A6.1. - Mathematical Modeling
A6.4.1. - Deterministic control
A6.4.3. - Observability and Controlability
A6.4.4. - Stability and Stabilization

Other Research Topics and Application Domains:

B1.2.1. - Understanding and simulation of the brain and the nervous system
B2.6. - Biological and medical imaging
B9.4.2. - Mathematics
B9.4.3. - Physics

1. Personnel

Research Scientists

Mario Sigalotti [Team leader, Inria, Researcher, until Jun 2017, HDR]
Ugo Boscain [CNRS, Senior Researcher, until Jun 2017, HDR]

Post-Doctoral Fellows

Francesco Boarotto [Ecole polytechnique, until Jun 2017]
Valentina Franceschi [Inria, until Jun 2017]

PhD Students

Nicolas Augier [Ecole polytechnique, until Jun 2017]
Mathieu Kohli [Ecole polytechnique, until Jun 2017]
Jakub Orlowski [Centrale-Supélec, until Jun 2017]
Ludovic Sacchelli [Ecole polytechnique, until Jun 2017]

Intern

Gontran Lance [Inria, from May 2017 until Jun 2017]

Administrative Assistant

Tiffany Caristan [Inria]

2. Overall Objectives

2.1. Overall Objectives

Motion planning is not only a crucial issue in control theory, but also a widespread task of all sort of human activities. The aim of the project-team is to study the various aspects preceding and framing *motion planning*: accessibility analysis (determining which configurations are attainable), criteria to make choice among possible trajectories, trajectory tracking (fixing a possibly unfeasible trajectory and following it as closely as required), performance analysis (determining the cost of a tracking strategy), design of implementable algorithms, robustness of a control strategy with respect to computationally motivated discretizations, etc. The viewpoint that we adopt comes from geometric control: our main interest is in qualitative and intrinsic properties and our focus is on trajectories (either individual ones or families of them).

The main application domain of GECO is *quantum control*. The importance of designing efficient transfers between different atomic or molecular levels in atomic and molecular physics is due to its applications to photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing.

A second application area concerns the control interpretation of phenomena appearing in *neurophysiology*. It studies the modeling of the mechanisms supervising some biomechanics actions or sensorial reactions such as image reconstruction by the primary visual cortex, eyes movement and body motion. All these problems can be seen as motion planning tasks accomplished by the brain.

As a third applicative domain we propose a system dynamics approach to *switched systems*. Switched systems are characterized by the interaction of continuous dynamics (physical system) and discrete/logical components. They provide a popular modeling framework for heterogeneous aspects issuing from automotive and transportation industry, energy management and factory automation.

3. Research Program

3.1. Geometric control theory

The main research topic of the project-team is **geometric control**, with a special focus on **control design**. The application areas that we target are control of quantum mechanical systems, neurogeometry and switched systems.

Geometric control theory provides a viewpoint and several tools, issued in particular from differential geometry, to tackle typical questions arising in the control framework: controllability, observability, stabilization, optimal control... [9], [43] The geometric control approach is particularly well suited for systems involving nonlinear and nonholonomic phenomena. We recall that nonholonomicity refers to the property of a velocity constraint that is not equivalent to a state constraint.

The expression **control design** refers here to all phases of the construction of a control law, in a mainly open-loop perspective: modeling, controllability analysis, output tracking, motion planning, simultaneous control algorithms, tracking algorithms, performance comparisons for control and tracking algorithms, simulation and implementation.

We recall that

- **controllability** denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law ;
- **output tracking** refers to a control strategy aiming at keeping the value of some functions of the state arbitrarily close to a prescribed time-dependent profile. A typical example is **configuration tracking** for a mechanical system, in which the controls act as forces and one prescribes the position variables along the trajectory, while the evolution of the momenta is free. One can think for instance at the lateral movement of a car-like vehicle: even if such a movement is unfeasible, it can be tracked with arbitrary precision by applying a suitable control strategy;
- **motion planning** is the expression usually denoting the algorithmic strategy for selecting one control law steering the system from a given initial state to an attainable final one;
- **simultaneous control** concerns algorithms that aim at driving the system from two different initial conditions, with the same control law and over the same time interval, towards two given final states (one can think, for instance, at some control action on a fluid whose goal is to steer simultaneously two floating bodies.) Clearly, the study of which pairs (or n-uples) of states can be simultaneously connected thanks to an admissible control requires an additional controllability analysis with respect to the plain controllability mentioned above.

At the core of control design is then the notion of motion planning. Among the motion planning methods, a preeminent role is played by those based on the Lie algebra associated with the control system ([63], [50], [56]), those exploiting the possible flatness of the system ([37]) and those based on the continuation method ([75]). Optimal control is clearly another method for choosing a control law connecting two states, although it generally introduces new computational and theoretical difficulties.

Control systems with special structure, which are very important for applications are those for which the controls appear linearly. When the controls are not bounded, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a smoothly varying norm (representing a cost of the control), the resulting geometrical structure is called *sub-Riemannian*. Sub-Riemannian geometry thus appears as the underlying geometry of the nonholonomic control systems, playing the same role as Euclidean geometry for linear systems. As such, its study is fundamental for control design. Moreover its importance goes far beyond control theory and is an active field of research both in differential geometry ([62]), geometric measure theory ([38], [13]) and hypoelliptic operator theory ([25]).

Other important classes of control systems are those modeling mechanical systems. The dynamics are naturally defined on the tangent or cotangent bundle of the configuration manifold, they have Lagrangian or Hamiltonian structure, and the controls act as forces. When the controls appear linearly, the resulting model can be seen somehow as a second-order sub-Riemannian structure (see [30]).

The control design topics presented above naturally extend to the case of distributed parameter control systems. The geometric approach to control systems governed by partial differential equations is a novel subject with great potential. It could complement purely analytical and numerical approaches, thanks to its more dynamical, qualitative and intrinsic point of view. An interesting example of this approach is the paper [10] about the controllability of Navier–Stokes equation by low forcing modes.

4. Application Domains

4.1. Quantum control

The issue of designing efficient transfers between different atomic or molecular levels is crucial in atomic and molecular physics, in particular because of its importance in those fields such as photochemistry (control by laser pulses of chemical reactions), nuclear magnetic resonance (NMR, control by a magnetic field of spin dynamics) and, on a more distant time horizon, the strategic domain of quantum computing. This last application explicitly relies on the design of quantum gates, each of them being, in essence, an open loop control law devoted to a prescribed simultaneous control action. NMR is one of the most promising techniques for the implementation of a quantum computer.

Physically, the control action is realized by exciting the quantum system by means of one or several external fields, being them magnetic or electric fields. The resulting control problem has attracted increasing attention, especially among quantum physicists and chemists (see, for instance, [68], [73]). The rapid evolution of the domain is driven by a multitude of experiments getting more and more precise and complex (see the recent review [29]). Control strategies have been proposed and implemented, both on numerical simulations and on physical systems, but there is still a large gap to fill before getting a complete picture of the control properties of quantum systems. Control techniques should necessarily be innovative, in order to take into account the physical peculiarities of the model and the specific experimental constraints.

The area where the picture got clearer is given by finite dimensional linear closed models.

- **Finite dimensional** refers to the dimension of the space of wave functions, and, accordingly, to the finite number of energy levels.
- **Linear** means that the evolution of the system for a fixed (constant in time) value of the control is determined by a linear vector field.
- **Closed** refers to the fact that the systems are assumed to be totally disconnected from the environment, resulting in the conservation of the norm of the wave function.

The resulting model is well suited for describing spin systems and also arises naturally when infinite dimensional quantum systems of the type discussed below are replaced by their finite dimensional Galerkin approximations. Without seeking exhaustiveness, let us mention some of the issues that have been tackled for finite dimensional linear closed quantum systems:

- controllability [11],
- bounds on the controllability time [7],
- STIRAP processes [78],
- simultaneous control [51],
- optimal control ([47], [20], [31]),
- numerical simulations [57].

Several of these results use suitable transformations or approximations (for instance the so-called rotating wave) to reformulate the finite-dimensional Schrödinger equation as a sub-Riemannian system. Open systems have also been the object of an intensive research activity (see, for instance, [12], [48], [69], [26]).

In the case where the state space is infinite dimensional, some optimal control results are known (see, for instance, [16], [27], [44], [17]). The controllability issue is less understood than in the finite dimensional setting, but several advances should be mentioned. First of all, it is known that one cannot expect exact controllability on the whole Hilbert sphere [77]. Moreover, it has been shown that a relevant model, the quantum oscillator, is not even approximately controllable [70], [60]. These negative results have been more recently completed by positive ones. In [18], [19] Beauchard and Coron obtained the first positive controllability result for a quantum particle in a 1D potential well. The result is highly nontrivial and is based on Coron's return method (see [33]). Exact controllability is proven to hold among regular enough wave functions. In particular, exact controllability among eigenfunctions of the uncontrolled Schrödinger operator can be achieved. Other important approximate controllability results have then been proved using Lyapunov methods [59], [64], [45]. While [59] studies a controlled Schrödinger equation in $\mathbb{R}$ for which the uncontrolled Schrödinger operator has mixed spectrum, [64], [45] deal mainly with general discrete-spectrum Schrödinger operators.

In all the positive results recalled in the previous paragraph, the quantum system is steered by a single external field. Different techniques can be applied in the case of two or more external fields, leading to additional controllability results [36], [23].

The picture is even less clear for nonlinear models, such as Gross–Pitaevski and Hartree–Fock equations. The obstructions to exact controllability, similar to the ones mentioned in the linear case, have been discussed in [42]. Optimal control approaches have also been considered [15], [28]. A comprehensive controllability analysis of such models is probably a long way away.

4.2. Neurophysiology

At the interface between neurosciences, mathematics, automatics and humanoid robotics, an entire new approach to neurophysiology is emerging. It arouses a strong interest in the four communities and its development requires a joint effort and the sharing of complementary tools.

A family of extremely interesting problems concerns the understanding of the mechanisms supervising some sensorial reactions or biomechanics actions such as image reconstruction by the primary visual cortex, eyes movement and body motion.

In order to study these phenomena, a promising approach consists in identifying the motion planning problems undertaken by the brain, through the analysis of the strategies that it applies when challenged by external inputs. The role of control is that of a language allowing to read and model neurological phenomena. The control algorithms would shed new light on the brain's geometric perception (the so-called neurogeometry [66]) and on the functional organization of the motor pathways.

- A challenging problem is that of the understanding of the mechanisms which are responsible for the process of image reconstruction in the primary visual cortex V1.

The visual cortex areas composing V1 are notable for their complex spatial organization and their functional diversity. Understanding and describing their architecture requires sophisticated modeling tools. At the same time, the structure of the natural and artificial images used in visual psychophysics can be fully disclosed only using rather deep geometric concepts. The word “geometry" refers here to the internal geometry of the functional architecture of visual cortex areas (not to the geometry of the Euclidean external space). Differential geometry and analysis both play a fundamental role in the description of the structural characteristics of visual perception.

A model of human perception based on a simplified description of the visual cortex V1, involving geometric objects typical of control theory and sub-Riemannian geometry, has been first proposed by Petitot ([67]) and then modified by Citti and Sarti ([32]). The model is based on experimental observations, and in particular on the fundamental work by Hubel and Wiesel [41] who received the Nobel prize in 1981.

In this model, neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli arriving at a given point of the retina and oriented along a given direction. The retina is modeled by the real plane, while the directions at a given point are modeled by the projective line. The fiber bundle having as base the real plane and as fiber the projective line is called the *bundle of directions of the plane*.

From the neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them sensitive to stimuli arriving at a given point, oriented along any direction. In the same hypercolumn, relative to a point of the plane, we also find neurons that are sensitive to other stimuli properties, such as colors. Therefore, in this model the visual cortex treats an image not as a planar object, but as a set of points in the bundle of directions of the plane. The reconstruction is then realized by minimizing the energy necessary to activate orientation columns among those which are not activated directly by the image. This gives rise to a sub-Riemannian problem on the bundle of directions of the plane.

- Another class of challenging problems concern the functional organization of the motor pathways.

  The interest in establishing a model of the motor pathways, at the same time mathematically rigorous and biologically plausible, comes from the possible spillovers in robotics and neurophysiology. It could help to design better control strategies for robots and artificial limbs, yielding smoother and more progressive movements. Another underlying relevant societal goal (clearly beyond our domain of expertise) is to clarify the mechanisms of certain debilitating troubles such as cerebellar disease, chorea and Parkinson’s disease.

  A key issue in order to establish a model of the motor pathways is to determine the criteria underlying the brain’s choices. For instance, for the problem of human locomotion (see [14]), identifying such criteria would be crucial to understand the neural pathways implicated in the generation of locomotion trajectories.

  A nowadays widely accepted paradigm is that, among all possible movements, the accomplished ones satisfy suitable optimality criteria (see [76] for a review). One is then led to study an inverse optimal control problem: starting from a database of experimentally recorded movements, identify a cost function such that the corresponding optimal solutions are compatible with the observed behaviors.

  Different methods have been taken into account in the literature to tackle this kind of problems, for instance in the linear quadratic case [46] or for Markov processes [65]. However all these methods have been conceived for very specific systems and they are not suitable in the general case. Two approaches are possible to overcome this difficulty. The direct approach consists in choosing a cost function among a class of functions naturally adapted to the dynamics (such as energy functions) and to compare the solutions of the corresponding optimal control problem to the experimental data. In particular one needs to compute, numerically or analytically, the optimal trajectories and to choose suitable criteria (quantitative and qualitative) for the comparison with observed trajectories. The inverse approach consists in deriving the cost function from the qualitative analysis of the data.

4.3. Switched systems

Switched systems form a subclass of hybrid systems, which themselves constitute a key growth area in automation and communication technologies with a broad range of applications. Existing and emerging areas include automotive and transportation industry, energy management and factory automation. The notion of hybrid systems provides a framework adapted to the description of the heterogeneous aspects related to the interaction of continuous dynamics (physical system) and discrete/logical components.

The characterizing feature of switched systems is the collective aspect of the dynamics. A typical question is that of stability, in which one wants to determine whether a dynamical system whose evolution is influenced by a time-dependent signal is uniformly stable with respect to all signals in a fixed class ([53]).

The theory of finite-dimensional hybrid and switched systems has been the subject of intensive research in the last decade and a large number of diverse and challenging problems such as stabilizability, observability, optimal control and synchronization have been investigated (see for instance [74], [54]).

The question of stability, in particular, because of its relevance for applications, has spurred a rich literature. Important contributions concern the notion of common Lyapunov function: when there exists a Lyapunov function that decays along all possible modes of the system (that is, for every possible constant value of the signal), then the system is uniformly asymptotically stable. Conversely, if the system is stable uniformly with respect to all signals switching in an arbitrary way, then a common Lyapunov function exists [55]. In the *linear* finite-dimensional case, the existence of a common Lyapunov function is actually equivalent to the global uniform exponential stability of the system [61] and, provided that the admissible modes are finitely many, the Lyapunov function can be taken polyhedral or polynomial [21], [22], [34]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see [35] and references therein). Algebraic approaches to prove the stability of switched systems under arbitrary switching, not relying on Lyapunov techniques, have been proposed in [52], [8].

Other interesting issues concerning the stability of switched systems arise when, instead of considering arbitrary switching, one restricts the class of admissible signals, by imposing, for instance, a dwell time constraint [40].

Another rich area of research concerns discrete-time switched systems, where new intriguing phenomena appear, preventing the algebraic characterization of stability even for small dimensions of the state space [49]. It is known that, in this context, stability cannot be tested on periodic signals alone [24].

Finally, let us mention that little is known about infinite-dimensional switched system, with the exception of some results on uniform asymptotic stability ([58], [71], [72]) and some recent papers on optimal control ([39], [79]).

5. Highlights of the Year

5.1. Highlights of the Year

GECO has ended in June 2017, after being evaluated earlier in the year. A new team, including all former members of GECO, has started in July 2017 in the Inria Paris center. Its name is CAGE, for *Control And GEometry*.

6. New Software and Platforms

6.1. IRHD

Image Reconstruction via Hypoelliptic Diffusion

FUNCTIONAL DESCRIPTION: IRHD is a software for reconstruction of corrupted and damaged images. One of the main features of the algorithm on which the software is based is that it does not require any information about the location and character of the corrupted places. Another important advantage is that this method is massively parallelizable, this allows to work with sufficiently large images. Theoretical background of the presented method is based on the model of geometry of vision due to Petitot, Citti and Sarti. The main step is numerical solution of the equation of 3D hypoelliptic diffusion. IRHD is based on Fortran.

- Contact: Mario Sigalotti

7. New Results

7.1. New results

Let us list some the new results in sub-Riemannian geometry and hypoelliptic diffusion obtained by GECO's members.

- On a sub-Riemannian manifold we define two type of Laplacians. The macroscopic Laplacian Δ_ω, as the divergence of the horizontal gradient, once a volume ω is fixed, and the microscopic Laplacian, as the operator associated with a geodesic random walk. In [1] we consider a general class of random walks, where all sub-Riemannian geodesics are taken in account. This operator depends only on the choice of a complement c to the sub-Riemannian distribution, and is denoted L_c. We address the problem of equivalence of the two operators. This problem is interesting since, on equiregular sub-Riemannian manifolds, there is always an intrinsic volume (e.g. Popp's one P) but not a canonical choice of complement. The result depends heavily on the type of structure under investigation:
  - On contact structures, for every volume ω, there exists a unique complement c such that $\Delta_\omega = L_c$.
  - On Carnot groups, if H is the Haar volume, then there always exists a complement c such that $\Delta_H = L_c$. However this complement is not unique in general.
  - For quasi-contact structures, in general, $\Delta_P \neq L_c$ for any choice of c. In particular, L_c is not symmetric w.r.t. Popp's measure. This is surprising especially in dimension 4 where, in a suitable sense, Δ_P is the unique intrinsic macroscopic Laplacian.

  A crucial notion that we introduce here is the N-intrinsic volume, i.e. a volume that depends only on the set of parameters of the nilpotent approximation. When the nilpotent approximation does not depend on the point, a N-intrinsic volume is unique up to a scaling by a constant and the corresponding N-intrinsic sub-Laplacian is unique. This is what happens for dimension smaller or equal than 4, and in particular in the 4-dimensional quasi-contact structure mentioned above.
- In sub-Riemannian geometry the coefficients of the Jacobi equation define curvature-like invariants. We show in [4] that these coefficients can be interpreted as the curvature of a canonical Ehresmann connection associated to the metric, first introduced by Zelenko and Li. We show why this connection is naturally nonlinear, and we discuss some of its properties.
- In [6] we study the cut locus of the free, step two Carnot groups $\mathbb{G}_k$ with k generators, equipped with their left-invariant Carnot-Carathéodory metric. In particular, we disprove the conjectures on the shape of the cut loci proposed by O. Myasnichenko, by exhibiting sets of cut points $C_k \subset \mathbb{G}_k$ which, for $k \geq 4$, are strictly larger than conjectured ones. While the latter were, respectively, smooth semi-algebraic sets of codimension $\Theta(k^2)$ and semi-algebraic sets of codimension $\Theta(k)$, the sets C_k are semi-algebraic and have codimension 2, yielding the best possible lower bound valid for all k on the size of the cut locus of $\mathbb{G}_k$. Furthermore, we study the relation of the cut locus with the so-called abnormal set. Finally, and as a straightforward consequence of our results, we derive an explicit lower bound for the small time heat kernel asymptotics at the points of C_k. The question whether C_k coincides with the cut locus for $k \geq 4$ remains open.

New results on complex systems with hybrid or switched components are the following.

- In [2] we address the exponential stability of a system of transport equations with intermittent damping on a network of $N \geq 2$ circles intersecting at a single point O. The N equations are coupled through a linear mixing of their values at O, described by a matrix M. The activity of the intermittent damping is determined by persistently exciting signals, all belonging to a fixed class. The main result is that, under suitable hypotheses on M and on the rationality of the ratios between the lengths of the circles, such a system is exponentially stable, uniformly with respect to the persistently exciting signals. The proof relies on an explicit formula for the solutions of this system, which allows one to track down the effects of the intermittent damping.
- In [3] we study the relative controllability of linear difference equations with multiple delays in the state by using a suitable formula for the solutions of such systems in terms of their initial conditions, their control inputs, and some matrix-valued coefficients obtained recursively from the matrices defining the system. Thanks to such formula, we characterize relative controllability in time T in terms of an algebraic property of the matrix-valued coefficients, which reduces to the usual Kalman controllability criterion in the case of a single delay. Relative controllability is studied for solutions in the set of all functions and in the function spaces L^p and C^k. We also compare the relative controllability of the system for different delays in terms of their rational dependence structure, proving that relative controllability for some delays implies relative controllability for all delays that are “less rationally dependent” than the original ones, in a sense that we make precise. Finally, we provide an upper bound on the minimal controllability time for a system depending only on its dimension and on its largest delay.

Finally, a new contribution has been proposed in the domain of the control of quantum systems. More precisely, in [5] we consider the bilinear Schrödinger equation with discrete-spectrum drift. We show, for $n \in \mathbb{N}$ arbitrary, exact controllability in projections on the first n given eigenstates. The controllability result relies on a generic controllability hypothesis on some associated finite-dimensional approximations. The method is based on Lie-algebraic control techniques applied to the finite-dimensional approximations coupled with classical topological arguments issuing from degree theory.

8. Partnerships and Cooperations

8.1. Regional Initiatives

- Starting from the end of 2015, we have been funded by PGMO (Gaspard Monge Program for Optimisation and operational research) through a grant on Geometric Optimal Control. The grant is coordinated by Mario Sigalotti.

8.2. National Initiatives

8.2.1. ANR

The ANR SRGI starts at the end of 2015, for a duration of four years. GECO is one of one of the partners of the ANR. The national coordinator is Emmanuel Trélat (UPMC) and the local one Ugo Boscain.

SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.

8.2.2. Other initiatives

Ugo Boscain and Mario Sigalotti are members of the project DISQUO of the program Inphyniti of the CNRS. Coordinator: Thomas Chambrion (Nancy).

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

Program: ERC Proof of Concept

Project acronym: ARTIV1

Project title: An artificial visual cortex for image processing

Duration: From April 2017 to September 2018.

Coordinator: Ugo Boscain

Abstract: The ERC starting grant GECOMETHODS, on which this POC is based, tackled problems of diffusion equations via geometric control methods. One of the most striking achievements of the project has been the development of an algorithm of image reconstruction based mainly on non-isotropic diffusion. This algorithm is bio-mimetic in the sense that it replicates the way in which the primary visual cortex V1 of mammals processes the signals arriving from the eyes. It has performances that are at the state of the art in image processing. These results together with others obtained in the ERC project show that image processing algorithms based on the functional architecture of V1 can go very far. However, the exceptional performances of the primary visual cortex V1 rely not only on the particular algorithm used, but also on the fact that such algorithm ?runs? on a dedicated hardware having the following features: 1. an exceptional level of parallelism; 2. connections that are well adapted to transmit information in a non-isotropic way as it is required by the algorithms of image reconstruction and recognition. The idea of this POC is to create a dedicated hardware (called ARTIV1) emulating the functional architecture of V1 and hence having on one hand a huge degree of parallelism and on the other hand connections among the CPUs that reflect the non-isotropic structure of the visual cortex V1. Such a hardware that we plan to build as an integrated circuit with an industrial partner will be a veritable artificial visual cortex. It will be fully programmable and it will be able to perform many biomimetic image processing tasks that we expect to be exceptionally performant. ARTIV1 will come to the marked accompanied by some dedicated software for image reconstruction and image recognition. However we expect that other applications will be developed by customers, as for instance softwares for optical flow estimation or for sound processing.

8.4. International Initiatives

8.4.1. Informal International Partners

SISSA (Scuola Internazionale Superiore di Studi Avanzati), Trieste, Italy.

Sector of Functional Analysis and Applications, Geometric Control group. Coordinator: Andrei A. Agrachev.

We collaborate with the Geometric Control group at SISSA mainly on subjects related with sub-Riemannian geometry. Thanks partly to our collaboration, SISSA has established an official research partnership with École Polytechnique.

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Andrei Agrachev (SISSA, Italy) has been visiting the GECO team for one year, ending in June 2017.

8.5.1.1. Internships

Gontran Lance has made an internship in GECO, under the supervision of Mario Sigalotti and Emmanuel Trélat on the turnpike phenomenon in the orbital transfer problem.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

Ugo Boscain was organizer of the conference “Mathematical Control Theory, with a special session in honor of Gianna Stefani”, Porquerolles, 27–30 June.

9.1.2. Journal

9.1.2.1. Member of the Editorial Boards

- Ugo Boscain is Associate Editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing Editor of Journal of Dynamical and Control Systems
- Mario Sigalotti is Associate Editor of Journal of Dynamical and Control Systems
- Mario Sigalotti is Associate Editor of ESAIM Control, Optimisation and Calculus of Variations
- Ugo Boscain is Associate Editor of Mathematical Control and Related Fields
- Ugo Boscain is Associate editor of Analysis and Geometry in Metric Spaces

9.1.3. Invited Talks

- Mario Sigalotti gave an invited talk at the conference “Mathematical Control Theory, with a special session in honor of Gianna Stefani”, Porquerolles, France, June 2017.
- Mario Sigalotti gave an invited talk at groupe de travail “Contrôle”, Laboratoire Jacques-Louis Lions, Paris, France, January 2017.

9.1.4. Research Administration

- Mario Sigalotti is member of the IFAC technical committee “Distributed Parameter Systems”.
- Mario Sigalotti was member of the steering committee of the *Institut pour le Contrôle et la Décision* of the Idex Paris-Saclay up to June 2017.

9.2. Teaching - Supervision - Juries

9.2.1. Supervision

- PhD in progress: Ludovic Sacchelli, “Sub-Riemannian geometry, hypoelliptic operators, geometry of vision", started in September 2015, supervisors: Ugo Boscain, Mario Sigalotti.
- PhD in progress: Nicolas Augier, “Contrôle adiabatique des systèmes quantiques", started in September 2016, supervisors: Ugo Boscain, Mario Sigalotti.
- PhD in progress: Mathieu Kohli, “Volume and curvature in sub-Riemannian geometry", started in September 2016, supervisors: Davide Barilari, Ugo Boscain.
- PhD in progress: Jakub Orłowski, “Modeling and steering brain oscillations based on in vivo optogenetics data", started in September 2016, supervisors: Antoine Chaillet, Alain Destexhe, and Mario Sigalotti.

10. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

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Other Publications

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