



IN PARTNERSHIP WITH:
CNRS

**Université de Pau et des Pays de
l'Adour**

Activity Report 2017

Project-Team MAGIQUE-3D

Advanced 3D Numerical Modeling in Geophysics

IN COLLABORATION WITH: Laboratoire de mathématiques et de leurs applications (LMAP)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
**Earth, Environmental and Energy
Sciences**

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Project-Team MAGIQUE-3D

Creation of the Project-Team: 2007 July 01

Keywords:

Computer Science and Digital Science:

- A6. - Modeling, simulation and control
- A6.1. - Mathematical Modeling
- A6.1.1. - Continuous Modeling (PDE, ODE)
- A6.1.4. - Multiscale modeling
- A6.1.5. - Multiphysics modeling
- A6.2. - Scientific Computing, Numerical Analysis & Optimization
- A6.2.1. - Numerical analysis of PDE and ODE
- A6.2.7. - High performance computing

Other Research Topics and Application Domains:

- B3. - Environment and planet
- B3.3. - Geosciences
- B3.3.1. - Earth and subsoil
- B4. - Energy
- B4.1. - Fossile energy production (oil, gas)
- B5.5. - Materials

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2. Overall Objectives

2.1. General setting

MAGIQUE-3D is a joint project-team between Inria and the Department of Applied Mathematics (LMA) of the University of Pau, in partnership with CNRS. The mission of MAGIQUE-3D is to develop and validate efficient solution methodologies for solving complex three-dimensional geophysical problems, with a particular emphasis on problems arising in seismic imaging, in response to the local industrial and community needs. Indeed, as it is well known, the region of Pau has long-standing tradition in the Geosciences activities. However, in spite of the recent significant advances in algorithmic considerations as well as in computing platforms, the solution of most real-world problems in this field remains intractable. Hence, there is a scientific need of pressing importance to design new numerical methods for solving efficiently and accurately wave propagation problems defined in strongly heterogeneous domains.

MAGIQUE-3D program possesses an exceptional combination that is a prerequisite for accomplishing its mission: the investigator backgrounds, research interests, and technical skills complement to form a research team with a potential for significant impact on the computational infrastructure of geophysical sciences. The research record of MAGIQUE-3D group covers a large spectrum of accomplishments in the field of wave propagation including (a) the design, validation, and performance assessment of a class of DG-methods for solving efficiently high frequency wave problems, (b) the construction, convergence analysis, and performance assessment of various absorbing-type boundary conditions that are key ingredients for solving problems in infinite domains, and (c) the development of asymptotic models that are the primary candidate in the presence of heterogeneities that are small compared to the wave length. MAGIQUE-3D has built strong collaborations and partnerships with various institutions including (a) local industry (TOTAL), (b) national research centers (ONERA and CEA), and (c) international academic partnerships (e.g. Interdisciplinary Research Institute for the Sciences (IRIS) at California State University, Northridge, USA; University of Pays Basque at Bilbao, Spain; University of Novosibirsk, Russia).

3. Research Program

3.1. Introduction

Probing the invisible is a quest that is shared by a wide variety of scientists such as archaeologists, geologists, astrophysicists, physicists, etc... Magique-3D is involved in Geophysical imaging which aims at understanding

the internal structure of the Earth from the propagation of waves. Both qualitative and quantitative information are required and two geophysical techniques can be used: **seismic reflection** and **seismic inversion**. Seismic reflection provides a qualitative description of the subsurface from reflected seismic waves by indicating the position of the reflectors while seismic inversion transforms seismic reflection data into a quantitative description of the subsurface. Both techniques are inverse problems based upon the numerical solution of wave equations. Oil and Gas explorations have been pioneering application domains for seismic reflection and inversion and even if numerical seismic imaging is computationally intensive, oil companies promote the use of numerical simulations to provide synthetic maps of the subsurface. This is due to the tremendous progresses of scientific computing which have pushed the limits of existing numerical methods and it is now conceivable to tackle realistic 3D problems. However, mathematical wave modeling has to be well-adapted to the region of interest and the numerical schemes which are employed to solve wave equations have to be both accurate and scalable enough to take full advantage of parallel computing. Today, geophysical imaging tackles more and more realistic problems and we can contribute to this task by improving the modeling and by deriving advanced numerical methods for solving wave problems.

Magique-3D proposes to organize its research around three main axes:

1. Mathematical modeling of multi-physics involving wave equations;
2. Supercomputing for Helmholtz problems;
3. Construction of high-order hybrid schemes.

These three research fields will be developed with the main objective of solving inverse problems dedicated to geophysical imaging.

3.2. Mathematical modeling of multi-physics involving wave equations

Wave propagation modeling is of great interest for many applications like oil and gas exploration, non destructive testing, medical imaging, etc. It involves equations which can be solved in time or frequency domain and their numerical approximation is not easy to handle, in particular when dealing with real-world problems. In both cases, the propagation domain is either infinite or its dimensions are much greater than the characteristic wavelength of the phenomenon of interest. But since wave problems are hyperbolic, the physical phenomenon can be accurately described by computing solutions in a bounded domain including the sources which have generated the waves. Until now, we have mainly worked on imaging techniques based on acoustic or elastic waves and we have developed advanced finite element software packages which are used by Total for oil exploration. Nevertheless, research on modeling must go on because there are simulations which can still not be performed because their computational cost is much too high. This is particularly true for complex tectonics involving coupled wave equations. We then propose to address the issue of coupling wave equations problems by working on the mathematical construction of reduced systems. By this way, we hope to improve simulations of elasto-acoustic and electro-seismic phenomena and then, to perform numerical imaging of strongly heterogeneous media. Even in the simplest situation where the wavelengths are similar (elasto-acoustic coupling), the dimension of the discrete coupled problem is huge and it is a genuine issue in the prospect of solving 3D inverse problems.

The accurate numerical simulation of full wave problems in heterogeneous media is computationally intensive since it needs numerical schemes based on grids. The size of the cells depends on the propagation velocity of waves. When coupling wave problems, conversion phenomena may occur and waves with very different propagation velocity coexist. The size of the cells is then defined from the smallest velocity and in most of the real-world cases, the computational cost is crippling. Regarding existing computing capabilities, we propose to derive intermediate models which require less computational burden and provide accurate solutions for a wide-ranging class of problems including Elasto-acoustics and Electro-seismology.

When it comes to mathematical analysis, we have identified two tasks which could help us simulate realistic 3D multi-physics wave problems and which are in the scope of our savoir-faire. They are construction of approximate and multiscale models which are different tasks. The construction of approximate problems aims at deriving systems of equations which discrete formulation involves middle-sized matrices and in general, they are based on high frequency hypothesis. Multiscale models are based on a rigorous analysis involving a small parameter which does not depend on the propagation velocity necessarily.

Recently, we have conducted research on the construction of approximate models for offshore imaging. Elastic and acoustic wave equations are coupled and we investigate the idea of eliminating the computations inside water by introducing equivalent interface conditions on the sea bottom. We apply an On-Surface-Radiation-Condition (OSRC) which is obtained from the approximation of the acoustic Dirichlet-to-Neumann (DtN) operator [74], [53]. To the best of our knowledge, OSRC method has never been used for solving reduced coupling wave problems and preliminary promising results are available at [56]. We would like to investigate this technique further because we could form a battery of problems which can be solved quickly. This would provide a set of solutions which we could use as initial guess for solving inverse problems. But we are concerned with the performance of the OSRC method when wave conversions with different wavelengths occur. Anyway, the approximation of the DtN operator is not obvious when the medium is strongly heterogeneous and multiscale analysis might be more adapted. For instance, according to existing results in Acoustics and Electromagnetism for the modeling of wire antennas [65], multiscale analysis should turn out to be very efficient when the propagation medium includes well logs, fractures and faults which are very thin structures when compared to the wavelength of seismic waves. Moreover, multiscale analysis should perform well when the medium is strongly oscillating like porous media. It could thus provide an alternative to homogenization techniques which can be applied only when the medium is periodic. We thus propose to develop reduced multi-scale models by performing rigorous mathematical procedure based on regular and singular multiscale analysis. Our approach distinguishes itself from others because it focuses on the numerical representation of small structures by time-dependent problems. This could give rise to the development of new finite element methods which would combine DG approximations with XFEM (Extended Finite Element Method) which has been created for the finite element treatment of thin structures like cracks.

But Earth imaging must be more than using elasto-acoustic wave propagation. Electromagnetic waves can also be used and in collaboration with Prof. D. Pardo (Iker Basque Foundation and University of Bilbao), we conduct researches on passive imaging to probe boreholes. Passive imaging is a recent technique of imaging which uses natural electromagnetic fields as sources. These fields are generated by hydromagnetic waves propagating in the magnetosphere which transform into electromagnetic waves when they reach the ionosphere. This is a mid-frequency imaging technique which applies also to mineral and geothermal exploration, to predict seismic hazard or for groundwater monitoring. We aim at developing software package for resistivity inversion, knowing that current numerical methods are not able to manage 3D inversion. We have obtained results based on a Petrov-Galerkin approximation [50], but they are limited to 2D cases. We have thus proposed to reduce the 3D problem by using 1D semi-analytic approximation of Maxwell equations [79]. This work has just started in the framework of a PhD thesis and we hope that it will give us the possibility of imaging 3D problems.

Magique-3D would like to expand its know-how by considering electro-seismic problems which are in the scope of coupling electromagnetic waves with seismic waves. Electro-seismic waves are involved in porous media imaging which is a tricky task because it is based on the coupling of waves with very different wavelengths described by Biot equations and Maxwell equations. Biot equations govern waves in saturated porous media and they represent a complex physical phenomenon involving a slow wave which is very difficult to simulate numerically. In [72], interesting results have been obtained for the simulation of piezoelectric sensors. They are based on a quasi-static approximation of the Maxwell model coupled with Elastodynamics. Now, we are concerned with the capability of using this model for Geophysical Imaging and we believe that the derivation and/or the analysis of suitable modelings is necessary. Collaborations with Geophysicists are thus mandatory in the prospect of using both experimental and numerical approaches. We would like to collaborate with Prof. C. Bordes and Prof. D. Brito (Laboratory of Complex Fluids and their Reservoirs, CNRS and University of Pau) who have efficient experimental devices for the propagation of electromagnetic waves inside saturated porous media [55]. This collaboration should be easy to organize since Magique-3D has a long-term experience in collaborating with geophysicists. We then believe that we will not need a lot of time to get joint results since we can use our advanced software packages Hou10ni and Montjoie and our colleagues have already obtained data. Electro-seismology is a very challenging research domain for us and we would like to enforce our collaborations with IsTerre (Institute of Earth Science, University of Grenoble) and for that topic with Prof. S. Garambois who is an expert in Electro-seismology [81], [82], [69], [70]. A joint research program could gather Geophysicists from the University of Pau and from IsTerre and Magique-3D.

In particular, it would be interesting to compare simulations performed with Hou10ni, Montjoie, with the code developed by Prof. S. Garambois and to use experimental simulations for validation.

3.3. Supercomputing for Helmholtz problems

Probing invisible with harmonic equations is a need for many scientists and it is also a topic offering a wealth of interesting problems for mathematicians. It is well-known that Helmholtz equations discretization is very sensitive to the frequency scale which can be wide-ranging for some applications. For example, depth imaging is searching for deeper layers which may contain hydrocarbons and frequencies must be of a few tens of Hertz with a very low resolution. If it is to detect hidden objects, the depth of the explored region does not exceed a few tens of meters and frequencies close to the kiloHertz are used. High performing numerical methods should thus be stable for a widest as possible frequency range. In particular, these methods should minimize phenomena of numerical pollution that generate errors which increase faster with frequency than with the inverse of space discretization step. As a consequence, there is a need of mesh refinement, in particular at high frequency.

During the period 2010-2014, the team has worked extensively on high order discontinuous Galerkin (DG) methods. Like standard Finite Element Methods, they are elaborated with polynomial basis functions and they are very popular because they are defined locally for each element. It is thus easy to use basis polynomial functions with different degrees and this shows the perfect flexibility of the approximation in case of heterogeneous media including homogeneous parts. Indeed, low degree basis functions can be used in heterogeneous regions where a fine grid is necessary while high degree polynomials can be used for coarse elements covering homogeneous parts. In particular, Magique-3D has developed Hou10ni that solves harmonic wave equations with DG methods and curved elements. We found that both the effects of pollution and dispersion, which are very significant when a conventional finite element method is used, are limited [57]. However, bad conditioning is persisting and reliability of the method is not guaranteed when the coefficients vary considerably. In addition, the number of unknowns of the linear system is too big to hope to solve a realistic 3D problem. So it is important to develop approximation methods that require fewer degrees of freedom. Magique-3D wishes to invest heavily in the development of new approximation methods for harmonic wave equations. It is a difficult subject for which we want to develop different tasks, in collaboration with academic researchers with whom we are already working or have established contacts. Research directions that we would like to follow are the following.

First, we will continue our long-term collaboration with Prof. Rabia Djellouli. We want to continue to work on hybrid finite element methods that rely on basis functions composed of plane waves and polynomials. These methods have demonstrated good resistance to the phenomenon of numerical pollution [51], [52], but their capability of solving industrial problems has not been illustrated. This is certainly due to the absence of guideline for choosing the plane waves. We are thus currently working on the implementation of a methodology that makes the choice of plane waves automatic for a given simulation (fixed propagation domain, data source, etc.). This is up-front investigation and there is certainly a lot of remaining work before being applied to geophysical imaging. But it gives the team the opportunity to test new ideas while remaining in contact with potential users of the methods.

Then we want to work with Prof. A. Bendali on developing methods of local integral equations which allow calculation of numerical fluxes on the edges of elements. One could then use these fluxes in a DG method for reconstructing the solution throughout the volume of calculation. This research is motivated by recent results which illustrate the difficulties of the existing methods which are not always able to approximate the propagating modes (plane waves) and the evanescent modes (polynomials) that may coexist, especially when one considers realistic applications. Integral equations are direct tools for computing fluxes and they are known for providing very good accuracy. They thus should help to improve the quality of approximation of DG methods which are fully flux-dependent. In addition, local integral equations would limit calculations at the interfaces, which would have the effect of limiting the number of unknowns generally high, especially for DG methods. Again, it is a matter of long-term research which success requires a significant amount of mathematical analysis, and also the development of non-trivial code.

To limit the effects of pollution and dispersion is not the only challenge that the team wants to tackle. Our experience alongside Total has made us aware of the difficulties in constructing meshes that are essential to achieve our simulations. There are several teams at Inria working on mesh generation and we are in contact with them, especially with Gamma3 (Paris-Rocquencourt Research Center). These teams develop meshes increasingly sophisticated to take account of the constraints imposed by realistic industrial benchmarks. But in our opinion, issues which are caused by the construction of meshes are not the only downside. Indeed, we have in mind to solve inverse problems and in this case it is necessary to mesh the domain at each iteration of Newton-type solver. It is therefore interesting to work on methods that either do not use mesh or rely on meshes which are very easy to construct. Regarding meshless methods, we have begun a collaboration with Prof. Djellouli which allowed us to propose a new approach called Mesh-based Frontier Free Formulation (MF3). The principle of this method is the use of fundamental solutions of Helmholtz equations as basic functions. One can then reduce the volumic variational formulation to a surfacic variational formulation which is close to an integral equation, but which does not require the calculation of singularities. The results are very promising and we hope to continue our study in the context of the application to geophysical imaging. An important step to validate this method will be particularly its extension to 3D because the results we have achieved so far are for 2D problems.

Keeping in mind the idea of limiting the difficulties of mesh, we want to study the method of virtual elements. This method attracts us because it relies on meshes that can be made of arbitrarily-shaped polygon and meshes should thus be fairly straightforward. Existing works on the subject have been mainly developed by the University of Pavia, in collaboration with Los Alamos National Laboratory [54], [61], [60], [58], [62]. None of them mentions the feasibility of the method for industrial applications and to our knowledge, there are no results on the method of virtual elements applied to the wave equations. First, we aim at applying the method described in [59] to the scalar Helmholtz equation and explore opportunities to use discontinuous elements within this framework. Then hp-adaptivity could be kept, which is particularly interesting for wave propagation in heterogeneous media.

DG methods are known to require a lot of unknowns that can exceed the limits accepted by the most advanced computers. This is particularly true for harmonic wave equations that require a large number of discretization points, even in the case of a conventional finite element method. We therefore wish to pursue a research activity that we have just started in collaboration with the project-team Nachos (Sophia-Antipolis Méditerranée Research Center). In order to reduce the number of degrees of freedom, we are interested in "hybrid mixed" Discontinuous Galerkin methods that provides a two-step procedure for solving the Helmholtz equations [73], [77], [76]. First, Lagrange multipliers are introduced to represent the flux of the numerical solution through the interface (edge or face) between two elements. The Lagrange multipliers are solution to a linear system which is constructed locally element by element. The number of degrees of freedom is then strongly reduced since for a standard DG method, there is a need of considering unknowns including volumetric values inside the element. And obviously, the gain is even more important when the order of the element is high. Next, the solution is reconstructed from the values of the multipliers and the cost of this step is negligible since it only requires inverting small-sized matrices. We have obtained promising results in the framework of the PhD thesis of Marie Bonnasse-Gahot and we want to apply it to the simulation of complex phenomena such as the 3D viscoelastic wave propagation.

Obviously, the success of all these works depends on our ability to consider realistic applications such as wave propagation in the Earth. And in these cases, it is quite possible that even if we manage to develop accurate less expensive numerical methods, the solution of inverse problems will still be computationally intensive. It is thus absolutely necessary that we conduct our research by taking advantage of the latest advances in high-performance computing. We have already initiated discussions with the project team HIEPACS (Bordeaux Sud-Ouest research Center) to test the performance of the latest features of Mumps <http://mumps.enseciht.fr/>, such as Low Rank Approximation or adaptation to hybrid CPU / GPU architectures and to Intel Xeon Phi, on realistic test cases. We are also in contact with the team Algorithm at Cerfacs (Toulouse) for the development of local integral equations solvers. These collaborations are essential for us and we believe that they will be decisive for the simulation of three-dimensional elasto-dynamic problems. However, our scientific contribution will be limited in this area because we are not experts in HPC.

3.4. Hybrid time discretizations of high-order

Most of the meshes we consider are composed of cells greatly varying in size. This can be due to the physical characteristics (propagation speed, topography, ...) which may require to refine the mesh locally, very unstructured meshes can also be the result of dysfunction of the mesher. For practical reasons which are essentially guided by the aim of reducing the number of matrix inversions, explicit schemes are generally privileged. However, they work under a stability condition, the so-called Courant Friedrichs Lewy (CFL) condition which forces the time step being proportional to the size of the smallest cell. Then, it is necessary to perform a huge number of iterations in time and in most of the cases because of a very few number of small cells. This implies to apply a very small time step on grids mainly composed of coarse cells and thus, there is a risk of creating numerical dispersion that should not exist. However, this drawback can be avoided by using low degree polynomial basis in space in the small meshes and high degree polynomials in the coarse meshes. By this way, it is possible to relax the CFL condition and in the same time, the dispersion effects are limited. Unfortunately, the cell-size variations are so important that this strategy is not sufficient. One solution could be to apply implicit and unconditionally stable schemes, which would obviously free us from the CFL constraint. Unfortunately, these schemes require inverting a linear system at each iteration and thus needs huge computational burden that can be prohibitive in 3D. Moreover, numerical dispersion may be increased. Then, as second solution is the use of local time stepping strategies for matching the time step to the different sizes of the mesh. There are several attempts [66], [63], [80], [75], [68] and Magique 3D has proposed a new time stepping method which allows us to adapt both the time step and the order of time approximation to the size of the cells. Nevertheless, despite a very good performance assessment in academic configurations, we have observed to our detriment that its implementation inside industrial codes is not obvious and in practice, improvements of the computational costs are disappointing, especially in a HPC framework. Indeed, the local time stepping algorithm may strongly affect the scalability of the code. Moreover, the complexity of the algorithm is increased when dealing with lossy media [71].

Recently, Dolean *et al* [67] have considered a novel approach consisting in applying hybrid schemes combining second order implicit schemes in the thin cells and second order explicit discretization in the coarse mesh. Their numerical results indicate that this method could be a good alternative but the numerical dispersion is still present. It would then be interesting to implement this idea with high-order time schemes to reduce the numerical dispersion. The recent arrival in the team of J. Chabassier should help us to address this problem since she has the expertise in constructing high-order implicit time scheme based on energy preserving Newmark schemes [64]. We propose that our work be organized around the two following tasks. The first one is the extension of these schemes to the case of lossy media because applying existing schemes when there is attenuation is not straightforward. This is a key issue because there is artificial attenuation when absorbing boundary conditions are introduced and if not, there are cases with natural attenuation like in visco-elastic media. The second one is the coupling of high-order implicit schemes with high-order explicit schemes. These two tasks can be first completed independently, but the ultimate goal is obviously to couple the schemes for lossy media. We will consider two strategies for the coupling. The first one will be based on the method proposed by Dolean *et al*, the second one will consist in using Lagrange multiplier on the interface between the coarse and fine grids and write a novel coupling condition that ensures the high order consistency of the global scheme. Besides these theoretical aspects, we will have to implement the method in industrial codes and our discretization methodology is very suitable for parallel computing since it involves Lagrange multipliers. We propose to organize this task as follows. There is first the crucial issue of a systematic distribution of the cells in the coarse/explicit and in the fine/implicit part. Based on our experience on local time stepping, we claim that it is necessary to define a criterion which discriminates thin cells from coarse ones. Indeed, we intend to develop codes which will be used by practitioners, in particular engineers working in the production department of Total. It implies that the code will be used by people who are not necessarily experts in scientific computing. Considering real-world problems means that the mesh will most probably be composed of a more or less high number of subsets arbitrarily distributed and containing thin or coarse cells. Moreover, in the prospect of solving inverse problems, it is difficult to assess which cells are thin or not in a mesh which varies at each iteration.

Another important issue is the load balancing that we can not avoid with parallel computing. In particular, we will have to choose one of these two alternatives: dedicate one part of processors to the implicit computations and the other one to explicit calculus or distribute the resolution with both schemes on all processors. A collaboration with experts in HPC is then mandatory since we are not expert in parallel computing. We will thus continue to collaborate with the team-projects Hiepac and Runtime with whom we have a long-term experience of collaborations.

In the future, we aim at enlarging the application range of implicit schemes. The idea will be to use the degrees of freedom offered by the implicit discretization in order to tackle specific difficulties that may appear in some systems. For instance, in systems involving several waves (as P and S waves in porous elastic media, or coupled wave problems as previously mentioned) the implicit parameter could be adapted to each wave and optimized in order to reduce the computational cost. More generally, we aim at reducing numeric bottlenecks by adapting the implicit discretization to specific cases.

4. Application Domains

4.1. Seismic Imaging

The main objective of modern seismic processing is to find the best representation of the subsurface that can fit the data recorded during the seismic acquisition survey. In this context, the seismic wave equation is the most appropriate mathematical model. Numerous research programs and related publications have been devoted to this equation. An acoustic representation is suitable if the waves propagate in a fluid. But the subsurface does not contain fluids only and the acoustic representation is not sufficient in the general case. Indeed the acoustic wave equation does not take some waves into account, for instance shear waves, turning waves or the multiples that are generated after several reflections at the interfaces between the different layers of the geological model. It is then necessary to consider a mathematical model that is more complex and resolution techniques that can model such waves. The elastic or viscoelastic wave equations are then reference models, but they are much more difficult to solve, in particular in the 3D case. Hence, we need to develop new high-performance approximation methods.

Reflection seismics is an indirect measurement technique that consists in recording echoes produced by the propagation of a seismic wave in a geological model. This wave is created artificially during seismic acquisition surveys. These echoes (i.e., reflections) are generated by the heterogeneities of the model. For instance, if the seismic wave propagates from a clay layer to sand, one will observe a sharp reflected signal in the seismic data recorded in the field. One then talks about reflection seismics if the wave is reflected at the interface between the two media, or talks about seismic refraction if the wave is transmitted along the interface. The arrival time of the echo enables one to locate the position of this transition, and the amplitude of the echo gives information on some physical parameters of the two geological media that are in contact. The first petroleum exploration surveys were performed at the beginning of the 1920's and for instance, the Orchard Salt Dome in Texas (USA) was discovered in 1924 by the seismic-reflection method.

4.2. Imaging complex media with ultrasonic waves

The acoustic behavior of heterogeneous or composite materials attracts considerable excitement. Indeed, their acoustic response may be extremely different from the single constituents responses. In particular, dispersions of resonators in a matrix are the object of large research efforts, both experimentally and theoretically. However it is still a challenge to dispose of numerical tools with sufficient abilities to deal with the simulation and imaging of such materials behavior. Indeed, not only acoustic simulations are very time-consuming, but they have to be performed on realistic enough solution domains, i.e. domains which capture well enough the structural features of the considered materials.

This collaboration with I2M, University of Bordeaux aims at addressing this type of challenges by developing numerical and experimental tools in order to understand the propagation of ultrasonic waves in complex media, image these media, and in the future, help design composite materials for industrial purposes.

4.3. Helioseismology

This collaboration with the Max Planck Institute for Solar System, Göttingen, Germany, which started in 2014, aims at designing efficient numerical methods for the wave propagation problems that arise in helioseismology in the context of inverse problems. The final goal is to retrieve information about the structure of the Sun i.e. inner properties such as density or pressure via the inversion of a wave propagation problem. Acoustic waves propagate inside the Sun which, in a first approximation and regarding the time scales of physical phenomena, can be considered as a moving fluid medium with constant velocity of motion. Some other simplifications lead to computational saving, such as supposing a radial or axisymmetric geometry of the Sun. Aeroacoustic equations must be adapted and efficiently solved in this context, this has been done in the finite elements code Montjoie. In other situations, a full 3D simulation is required and demands large computational resources. Ultimately, we aim at modeling the coupling with gravity potential and electromagnetic waves (MHD equations) in order to be able to better understand Sun spots.

5. New Software and Platforms

5.1. Elasticus

KEYWORDS: Discontinuous Galerkin - Acoustic equation - Elastodynamic equations - Elastoacoustic - 2D - 3D - Time Domain

SCIENTIFIC DESCRIPTION: Elasticus simulate acoustic and elastic wave propagation in 2D and in 3D, using Discontinuous Galerkin Methods. The space discretization is based on two kind of basis functions, using Lagrange or Jacobi polynomials. Different kinds of fluxes (upwind and centered) are implemented, coupled with RK2 and RK4 time schemes.

FUNCTIONAL DESCRIPTION: Elasticus is a sequential library, independent of Total platform and developed in Fortran, to simulate wave propagation in geophysical environment, based on a DG method. It is meant to help PhD students and post-doctoral fellows to easily implement their algorithms in the library. Thus, readability of the code is privileged to optimization of its performances. Developed features should be easily transferred in the computing platform of Total. Elasticus manages arbitrary orders for the spatial discretization with DG method.

NEWS OF THE YEAR: In 2017, we implemented the coupling between triangles and quadrangles, and we started the implementation of coupling between Discontinuous Galerkin methods and Spectral Element methods

- Participants: Julien Diaz, Lionel Boillot and Simon Ettouati
- Partner: TOTAL
- Contact: Julien Diaz

5.2. Hou10ni

KEYWORDS: 2D - 3D - Elastodynamic equations - Acoustic equation - Elastoacoustic - Frequency Domain - Time Domain - Discontinuous Galerkin

SCIENTIFIC DESCRIPTION: Hou10ni simulates acoustic and elastic wave propagation in time domain and in harmonic domain, in 2D and in 3D. It is also able to model elasto acoustic coupling. It is based on the second order formulation of the wave equation and the space discretization is achieved using Interior Penalty Discontinuous Galerkin Method. Recently, the harmonic domain solver has been extended to handle Hybridizable Discontinuous Galerkin Methods.

FUNCTIONAL DESCRIPTION: This software simulates the propagation of waves in heterogeneous 2D and 3D media in time-domain and in frequency domain. It is based on an Interior Penalty Discontinuous Galerkin Method (IPDGM) and allows for the use of meshes composed of cells of various order (p-adaptivity in space).

NEWS OF THE YEAR: In 2017, we have completed the implementation of hybridizable DG for 3D anisotropic elastic, and we have coupled Hou10ni with Maphys (developed by Inria team project Hiepac). We have begun scalability tests and performance comparison of Hou10ni/Mumps vs Hou10ni/Maphys, in the framework of the european project HPC4E.

- Participants: Conrad Hillairet, Elodie Estecahandy, Julien Diaz, Lionel Boillot and Marie Bonnasse Gahot
- Contact: Julien Diaz
- Publications: Hybridizable discontinuous Galerkin method for the two-dimensional frequency-domain elastic wave equations - Convergence of seismic full waveform inversion and extension to Cauchy data - Convergence Analysis for Seismic Full Waveform Inversion - Stability and convergence analysis for seismic depth imaging using FWI - On the use of a laser ablation as a laboratory seismic source - Towards Energy-Efficient Storage Servers - Equivalent Robin Boundary Conditions for Acoustic and Elastic Media - Comparison of solvers performance when solving the 3D Helmholtz elastic wave equations over the Hybridizable Discontinuous Galerkin method - Comparison of solvers performance when solving the 3D Helmholtz elastic wave equations using the Hybridizable Discontinuous Galerkin method - Resolution strategy for the Hybridizable Discontinuous Galerkin system for solving Helmholtz elastic wave equations - Seismic imaging in laboratory trough laser Doppler vibrometry - Absorbing Boundary Conditions for 3D Elastic TTI Modeling, Application to Time-Based and Time-Harmonic Simulations - Shape and material parameter reconstruction of an isotropic or anisotropic solid immersed in a fluid - Modelling and advanced simulation of wave propagation phenomena in 3D geophysical media. - Multi-level explicit local time-stepping methods for second-order wave equations - Absorbing Boundary Conditions for 3D elastic TTI modeling - Modeling of elastic Helmholtz equations by hybridizable discontinuous Galerkin method (HDG) for geophysical applications - Performance Assessment on Hybridizable Dg Approximations for the Elastic Wave Equation in Frequency Domain - High-Order IPDG Approximations for Elasto-Acoustic Problems - High-order Discontinuous Galerkin approximations for elasto-acoustic scattering problems - Modelling of seismic waves propagation in harmonic domain by hybridizable discontinuous Galerkin method (HDG) - Absorbing Boundary Conditions for 3D Tilted Transverse Isotropic media - Performance comparison between hybridizable DG and classical DG methods for elastic waves simulation in harmonic domain - Polynomial speeds in a Discontinuous Galerkin code - Hybridizable Discontinuous Galerkin method for the simulation of the propagation of the elastic wave equations in the frequency domain - Discontinuous Galerkin methods for the simulation of the propagation of the elastic wave equations in the frequency domain - High order discontinuous Galerkin methods for time-harmonic elastodynamics - Hybridizable discontinuous Galerkin method for the two-dimensional frequency-domain elastic wave equations - Efficient DG-like formulation equipped with curved boundary edges for solving elasto-acoustic scattering problems - Numerical schemes for the simulation of seismic wave propagation in frequency domain - Performance analysis of DG and HDG methods for the simulation of seismic wave propagation in harmonic domain - Hybridizable Discontinuous Galerkin method for solving Helmholtz elastic wave equations - Discontinuous Galerkin methods for solving Helmholtz elastic wave equations for seismic imaging - Performance comparison of HDG and classical DG method for the simulation of seismic wave propagation in harmonic domain - Contributions to the mathematical modeling and to the parallel algorithmic for the optimization of an elastic wave propagator in anisotropic media - Contribution to the mathematical analysis and to the numerical solution of an inverse elasto-acoustic scattering problem
- URL: <https://team.inria.fr/magique3d/software/hou10ni/>

5.3. MONTJOIE

KEYWORDS: High order finite elements - Edge elements - Aeroacoustics - High order time schemes

SCIENTIFIC DESCRIPTION: Montjoie is designed for the efficient solution of time-domain and time-harmonic linear partial differential equations using high-order finite element methods. This code is mainly written for quadrilateral/hexahedral finite elements, partial implementations of triangular/tetrahedral elements are provided. The equations solved by this code, come from the "wave propagation" problems, particularly acoustic, electromagnetic, aeroacoustic, elastodynamic problems.

FUNCTIONAL DESCRIPTION: Montjoie is a code that provides a C++ framework for solving partial differential equations on unstructured meshes with finite element-like methods (continuous finite element, discontinuous Galerkin formulation, edge elements and facet elements). The handling of mixed elements (tetrahedra, prisms, pyramids and hexahedra) has been implemented for these different types of finite elements methods. Several applications are currently available : wave equation, elastodynamics, aeroacoustics, Maxwell's equations.

- Participants: Gary Cohen, Juliette Chabassier, Marc Duruflé and Morgane Bergot
- Contact: Marc Duruflé
- URL: <http://montjoie.gforge.inria.fr/>

5.4. tmodeling-DG

Time-domain Wave-equation Modeling App

KEYWORDS: 2D - 3D - Elastoacoustic - Elastodynamic equations - Discontinuous Galerkin - Time Domain
SCIENTIFIC DESCRIPTION: tmodeling-DG simulate acoustic and elastic wave propagation in 2D and in 3D, using Discontinuous Galerkin Methods. The space discretization is based on two kind of basis functions, using Lagrange or Jacobi polynomials. Different kinds of fluxes (upwind and centered) are implemented, coupled with RK2 and RK4 time schemes.

FUNCTIONAL DESCRIPTION: tmodelling-DG is the follow up to DIVA-DG that we develop in collaboration with our partner Total. Its purpose is more general than DIVA-DG and should contains various DG schemes, basis functions and time schemes. It models wave propagation in acoustic media, elastic (isotropic and TTI) media and elasto-acoustic media, in two and three dimensions.

NEWS OF THE YEAR: In 2017, we have completed the implementation of Lagrange and Jacobi polynomials and we have released the 3D elastodynamic version and the 3D elasto-acoustic coupling.

- Participants: Julien Diaz, Lionel Boillot and Simon Ettouati
- Partner: TOTAL
- Contact: Julien Diaz

5.5. fmodeling

Frequency-domain Wave-equation Modeling App (fModeling)

KEYWORDS: Discontinuous Galerkin - Frequency Domain - 2D - 3D - Elastodynamic equations - Acoustic equation

SCIENTIFIC DESCRIPTION: FModelling simulates acoustic and elastic wave propagation in frequency domain, in 2D and in 3D, using Discontinuous Galerkin Methods and Hybridizable Discontinuous Galerkin Methods. The space discretization is based on Lagrange or Jacobi polynomials. Different kinds of fluxes (upwind and centered) are implemented, coupled with two linear solvers (Mumps and Maphys).

FUNCTIONAL DESCRIPTION: fmodeling is developed in partnership with Total in the context of the Depth Imaging Partnership (DIP). It is the equivalent of Tmodeling for frequency domain. The software deals with wave equation in the frequency domain and solves the forward problem using Discontinuous Galerkin methods or Hybridizable Discontinuous Galerkin Methods. In particular, acoustic and elastic (isotropic and TTI) media are considered in two and three dimensions. It is planned to implement and to test various kind of basis function and to couple the code with various linear solvers (such as Mumps or Maphys). The software is coupled to the Inversion solver of Total to conduct Seismic Imaging using iterative minimization with the Full Waveform Inversion method.

NEWS OF THE YEAR: In 2017, we completed the implementation of Hybridizable Discontinuous Galerkin Methods and we started the validation of the code in an industrial context. We have also coupled the code with the Full Waveform Inversion solver of Total.

- Partner: TOTAL
- Contact: Julien Diaz

6. New Results

6.1. Seismic Imaging and Inverse Problems

6.1.1. *Mathematical determination of the Fréchet derivative with respect to the domain for a fluid-structure scattering problem. Case of polygonal-shaped domains.*

Participants: H el ene Barucq, Elodie Estecahandy.

The characterization of the Fr echet derivative of the elasto-acoustic scattered field with respect to Lipschitz continuous polygonal domains is established. The considered class of domains is of practical interest since two-dimensional scatterers are always transformed into polygonal-shaped domains when employing finite element methods for solving direct and inverse scattering problems. The obtained result indicates that the Fr echet derivative with respect to the scatterer of the scattered field is the solution of the same elasto-acoustic scattering problem but with additional right-hand side terms in the transmission conditions across the fluid-structure interface. This characterization has the potential to advance the state-of-the-art of the solution of inverse obstacle problems.

This work has been done in collaboration with Prof. Rabia Djellouli (California State University at Northridge) and has been accepted for publication in Siam Journal of Applied Mathematics [16].

6.1.2. *Shape-reconstruction and parameter identification of an elastic object immersed in a fluid*

Participants: Izar Azpiroz Iragorri, H el ene Barucq, Julien Diaz, Kevin Lagnoux.

We have developed a procedure to reconstruct the shape and material parameters of an elastic obstacle immersed in a fluid medium from some external measurements given by the so called far-field pattern. It is a nonlinear and ill-posed problem which is solved by applying a Newton-like iterative method involving the Fr echet derivatives of the scattered field. These derivatives express the sensitivity of the scattered field with respect to the parameters of interest. They are defined as the solution of boundary value problems which differ from the direct one only at the right-hand sides level. We have been able to establish the well-posedness of each problem in the case of a regular obstacle and it would be interesting in the near future to extend those results to the case of scatterers with polygonal boundaries. It requires to work with less regular Sobolev spaces for which the definition of traces is not obvious. We have also provided an analytical representation of the Fr echet derivatives in the case of a circle. This provides a way of validating the numerical experiments and it would be interesting to obtain their expression in the case of elliptical scatterers or spherical ones. It is worth mentioning that this work has been done only in the case of isotropic media and it would be interesting to extend it to anisotropic media as well. It requires to establish analytic representations of the scattered field in anisotropic media which is more difficult because it involves more parameters.

We have studied the response of the data to the different parameters. It turns out that the sensitivity of the far field pattern is very different regarding the shape or the material parameters. We have delivered a sensitivity analysis which has been essential for understanding that the reconstruction of the material parameters is conditioned by the recovering of the shape parameters. This makes the full reconstruction very difficult and sometimes unstable. In particular, in the case of a disk-shaped obstacle, when addressing the role of the frequency in the reconstruction, we have been faced to the issue of the existence of Jones modes which had been already observed by Elodie Estecahandy in her PhD thesis. Next, we have introduced a series of numerical experiments that have been performed by applying two algorithms which propose two strategies of full reconstruction regarding the material parameters are retrieved simultaneously with the shape or not. It turns out that both work similarly delivering the same level of accuracy but the simultaneous reconstruction requires less iterations. We have thus opted for retrieving all the parameters simultaneously. Since realistic configurations include noisy data, we have performed some simulations for the reconstruction of the shape along with the Lamé coefficients for different noise levels. Other interesting experiments have been carried out using a multistage procedure where the parameters of interest are the density of the solid interior, the shape of the obstacle and its position. We have considered the case of Limited Aperture Data in back-scattering configurations, using multiple incident plane waves, mimicing a physical disposal of non-destructive testing. This is an encouraging ongoing work which deserves to be completed by considering a wide range of examples including more general geometries of the scatterer. It should also be extended by dealing with limited aperture data using only one incident wave (which will probably require multiple frequency data).

These results have been obtained in collaboration with Rabia Djellouli (California State University at Northridge, USA) and were presented to the Waves 2017 conference.

6.1.3. Shape-reconstruction and parameter identification of an anisotropic elastic object immersed in a fluid

Participants: Izar Azpiroz Irigorri, H el ene Barucq, Julien Diaz.

We extended the solution methodology for reconstructing the shape and material parameters of an elastic obstacle (see 6.1.2) to the case of anisotropic media. This is a very challenging case which still deserves further works. We have obtained some results but since the impact of some of the anisotropic parameters on the FFP is even weaker than the Lam e coefficients, the reconstruction of these parameters together with the shape parameters requires several frequencies and carefully adapted regularization parameters. It is in particular difficult to retrieve the Thomsen parameters ϵ and δ because their reconstruction requires to have an accurate adjustment on the rest of material and shape parameters. The recovery process is thus computationally intensive and some efforts should be done in the near future to decrease the computational costs. We were able to recover all the anisotropic parameters when the shape were assumed to be known. However, when trying to recover both shape and material parameters, we could only recover the shape and some of the physical parameters (namely the three most important ones : the density and the two velocities V_p and V_s). We should now find a way to determine all the Thomsen parameters together with the shape. Then, we will have to deal with more complex media such as TTI media (this will add the angle of anisotropy as additional parameter). The last step will be to consider general anisotropy, which could be done by recovering each element of the elastic stiffness tensor. This is simple to implement, since the derivative of the stiffness tensor with respect to one of its component is easily computable (it is a tensor composed of zeroes and ones). However, the stability of the reconstruction is not guaranteed, since we will strongly increase the number of components to be retrieved.

These results have been obtained in collaboration with Rabia Djellouli (California State University at Northridge, USA).

6.1.4. Mathematical analysis and solution methodology for a class of inverse spectral problems arising in the design of optical waveguides

Participant: H el ene Barucq.

We analyze mathematically the problem of determining refractive index profiles from some desired/measured guided waves propagating in optical fibers. We establish the uniqueness of the solution of this inverse spectral problem assuming that only one guided mode is known. Then, we propose an iterative computational procedure for solving numerically the considered inverse spectral problem. Numerical results are presented to illustrate the potential of the proposed regularized Newton algorithm to efficiently and accurately retrieve the refractive index profiles even when the guided mode measurements are highly noisy.

This work has been submitted for publication in a peer-reviewed journal. It has been done in collaboration with Rabia Djellouli (California State University at Northridge, USA) and Chokri Bekkey (University of Monastir, Tunisia)

6.1.5. *Time-harmonic seismic inverse problem with Cauchy data*

Participant: Florian Faucher.

This work is a collaboration with Giovanni Alessandrini (Università di Trieste), Maarten V. de Hoop (Rice University), Romina Gaburro (University of Limerick) and Eva Sincich (Università di Trieste).

We study the performance of Full Waveform Inversion (FWI) from time-harmonic Cauchy data via conditional well-posedness driven iterative regularization. The Cauchy data can be obtained with dual sensors measuring the pressure and the normal velocity. We define a novel misfit functional which, adapted to the Cauchy data, allows the independent location of experimental and computational sources. The conditional well-posedness is obtained for a hierarchy of subspaces in which the inverse problem with partial data is Lipschitz stable. Here, these subspaces yield piecewise linear representations of the wave speed on given domain partitions. Domain partitions can be adaptively obtained through segmentation of the gradient. The domain partitions can be taken as a coarsening of an unstructured tetrahedral mesh associated with a finite element discretization of the Helmholtz equation. We illustrate the effectiveness of the iterative regularization through computational experiments with data in dimension three. In comparison with earlier work, the Cauchy data do not suffer from eigenfrequencies in the configurations.

The resulting paper is [47] and is also connected to the following conference presentations, [36], [27].

6.1.6. *Quantitative Convergence of Full Waveform Inversion in the Frequency Domain*

Participants: H el ene Barucq, Florian Faucher.

This work is a collaboration with Guy Chavent (Inria Rocquencourt).

We study the convergence of the inverse problem associated with the frequency domain wave equations for the recovery of subsurface parameters. The numerical method selected for the resolution is the Full Waveform Inversion (FWI), which designs an iterative minimization algorithm. We study the convergence of the scheme in the context of least squares minimization. We establish numerical estimates based on the Fr echet derivatives for the radius of curvature and the deflection. We quantify the (complex) frequency progression to select to foster the convergence, and illustrate the effect of the subsurface geometry. From the curvature estimates, we also provide an insight of the robustness with noise depending on the situation. We supplement the numerical analysis with numerical experiments to demonstrate the results.

The results have been presented in the following conference, [36], [27], [26], [25].

6.1.7. *Contributions to seismic full waveform inversion for time harmonic wave equations: stability estimates, convergence analysis, numerical experiments involving large scale optimization algorithms*

Participants: H el ene Barucq, Florian Faucher.

In this project, we investigate the recovery of subsurface Earth parameters. We consider the seismic imaging as a large scale iterative minimization problem, and deploy the Full Waveform Inversion (FWI) method. The reconstruction is based on the wave equations because the characteristics of the measurements indicate the nature of the medium in which the waves propagate. First, the natural heterogeneity and anisotropy of the Earth require numerical methods that are adapted and efficient to solve the wave propagation problem. In this study, we have decided to work with the harmonic formulation, i.e., in the frequency domain.

The inverse problem is then established in order to frame the seismic imaging. It is a nonlinear and ill-posed inverse problem by nature, due to the limited available data, and the complexity of the subsurface characterization. However, we obtain a conditional Lipschitz-type stability in the case of piecewise constant model representation. We derive the lower and upper bound for the underlying stability constant, which allows us to quantify the stability with frequency and scale. It is of great use for the underlying optimization algorithm involved to solve the seismic problem. We review the foundations of iterative optimization techniques and provide the different methods that we have used in this project. The Newton method, due to the numerical cost of inverting the Hessian, may not always be accessible. We propose some comparisons to identify the benefits of using the Hessian, in order to study what would be an appropriate procedure regarding the accuracy and time. We study the convergence of the iterative minimization method, depending on different aspects such as the geometry of the subsurface, the frequency, and the parametrization. In particular, we quantify the frequency progression, from the point of view of optimization, by showing how the size of the basin of attraction evolves with frequency.

Following the convergence and stability analysis of the problem, the iterative minimization algorithm is conducted via a multi-level scheme where frequency and scale progress simultaneously. We perform a collection of experiments, including acoustic and elastic media, in two and three dimensions. The perspectives of attenuation and anisotropic reconstructions are also introduced.

6.1.8. *Quantitative localization of small obstacles with single-layer potential fast solvers*

Participants: H el ene Barucq, Florian Faucher, Ha Pham.

In this work, we numerically study the inverse problem of locating small circular obstacles in a homogeneous medium using noisy backscattered data collected at several frequencies. The main novelty of our work is the implementation of a single-layer potential based fast solver (called FSSL) in a Full-Waveform inversion procedure, to give high quality reconstruction with low-time cost. The efficiency of FSSL was studied in our previous works. We show reconstruction results with up to 12 obstacles in structured or random configurations with several initial guesses, all allowed to be far and different in nature from the target. This last assumption is not expected in results using nonlinear optimization schemes in general. For results with 6 obstacles, we also investigate several optimization methods, comparing between nonlinear gradient descent and quasi-Newton, as well as their convergence with different line search algorithms.

The resulting research report is [45].

6.2. Mathematical modeling of multi-physics involving wave equations

6.2.1. *Atmospheric Radiation Boundary Conditions for the Helmholtz Equation*

Participants: H el ene Barucq, Juliette Chabassier, Marc Durufl e.

An article is to be published in M2AN, see [14]. This work offers some contributions to the numerical study of acoustic waves propagating in the Sun and its atmosphere. The main goal is to provide boundary conditions for outgoing waves in the solar atmosphere where it is assumed that the sound speed is constant and the density decays exponentially with radius. Outgoing waves are governed by a Dirichlet-to-Neumann map which is obtained from the factorization of the Helmholtz equation expressed in spherical coordinates. For the purpose of extending the outgoing wave equation to axisymmetric or 3D cases, different approximations are implemented by using the frequency and/or the angle of incidence as parameters of interest. This results in boundary conditions called Atmospheric Radiation Boundary Conditions (ARBC) which are tested in ideal and realistic configurations. These ARBCs deliver accurate results and reduce the computational burden by a factor of two in helioseismology applications. This work has been done in collaboration with Laurent Gizon and Michael Legu ebe (Max-Planck-Institut f ur Sonnensystemforschung, G ttingen, Germany).

6.2.2. *Atmospheric radiation boundary conditions for high frequency waves in time-distance helioseismology*

Participants: H el ene Barucq, Juliette Chabassier, Marc Durufl e.

An article has been published in *Astronomy and Astrophysics* [22]. The temporal covariance between seismic waves measured at two locations on the solar surface is the fundamental observable in time-distance helioseismology. Above the acoustic cutoff frequency (5.3 mHz), waves are not trapped in the solar interior and the covariance function can be used to probe the upper atmosphere. We wish to implement appropriate radiative boundary conditions for computing the propagation of high-frequency waves in the solar atmosphere. We consider the radiative boundary conditions recently developed by Barucq et al. (2017) for atmospheres in which sound-speed is constant and density decreases exponentially with radius. We compute the cross-covariance function using a finite element method in spherical geometry and in the frequency domain. The ratio between first-and second-skip amplitudes in the time-distance diagram is used as a diagnostic to compare boundary conditions and to compare with observations. We find that a boundary condition applied 500 km above the photosphere and derived under the approximation of small angles of incidence accurately reproduces the 'infinite atmosphere' solution for high-frequency waves. When the radiative boundary condition is applied 2 Mm above the photosphere, we find that the choice of atmospheric model affects the time-distance diagram. In particular, the time-distance diagram exhibits double-ridge structure when using a VAL atmospheric model. This is a collaboration with Damien Fournier, Laurent Gizon, Chris Hanson and Michael Leguèbe (Max-Planck-Institut für Sonnensystemforschung, Göttingen, Germany).

6.2.3. *Computational helioseismology in the frequency domain: acoustic waves in axisymmetric solar models with flows*

Participants: H el ene Barucq, Juliette Chabassier, Marc Durufl e.

An article has been published in *Astronomy and Astrophysics* [23]. Context. Local helioseismology has so far relied on semi-analytical methods to compute the spatial sensitivity of wave travel times to perturbations in the solar interior. These methods are cumbersome and lack flexibility. Aims. Here we propose a convenient framework for numerically solving the forward problem of time-distance helioseismology in the frequency domain. The fundamental quantity to be computed is the cross-covariance of the seismic wavefield. Methods. We choose sources of wave excitation that enable us to relate the cross-covariance of the oscillations to the Green's function in a straightforward manner. We illustrate the method by considering the 3D acoustic wave equation in an axisymmetric reference solar model, ignoring the effects of gravity on the waves. The symmetry of the background model around the rotation axis implies that the Green's function can be written as a sum of longitudinal Fourier modes, leading to a set of independent 2D problems. We use a high-order finite-element method to solve the 2D wave equation in frequency space. The computation is embarrassingly parallel, with each frequency and each azimuthal order solved independently on a computer cluster. Results. We compute travel-time sensitivity kernels in spherical geometry for flows, sound speed, and density perturbations under the first Born approximation. Convergence tests show that travel times can be computed with a numerical precision better than one millisecond, as required by the most precise travel-time measurements. Conclusions. The method presented here is computationally efficient and will be used to interpret travel-time measurements in order to infer, e.g., the large-scale meridional flow in the solar convection zone. It allows the implementation of (full-waveform) iterative inversions, whereby the axisymmetric background model is updated at each iteration. This work is a collaboration with Aaron Birch, Damien Fournier, Laurent Gizon, Chris Hanson, Michael Legu e and Emanuele Papini(Max-Planck-Institut f ur Sonnensystemforschung, G ttingen, Germany) and with Thorsten Hohage (G ttingen University, Germany).

6.2.4. *The virtual workshop : towards versatile optimal design of musical wind instruments for the makers*

Participants: Juliette Chabassier, Robin Tournemene.

Our project aims at proposing optimization solutions for wind instrument making. Our approach is based on a strong interaction with makers and players, aiming at defining interesting criteria to optimize from their point of view. After having quantified those criteria under the form of a cost function and a design parameters space, we wish to implement state-of-the-art numerical methods (finite elements, full waveform inversion, neuronal networks, diverse optimization techniques...) that are versatile (in terms of models, formulations, couplings...) in order to solve the optimization problem. More precisely, we wish to take advantage of the fact that sound

waves in musical instruments satisfy the laws of acoustics in pipes (PDE), which gives us access to the full waveform inversion technique, usable in harmonic or temporal regime. The methods that we want to use are attractive because they depend on the chosen criterion, and they are easily adaptable to various physical situations (multimodal decomposition in the pipe, coupling with the embouchure, ...), which can therefore be modified a posteriori. The goal is to proceed iteratively between instrument making and optimal design (the virtual workshop) in order to get close to tone quality related and playability criteria.

6.2.5. Energy based model and simulation in the time domain of linear acoustic waves in a radiating pipe

Participants: Juliette Chabassier, Robin Tournemenne.

We model in the time domain linear acoustic waves in a radiating pipe without damping. The acoustic equations system is formulated in flow and pressure, which leads to a first order space time equations system. The radiation condition is also written as a first order in time equation, and is parametrized by two real coefficients. Moreover, an auxiliary variable is introduced at the radiating boundary. The choice of this variable is adapted to the considered source type in order to ensure the model stability by energy techniques, under some conditions on the radiating condition. We then propose a stable space time explicit discretization, which ensures the dissipation of a discrete energy. The novelty of the discretization lies, on the one hand, in the variational nature of the space approximation (which leads to arbitrary order finite elements with no required matrix inversion), and on the other hand, on the definition of the auxiliary variable for any acoustic source type (which leads to the decay of a well defined energy). Finally, we quantify the frequential domain of validity of the used radiation condition by comparison with theoretical and experimental models of the literature. This is a collaboration with Morgane Bergot (Université Claude Bernard, Lyon 1).

6.2.6. Computation of the entry impedance of a dissipative radiating pipe

Participants: Juliette Chabassier, Robin Tournemenne.

Modeling the entry impedance of wind instruments pipes is essential for sound synthesis or instrument qualification. We study this modeling with the finite elements method in one dimension (FEM1D) and with the more classically used transfer matrix method (TMM). The TMM gives an analytical formula of the entry impedance depending on the bore (intern geometry of the instrument) defined as a concatenation of simple elements (cylinders, cones, etc). The FEM1D gives the entry impedance for any instrument geometry. The main goals of this work are to assess the viability of the FEM1D and to study the approximations necessary for the TMM in dissipative pipes. First, lossless Weber's equation in one dimension is studied with arbitrary radiation conditions. In this context and for cylinders or cones, the TMM is exact. We verify that the error made with FEM1D for fine enough elements is as small as desired. When we consider viscothermal losses, the TMM does not solve the classical Kirchhoff model because two terms are supposed constant. In order to overcome this model approximation, simple elements, on which are based the TMM, are decomposed into much smaller elements. The FEM1D does not necessitate any model approximation, and it is possible to show that it solves the dissipative equation with any arbitrarily small error. With this in hand, we can quantify the TMM model approximation error.

6.2.7. Hybrid discontinuous finite element approximation for the elasto-acoustics.

Participants: Hélène Barucq, Julien Diaz, Elvira Shishenina.

Discontinuous Finite Element Methods (DG FEM) have proven their numerical accuracy and flexibility. However, numerically speaking, the high number of degrees of freedom required for computation makes them more expensive, compared to the standard techniques with continuous approximation. Among the different variational approaches to solve boundary value problems there exists a distinct family of methods, based on the use of trial functions in the form of exact solutions of the governing equations. The idea was first proposed by Trefftz in 1926, and since then it has been largely developed and generalized. By its definition, Trefftz-DG methods reduce numerical cost, since the variational formulation contains the surface integrals only. Thus, it makes possible exploration of the meshes with different geometry, in order to create more realistic application. Trefftz-type approaches have been widely used for time-harmonic problems, while their implementation is still

limited in time domain. The particularity of Trefftz-DG methods applied to the time-dependent formulations consists in the use of space-time meshes. Even though it creates another computational difficulty, due to a dense form of the matrix, which represents the global linear system, the inversion of the full "space-time" matrix can be reduced to the inversion of one block-diagonal matrix, which corresponds to the interactions in time. In the present work, we develop a theory for solving the coupled elasto-acoustic wave propagation system. We study well-posedness of the problem, based on the error estimates in mesh-dependent norms. We consider a space-time polynomial basis for numerical discretization. The obtained numerical results are validated with analytical solutions. Regarding the advantages of the method, following properties have been proven by the numerical tests: high flexibility in the choice of basis functions, better order of convergence, low dispersion. These results have been obtained in collaboration with Henri Calandra (TOTAL) and have been published in a research report [43]. A paper has been submitted and a second one is being prepared.

6.2.8. Construction of stabilized high-order hybrid Galerkin schemes.

Participants: H el ene Barucq, Aur elien Citrain, Julien Diaz.

We have compared the performance of Discontinuous Galerkin Methods and Spectral Element Methods on academic benchmark and on realistic geophysical model in two dimensions. We have shown that, for a given accuracy, SEM on quadrilateral meshes could be 10 times faster than DGm, which justifies our strategy to consider SEM wherever it is possible to use quadrilateral/hexahedral cells. These first results have been presented in Matthias conference. Then, we have considered the SEM/DG coupling proposed for electromagnetics in [78] and we have implemented it in our acoustics code. We are now analyzing the performance of this strategy and we are extending it to deal with elastodynamic and elasto-acoustic coupling. The following steps will be the extension of the analysis to 3D dimensional problems and the application to realistic test case. The main bottleneck is obviously to the definition of an efficient strategy to couple tetrahedra and hexahedra. Indeed, if in the 2D case, the edges of both triangles and quadrilaterals are all segments the faces of tetrahedra are triangle while the faces of hexahedra are quadrilaterals. Hence, in 2D it sufficed to define integration on segment, while in 3D it will be necessary to consider integration of various polygon resulting of the intersection of triangle and quadrilaterals. Once this strategy is defined and implemented, we expect to be able to reduce the computational of the platform that we develop jointly with Total by a factor between 5 and 10. These results have been obtained in collaboration with Henri Calandra (TOTAL) and Christian Gout (INSA Rouen).

6.2.9. Modeling of dissipative porous media.

Participants: Juliette Chabassier, Julien Diaz, Fatima Jabiri.

In this work we have considered the modeling of 1D acoustic wave propagation coupled with visco-thermal losses that occur in porous media. We have proposed a family of dissipative models from which we have been able to obtain a quasi-constant quality factor (which is an indicator of the dissipation as a function of the frequency). We have derived stability conditions on the parameters of the model thanks to an energy analysis and we have rewritten the problem of designing a quasi-constant quality factor as a constrained least-square optimization problem. The parameters to optimize are the parameters of the family of dissipative models and the constraints are the stability of the final model. We are now considering the extension of the family to more general formulations and to heterogeneous media, before tackling multidimensional problems. These results have been obtained during the Master internship of Fatima Jabiri, in collaboration with S ebastien Imperiale (Inria Project-Team M3DISIM)

6.2.10. Asymptotic models for the electric potential across a highly conductive casing.

Application to the field of resistivity measurements.

Participants: H el ene Barucq, Aralar Erdozain, Victor P eron.

A configuration that involves a steel-cased borehole is analyzed, where the casing that covers the borehole is considered as a highly conductive thin layer. Asymptotic techniques are presented as the suitable tool for deriving reduced problems capable of dealing with the numerical issues caused by the casing when applying the traditional numerical methods. The derivation of several reduced models is detailed by employing two different approaches, each of them leading to different classes of models. The stability and convergence of these models is studied and uniform estimates are proved. The theoretical orders of convergence are supported by numerical results obtained with the finite element method. We develop an application to the field of resistivity measurements. The second derivative of the potential which solves a reduced model has been employed to recover the resistivity of rock formations. These results are in accordance with an experiment of Kaufmann for the reference solution and have been obtained in collaboration with David Pardo (UPV/EHU).

6.2.11. Semi-Analytical Solutions for the Electric Potential across a Highly Conductive Casing.

Participants: H el ene Barucq, Aralar Erdozain, Victor P eron.

A transmission problem for the electric potential is considered, where one part of the domain is a high-conductive casing. Semi-analytical solutions are derived for several asymptotic models. These asymptotic models are designed to replace the casing by appropriate impedance conditions in order to avoid numerical instabilities. A decomposition in Fourier series of the solution to these asymptotic models is characterized. As an application we reproduced successfully the experiment of Kaufmann, using his same parameters, but computing with a fourth order asymptotic model. This experiment allows to recover the resistivity of rock formations employing a second derivative of the potential along the vertical direction. These results have been obtained in collaboration with Ignacio Muga (Pontificia Universidad Catolica of Valparaiso).

6.2.12. Asymptotic modeling of the electromagnetic wave scattering problem by a small sphere perfectly conducting.

Participants: Justine Labat, Victor P eron, S ebastien Tordeux.

In the context of non-destructive testing in medical imaging or civil engineering, the detection of small heterogeneities can be a difficult task in three dimensional domains. The complexity for solving numerically the direct problem both in terms of computation time and memory cost is due to the small size of obstacles in comparison with the incident wavelength and the large size of the domain of interest. Then the fine mesh size makes unsuitable or too expensive the use of classical numerical methods type continuous and discontinuous finite element methods or boundary element methods. The use of reduced models allows to get an approximation of the exact solution at a certain accuracy with a lower cost. We develop a Matched Asymptotic Expansions method to solve a time-harmonic electromagnetic scattering problem by a small sphere. This method allows to replace the scatterer by an equivalent asymptotic point source. In practice, it consists in defining an approximate solution using multi-scale expansions over far and near fields, related in a matching area. When the scatterer is a sphere, we make explicit the asymptotic expansions until the second order of approximation, relatively to the sphere radius. Numerical results make evident the convergence rate with respect to the sphere radius. Reference solutions are analytical solutions computed thanks to Montjoie Code. This work has been presented in the *Caleta Numerica* seminar, Pontificia Universidad Catolica of Valparaiso, Chili [48].

6.2.13. Asymptotic Models and Impedance Conditions for Highly Conductive Sheets in the Time-Harmonic Eddy Current Model.

Participant: Victor P eron.

This work is concerned with the time-harmonic eddy current problem for a medium with a highly conductive thin sheet. We present asymptotic models and impedance conditions up to the second order of approximation for the electromagnetic field. The conditions are derived asymptotically for vanishing sheet thickness ϵ where the skin depth is scaled like ϵ . The first order condition is the perfect electric conductor boundary condition. The second order condition turns out to be a Poincar e-Steklov map between tangential components of the magnetic field and the electric field [49]. Numerical experiments have been performed to assess the accuracy

of the second order model. Complementary simulations will be conducted to study the robustness with respect to the sheet conductivity and the convergence of the modelling error. These results have been obtained in collaboration with Mohammad Issa and Ronan Perrussel (LAPLACE, CNRS/IMPT/UPS, Univ. de Toulouse) and this work has been presented in the international conference ACOMEN 2017 [33].

6.2.14. Numerical robustness of single-layer method with Fourier basis for multiple obstacle acoustic scattering in homogeneous media

Participants: H el ene Barucq, Juliette Chabassier, Ha Pham, S ebastien Tordeux.

We investigate efficient methods to simulate the multiple scattering of obstacles in homogeneous media. With a large number of small obstacles on a large domain, optimized pieces of software based on spatial discretization such as Finite Element Method (FEM) or Finite Difference lose their robustness. As an alternative, we work with an integral equation method, which uses single-layer potentials and truncation of Fourier series to describe the approximate scattered field. In the theoretical part of the paper, we describe in detail the linear systems generated by the method for impenetrable obstacles, accompanied by a well-posedness study. For the numerical performance study, we limit ourselves to the case of circular obstacles. We first compare and validate our codes with the highly optimized FEM-based software Montjoie. Secondly, we investigate the efficiency of different solver types (direct and iterative of type GMRES) in solving the dense linear system generated by the method. We observe the robustness of direct solvers over iterative ones for closely-spaced obstacles, and that of GMRES with Lower–Upper Symmetric Gauss–Seidel and Symmetric Gauss–Seidel preconditioners for far-apart obstacles.

This work has been published in the journal *Wave Motion*, [15] and is also connected to the following conference presentations, [31], [41].

6.2.15. A study of the Numerical Dispersion for the Continuous Galerkin discretization of the one-dimensional Helmholtz equation

Participants: H el ene Barucq, Ha Pham, S ebastien Tordeux.

This work is a collaboration with Henri Calandra (TOTAL).

Although true solutions of Helmholtz equation are non-dispersive, their discretizations suffer from a phenomenon called numerical dispersion. While the true phase velocity is constant, the numerical one changes with the discretization scheme, order and mesh size. In our work, we study the dispersion associated with classical finite element. For arbitrary order of discretization, without using an Ansatz, we construct the numerical solution on the whole \mathbb{R} , and obtain an asymptotic expansion for the phase difference between the exact wavenumber and the numerical one. We follow an approach analogous to that employed in the construction of true solutions at positive wavenumbers, which involves Z-transform, contour deformation and limiting absorption principle. This perspective allows us to identify the numerical wavenumber with the angle of analytic poles. Such an identification is useful since the latter (analytic poles) can be numerically evaluated by an algorithm, which then yields the value of numerical wavenumber.

This work is detailed in the research report [44].

6.3. Supercomputing for Helmholtz problems

6.3.1. Numerical libraries for hybrid meshes in a discontinuous Galerkin context

Participants: H el ene Barucq, Lionel Boillot, Aur elien Citrain, Julien Diaz.

Elasticus team code has been designed for triangles and tetrahedra mesh cell types. The first part of this work was dedicated to add quadrangle libraries and then to extend them to hybrid triangles-quadrangles (so in 2D). This implied to work on polynomials to form functions basis for the (discontinuous) finite element method, to finally be able to construct reference matrices (mass, stiffness, ...).

A complementary work has been done on mesh generation. The goal was to encircle an unstructured triangle mesh, obtained by third-party softwares, with a quadrangle mesh layer. At first, we built scripts to generate structured triangle meshes, quadrangle meshes and hybrid meshes (triangles surrounded by quadrangles). We are now able to couple unstructured triangle mesh with structured quadrangle mesh, and we are now working on the implementation of the coupling between Discontinuous Galerkin methods (for the triangles) and Spectral Element methods (for the quadrangles).

6.3.2. Hybridizable Discontinuous Galerkin methods for solving the elastic Helmholtz equations

Participants: Marie Bonnasse-Gahot, Julien Diaz.

The advantage of performing seismic imaging in frequency domain is that it is not necessary to store the solution at each time step of the forward simulation. Unfortunately, the drawback of the Helmholtz equations, when considering 3D realistic elastic cases, lies in solving large linear systems. This represents today a challenging task even with the use of High Performance Computing (HPC). To reduce the size of the global linear system, we developed a Hybridizable Discontinuous Galerkin method (HDGm). It consists in expressing the unknowns of the initial problem in function of the trace of the numerical solution on each face of the mesh cells. In this way the size of the matrix to be inverted only depends on the number of degrees of freedom on each face and on the number of the faces of the mesh, instead of the number of degrees of freedom on each cell and on the number of the cells of the mesh as we have for the classical Discontinuous Galerkin methods (DGm). The solution to the initial problem is then recovered thanks to independent elementwise calculation. These results have been published in [18]. This is a collaboration with Henri Calandra (Total) and Stéphane Lanteri (Inria Project Team Nachos)

6.3.3. Scalability of linear solvers for Hybridizable Discontinuous Galerkin methods

Participants: Marie Bonnasse-Gahot, Julien Diaz.

We coupled our HDG code with tested two linear solvers: a parallel sparse direct solver MUMPS (MULTifrontal Massively Parallel sparse direct Solver) and a hybrid solver MaPHyS (Massively Parallel Hybrid Solver) which combines direct and iterative methods. In the framework of the european project HPC4E, we analyzed the scalability of the two solvers on the platform Plafrim We compared the performances of the two solvers when solving 3D elastic waves propagation over HDGm. These comparisons were presented at the 2017 EAGE Workshop on High Performance Computing for Upstream and at MATHIAS 2017 conferences. This is a collaboration with Henri Calandra (Total), Luc Giraud, Mathieu Kuhn (Inria Project-Team Hiepac) and Stéphane Lanteri (Inria Project Team Nachos).

6.4. Hybrid time discretizations of high-order

6.4.1. Construction and analysis of a fourth order, energy preserving, explicit time discretization for dissipative linear wave equations.

Participants: Juliette Chabassier, Julien Diaz, Anh-Tuan Ha.

We submitted a paper to M2AN. This paper deals with the construction of a fourth order, energy preserving, explicit time discretization for dissipative linear wave equations. This scheme is obtained by replacing the inversion of a matrix, that comes naturally after using the technique of the Modified Equation on the second order Leap Frog scheme applied to dissipative linear wave equations, by an explicit approximation of its inverse. The stability of the scheme is studied first using an energy analysis, then an eigenvalue analysis. Numerical results in 1D illustrate the good behavior regarding space/time convergence and the efficiency of the newly derived scheme compared to more classical time discretizations. A loss of accuracy is observed for non smooth profiles of dissipation, and we propose an extension of the method that fixes this issue. Finally, we assess the good performance of the scheme for a realistic dissipation phenomenon in Lorentz's materials. This work has been done in collaboration with Sébastien Imperiale (Inria Project-Team M3DISIM) and Alain Anh-Tuan Ha (Internship at Magique 3D in 2016).

6.4.2. Higher-order optimized explicit Runge-Kutta schemes for linear ODEs

Participants: H el ene Barucq, Marc Durufl e, Mamadou N’Diaye.

In this work, we have constructed optimized explicit Runge-Kutta schemes for linear ODEs that we called Linear-ERK. These schemes can be applied to the following ODE

$$M_h \frac{dU}{dt} = K_h U + F(t)$$

where M_h is the mass matrix, K_h the stiffness matrix and $F(t)$ a source term. Linear-ERK schemes are constructed using polynomial stability functions which are obtained by maximizing the CFL number. We have considered a polynomial stability function based on the Taylor series expansion of an exponential function. Then, we have added extra terms beyond the terms of the Taylor expansion without changing the order of accuracy. The coefficients of those extra terms have been computed by optimizing the CFL number such that the stability region of the developed scheme includes a typical spectrum. This spectrum has been obtained by computing eigenvalues of the matrix $M_h^{-1}K_h$ for the wave equation solved on a square with Hybrid Discontinuous Formulation (HDG). The optimization is performed by using the algorithm developed by D. Ketcheson and coworkers. By proceeding this way, we have obtained optimized explicit schemes up to order 8. We have also determined the CFL number and the efficiency on the typical spectrum for each explicit scheme. We have provided algorithms to implement these schemes and numerical results to compare them.

This work is a chapter of the thesis defended by Mamadou N’diaye on December 8, 2017, under the joint supervision of H el ene Barucq and Marc Durufl e.

6.4.3. High-order locally implicit time schemes for linear ODEs

Participants: H el ene Barucq, Marc Durufl e, Mamadou N’Diaye.

In this work we have proposed a method that combines optimized explicit schemes and implicit schemes to form locally implicit schemes for linear ODEs, including in particular ODEs coming from the space discretization of wave propagation phenomena. This method can be applied to the following ODE

$$M_h \frac{dU}{dt} = K_h U + F(t)$$

Like in the local time-stepping developed by Grote and co-workers, the computational domain is split into a fine region and a coarse region. The matrix A_h is given as

$$A_h = M_h^{-1}K_h = A_h P + A_h(I - P)$$

where P is the projector on the fine region of the computational domain. Then the proposed locally implicit method is obtained from the combination of the A-stable implicit schemes we have developed in 2016 (Pad e schemes or Linear-SDIRK schemes detailed in [17]) on the fine region and explicit schemes with optimal CFL number in the coarse region. The developed method has been used to solve the acoustic wave equation and we have checked the convergence in time of these schemes for order 4, 6 and 8.

This work has been presented at the Mathias annual Total seminar and is a chapter of the thesis defended by Mamadou N’diaye on December 8, 2017, under the joint supervision of H el ene Barucq and Marc Durufl e.

7. Bilateral Contracts and Grants with Industry

7.1. Contracts with TOTAL

- Depth Imaging Partnership (DIP)
Period: 2014 May - 2019 April , Management: Inria Bordeaux Sud-Ouest, Amount: 120000 euros/year.
- Approximations hybrides par éléments finis discontinus pour l'élasto-acoustique
Period: 2016 November - 2018 October, Management: Inria Bordeaux Sud-Ouest, Amount: 165000 euros.
- Méthodes d'inversion sismique dans le domaine fréquentiel
Period: 2014 October - 2017 December , Management: Inria Bordeaux Sud-Ouest, Amount: 180000 euros.
- Portage de méthodes numériques de simulation de phénomènes complexes sur des architectures exascales
Period: 2016 January - 2017 December , Management: Inria Bordeaux Sud-Ouest, Amount: 150000 euros.
- Utilisation d'images 3D DRP à différentes échelles et résolutions pour vérifier l'applicabilité des problèmes acoustiques
Period: 2017 November - 2019 October, Management: Inria Bordeaux Sud-Ouest, Amount: 170000 euros.
- Petrophysics in pre-salt carbonate rocks
Period: 2017 December - 2019 November, Management: Inria Bordeaux Sud-Ouest, Amount: 190000 euros.

8. Partnerships and Cooperations

8.1. Regional Initiatives

8.1.1. *Partnership with I2M in Bordeaux supported by Conseil Régional d'Aquitaine*

Title: Imaging complex materials.

Coordinator: Hélène Barucq

Other partners: I2M CNRS Université Bordeaux I

The detection, localization and monitoring of the defect evolution in composite materials, concrete and more generally heterogeneous materials is a challenging problem for Aeronautics and energy production. It is already possible to localize defects in homogeneous materials by using methods based on ultrasonic inspection and sometimes, they are usable in particular heterogeneous materials, most of the time in 2D. Classical methods rely on the correspondence between the distance and the propagation time of the wave traveling between the defect and the receivers. In complex media, such a correspondence may be lapsed, for instance when the velocity depends on the frequency (dispersion) or of the propagation direction (anisotropy). The defect signature can also be embedded in the acoustic field sent by the structure (multiple reflections). The complexity of the propagation in heterogeneous materials makes then difficult the accurate localization of the defect, in particular in 3D.

Topological imaging techniques can be applied to heterogeneous media. They can find the positions of defects from two simulations performed in a safe experimental medium. They have been developed at I2M laboratory to carry on 2D single/multi mode inspection in isotropic and anisotropic waveguides. They have also been applied to a highly reflecting medium observed with a single sensor. The objective of this work is to extend the technique to 3D problems. In particular, we are going to handle detection in composite plates and in highly heterogeneous media including a collection of small scatterers.

This project is supported by the Conseil Régional d'Aquitaine, for a duration of 2 years.

8.2. National Initiatives

8.2.1. Depth Imaging Partnership

Magique-3D maintains active collaborations with Total. In the context of Depth Imaging, Magique-3D coordinates research activities dealing with the development of high-performance numerical methods for solving wave equations in complex media. This project has involved 2 other Inria Team-Projects (Hiepac and Nachos) which have complementary skills in mathematics, computing and in geophysics. DIP is fully funded by Total by the way of an outline agreement with Inria .

In 2014, the second phase of DIP has begun. Lionel Boillot has been hired as engineer to work on the DIP platform. Six PhD students have defended their PhD since 2014 and they are now post-doctoral researchers or engineers in Europe. DIP is currently employing 2 PhD students and one post-doctoral researcher.

8.2.2. ANR Num4Sun

The ANR has launched a specific program for supporting and promoting applications to European or more generally International projects. Magique-3D has been selected in 2016 after proposing a project to be applied as a FET project on the occasion of a call that will open in 2017 April. This project will gather researchers of the MPS (<https://www.mps.mpg.de/en>), of the BSC (<https://www.bsc.es/>), of the BCAM (<http://www.bcamath.org/en/>), of Heriot-Watt University (<https://www.hw.ac.uk/>) and Inria teams.

A kick-off meeting has been held in November 2016 in Strasbourg and a second one in Paris in July 2017. Thanks to this support, we have submitted a ETPHPC proposal in September 2017 The project is funded for 18 months starting from August 2016. The funding amounts 30000€.

8.2.3. ANR NonLocalDD

Magique 3-D is a partner of the ANR project entitled "Non Local Domain Decomposition Methods in Electromagnetics" that begins in october 2015. The aim of this project is to develop domain decomposition methods for the efficient solution of acoustics and Maxwell's equation either with boundary integral equations or finite element volume method. To obtain an exponential convergence of the iterative solution, non-local operators are studied and optimized to achieve a faster convergence. A post-doctoral student Marcella Bonazzoli has been hired by Magique 3-D in 2017 to study multi-domain integral equations for wave propagation. This student is supervised by Xavier Claeys, a partner of the NonLocalDD ANR project.

8.3. European Initiatives

8.3.1. FP7 & H2020 Projects

8.3.1.1. GEAGAM

Title: Geophysical Exploration using Advanced Galerkin Methods

Program: H2020

Duration: January 2015 - December 2017

Coordinator: Universidad Del Pais Vasco (EHU UPV)

Partners:

Bcam - Basque Center for Applied Mathematics Asociacion (Spain)

Barcelona Supercomputing Center - Centro Nacional de Supercomputacion (Spain)

Total S.A. (France)

Universidad Del Pais Vasco Ehu Upv (Spain)

Pontificia Universidad Catolica de Valparaiso (Chile)

Universidad de Chile (Chile)

Universidad Tecnica Federico Santa Maria (Chile)

University of Texas at Austin (USA)

Inria contact: Hélène BARUCQ

The main objective of this Marie Curie RISE action is to improve and exchange interdisciplinary knowledge on applied mathematics, high performance computing, and geophysics to be able to better simulate and understand the materials composing the Earth's subsurface. This is essential for a variety of applications such as CO₂ storage, hydrocarbon extraction, mining, and geothermal energy production, among others. All these problems have in common the need to obtain an accurate characterization of the Earth's subsurface, and to achieve this goal, several complementary areas will be studied, including the mathematical foundations of various high-order Galerkin multiphysics simulation methods, the efficient computer implementation of these methods in large parallel machines and GPUs, and some crucial geophysical aspects such as the design of measurement acquisition systems in different scenarios. Results will be widely disseminated through publications, workshops, post-graduate courses to train new researchers, a dedicated webpage, and visits to companies working in the area. In that way, we will perform an important role in technology transfer between the most advanced numerical methods and mathematics of the moment and the area of applied geophysics.

8.3.1.2. *HPC4E*

Title: HPC for Energy

Program: H2020

Duration: December 2015 - November 2017

Coordinator: Barcelona Supercomputing Center

Partners:

Centro de Investigaciones Energeticas, Medioambientales Y Tecnologicas-Ciemat (Spain)

Iberdrola Renovables Energia (Spain)

Repsol (Spain)

Lancaster University (United Kingdom)

Total S.A. (France)

Fundação Coordenação de Projetos, Pesquisas e Estudos Tecnológicos, (Brazil)

National Laboratory for Scientific Computation, (Brazil)

Instituto Tecnológico de Aeronáutica, (Brazil)

Petrobras, (Brazil)

Universidade Federal do Rio Grande do Sul, (Brazil)

Universidade Federal de Pernambuco, (Brazil)

Inria contact: Stéphane Lanteri

This project aims to apply the new exascale HPC techniques to energy industry simulations, customizing them, and going beyond the state-of-the-art in the required HPC exascale simulations for different energy sources: wind energy production and design, efficient combustion systems for biomass-derived fuels (biogas), and exploration geophysics for hydrocarbon reservoirs. For wind energy industry HPC is a must. The competitiveness of wind farms can be guaranteed only with accurate wind resource assessment, farm design and short-term micro-scale wind simulations to forecast the daily power production. The use of CFD LES models to analyse atmospheric flow in a wind farm capturing turbine wakes and array effects requires exascale HPC systems. Biogas, i.e. biomass-derived fuels by anaerobic digestion of organic wastes, is attractive because of its wide availability, renewability and reduction of CO₂ emissions, contribution to diversification of energy supply, rural development, and it does not compete with feed and food feedstock. However, its use

in practical systems is still limited since the complex fuel composition might lead to unpredictable combustion performance and instabilities in industrial combustors. The next generation of exascale HPC systems will be able to run combustion simulations in parameter regimes relevant to industrial applications using alternative fuels, which is required to design efficient furnaces, engines, clean burning vehicles and power plants. One of the main HPC consumers is the oil & gas (O&G) industry. The computational requirements arising from full wave-form modelling and inversion of seismic and electromagnetic data is ensuring that the O&G industry will be an early adopter of exascale computing technologies. By taking into account the complete physics of waves in the subsurface, imaging tools are able to reveal information about the Earth's interior with unprecedented quality.

8.4. International Initiatives

8.4.1. Inria International Partners

8.4.1.1. Declared Inria International Partners

8.4.1.1.1. MAGIC2

Title: Advance Modeling in Geophysics

International Partner (Institution - Laboratory - Researcher):

California State University at Northridge (United States) - Department of Mathematics -
Djellouli Rabia

The Associated Team MAGIC was created in January 2006 and renewed in January 2009. At the end of the program in December 2011, the two partners, MAGIQUE-3D and the California State University at Northridge (CSUN) decided to continue their collaboration and obtained the "Inria International Partner" label in 2013.

See also: <https://project.inria.fr/magic/>

The ultimate objective of this research collaboration is to develop efficient solution methodologies for solving inverse problems arising in various applications such as geophysical exploration, underwater acoustics, and electromagnetics. To this end, the research program will be based upon the following three pillars that are the key ingredients for successfully solving inverse obstacle problems. 1) The design of efficient methods for solving high-frequency wave problems. 2) The sensitivity analysis of the scattered field to the shape and parameters of heterogeneities/scatterers. 3) The construction of higher-order Absorbing Boundary Conditions.

In the framework of Magic2, Izar Azpiroz visited CSUN in May 2017 and Rabia Djellouli (CSUN) visited Magique 3D in December 2017

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Rabia Djellouli (CSUN) visited Magique 3D in December 2017.
- Damien Fournier (MPS) visited Magique 3D in October 2017.
- Morgane Bergot (Univ Lyon) visited Magique 3D in November 2017.

8.5.2. Visits to International Teams

8.5.2.1. Research Stays Abroad

- In the framework of the European project Geagam, Izar Azpiroz and Justine Labat visited Ignacio Muga, PUCV, Chile, in April 2017.
- In the framework of the International Partnership Magic2, Izar Azpiroz visited Rabia Djellouli, CSUN (California State University at Northridge), USA, in May 2017.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

- Hélène Barucq, Julien Diaz and Sébastien Tordeux organized the conference in honor of Abderrahmane Bendali in Pau, December 12th-14th, 2017, <https://project.inria.fr/bendali/>
- Julien Diaz co-organized the conference in honor of Patrick Joly in Saclay, August 28th-30th 2017, in Saclay <https://wavesjoly60.inria.fr/>

9.1.1.2. Member of the Conference Program Committees

- Hélène Barucq and Julien Diaz were members of the scientific committee of Waves 2017 <https://cceeevents.umn.edu/waves-2017>

9.1.2. Journal

9.1.2.1. Reviewer - Reviewing Activities

Members of Magique 3D have been reviewers for the following journals:

- Applied Numerical Mathematics
- Mathematics and Computers in Simulation
- International Journal for Numerical Methods in Engineering
- Geophysical Journal International
- IMA Journal of Numerical Analysis
- SIAM Journal on Scientific Computing
- Computers and Mathematics with Applications
- Journal of Mathematical Analysis and Applications
- Journal of Computational Physics
- Journal of the Acoustical Society of America

9.1.3. Scientific Expertise

- Julien Diaz was expert for the evaluation of Millennium Science Initiative project for the government of Chile.

9.1.4. Research Administration

- Hélène Barucq has been the chairwoman of the local jury of Inria competitive selection for Young Graduate Scientists (CR2) in Bordeaux. She has been part of a working group dealing with the new strategic plan of Inria. In January 2017, she has been appointed chairwoman of the committee created by the regional council of Nouvelle Aquitaine. She is in charge of the scientific evaluation of research projects in Mathematics, Informatics, Electronics, Optics. She is the scientific head of the project DIP since its creation in 2009.
- Juliette Chabassier is member of the Inria BSO Young Researcher Committee and of the Inria BSO Center Committee. She is member of the Workgroup for sustainable development at Inria Bordeaux Sud-Ouest.
- Julien Diaz is elected member of the Inria Technical Committee and of the Inria Administrative and Scientific Boards. He is appointed member of the CDT (Commission de Développement Technologique)
- Mamadou N'Diaye is member of the Center Committee of Inria Bordeaux Sud-Ouest.

- Victor Péron is appointed member of the CJC (Commission Jeunes Chercheurs) of Inria Bordeaux Sud-Ouest.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master : Julien Diaz, Transformées, 24h Eq. TD, M1, EISTIA, France
 Licence : Justine Labat, Algèbre 1, 19,5h Eq. TD, L1, UPPA, France
 Licence : Justine Labat, Algèbre linéaire, 19,5h Eq. TD, L1, UPPA, France
 Licence : Justine Labat, Introduction aux Probabilités, 19,5h Eq. TD, L2, UPPA, France
 Licence : Victor Péron, Analyse 2, 39 Eq. TD, L1, UPPA, France
 Licence : Victor Péron, Mathématiques appliquées, 15 Eq. TD, L1, UPPA, France
 Licence : Victor Péron, Courbes et calcul intégral, 19.5 Eq. TD, L2, UPPA, France
 Licence : Victor Péron, Analyse numérique des systèmes lin., 48.75 Eq. TD, L3, UPPA, France
 Master : Victor Péron et Sébastien Tordeux, Analyse num. des EDP 1: différences finies, 75 eq. TD, Master1, UPPA, FRANCE
 Master : Victor Péron et Sébastien Tordeux, Introduction aux phénomènes de propagation d'ondes, 38 eq. TD, Master 2, UPPA, FRANCE
 Master : Victor Péron et Sébastien Tordeux, Méthodes asymptotiques, 35 eq. TD, Master 2, UPPA, FRANCE

9.2.2. Supervision

HDR : Victor Péron, Asymptotic models for acoustic, elastic and electromagnetic media. Corner and edge asymptotics for elliptic systems, Université de Pau et des Pays de l'Adour, December 6th, 2017 [12].
 PhD : Vincent Darrigrand, Etude d'erreur pour des problèmes d'Helmholtz approchés par des techniques de Petrov-Galerkin , September 1st 2017, Hélène Barucq and David Pardo.
 PhD : Florian Faucher, Méthodes d'inversion sismique dans le domaine fréquentiel , November 29th 2017, Hélène Barucq.
 PhD : Mamadou N'Diaye, Analyse et développement de schémas temporels hybrides pour les équations hyperboliques du premier ordre, December 8th 2017, Hélène Barucq and Marc Duruflé.
 PhD in progress : Aurélien Citrain, Déformation 3D de maillages en imagerie sismique, Méthodes d'inversion sismique dans le domaine fréquentiel , October 2016, Hélène Barucq and Christian Gout.
 PhD in progress : Izar Azpiroz Iragorri, Approximation des problèmes d'Helmholtz couplés sur maillages virtuels , October 2014, Hélène Barucq, Julien Diaz and Rabia Djellouli (CSUN).
 PhD in progress : Justine Labat, Diffraction of an electromagnetic wave by small obstacles, Université de Pau et des Pays de l'Adour, October 2016, Victor Péron and Sébastien Tordeux
 PhD in progress : Hamza Alaoui Hafidi, Imagerie ultrasonore tridimensionnelle dans les milieux hétérogènes complexes, October 2015, Encadrement : Marc Deschamps, Michel Castaings, Eric Ducasse, Samuel Rodriguez (I2M), Hélène Barucq, Marc Duruflé, Juliette Chabassier (Magique 3D).
 PhD in progress : Pierre Jacquet, ,October 2015, Hélène Barucq and Julien Diaz.
 PhD in progress : Chengyi Shen, Approches expérimentale et numérique de la propagation d'ondes sismiques dans les roches carbonatées, October 2016, Julien Diaz and Daniel Brito (LFC).
 PhD in progress : Elvira Shishenina, Approximations hybrides par éléments finis et éléments virtuels discontinus pour l'élasto-acoustique, October 2015, Hélène Barucq and Julien Diaz.

PhD in progress : Alexandre Gras, Hybrid resonance for sensing applications, IOGS, October 2017, Encadrement : Philippe Lalanne(IOGS), Marc Duruflé, Hélène Barucq (Magique 3D)

Master 1 internship : Kevin Lagnoux, Détermination des paramètres physiques et de forme d'un objet dans un milieu élasto-acoustique, Université Toulouse III, Sept. 2017.

Master 1 internship : Fatima Ezzahra, Formulation de modèles de dissipation dans les milieux poreux en géophysique, École des Mines de Saint-Étienne, Sept. 2017.

9.2.3. *Juries*

- Hélène Barucq : Laure Pesudo (Université Paris Saclay) "Une méthode hybride couplant la méthode des équations intégrales et la méthode des rayons en vue d'applications au contrôle non destructif ultrasonore", PhD thesis, October 8th 2017
- Hélène Barucq : Victor Péron (Université de Pau et des Pays de l'Adour) "Analyse asymptotique et calcul scientifique pour des applications en physique", HDR, December 6th 2017
- Hélène Barucq (reviewer): Sébastien Pernet (Université Paul Sabatier, Toulouse) "Quelques méthodes performantes pour la simulation des phénomènes de propagation et de diffraction d'ondes", HDR, December 11th 2017
- Hélène Barucq (reviewer): Marius Albrand (Université Paul Sabatier, Toulouse) "Etude d'une solution d'évaluation des constantes diélectriques du béton d'ouvrages à risque par une approche problème inverse en électromagnétisme", PhD thesis, December 18th 2017
- Julien Diaz (reviewer): Mohamed Lakhel (Université Paris Saclay) "Méthodes d'inversion pour la reconstruction de mines enfouies à partir de mesures d'antennes radar.", PhD thesis, June 22nd 2017
- Julien Diaz : Vincent Darrigrand (University of Basque Country/Université de Pau et des Pays de l'Adour) "Goal-oriented adaptivity using unconventional error representations", PhD thesis, September 1st 2017
- Julien Diaz : Octavio Castillo Reyes (UPC Universitat Politecnica de Catalunya) "Edge-Elements Formulation of 3D CSEM in Geophysics: A Parallel Approach.", PhD thesis, October 23th 2017
- Julien Diaz : Mamadou N'Diaye (Université de Pau et des Pays de l'Adour) "Étude et développement de méthodes numériques d'ordre élevé pour la résolution des équations différentielles ordinaires (EDO), applications à la résolution des équations d'ondes acoustiques et électromagnétiques.", PhD thesis, December 8th 2017
- Juliette Chabassier : Jin Jack Tan (Université Paris Saclay) "Piano acoustics: string's double polarisation and piano source identification", PhD thesis, November 15th 2017.

9.3. Popularization

- Juliette Chabassier shared her experience as a scientist in the collège de Lussac in March 2017.
- Juliette Chabassier participated in scientific "speed datings" during the "Filles et Maths" day in April 2017.
- Juliette Chabassier shared her experience as a scientist during "Printemps de la Mixité" in May 2017.
- Juliette Chabassier presented a talk around mathematics in music in Pau in April 2017.
- Juliette Chabassier co-organized a series of three conferences around the theme of women in informatics in 2017.
- Juliette Chabassier gave a talk about mathematics in music in Bordeaux during the "découvreuses anonymes" exposition in November 2017.
- Juliette Chabassier presented a workshop around mathematics in music in November 2017.
- Juliette Chabassier gave a pitch of science during the national event "50 ans Inria" in November 2017.

10. Bibliography

Major publications by the team in recent years

- [1] H. BARUCQ, A. BENDALI, M. FARES, V. MATTESI, S. TORDEUX. *A Symmetric Trefftz-DG formulation based on a local boundary element method for the solution of the Helmholtz equation*, in "Journal of Computational Physics", October 2016 [DOI : 10.1016/J.JCP.2016.09.062], <https://hal.archives-ouvertes.fr/hal-01395861>
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- [3] E. BERETTA, M. V. DE HOOP, F. FAUCHER, O. SCHERZER. *Inverse Boundary Value Problem For The Helmholtz Equation: Quantitative Conditional Lipschitz Stability Estimates*, in "SIAM Journal on Mathematical Analysis", November 2016, vol. 48, n^o 6, pp. 3962 - 3983 [DOI : 10.1137/15M1043856], <https://hal.archives-ouvertes.fr/hal-01407446>
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- [7] J. DIAZ, V. PÉRON. *Equivalent Robin Boundary Conditions for Acoustic and Elastic Media*, in "Mathematical Models and Methods in Applied Sciences (M3AS)", 2016, <https://hal.inria.fr/hal-01254194>
- [8] L. GIZON, H. BARUCQ, M. DURUFLÉ, C. HANSON, M. LEGUÈBE, A. BIRCH, J. CHABASSIER, D. FOURNIER, T. HOHAGE, E. PAPINI. *Computational helioseismology in the frequency domain: acoustic waves in axisymmetric solar models with flows*, 2016, to appear, <https://hal.archives-ouvertes.fr/hal-01403332>

Publications of the year

Doctoral Dissertations and Habilitation Theses

- [9] V. DARRIGRAND. *Goal-oriented adaptivity using unconventional error representations*, Universidad del Pais Vasco & Université de Pau et des Pays de l'Adour, 2017
- [10] F. FAUCHER. *Contributions to seismic full waveform inversion for time harmonic wave equations: stability estimates, convergence analysis, numerical experiments involving large scale optimization algorithms*, Université de Pau et des Pays de l'Adour, 2017
- [11] M. N'DIAYE. *On the study and development of high-order time integration schemes for ODEs applied to acoustic and electromagnetic wave propagation problems*, Université de Pau et des Pays de l'Adour, 2017

- [12] V. PÉRON. *Asymptotic Models for Acoustic, Elastic and Electromagnetic Media. Corner and Edge Asymptotics for Elliptic Systems*, Université de Pau et des Pays de l'Adour, December 2017, Habilitation à diriger des recherches, <https://hal.inria.fr/tel-01660795>

Articles in International Peer-Reviewed Journals

- [13] H. BARUCQ, A. BENDALI, M. FARES, V. MATTESI, S. TORDEUX. *A Symmetric Trefftz-DG formulation based on a local boundary element method for the solution of the Helmholtz equation*, in "Journal of Computational Physics", February 2017, vol. 330, pp. 1069-1092 [DOI : 10.1016/j.jcp.2016.09.062], <https://hal.archives-ouvertes.fr/hal-01395861>
- [14] H. BARUCQ, J. CHABASSIER, M. DURUFLÉ, L. GIZON, M. LEGUÈBE. *Atmospheric Radiation Boundary Conditions for the Helmholtz Equation*, in "ESAIM: Mathematical Modelling and Numerical Analysis", 2017, forthcoming, <https://hal.inria.fr/hal-01581834>
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in "Astronomy and Astrophysics - A&A", 2017, vol. 608, n^o A109, pp. 1-9 [DOI : 10.1051/0004-6361/201731283], <https://hal.archives-ouvertes.fr/hal-01622697>

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- [25] H. BARUCQ, H. CALANDRA, G. CHAVENT, M. V. DE HOOP, F. FAUCHER. *Stability and convergence analysis for seismic depth imaging using FWI*, in "Computational Inverse Problems for Partial Differential Equations Workshop", Oberwolfach, Germany, Computational Inverse Problems for Partial Differential Equations, May 2017, pp. 67–70, <https://hal.archives-ouvertes.fr/hal-01623952>
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- [30] H. BARUCQ, M. DURUFLÉ, M. N'DIAYE. *A-stable high-order implicit time schemes*, in "Waves 2017 International Conference on Mathematical and Numerical Aspects of Wave Propagation", Minneapolis, United States, May 2017, vol. 2017, <https://hal.inria.fr/hal-01535638>
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the Project Review, Geo-Mathematical Imaging Group at Rice University, Houston TX", Houston, TX, United States, Proceedings of the Project Review, Geo-Mathematical Imaging Group at Rice University, Houston TX, Proceedings of the Project Review, Geo-Mathematical Imaging Group at Rice University, Houston TX, April 2017, vol. 1, pp. 31–42, <https://hal.archives-ouvertes.fr/hal-01623949>

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- [43] H. BARUCQ, H. CALANDRA, J. DIAZ, E. SHISHENINA. *Space-Time Trefftz - Discontinuous Galerkin Approximation for Elasto-Acoustics*, Inria Bordeaux Sud-Ouest ; UPPA (LMA-Pau) ; Total E&P, October 2017, n^o RR-9104, <https://hal.inria.fr/hal-01614126>
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- [46] J. CHABASSIER, J. DIAZ, A. HA, S. IMPERIALE. *Construction and Analysis of a Fourth Order, Energy Preserving, Explicit Time Discretization for Dissipative Linear Wave Equations*, January 2018, working paper or preprint, <https://hal.archives-ouvertes.fr/hal-01690548>
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