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Project-Team REALOPT

Reformulations based algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB), Laboratoire Bordelais de Recherche en Informatique (LaBRI)

RESEARCH CENTER
Bordeaux - Sud-Ouest

THEME
Optimization, machine learning and statistical methods

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Project-Team REALOPT

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A8.1. - Discrete mathematics, combinatorics

A8.2. - Optimization

A8.7. - Graph theory

Other Research Topics and Application Domains:

B6.5. - Information systems

1. Personnel

Research Scientists

Olivier Beaumont [Inria, Senior Researcher, HDR]

Lionel Eyraud-Dubois [Inria, Researcher]

Vitor Nesello [ADERA, temporary Researcher, from Sep 2017, part-time]

Ruslan Sadykov [Inria, Researcher]

Faculty Members

Francois Vanderbeck [Team leader, Univ of Bordeaux, Professor, HDR]

Francois Clautiaux [Univ of Bordeaux, Professor, HDR]

Boris Detienne [Univ of Bordeaux Associate Professor]

Arnaud Pecher [Univ of Bordeaux, Professor, HDR]

Pierre Pesneau [Univ of Bordeaux, Associate Professor]

Post-Doctoral Fellows

Ayse Nur Arslan [Inria]

Yihong Gao [Inria, from Feb 2017]

Agnes Le Roux [Univ de Bordeaux, until Jan 2017]

Halil Sen [PGMO, until Oct 2017, Inria thereafter]

Jerome Clet-Ortega [Inria, from Mar 2017]

PhD Students

Thomas Bellitto [ENS]

Imen Ben Mohamed [Kedge Business School, Bordeaux]

Mohamed Benkirane [Cifre SNCF]

Rodolphe Griset [Cifre EDF]

Gael Guillot [Univ de Bordeaux, from Nov 2017]

Jeremy Guillot [Univ of Bordeaux, ATER]

Thomas Lambert [ENS Lyon, until Aug 2017]

Guillaume Marques [Univ of Bordeaux, from Sep 2017]

Quentin Viaud [Cifre Saint-Gobain]

Technical staff

Adeline Fonseca [Inria, from Sep 2017]

Guillaume Marques [Inria, until Aug 2017]

Issam Tahiri [Inria]

Interns

Yacine Al Najjar [Inria, from May 2017 until Jul 2017]

Adeline Fonseca [Inria, from Mar 2017 until Aug 2017]
Gael Guillot [Univ of Bordeaux, from Mar 2017 until Aug 2017]
Johan Leveque [Inria, from May 2017 until Jul 2017]

Administrative Assistant

Joelle Rodrigues [Inria]

Visiting Scientist

Orlando Rivera Letelier [University Alfonso Ibanez, Chile, until Feb 2017]

External Collaborators

Cedric Joncour [Univ Du Havre, Assistant Prof]
Sophie Michel [Univ Du Havre, Assistant Prof]
Walid Klibi [Kedge Business School, Bordeaux, Assistant Prof]
Gautier Stauffer [Institut polytechnique de Grenoble, Professor]
Alexis Toullat [Ertus Consulting]

2. Overall Objectives

2.1. Overall Objectives

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.

Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications along variable fixing based on reduced cost, Lagrangian decomposition methods to produce powerful relaxations, and Bender's decomposition to project the formulation , reducing the problem to the important decision variables, and to implement multi-level programming that models a hierarchy of decision levels or recourse decision in the case of data adjustment, primal heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum, with specific strong strategies for the selection on the sequence of fixings. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random (or dynamically adjusted) data by way of modeling expectation (stochastic approaches) or worst-case behavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

(i) Methodologies: To design tight formulations for specific problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. More broadly, to contribute to theoretical and methodological developments of exact approaches in combinatorial optimization, while extending the scope of applications.

(ii) Problem solving: To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact methods and heuristics.

(iii) Software platform: To provide prototypes of specific model solvers and generic software tools that build on our research developments, writing code that serves as the proof-of-concept of the genericity and efficiency of our approaches, while transferring our research findings to internal and external users.

3. Research Program

3.1. Introduction

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, decomposition and reformulation techniques in mixed integer programing, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems, High Performance and Cloud Computing.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [76]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$ with matrix $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\,x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\,x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition and reformulation approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition [7], [9], [8]. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use variable fixing techniques, primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...). Such methods are likely to produce good quality solutions only if the integer programming formulation is of top quality, i.e., if its LP relaxation provides a good approximation of the IP solution.

In the same line, variable fixing techniques, that are essential in reducing the size of large scale problems, rely on good quality approximations: either tight formulations or tight relaxation solvers (as a dynamic program combined with state space relaxation). Then if the dual bound derives when the variable is fixed to one exceeds the incubent solution value, the variable can be fixed to zero and hence removed from the problem. The process can be apply sequentially by refining the degree of relaxation.

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems. Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization account for the expected realization of random data and optimize an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behaviour of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

3.6. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Introduction

Our group has tackled applications in logistics, transportation and routing [33] [14] [75], [74], [70], [72], in production planning [92] and inventory control [70], [72], in network design and traffic routing [53], [62], [68], [95], [50], [63], [80], [88], in cutting and placement problems [77], [78], [89], [90], [91], [93], and in scheduling [86], [81], [46], and in High Performance and Cloud Computing [20] [40] [41] [34] [48] [47].

4.2. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form “at least two paths link each pair of terminals”. We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [62]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today’s optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronic system installation cost. We developed and compared several decomposition approaches [97], [96], [95] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [94]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [68], [69].

We studied several time dependent formulations for the unit demand vehicle routing problem [55], [54]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [87] simplifying results from Baiou and Barahona.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

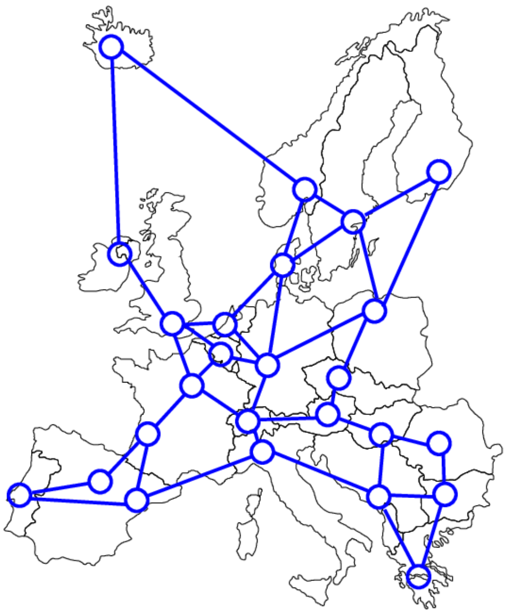

Figure 1. Design of a SDH/SONET european network where demands are multiplexed.

For the problem of routing freight railcars, we proposed two algorithmes based on the column generation approach. These algorithmes have been tested on a set of real-life instances coming from a real Russian freight transportation company. Our algorithms have been faster on these instances than the current solution approach being used by the company.

4.3. Packing and Covering Problems

Realopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [85]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developped a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [44] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem [72]: we faced a multiple-class integer knapsack problem with setups [71] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [65], [64], [67], [66]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2D knapsack problem that uses our 2D packing feasibility check. We are currently developping exact optimization tools for glass-cutting problems in a collaboration with Saint-Gobain [35]. This 2D-3stage-Guillotine cut problems are very hard to solve given the scale of the instance we have to deal with. Moreover one has to issue cutting patterns that avoid the defaults that are present in the glass sheet that are used as raw material. There are extra sequencing constraints regarding the production that make the problem even more complex.
We have also organized a european challenge on packing with society Renault: see http://challenge-esicup-2015.org/. This challenge is about loading trucks under practical constraints.

4.4. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a branch-and-price algorithm where periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem, and the global planning of customer visits is coordinated by the master program [73]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [75].

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [79] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximize the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [61] and exact algorithms [46]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [45]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [81]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [83]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [84], [82].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [43], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [42] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [49] provide demonstrably stronger formulations for some problem classes than any previously proposed. We are now working on planning phytosanitary treatments in vineries.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [57], [56], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [51]. We considered train timetabling problems and their re-optimization after a perturbation in the network [59], [58]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF [26], [27], [28]. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem [60]. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge adressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem [52].

4.5. Resource Allocation for High Performance and Cloud Computing

In the context of numerical simulations on high performance machines, optimizing data locality and resource usage is very important for faster execution times and lower energy consumption. This optimization can be seen as a special case of scheduling problem on parallel resource, with several challenges. First, instances are typically large: a large matrix factorization (with 50×50 blocks) involves about $30 \cdot 10^3$ tasks. Then, HPC platforms consist of heterogeneous and unrelated resources, what is known to make scheduling problems hard to approximate. Finally, due to co-scheduling effects and shared communication resources, it is not realistic to accurately model the exact duration of tasks. All these observations make it impossible to rely on static optimal solutions, and HPC applications have gone from simple generic static allocations to runtime dynamic scheduling strategies that make their decisions based on the current state of the platform (the location of input data), the expected transfer and running times for the tasks, and some affinity and priority information that have possibly been computed offline. In this context, we are strongly involved in the design of scheduling strategies for the StarPU runtime, with two goals: proving that it is possible to design approximation algorithms whose complexity is extremely small (typically sub-linear in the number of ready tasks), and show that they can be used in practice with good performance results. We are pursuing collaborations both with teams developing the StarPU system (Storm) by designing algorithms for the generic scheduling problems [20], and with teams developing linear algebra algorithms over the runtime (Hiepacs), by proposing specialized algorithms for specific cases. For example, in the case of linear algebra applications on heterogeneous platforms, we have considered the combinatorial optimization problem associated to matrix multiplication, that is amenable to partitioning the unit square into zones of prescribed areas while minimizing the overall size of the boundaries. We have improved the best known approximation ratio to 1.15 in [40] and we have shown that the resulting distribution schemes can indeed be used to design efficient implementations using StarPU in [41].

In the context of Cloud Computing platforms and Data Science, we are interested in resource allocation and data placement strategies. For BigData applications running on data centers platforms, data locality is the largest contributing factor to application performance. In practice, allocation decisions are made at runtime based on strategies that tend to favor local tasks. Our goal is to assess and to improve the efficiency of these runtime strategies. In particular, we have proven that the problem of maximizing locality when allocating map tasks is amenable to a graph orientation and a semi matching problem, what enabled us to assess the efficiency of classical MapReduce allocation algorithm in [34] for the map phase and to propose a low cost algorithm to compute optimal allocation schemes. We also consider more generic VM placement problem for large-scale datacenters. This placement is often handled with naive greedy rules, whereas it is possible to propose online or offline efficient allocation algorithms, for example to optimize the reliability of all applications in a platform with faults [48] or to co-locate applications with compatible periodic load variations [47].

5. Highlights of the Year

5.1. Highlights of the Year

Olivier Beaumont was the Track Chair of the Algorithm Track of Super Computing 2017 (November, Denver, USA); “The International Conference for High Performance Computing, Networking, Storage and Analysis” https://sc17.supercomputing.org. SuperComputing is the major international conference on High Performance Computing.

We have contributed to the JULIA mathematical programming ecosystem by providing tools to decompose a mixed integer programming model into blocks. This makes it very convenient to model Benders or Dantzig-Wolfe decomposition using JUMP and to compare different decomposition for a given problem formulation.

Our generic software platform BaPCod is now giving rise to specific branches for classes of applications. The first such release concerns the classic benchmark Vehicle Routing Problem variants that arise in logistics. The methods that are build in the platform emerge from our collaboration with our Brazilian partners of the SAMBA associated team. For their anterior work, our partners have received the 2017 best paper award from the prestigious journal “Mathematical Programming Computation”. With the new version that is built under BaPCod, we have managed to solve to optimality many more open instances of classic and very competitive Vehicle Routing Problem with Time Windows [37]. This study has been an opportunity to improve significantly the performance on the generic Branch-Cut-and-Price platform and to highlight the interests of such generic methodologies.

6. New Software and Platforms

6.1. BaPCod

A generic Branch-And-Price-And-Cut Code
KEYWORDS: Column Generation - Branch-and-Price - Branch-and-Cut - Mixed Integer Programming - Mathematical Optimization - Benders Decomposition - Dantzig-Wolfe Decomposition - Extended Formulation
FUNCTIONAL DESCRIPTION: BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithms, Benders approaches, network flow and dynamic programming algorithms. These methods can be combined in several hybrid algorithms to produce exact or approximate solutions (primal solutions with a bound on the deviation to the optimum).

- Participants: Artur Alves Pessoa, Boris Detienne, Eduardo Uchoa Barboza, Franck Labat, François Clautiaux, Francois Vanderbeck, Halil Sen, Issam Tahiri, Michael Poss, Pierre Pesneau, Romain Leguay and Ruslan Sadykov
- Partners: Université de Bordeaux - CNRS - IPB - Universidade Federal Fluminense
- Contact: Francois Vanderbeck
- URL: https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/BaPCod

6.2. WineryPlanning

- Participants: Agnes Le Roux, Alexis Toullat, Francois Vanderbeck, Issam Tahiri and Ruslan Sadykov
- Contact: Francois Vanderbeck

6.3. ORTOJ

Operation Research Tools Under Julia

KEYWORDS: Modeling - Processing - Dashboard

FUNCTIONAL DESCRIPTION: This set of tools currently includes : 1) BlockJuMP.jl: extension of JuMP to model decomposable mathematical programs (using either Benders or Dantzig-Wolfe decomposition paradign) 2) Scanner.jl: a default data parser to ease the reading of the input data in the form that they are often encountered in operational research. 3) BenchmarkUtils.jl: Tools to ease the setup of numerical experiments to benchmark algorithmic feature performances. The test automation permits to quickly calibrate the parameters of an arbitrary algorithm control function.

- Participants: Francois Vanderbeck, Guillaume Marques, Issam Tahiri and Ruslan Sadykov
- Contact: Issam Tahiri

7. New Results

7.1. Improving Branch-and-Price Methods

We have made progress on stabilization techniques and math-heuristics that are essential components for generic Branch-and-Price methods.

The convergence of a column generation algorithm can be improved in practice by using stabilization techniques. Smoothing and proximal methods based on penalizing the deviation from the incumbent dual solution have become standards of the domain. Interpreting column generation as cutting plane strategies in the dual problem, we have analyzed [15] the mechanisms on which stabilization relies. In particular, the link is established between smoothing and in-out separation strategies to derive generic convergence properties. For penalty function methods as well as for smoothing, we describe proposals for parameter self-adjusting schemes. Such schemes make initial parameter tuning less of an issue as corrections are made dynamically. Such adjustments also allow to adapt the parameters to the phase of the algorithm. Extensive test reports validate our self-adjusting parameter scheme and highlight their performances. Our results also show that using smoothing in combination with penalty function yields a cumulative effect on convergence speed-ups.

Math heuristics have become an essential component in mixed integer programming (MIP) solvers. Extending MIP based heuristics, we have studied [17] generic procedures to build primal solutions in the context of a branch-and-price approach. As the Dantzig-Wolfe reformulation of a problem is typically tighter than that of the original compact formulation, heuristics based on rounding its linear programing (LP) solution can be more competitive. We focus on the so-called diving methods that used re-optimization after each LP rounding. We explore combination with diversification- intensification paradigms such as Limited Discrepancy Search, sub-MIPing, relaxation induced neighbourhood search, local branching, and strong branching. The dynamic generation of variables inherent to a column generation approach requires specific adaptation of heuristic paradigms. We manage to use simple strategies to get around these technical issues. Our numerical results on generalized assignment, cutting stock, and vertex coloring problems sets new benchmarks, highlighting the performance of diving heuristics as generic procedures in a column generation context and producing better solutions than state-of-the-art specialized heuristics in some cases.

7.2. Aggregation Techniques

We have developed [13] a general solution framework based on aggregation techniques to solve NP-Hard problems that can be formulated as a circulation model with specific side constraints. The size of the extended Mixed Integer Linear Programming formulation is generally pseudo-polynomial. To efficiently solve exactly these large scale models, we propose a new iterative aggregation and disaggregation algorithm. At each iteration, it projects the original model onto an aggregated one, producing an approximate model. The process iterates to refine the current aggregated model until the optimality is proved.

The computational experiments on two hard optimization problems (a variant of the vehicle routing problem and the cutting-stock problem) show that a generic implementation of the proposed framework allows us to outperform previous known methods.

We have applied this aggregation method to reduce the size of column generation (CG) models for covering problems in which the feasible subsets depend on a resource constraint [16]. The aggregation relies on a correlation between the resource consumption of the elements and the corresponding optimal dual values. The resulting aggregated dual model is a restriction of the original one, and it can be rapidly optimized to obtain a feasible dual solution. A primal bound can also be obtained by restricting the set of columns to those saturated by the dual feasible solution obtained by aggregation. The convergence is realized by iterative disaggregation until the gap is closed by the bounds. Computational results show the usefulness of our method for different cutting-stock problems. An important advantage is the fact that it can produce high-quality dual bounds much faster than the traditional Lagrangian bound used in stabilized column generation.

We have developed an algorithm for the exact solution of the Temporal Knapsack Problem [29], [24]. We proposed a dynamic programming formulation for the problem, whose size is exponential in the size of the input data. To cope with the curse of dimensionality, we based our algorithm on the Successive Sublimation Dynamic Programming method. We generalized it to allow more precise aggregation of the state space of the dynamic program. Several application-specific feasibility tests and dominance relations, based on aggregated information, are used to derive an efficient implementation of the method. The algorithms compares favorably with the literature, solving several open instances.

7.3. Revisiting Benders Decomposition & Enhancing the Algorithm

In Benders decomposition approach to mixed integer programs, the optimization is carried in two stages: key first-stage decision variables are optimized using a polyhedral approximation of the full-blown problem projection, then a separation problem expressed in the second-stage variables is solved to check if the current first-stage solution is truly feasible, and otherwise, it produces a violated inequality. Such cutting-plane algorithms suffer from several drawbacks and may have very bad convergence rates. We have reviewed [98] the battery of approaches that have been proposed in the literature to address these drawbacks and to speed-up the algorithm. Our contribution consists in explaining these techniques in simple terms and unified notations, showing that in several cases, different proposals of the literature boil down to the same key ideas. We classify methods into specific initialization mode, stabilization techniques, strategies to select the separation point, and cut generation strategies. We have contributed to enhance convergence of Benders cutting plane algorithm by a mixture of smoothing techniques and proximal approaches. Our numerical benchmarking is still on going [18].

7.4. Routing Problems

Given a directed graph $G = (V, A)$, a cost function c associated with the arcs of A, and a set of precedence constraints $B \subset V \times V$, the Precedence Constrained Asymmetric Traveling Salesman Problem (PCATSP) seeks for a minimum cost Hamiltonian circuit, starting at node 1, and such that for each $(i, j) \in B$, the node i is visited before node j. There are many ways of modelling the ATSP and several for the PCATSP. In [14], [25] we present new formulations for the two problems that can be viewed as resulting from combining precedence variable based formulations with network flow based formulations. Indeed, the former class of formulations permits to integrate linear ordering constraints. The motivating formulation for this work is a complicated and "ugly" formulation that results from the separation of generalized subtour elimination constraints presented. This so called "ugly" formulation exhibits, however, one interesting feature, namely the "disjoint subpaths" property that is further explored to create more complicated formulations that combine two (or three) "disjoint path" network flow based formulations and have a stronger linear programming bound. Some of these stronger formulations are related to the ones presented for the PCATSP and can be viewed as generalizations in the space of the precedence based variables. Several sets of projected inequalities in the space of the arc and precedence variables are obtained by projection from these network flow based formulations. Computational results for the ATSP and PCATSP evaluate the quality of the new models and inequalities.

In [36] we deal with the Minimum Latency Problem (MLP), another variant of the well-known Traveling Salesman Problem in which the objective is to minimize the sum of waiting times of customers. This problem arises in many applications where customer satisfaction is more important than the total time spent by the server. This paper presents a novel branch-and-price algorithm for MLP that strongly relies on new features for the *ng*-path relaxation, namely: (1) a new labeling algorithm with an enhanced dominance rule named multiple partial label dominance; (2) a generalized definition of *ng*-sets in terms of arcs, instead of nodes; and (3) a strategy for decreasing *ng*-set sizes when those sets are being dynamically chosen. Also, other elements of efficient exact algorithms for vehicle routing problems are incorporated into our method, such as reduced cost fixing, dual stabilization, route enumeration and strong branching. Computational experiments over TSPLIB instances are reported, showing that several instances not solved by the current state-of-the-art method can now be solved.

In [37], [31] we consider the Resource Constrained Shortest Path problem arising as a subproblem in state-of-the-art Branch-Cut-and-Price algorithms for vehicle routing problems. We propose a variant of the bi-directional label correcting algorithm in which the labels are stored and extended according to so-called bucket graph. Such organization of labels helps to decrease significantly the number of dominance checks and the running time of the algorithm. We also show how the forward/backward route symmetry can be exploited and how to filter the bucket graph using reduced costs. The proposed algorithm can be especially beneficial for vehicle routing instances with large vehicle capacity and/or with time constraints. Computational experiments were performed on instances from the distance constrained vehicle routing problem, including multi-depot and site-depended variants, on the vehicle routing problem with time windows, and on the "nightmare" instances of the heterogeneous fleet vehicle routing problem. Very significant improvements over the best algorithms in the literature were achieved and many instances could be solved for the first time.

We also considered a family of Vehicle Routing Problem (VRP) variants that generalize the classical Capacitated VRP by taking into account the possibility that vehicles differ by capacity, costs, depot allocation, or even by the subset of customers that they can visit. In [33], [30], [23] we propose a branch-cut-and-price algorithm that adapts advanced features found in the best performing exact algorithms for homogeneous fleet VRPs. The original contributions include: (i) the use of Extended Capacity Cuts, defined over a pseudo-polynomially large extended formulation, together with Rank-1 Cuts, defined over the Set Partitioning Formulation; (ii) the concept of vehicle-type dependent memory for Rank-1 Cuts; and (iii) a new family of lifted Extended Capacity Cuts that takes advantage of the vehicle-type dependent route enumeration. The algorithm was extensively tested in instances of the literature and was shown to be significantly better than previous exact algorithms, finding optimal solutions for many instances with up to 200 customers and also for some larger instances. Several new best solutions were found too.

7.5. Machine Scheduling Problems

In [21] we consider the unrelated parallel machine scheduling problem with setup times to minimize a general objective function. In this work we present a novel exact algorithm that is capable of solving this problem $R|r_j, s_{ij}^k| \sum f_j(C_j)$ and the large class of problems that can be derived as particular cases from it. The proposed algorithm consists of a branch-cut-and-price approach that combines several features such as non-robust cuts, strong branching, reduced cost fixing and dual stabilization. To our knowledge, this is the first exact algorithm for unrelated machines with earliness and/or tardiness criteria that can solve consistently instances with more than 20 jobs. We report improved bounds for instances of problems $R|r_j, s_{ij}^k| \sum w'_j E_j + w_j T_j$ and $R|| \sum w'_j E_j + w_j T_j$ with up to 80 and 120 jobs, respectively.

7.6. Scheduling Strategies for Runtime Systems

We consider the design of low cost but guaranteed approximation algorithms in the context of the runtime StarPU in [20]. In High Performance Computing, heterogeneity is now the norm with specialized accelerators like GPUs providing efficient computational power. The added complexity has led to the development of task-based runtime systems, which allow complex computations to be expressed as task graphs, and rely on scheduling algorithms to perform load balancing between all resources of the platforms. Developing good scheduling algorithms , even on a single node, and analyzing them can thus have a very high impact on the performance of current HPC systems. The special case of two types of resources (namely CPUs and GPUs) is of practical interest. HeteroPrio is such an algorithm which has been proposed in the context of fast multipole computations, and then extended to general task graphs with very interesting results. In this paper, we provide a theoretical insight on the performance of HeteroPrio, by proving approximation bounds compared to the optimal schedule in the case where all tasks are independent and for different platform sizes. Interestingly, this shows that spoliation allows to prove approximation ratios for a list scheduling algorithm on two unrelated resources, which is not possible otherwise. We also establish that almost all our bounds are tight. Additionally, we provide an experimental evaluation of HeteroPrio on real task graphs from dense linear algebra computation, which highlights the reasons explaining its good practical performance.

7.7. Matrix Partitioning for Parallel Computing on Heterogeneous Platforms

We consider the combinatorial optimization problem that arises in the context of matrix multiplication in [40]. The problem of partitioning a matrix into a set of sub-matrices has received increased attention recently and is crucial when considering dense linear algebra and kernels with similar communication patterns on heterogeneous platforms. The problem of load balancing and minimizing communication is traditionally reducible to an optimization problem that involves partitioning a square into rectangles. This problem has been proven to be NP-Complete for an arbitrary number of partitions. In this paper, we present recent approaches that relax the restriction that all partitions be rectangles. The first approach uses an original mathematical technique to find the exact optimal partitioning. Due to the complexity of the technique, it has been developed for a small number of partitions only. However, even at a small scale, the optimal partitions found by this approach are often non-rectangular and sometimes non-intuitive. The second approach is the study of approximate partitioning methods by recursive partitioning algorithms. In particular we use the work on optimal partitioning to improve pre-existing algorithms. In this paper we discuss the different perspectives it opens and present two algorithms, SNRPP which is a sqrt(3/2) approximation, and NRPP which is a 2/sqrt(3) approximation. While sub-optimal, this approach works for an arbitrary number of partitions. We use the first exact approach to analyse how close to the known optimal solutions the NRRP algorithm is for small numbers of partitions. In order to validate above approach, we consider in [41] how to allocate data when performing matrix multiplication on a heterogeneous node, with multicores and GPUs. Classical (cyclic) allocations designed for homogeneous settings are not appropriate, but the advent of task-based runtime systems makes it possible to use more general allocations. Previous theoretical work has proposed square and cube partitioning algorithms aimed at minimizing data movement for matrix multiplication. We propose techniques to adapt these continuous square partitionings to allocating discrete tiles of a matrix, and strategies to adapt the static allocation at run-time. We use these techniques in an implementation of Matrix Multiplication based on the StarPU runtime system, and we show through extensive experiments that this implementation allows to consistently obtain a lower communication volume while improving slightly the execution time, compared to standard state-of-the-art dynamic strategies.

7.8. Convergence between HPC and Data Science

We consider the use of replication when scheduling independent identical tasks in [34]. MapReduce is a well-know framework for distributing data-processing computations onto parallel clusters. In MapReduce, a large computation is broken into small tasks that run in parallel on multiple machines, and scales easily to very large clusters of inexpensive commodity computers. Before the Map phase, the original dataset is split into data chunks that are replicated (a constant number of times, usually 3) and distributed randomly onto computing nodes. During the Map phase, local tasks (i.e., tasks whose data chunks are stored locally) are assigned in priority when processors request tasks. In this paper, we provide the first complete theoretical analysis of data locality in the Map phase of MapReduce, and more generally, for bag-of-tasks applications that behave like MapReduce. We prove that if tasks are homogeneous (in terms of processing time), as soon as the replication factor is larger than 2, FindAssignment, a matching based algorithm, achieves a quasi-perfect makespan (i.e., optimal up to an additive constant of one step) using a sophisticated matching algorithm. Above result is proved with high probability when the number of tasks becomes arbitrarily large, and we therefore complement theoretical results with simulations that corroborate them even for small number of tasks. We also show that the matching-based approach leads to an improvement of data locality during the Map phase and therefore decreases the amount of communications needed to achieve perfect makespan, compared to the classical MapReduce greedy approach. In the context of the convergence between HPC and Data Science, we investigate the use of Burst Buffers for HPC applications in [38]. Burst-Buffers are high throughput, small size intermediate storage systems typically based on SSDs or NVRAM that are designed to be used as a potential buffer between the computing nodes of a supercomputer and its main storage system consisting of hard drives. Their purpose is to absorb the bursts of I/O that many HPC applications experience (for example for saving checkpoints or data from intermediate results). In this paper, we propose a probabilistic model for evaluating the performance of Burst-Buffers. From a model of application and a data management strategy, we build a Markov chain based model of the system, that allows to quickly answer issues about dimensioning of the system: for a given set of applications, and for a given Burst-Buffer size and bandwidth, how often does the buffer overflow? We also provide extensive simulation results to validate our modeling approach.

7.9. Network Design Problems

The delivery of freight from manufacturing platforms to demand zones is often managed through one or more intermediate locations where storing, merging, transshipment and consolidation activities are performed. In [22], we design a Two-Echelon Distribution Network that helps synchronise different flows of product. Under demand uncertainty, our model integrates decisions on the locations and the size of second echelon facilities an decisions on the flows assignment between the echelons, and on delivery routes to serve the demand zones.

7.10. Two-dimensional Guillotine Cutting Problems

The two-dimensional knapsack problem consists in packing a set of small rectangular items into a given large rectangle while maximizing the total reward associated with selected items. In [13], we restrict our attention to packings that emanate from a k-stage guillotine-cut process. We introduce a generic model where a knapsack solution is represented by a flow in a directed acyclic hypergraph. This hypergraph model derives from a forward labeling dynamic programming recursion that enumerates all non-dominated feasible cutting patterns. To reduce the hypergraph size, we make use of further dominance rules and a filtering procedure based on Lagrangian reduced costs fixing of hyperarcs. Our hypergraph model is (incrementally) extended to account for explicit bounds on the number of copies of each item. Our exact forward labeling algorithm is numerically compared to solving the max-cost flow model in the base hyper-graph with side constraints to model production bounds. Benchmarks are reported on instances from the literature and on datasets derived from a real-world application.

Also we consider a variant of two-dimensional guillotine cutting-stock problem that arises when different bills of order (or batches) are considered consecutively. The raw material leftover of the last cutting pattern is not counted as waste as it can be reused for cutting the next batch. The objective is thus to maximize the length of the leftover. In [35], [32] we propose a diving heuristic based on a Dantzig-Wolfe reformulation solved by column generation in which the pricing problem is solved using dynamic programming (DP). This DP generates so-called non-proper columns, *i.e.* cutting patterns that cannot participate in a feasible integer solution of the problem. We show how to adapt the standard diving heuristic to this “non-proper” case while keeping its effectiveness. We also introduce the partial enumeration technique, which is designed to reduce the number of non-proper patterns in the solution space of the dynamic program. This technique helps to strengthen the lower bounds obtained by column generation and improve the quality of solutions found by the diving heuristic. Computational results are reported and compared on classical benchmarks from the literature as well as on new instances inspired from industrial data. According to these results, proposed diving algorithms outperform constructive and evolutionary heuristics.

7.11. On sets avoiding distance 1

In a joint work with C. Bachoc, T. Bellitto and P. Moustrou [39], we consider the maximum density of sets avoiding distance 1 in $\mathbb{R}^n$. Let $||.||$ be a norm of R^n and $G_{||.||}$ be the so-called unit distance graph with the points of R^n as vertex set and for edge set, the set of pairs $\{x,y\}$ such that $||x-y||=1$. An independent set of $G_{||.||}$ is said to avoid distance 1.

Let $||.||_E$ denote the Euclidean norm. For $n=2$, the chromatic number of $G_{||.||_E}$ is still wide open: it is only known that $4 \leq \chi\left(G_{||.||_E}\right) \leq 7$ (Nelson, Isbell 1950). The *measurable* chromatic number χ_m of the graph $G_{||.||}$ is the minimal number of *measurable* stable sets of $G_{||.||}$ needed to cover all its vertices. Obviously, we have $\chi\left(G_{||.||_E}\right) \leq \chi_m\left(G_{||.||_E}\right)$. For $n=2$, $5 \leq \chi_m\left(G_{||.||_E}\right)$ (Falconer 1981).

Let $m_1\left(G_{||.||}\right)$ denote the maximum density of a measurable set avoiding distance 1. We have $\frac{1}{m_1\left(G_{||.||}\right)} \leq \chi_m\left(G_{||.||}\right)$. We study the maximum density m_1 for norms defined by polytopes: if P is a centrally symmetric polytope and x is a point of R^n, $||x||_P$ is the smallest positive real t such that $x \in tP$. Polytope norms include some usual norms such as the L^1 and L^∞ norms.

If P tiles the space by translation, then it is easy to see that $m_1\left(G_{||.||_P}\right) \geq \frac{1}{2^n}$. C. Bachoc and S. Robins conjectured that equality always holds. We show that this conjecture is true for $n=2$ and for some polytopes in higher dimensions.

7.12. Separating Codes and Traffic Monitoring

The paper [12] studies the problem of traffic monitoring which consists of differentiating a set of walks on a directed graph by placing sensors on as few arcs as possible. The problem of characterising a set of individuals by testing as few attributes as possible is already well-known, but traffic monitoring presents new challenges that the previous models of separation fall short from modelling such as taking into account the multiplicity and order of the arcs in a walk. We introduce a new and stronger model of separation based on languages that generalises the traffic monitoring problem. We study three subproblems with practical applications and develop methods to solve them by combining integer linear programming, separating codes and language theory.

8. Bilateral Contracts and Grants with Industry

8.1. Collaboration with EDF on robust maintenance planning

Our project with EDF concerns the optimization of the long term energy production planning, allowing for nuclear power plants maintenance. The challenges are to handle the large-scale instance of a five year planning and to handle the stochastic aspects of the problem: the stochastic variation of the electricity demand, the production capacity and the duration of maintenance period. The key decisions to be optimized are the dates of outages (for maintenance) and the level refuelling that determines the production of the year to come. We previously developed a column generation approach based on extended formulation which enables to solve within a few minutes a deterministic instance of the problem, which is within the time frame of the operational tools currently used by EDF. We now investigate stochastic and robust versions of the problem, where the duration of maintenance operations and the power demand are uncertain. Our approaches shall be evaluated on real life instances within a rolling horizon framework.

8.2. Collaboration with ERTUS on phytosanitary treatment planning

In planning winary operations (most importantly phytosanitary treatments on the wine tree) under wheather forcast uncertainty, one searches for solutions that remain feasible and “cheap” in case of perturbation in the data. We consider the planning and scheduling of the operations that arise over a one-year horizon. More precisely, the operations to be sheduled include tasks related to soil care, or grape tree care: cutting, line building, thinning out leaves, ..., and chemical treatments. The latter are a main focus of our study since one of the principal goals of better planning is to reduce the amount of chemical treatments by selecting the appropriate products and schemes, but also by spacing out treatements while guarantying a desease free vineyard with some confidence. Each of the scheduled tasks requires its own resource, so the planning also triggers equipement and raw products selection decisions. The objective is to minimize both equipment and product costs augmented by an evaluation of the hazard of chemical product use. The planning should be “robust” to seasonal variations on the proper time frame for scheduling tasks.

8.3. Collaboration with St-Gobain Recherche on glass cutting

Through the PhD of Quentin Viaud, we study a hard glass-cutting problem. The objective is to minimize the quantity of trim loss when rectangular pieces are cut from large rectangles. This first study has shown that our methodologies are able to cope with this problem for medium-size instances. Solving the problem with large instances is a scientific challenge that we will address in the a follow-up contract.

8.4. Collaboration with SNCF on timetable and rolling stock rotation planning

Our projet with SNCF concerns the optimisation of timetable and rolling stock rotation planning. The railway production planning process combines heterogeneous resources and is usually decomposed into different sequential sub-problems, beginning by line planning, timetabling, rolling stock rotations and crew scheduling. Our goal is to solvie the timetable and rolling stock problems in an integrated manner. Given a line planning and service requirement constraints, the problem is to produce a timetable for a set of trains and the objective is to minimize the cost of the railcars used. An originality of our approach is to deal with railcars composed of multiple units, which can be coupled or decoupled at some stations. The PhD thesis of Mohamed Benkirane is funded by this project.

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR

9.1.1.1. ANR Solhar (ANR-13-MONU-0007)

This project aims at studying and designing algorithms and parallel programming models for implementing direct methods for the solution of sparse linear systems on emerging computing platforms equipped with accelerators. This project proposes an innovative approach which relies on the efficiency and portability of runtime systems, such as the StarPU tool. The focus of RealOpt in this project is on the scheduling aspect. Indeed, executing a heterogeneous workload with complex dependencies on a heterogeneous architecture is a very challenging problem that demands the development of effective scheduling algorithms. These will be confronted with possibly limited views of dependencies among tasks and multiple, and potentially conflicting objectives, such as minimizing the makespan, maximizing the locality of data or, where it applies, minimizing the memory consumption.
See also: http://solhar.gforge.inria.fr/

9.2. International Initiatives

9.2.1. Inria International Partners

In the follow-up of our 6 year Inria Associate Team project SAMBA , we have set an important research collaboration with Brazil (Universidade Federal Fluminense, Pontificia Universidade Catolica do Rio de Janeiro) and Chile (Universidad Adolfo Ibanez). This results in joint publications and frequent visits, including long stay by research students.

9.3. International Research Visitors

9.3.1. Visits of International Scientists

- Teobaldo LEITE BULHOES, from Universidade Federal Fluminense (Niteroi, Brazil), visited the team from October 23rd to December 13th 2017.
- Orlando Rivera Letelier, from (Universidad Adolfo Ibanez, Chile, visited the team for January 2017.
- Eduardo UCHOA, from Universidade Federal Fluminense (Niteroi, Brazil), visited the team during two weeks from November 5th to 18th 2017.
- Xuding ZHU, from Zhejiang Normal University (Jinhua, China) visited the team during one month in June 2017.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

10.1.1.1. Member of the Organizing Committees

- Arnaud Pêcher, François Clautiaux and Pierre Pesneau have organized "Journées Graphes et Algorithmes", Bordeaux, Novembre 15-17, 2017

10.1.2. Scientific Events Selection

10.1.2.1. Chair of Conference Program Committees

- Olivier Beaumont is the Chair of the Algorithm Track of Super Computing 2017 (November, Denver, USA); “The International Conference for High Performance Computing, Networking, Storage and Analysis” https://sc17.supercomputing.org
- Olivier Beaumont is the Chair of the Algorithm Track of HIPC 2017 (December, Jaipur, India); “24th Ieee International Conference On High Performance Computing, Data, And Analytics” http://hipc.org
- Lionel Eyraud-Dubois is Chair of the “Cloud Computing and Data Center Management” track of I-SPAN 2017; the 14th International Symposium on Pervasive Systems, Algorithms, and Networks

10.1.2.2. Member of the Conference Program Committees

The team members are members of the following program committees:

- Francois Clautiaux, Arnaud Pecher, and Francois Vanderbeck: ROADEF 2017: French Operational Research Society Conference.
- Lionel Eyraud-Dubois and Olivier Beaumont: HiPC 2017: 24th IEEE International Conference on High Performance Computing, Data, and Analytics
- Lionel Eyraud-Dubois: REPPAR 2017: 4th International Workshop on Reproducibility in Parallel Computing

- Olivier Beaumont: Primary PC Member, IPDPS 2017, 31st IEEE International Parallel & Distributed Processing Symposium May 29 – June 2, 2017, Orlando, Florida USA http://www.ipdps.org/ipdps2017/index.html
- Olivier Beaumont: HeteroPar 2017 (a EuroPar workshop), Fifteenth International Workshop on Algorithms, Models and Tools for Parallel Computing on Heterogeneous Platforms, August 28th, 2017, Santiago de Compostela, Spain https://web.fe.up.pt/~heteropar2017/index.php?pane=home
- Olivier Beaumont: IPDPSW 2017, IPDPS 2017, 31st IEEE International Parallel & Distributed Processing Symposium May 29 – June 2, 2017, Orlando, Florida USA, PC member http://www.ipdps.org/ipdps2017/2017_call_for_workshops.html

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

- Olivier Beaumont is editor for IEEE Transactions on Parallel and Distributed Systems (TPDS)
- Francois Vanderbeck is Associate Editor for the EURO Journal on Computational Optimization
- Francois Clautiaux is Associate Editor for Mathematical Programming and Exact Methods in the journal ISTE “Recherche Opérationnelle”

10.1.3.2. Reviewer - Reviewing Activities

The team members are regular referees for the best journals of the field.

10.1.4. Invited Talks

Arnaud Pêcher: *Lovasz's theta function and perfect graphs*, “The beauty of discrete mathematics” workshop , Montréal, Canada, October 2017.

François Vanderbeck: *Revisiting Benders Decomposition*, Combinatorial Optimization and Applications Workshop, Edinburgh, Scotland, February 2017.

10.1.5. Leadership within the Scientific Community

Our group is actively preparing the triennal symposium of the international mathematical optimization society. We organize it in Bordeaux in July 2018. 2000 attendees are expected.

10.1.6. Scientific Expertise

- Olivier Beaumont is a member of the INCITE (math-comp track) panel
- Olivier Beaumont is an expert for the H2020-FET-OPEN-2016 projects

10.1.7. Research Administration

- Olivier Beaumont is the scientific deputy of Inria Bordeaux Sud-Ouest and a member of the Evaluation Committee of Inria ***Verify it!!***.
- François Vanderbeck is taking care of the team OptimAl (“Optimisation Mathématique Modèle Aléatoire et Statistique”) at the Mathematics Institute of Bordeaux.
- Arnaud Pêcher is the head of the Computer Science Department, IUT of Bordeaux.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence : A. Pêcher, Programmation Impérative, 10h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Conception Objet, 42h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Programmation objet en Java, 44h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Algorithmique Avancée, 32h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Assembleur, 24h, DUT, Université de Bordeaux, France

Licence : A. Pêcher, Programmation Mobile, 24h, DUT, Université de Bordeaux, France

Master : F. Clautiaux, Gestion des Opérations et Planification de la Production, 20h, M2, Université de Bordeaux, France

Master : F. Clautiaux, Flot et Combinatoire, 10h, M2, Institut Polytechniques de Bordeaux, France

Master : F. Clautiaux, Introduction à la Programmation en Variables Entières, 20h, M1, Université de Bordeaux, France

Master : F. Clautiaux, Projet d'optimisation pour l'insertion professionnelle, M2, Université de Bordeaux, France

Master : L. Eyraud-Dubois, Optimisation en Cloud Computing et Big Data, 15h, M2, Université de Bordeaux, France

Licence : Licence : P. Pesneau, Optimisation, 37h, L2, Université de Bordeaux, France

Licence : Licence : P. Pesneau, Programmation pour le calcul scientifique, 24h, L2, Université de Bordeaux, France

Licence : Licence : P. Pesneau, Recherche Opérationnelle, 24h, DUT, Université de Bordeaux, France

Licence : Master : P. Pesneau, Algorithmique et Programmation 1, 28h, M1, Université de Bordeaux, France

Licence : Master : P. Pesneau, Programmation linéaire, 29h, M1, Université de Bordeaux, France

Licence : Master : P. Pesneau, Optimisation dans les graphes (partie flots), 15h, M1, Université de Bordeaux, France

Master : O. Beaumont, Approximation et Big Data, 15h, M2, Université de Bordeaux, France

Master : O. Beaumont, Distributed Computing and Data Mining, 4h, M2, Institut National Polytechnique de Bordeaux, France

Licence : B. Detienne, Initiation à l'ingénierie en optimisation, 12h, L1, Université de Bordeaux, France

Licence : B. Detienne, Modèles et Méthodes d'Optimisation, 21h, L2, Université de Bordeaux, France

Licence : B. Detienne, Groupe de travail applicatif, 12h, L3, Université de Bordeaux, France

Master : B. Detienne, Optimisation continue, 43h, M1, Université de Bordeaux, France

Master : B. Detienne, Problèmes combinatoires et routage, 14h, M1, Université de Bordeaux, France

Master : B. Detienne, Problèmes combinatoires et routage, 14h, M1, Institut National Polytechnique de Bordeaux, France

Master : B. Detienne, Optimisation dans l'incertain, 58h, M2, Université de Bordeaux, France

Master : R. Sadykov, Introduction à la Programmation par Contraintes, 30h, M1, Université de Bordeaux, France

Master : I. Tahiri, Recherche Opérationnelle, 16h, M1, Institut National Polytechnique de Bordeaux, France

Master : F. Vanderbeck, Recherche Opérationnelle, 15h, M1, Institut National Polytechnique de Bordeaux, France

Master : F. Vanderbeck, Programmation Entière, 58h, M2, Université de Bordeaux, France

10.2.2. Supervision

PhD : Suraj Kumar, Scheduling of Dense Linear Algebra Kernels on Heterogeneous Resources [10], Université de Bordeaux, 12/04/2017, Olivier Beaumont (dir) and Lionel Eyraud-Dubois (co-dir)

PhD: Thomas Lambert, Placement de tâches et réplication de fichiers sur plates-formes parallèles [11], Université de Bordeaux, 8/09/2017, Olivier Beaumont (dir) and Lionel Eyraud-Dubois (co-dir)

PhD in progress : Jérémy Guillot, Optimisation de problèmes de partitionnement, from September 2014, François Clautiaux (dir) and Pierre Pesneau (dir).

PhD in progress : Quentin Viaud, Méthodes de programmation mathématiques pour des problèmes complexes de découpe, from January 2015, François Clautiaux (dir), Ruslan Sadykov (dir), and François Vanderbeck (co-dir)).

PhD in progress : Rodolphe Griset, Robust planning in Electricity production, from November 2015, Boris Detienne (dir) and François Vanderbeck (dir).

PhD in progress : Imen Ben Mohamed, Location routing problems, from October 2015, Walid Klibi (dir) and François Vanderbeck (dir).

PhD in progress : Thomas Bellitto, Infinite graphs, from September 2015, Arnaud Pêcher (dir) and Christine Bachoc (dir).

PhD in progress : Guillaume Marques, Planification de tournées de véhicules avec transbordement en logistique urbaine : approches basées sur les méthodes exactes de l'optimisation mathématique, from September 2017, François Vanderbeck (dir) and Ruslan Sadykov (co-dir).

PhD in progress : Gaël Guillot, Aggregation and disaggregation methods for hard combinatorial problems, from November 2017, François Clautiaux (dir) and Boris Detienne (dir).

10.2.3. Juries

- Lionel Eyraud-Dubois participated in the jury of Raphaël Bleuse, who defended on October 11, 2017, at Université de Grenoble Alpes.
- Olivier Beaumont: Evaluation (rapporteur) and President of the PhD thesis committee of Aymen Jlassi (University of Tours, France)
- Olivier Beaumont: Evaluation (rapporteur) of the PhD thesis committee of Orcun Yildiz (Ecole Normale Supérieure de Rennes, France)
- Olivier Beaumont: Member of the PhD thesis committee of Alexandre Perrot (University of Bordeaux, France)
- Olivier Beaumont: Member of the PhD thesis committee of Noel Gillet (University of Bordeaux, France)
- Francois Vanderbeck: Member of the PhD thesis committee of Nicolas HUIN (Inria Sophia)

10.3. Popularization

- François Clautiaux is a member of the board of AMIES, the French Agency for Interaction in Mathematics with Business and Society. AMIES is a national organization that aims to develop relations between academic research teams in mathematics and business, especially SMEs.
- Olivier Beaumont participated to "Unithé ou Café" (May 19, 2017), a local event dedicated to popular science on the topic of online algorithms.
- Olivier Beaumont participated to "La Fête de la Science" (October 3 and 4, 2017) on the computation of PageRank.

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Major publications by the team in recent years

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[3] F. CLAUTIAUX, S. HANAFI, R. MACEDO, E. VOGE, C. ALVES. *Iterative aggregation and disaggregation algorithm for pseudo-polynomial network flow models with side constraints*, in "European Journal of Operational Research", 2017, vol. 258, pp. 467 - 477 [*DOI :* 10.1016/J.EJOR.2016.09.051], https://hal.inria.fr/hal-01410170

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[10] S. KUMAR. *Scheduling of Dense Linear Algebra Kernels on Heterogeneous Resources*, Université de Bordeaux, April 2017, https://tel.archives-ouvertes.fr/tel-01538516

[11] T. LAMBERT. *On the Effect of Replication of Input Files on the Efficiency and the Robustness of a Set of Computations*, Université de Bordeaux, September 2017, https://tel.archives-ouvertes.fr/tel-01661588

Articles in International Peer-Reviewed Journals

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