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# Activity Report 2018

# Project-Team AROMATH

## Algebre geometrie Modelisation et Algorithmes

RESEARCH CENTER
**Sophia Antipolis - Méditerranée**

THEME
**Algorithmics, Computer Algebra and Cryptology**

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**Keywords:**

**Computer Science and Digital Science:**

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A8.3. - Geometry, Topology
A8.4. - Computer Algebra

**Other Research Topics and Application Domains:**

B9.5.1. - Computer science
B9.5.2. - Mathematics

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# 2. Overall Objectives

## 2.1. Overall Objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of "real" phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to "act" on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, ... are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects "at home" from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimisation loops where the numerical simulation results are used to improve the "performance" of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world...), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

# 3. Research Program

## 3.1. High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in CAGD, to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at the seams. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

## 3.2. Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally, as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods, which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One is based on linearization of algebraic problems based on “elimination of variables” or projection into a space of smaller dimension. Resultant theory provides strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progresses have been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis, which provides general tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients. An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute "only" the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

# 4. Application Domains

## 4.1. Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing ...Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axes machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

## 4.2. Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree $\leq 2$).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

# 5. New Software and Platforms

## 5.1. Platforms

### *5.1.1. Axl*

KEYWORDS: Algorithm , CAD , Numerical algorithm , Geometric algorithms

SCIENTIFIC DESCRIPTION

Axl is an algebraic geometric modeler that aims at providing “algebraic modeling” tools for the manipulation and computation with curves, surfaces or volumes described by semi-algebraic representations. These include parametric and implicit representations of geometric objects. Axl also provides algorithms to compute intersection points or curves, singularities of algebraic curves or surfaces, certified topology of curves and surfaces, etc. A plugin mechanism allows to extend easily the data types and functions available in the plateform.

FUNCTIONAL DESCRIPTION

Axl is a cross platform software to visualize, manipulate and compute 3D objects. It is composed of a main application and several plugins. The main application provides atomic geometric data and processes, a viewer based on VTK, a GUI to handle objects, to select data, to apply process on them and to visualize the results. The plugins provides more data with their reader, writer, converter and interactors, more processes on the new or atomic data. It is written in C++ and thanks to a wrapping system using SWIG, its data structures and algorithms can be integrated into C# programs, as well as Python. The software is distributed as a source package, as well as binary packages for Linux, MacOSX and Windows.

- Participants: Emmanouil Christoforou, Nicolas Douillet, Anaïs Ducoffe, Valentin Michelet, Bernard Mourrain, Meriadeg Perrinel, Stéphane Chau and Julien Wintz
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Collaboration with Elisa Berrini (MyCFD, Sophia), Tor Dokken (Gotools library, Oslo, Norway), Angelos Mantzaflaris (GISMO library, Linz, Austria), Laura Saini (Post-Doc GALAAD/Missler, TopSolid), Gang Xu (Hangzhou Dianzi University, China), Meng Wu (Hefei University of Technology, China).

### *5.1.2. Skelton*

KEYWORDS: Algorithm, Numerical algorithm, Geometric algorithm, Scaffolding, Implicit surface, Mesh generation

SCIENTIFIC DESCRIPTION

Skelton is a C++ library for skeleton-based modeling with convolution surfaces. It supports skeletons made of line segments and arcs of circle, including circular splines, and features an anisotropic extension to circular splines. The library can generate a quad dominant mesh that surrounds, and follows, the structure of the skeleton. The mesh is generated with an advanced scaffolding algorithm that works for skeletons of any topology.

FUNCTIONAL DESCRIPTION

Skelton is a multi-platform C++ library distributed as source code. It provides a class hierarchy for creating and manipulating geometric objects generated with anisotropic convolution. It can take the skeleton as an input file or defined directly by the user. The library supports an improved version of convolution surfaces and permits the design of anisotropic surfaces. The scaffolding algorithm can be used independently of the implicit surface definition. The library has a command line interface that controls most of the functionality. The user can subclass elements of the library for extended behavior.

- Participants: Alvaro Fuentes, Evelyne Hubert.
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# 6. New Results

## 6.1. Solving Polynomial Systems via Truncated Normal Forms

**Participant:** Bernard Mourrain.

In [12], we consider the problem of finding the isolated common roots of a set of polynomial functions defining a zero-dimensional ideal $I$ in a ring $R$ of polynomials over $\mathbb{C}$. We propose a general algebraic framework to find the solutions and to compute the structure of the quotient ring $R/I$ from the null space of a Macaulay-type matrix. The affine dense, affine sparse, homogeneous, and multihomogeneous cases are treated. In the presented framework, the concept of a border basis is generalized by relaxing the conditions on the set of basis elements. This allows for algorithms to adapt the choice of basis in order to enhance the numerical stability. We present such an algorithm and show numerical results.

This is a joint work with Simon Telen and Marc Van Barel (Univ. Leuven, Belgium)

## 6.2. On supersolvable and nearly supersolvable line arrangements

**Participant:** Alexandru Dimca.

In the paper [3], we introduce a new class of line arrangements in the projective plane, called nearly supersolvable, and show that any arrangement in this class is either free or nearly free. More precisely, we show that the minimal degree of a Jacobian syzygy for the defining equation of the line arrangement, which is a subtle algebraic invariant, is determined in this case by the combinatorics. When such a line arrangement is nearly free, we discuss the splitting types and the jumping lines of the associated rank two vector bundle, as well as the corresponding jumping points, introduced recently by S. Marchesi and J. Vallès.

Joint work with Gabriel Sticlaru (Faculty of Mathematics and Informatics, Ovidius University, Romania).

## 6.3. Computing the monodromy and pole order filtration on Milnor fiber cohomology of plane curves

**Participant:** Alexandru Dimca.

In the paper [4], we describe an algorithm computing the monodromy and the pole order filtration on the Milnor fiber cohomology of any reduced projective plane curve C. The relation to the zero set of Bernstein-Sato polynomial of the defining homogeneous polynomial for C is also discussed. When C has some non weighted homogeneous singularities, then we have to assume that a conjecture holds in order to get some of our results. In all the examples computed so far this conjecture holds.

Joint work with Gabriel Sticlaru (Faculty of Mathematics and Informatics, Ovidius University, Romania).

## 6.4. Invariant Algebraic Sets and Symmetrization of Polynomial Systems

**Participant:** Evelyne Hubert.

Assuming the variety of a polynomial set is invariant under a group action, we construct, in [9], a set of invariants that cut the same variety. The construction can be seen as a generalization of the previously known construction for finite groups. The result though has to be understood outside an invariant variety which is independent of the polynomial set considered. We introduce the symmetrizations of a polynomial that are polynomials in a generating set of rational invariants. The generating set of rational invariants and the symmetrizations are constructed w.r.t. a section of the orbits of the group action.

## 6.5. Rational invariants of even ternary forms under the orthogonal group

**Participant:** Evelyne Hubert.

In [8], we determine a generating set of rational invariants of minimal cardinality for the action of the orthogonal group $O_3$ on the space $R[x,y,z]_{2d}$ of ternary forms of even degree $2d$. The construction relies on two key ingredients: On the one hand, the Slice Lemma allows us to reduce the problem to determining the invariants for the action on a subspace of the finite subgroup $B_3$ of signed permutations. On the other hand, our construction relies in a fundamental way on specific bases of harmonic polynomials. These bases provide maps with prescribed $B_3$-equivariance properties. Our explicit construction of these bases should be relevant well beyond the scope of this paper. The expression of the $B_3$-invariants can then be given in a compact form as the composition of two equivariant maps. Instead of providing (cumbersome) explicit expressions for the $O_3$-invariants, we provide efficient algorithms for their evaluation and rewriting. We also use the constructed $B_3$-invariants to determine the $O_3$-orbit locus and provide an algorithm for the inverse problem of finding an element in $R[x,y,z]_{2d}$ with prescribed values for its invariants. These are the computational issues relevant in brain imaging.

This is a joint work with P. Görlach (Max Planck institute, Leipzig) and T. Papadopoulo (EPI Athena, Inria SAM)

## 6.6. Algorithms for Computing Cubatures Based on Moment Theory

**Participant:** Evelyne Hubert.

Quadrature is an approximation of the definite integral of a function by a weighted sum of function values at specified points, or nodes, within the domain of integration. Gaussian quadratures are constructed to yield exact results for any polynomials of degree $2r-1$ or less by a suitable choice of $r$ nodes and weights. Cubature is a generalization of quadrature in higher dimension. In [2] we elaborate algorithms to compute all minimal cubatures for a given domain and a given degree. We propose first an algorithm in symbolic computation to characterize all cubatures of a given degree with a fixed number of nodes. The determination of the nodes and weights is then left to the computation of the eigenvectors of the matrix identified at the characterization stage and can be performed numerically. The characterisation of cubatures on which our algorithms are based stems from moment theory. We formulate the results there in a basis independent way : rather than considering the moment matrix, the central object in moment problems, we introduce the underlying linear map from the polynomial ring to its dual, the Hankel operator. This makes natural the use of bases of polynomials other than the monomial basis, and proves to be computationally relevant, either for numerical properties or to exploit symmetry.

Joint work with M. Collowald, (previously Université Nice Sophia Antipolis).

## 6.7. Products of Euclidean Metrics and Applications to Proximity Questions among Curves

**Participants:** Ioannis Emiris, Ioannis Psarros.

In [18], we study Approximate Nearest Neighbor (ANN) search on 1-dimensional shapes. We start with distance functions between discretized curves in Euclidean space: they appear in a wide range of applications, from road segments and molecular backbones to time-series in general dimension. For p-products of Euclidean metrics, for any positive integer p, we design simple and efficient data structures for ANN, based on randomized projections, which are of independent interest. They serve to solve proximity problems under a notion of distance between discretized curves, which generalizes both discrete Fréchet and Dynamic Time Warping distances. These are the most popular and practical approaches to comparing such curves. We offer the first data structures and query algorithms for ANN with arbitrarily good approximation factor, at the expense of increasing space usage and preprocessing time over existing methods. Query time complexity is comparable or significantly improved by our algorithms.

## 6.8. Efficient Random-Walk Methods for Approximating Polytope Volume

**Participant:** Ioannis Emiris.

In [5] we experimentally study the fundamental problem of computing the volume of a convex polytope given as an intersection of linear inequalities. We implement and evaluate practical randomized algorithms for accurately approximating the polytope's volume in high dimensions (e.g. one hundred). To carry out this efficiently we experimentally correlate the effect of parameters, such as random walk length and number of sample points, on accuracy and runtime. Moreover, we exploit the problem's geometry by implementing an iterative rounding procedure, computing partial generations of random points and designing fast polytope boundary oracles. Our publicly available code is significantly faster than exact computation and more accurate than existing approximation methods. We provide volume approximations for the Birkhoff polytopes of order 11 to 15, whereas exact methods have only computed that for order 10.

This is a joint work with Vissarion Fisikopoulos (Oracle Corp., Athens, Greece).

## 6.9. Randomized Embeddings with Slack and High-Dimensional Approximate Nearest Neighbor

**Participants:** Evangelos Anagnostopoulos, Ioannis Emiris, Ioannis Psarros.

In [1], we study the approximate nearest neighbor problem (e-ANN) in high dimensional Euclidean space with methods beyond Locality Sensitive Hashing (LSH), which has polynomial dependence in the dimension, sublinear query time, but subquadratic space requirement. In particular, we introduce a new definition of "low-quality" embeddings for metric spaces. It requires that, for some query point q, there exists an approximate nearest neighbor among the pre-images of the k approximate nearest neighbors in the target space. Focusing on Euclidean spaces, we employ random projections in order to reduce the original problem to one in a space of dimension inversely proportional to k. The k approximate nearest neighbors can be efficiently retrieved by a data structure such as BBD-trees. The same approach is applied to the problem of computing an approximate near neighbor, where we obtain a data structure requiring linear space, and query time in $O(dn^{\rho})$, for $\rho{\approx}1{-}e^2/\log(1/e)$. This directly implies a solution for e-ANN, while achieving a better exponent in the query time than the method based on BBD-trees. Better bounds are obtained in the case of doubling subsets of $\ell^2$, by combining our method with r-nets. We implement our method in C++, and present experimental results in dimension up to 500 and $10^6$ points, which show that performance is better than predicted by the analysis. In addition, we compare our ANN approach to E2LSH, which implements LSH, and we show that the theoretical advantages of each method are reflected on their actual performance.

## 6.10. Practical Volume Computation of Structured Convex Bodies, and an Application to Modeling Portfolio Dependencies and Financial Crises

**Participants:** Ioannis Emiris, Apostolos Chalkis.

In [16], we examine volume computation of general-dimensional polytopes and more general convex bodies, defined as the intersection of a simplex by a family of parallel hyperplanes, and another family of parallel hyperplanes or a family of concentric ellipsoids. Such convex bodies appear in modeling and predicting financial crises. The impact of crises on the economy (labor, income, etc.) makes its detection of prime interest for the public in general and for policy makers in particular. Certain features of dependencies in the markets clearly identify times of turmoil. We describe the relationship between asset characteristics by means of a copula; each characteristic is either a linear or quadratic form of the portfolio components, hence the copula can be constructed by computing volumes of convex bodies. We design and implement practical algorithms in the exact and approximate setting, we experimentally juxtapose them and study the tradeoff of exactness and accuracy for speed. We analyze the following methods in order of increasing generality: rejection sampling relying on uniformly sampling the simplex, which is the fastest approach, but inaccurate for small volumes; exact formulae based on the computation of integrals of probability distribution functions, which are the method of choice for intersections with a single hyperplane; an optimized Lawrence sign decomposition method, since the polytopes at hand are shown to be simple with additional structure; Markov chain Monte Carlo algorithms using random walks based on the hit-and-run paradigm generalized to nonlinear convex bodies and relying on new methods for computing a ball enclosed in the given body, such as a second-order cone program; the latter is experimentally extended to non-convex bodies with very encouraging results. Our C++ software, based on CGAL and Eigen and available on github, is shown to be very effective in up to 100 dimensions. Our results offer novel, effective means of computing portfolio dependencies and an indicator of financial crises, which is shown to correctly identify past crises. (The views expressed are those of the authors and do not necessarily reflect official positions of the European Commission.)

This is a joint work with Ludovic Calées (EU JRC, Ispra, Italy), and Vissarion Fisikopoulos (Oracle Corp., Athens, Greece).

## 6.11. On the maximal number of real embeddings of spatial minimally rigid graphs

**Participants:** Ioannis Emiris, Evangelos Bartzos.

In [15], we study the number of embeddings of minimally rigid graphs in Euclidean space $R^D$, which is (by definition) finite, modulo rigid transformations, for every generic choice of edge lengths. Even though various approaches have been proposed to compute it, the gap between upper and lower bounds is still enormous. Specific values and its asymptotic behavior are major and fascinating open problems in rigidity theory. Our work considers the maximal number of real embeddings of minimally rigid graphs in $R^3$. We modify a commonly used parametric semi-algebraic formulation that exploits the Cayley-Menger determinant to minimize the *a priori* number of complex embeddings, where the parameters correspond to edge lengths. To cope with the huge dimension of the parameter space and find specializations of the parameters that maximize the number of real embeddings, we introduce a method based on coupler curves that makes the sampling feasible for spatial minimally rigid graphs. Our methodology results in the first full classification of the number of real embeddings of graphs with 7 vertices in $R^3$, which was the smallest open case. Building on this and certain 8-vertex graphs, we improve the previously known general lower bound on the maximum number of real embeddings in $R^3$.

This is a joint work with J. Legersky (JK University, Linz, Austria) and E. Tsigaridas (PolSys, Inria).

## 6.12. Curved Optimal Delaunay Triangulation

**Participant:** Laurent Busé.

Meshes with curvilinear elements hold the appealing promise of enhanced geometric flexibility and higher-order numerical accuracy compared to their commonly-used straight-edge counterparts. However, the generation of curved meshes remains a computationally expensive endeavor with current meshing approaches: high-order parametric elements are notoriously difficult to conform to a given boundary geometry, and enforcing a smooth and non-degenerate Jacobian everywhere brings additional numerical difficulties to the meshing of complex domains. In the paper [6], we propose an extension of Optimal Delaunay Triangulations (ODT) to curved and graded isotropic meshes. By exploiting a continuum mechanics interpretation of ODT instead of the usual approximation theoretical foundations, we formulate a very robust geometry and topology optimization of Bézier meshes based on a new simple functional promoting isotropic and uniform Jacobians throughout the domain. We demonstrate that our resulting curved meshes can adapt to complex domains with high precision even for a small count of elements thanks to the added flexibility afforded by more control points and higher order basis functions.

Joint work Leman Feng (ENPC), Pierre Alliez (EPI Titane), Hervé Delingette (EPI Asclepios) and Mathieu Desbrun (CalTech, USA),

## 6.13. Convolution surfaces with varying radius: Formulae for skeletons made of arcs of circles and line segments

**Participants:** Evelyne Hubert, Alvaro Javier Fuentes Suárez.

In [19], we develop closed form formulae for the computation of the defining fields of convolutions surfaces. The formulae are obtained for power inverse kernels with skeletons made of line segments or arcs of circle. To obtain the formulae we use Creative Telescoping and describe how this technique can be used for other families of kernels and skeleton primitives. We apply the new formulae to obtain convolution surfaces around $\mathcal{G}^1$ skeletons, some of them closed curves. We showcase how the use of arcs of circles greatly improves the visualization of the surface around a general curve compared with a segment based approach.

## 6.14. Scaffolding a Skeleton with Quadrangular Tubes

**Participant:** Evelyne Hubert.

The goal of [22] is to construct a quadrilateral mesh around a one-dimensional skeleton that is as coarse as possible, the "scaffold". A skeleton allows one to quickly describe a shape, in particular a complex shape of high genus. The constructed scaffold is then a potential support for the surface representation: it provides a topology for the mesh, a domain for parametric representation (a quad mesh is ideal for tensor product splines) or, together with the skeleton, a grid support on which to project an implicit surface that is naturally defined by the skeleton through convolution. We provide a constructive algorithm to derive a quad-mesh scaffold with topologically regular cross-sections (which are also quads), and no T-junctions. We show that this construction is optimal in the sense that no coarser quad mesh with topologically regular cross-sections may be constructed. Finally, we apply an existing rotation minimization algorithm along the skeleton branches, which produces a mesh with a natural edge flow along the shape.

This is joint work with A. Panotopoulou (Dartmouth College), E. Ross (MESH consultants), K. Welker (University of Trier), G. Morin (Intitut de Recherche en Informatique de Toulouse).

## 6.15. Scaffolding skeletons using spherical Voronoi diagrams: feasibility, regularity and symmetry

**Participants:** Evelyne Hubert, Alvaro Javier Fuentes Suárez.

Given a skeleton made of line segments, in [7] we describe how to obtain a coarse quad mesh of a surface that encloses tightly the skeleton and follows its structure - the scaffold. We formalize as an Integer Linear Program the problem of constructing an optimal scaffold that minimizes the total number of quads on the mesh. We prove the feasibility of the Integer Linear Program for any skeleton. In particular we can generate these scaffolds for skeletons with cycles. We additionally show how to obtain regular scaffolds, i.e. with the same number of quad patches around each line segment, and symmetric scaffolds that respect the symmetries of the skeleton. An application to polygonization of skeleton-based implicit surfaces is also presented.

## 6.16. Exact conversion from Bézier tetrahedra to Bézier hexahedra

**Participant:** Bernard Mourrain.

Modeling and computing of trivariate parametric volumes is an important research topic in the field of three-dimensional isogeometric analysis. In [13], we propose two kinds of exact conversion approaches from Bézier tetrahedra to Bézier hexahedra with the same degree by reparametrization technique. In the first method, a Bézier tetrahedron is converted into a degenerate Bézier hexahedron, and in the second approach, a non-degenerate Bézier tetrahedron is converted into four non-degenerate Bézier hexahedra. For the proposed methods, explicit formulae are given to compute the control points of the resulting tensor-product Bézier hexahedra. Furthermore, in the second method, we prove that tetrahedral spline solids with $C^k$-continuity can be converted into a set of tensor-product Bézier volumes with $G^k$-continuity. The proposed methods can be used for the volumetric data exchange problems between different trivariate spline representations in CAD/CAE. Several experimental results are presented to show the effectiveness of the proposed methods.

This is a joint work with Gang Xu (Hanghzou, China), Yaoli Jin (Hanghzou, China), Zhoufang Xiao (Hanghzou, China), Qing Wu (Hanghzou, China), Timon Rabczuk (Weimar, Germany).

## 6.17. Constructing IGA-suitable planar parameterization from complex CAD boundary by domain partition and global/local optimization

**Participant:** Bernard Mourrain.

In the paper [14], we propose a general framework for constructing IGA-suitable planar B-spline parameterizations from given complex CAD boundaries. Instead of the computational domain bounded by four B-spline curves, planar domains with high genus and more complex boundary curves are considered. Firstly, some pre-processing operations including Bézier extraction and subdivision are performed on each boundary curve in order to generate a high-quality planar parameterization; then a robust planar domain partition framework is proposed to construct high-quality patch-meshing results with few singularities from the discrete boundary formed by connecting the end points of the resulting boundary segments. After the topology information generation of quadrilateral decomposition, the optimal placement of interior Bézier curves corresponding to the interior edges of the quadrangulation is constructed by a global optimization method to achieve a patch-partition with high quality. Finally, after the imposition of $C^1$/$G^1$-continuity constraints on the interface of neighboring Bézier patches with respect to each quad in the quadrangulation, the high-quality Bézier patch parameterization is obtained by a local optimization method to achieve uniform and orthogonal iso-parametric structures while keeping the continuity conditions between patches. The efficiency and robustness of the proposed method are demonstrated by several examples which are compared to results obtained by the skeleton-based parameterization approach.

This is a joint work with Gang Xu (Hanghzou, China), Ming Li (Zhejiang, China), Timon Rabczuk (Weimar, Germany), Jinlan Xu (Hangzhou, China), Stéphane P.A. Bordas (Luxembourg).

## 6.18. A Classification Approach to Efficient Global Optimization in Presence of Non-Computable Domains

**Participant:** Elisa Berrini.

Gaussian-Process based optimization methods have become very popular in recent years for the global optimization of complex systems with high computational costs. These methods rely on the sequential construction of a statistical surrogate model, using a training set of computed objective function values, which is refined according to a prescribed infilling strategy. However, this sequential optimization procedure can stop prematurely if the objective function cannot be computed at a proposed point. Such a situation can occur when the search space encompasses design points corresponding to an unphysical configuration, an ill-posed problem, or a non-computable problem due to the limitation of numerical solvers. To avoid such a premature stop in the optimization procedure, we propose in [11] to use a classification model to learn non-computable areas and to adapt the infilling strategy accordingly. Specifically, the proposed method splits the training set into two subsets composed of computable and non-computable points. A surrogate model for the objective function is built using the training set of computable points, only, whereas a probabilistic classification model is built using the union of the computable and non-computable training sets. The classifier is then incorporated in the surrogate-based optimization procedure to avoid proposing new points in the non-computable domain while improving the classification uncertainty if needed. The method has the advantage to automatically adapt both the surrogate of the objective function and the classifier during the iterative optimization process. Therefore, non-computable areas do not need to be a priori known. The proposed method is applied to several analytical problems presenting different types of difficulty, and to the optimization of a fully nonlinear fluid-structure interaction system. The latter problem concerns the drag minimization of a flexible hydrofoil with cavitation constraints. The efficiency of the proposed method compared favorably to a reference evolutionary algorithm, except for situations where the feasible domain is a small portion of the design space.

This is joint work with Matthieu Sacher (IRENAV), Régis Duvigneau (ACUMES), Olivier Le Maitre (LIMSI), Mathieu Durand (K-Epsilon), Frédéric Hauville (IRENAV), Jacques-André Astolfi (IRENAV).

## 6.19. Compressions of a polycarbonate honeycomb

**Participant:** André Galligo.

In [21], the in-plane compressive response of a polycarbonate honeycomb with circular close-packed cells is considered first experimentally then analytically. Under quasi-static uniaxial compression, we observed behaviors strongly depending on the orientation: for one of the two main orientations the compression is homogeneous, while for the other the deformation localizes in a very narrow band of cells. More surprisingly, for not crushing but extreme compression, when the load is released, the deformation is reversed, the localization disappears and the polycarbonate returns to its original shape. In order to explain this strange phenomena, we develop a geometric model of this honeycomb together with an expression of the bending energy. We focus on a basic mechanical element made of an elastica triangle. We also compare our description with previous experimental studies and simulations made with similar material. Finally , to illustrate mathematically this type of behavior, we present a simple model for buckling deformations with two degrees of freedom.

This is a joint work with Jean Rajchenbach (LPMC, UCA) and Bernard Rousselet (JAD, UCA).

## 6.20. Modeling and Computation of a liquid-vapor bubble formation

**Participant:** André Galligo.

The Capillary Equation correctly predicts the curvature evolution and the length of a quasi-static vapour formation. It describes a two-phase interface as a smooth curve resulting from a balance of curvatures that are influenced by surface tension and hydrostatic pressures. The present work provides insight into the application of the Capillary Equation to the prediction of single nu-cleate site phase change phenomena. In an effort to progress towards an application of the Capillary Equation to boiling events, a procedure to generating a numerical solution, in which the computational expense is reduced, is reported in [20].

This is a joint work with Frédéric Lesage (LCPI, UCA), Sebastian Minjeaud (JAD, UCA).

## 6.21. Axl, a geometric modeler for semi-algebraic shapes

**Participants:** Emmanouil Christoforou, Bernard Mourrain.

In [17], we describe the algebraic-geometric modeling platform Axl, which provides tools for the manipulation, computation and visualisation of semi-algebraic models. This includes meshes, basic geometric objects such as spheres, cylinders, cones, ellipsoids, torus, piecewise polynomial parameterisations of curves, surfaces or volumes such as B-spline parameterisations, as well as algebraic curves and surfaces defined by polynomial equations. Moreover, Axl provides algorithms for processing these geometric representations, such as computing intersection loci (points, curves) of parametric models, singularities of algebraic curves or surfaces, certified topology of curves and surfaces, etc. We present its main features and describe its generic extension mechanism, which allows one to define new data types and new processes on the data, which benefit from automatic visualisation and interaction facilities. The application capacities of the software are illustrated by short descriptions of plugins on algebraic curves and surfaces and on splines for Isogeometric Analysis.

This is a joint work with Angelos Mantzaflaris (JKU, Austria), Julien Wintz (SED, Inria).

# 7. Partnerships and Cooperations

## 7.1. Regional Initiatives

Our team AROMATH participates to the VADER project for VIRTUAL MODELING of RESPIRATION, UCA Jedi, axis “Modélisation, Physique et Mathématique du vivant". http://benjamin.mauroy.free.fr/VADER.

## 7.2. European Initiatives

### *7.2.1. FP7 & H2020 Projects*

Program: Marie Skłodowska-Curie ITN

Project acronym: ARCADES

Project title: Algebraic Representations in Computer-Aided Design for complEx Shapes

Duration: January 2016 - December 2019

Coordinator: I.Z. Emiris (NKUA, Athens, Greece, and ATHENA Research Innovation Center)

Scientist-in-charge at Inria: L. Busé

Other partners: U. Barcelona (Spain), Inria Sophia Antipolis (France), J. Kepler University, Linz (Austria), SINTEF Institute, Oslo (Norway), U. Strathclyde, Glascow (UK), Technische U. Wien (Austria), Evolute GmBH, Vienna (Austria).

Webpage: http://arcades-network.eu/

Abstract: ARCADES aims at disrupting the traditional paradigm in Computer-Aided Design (CAD) by exploiting cutting-edge research in mathematics and algorithm design. Geometry is now a critical tool in a large number of key applications; somewhat surprisingly, however, several approaches of the CAD industry are outdated, and 3D geometry processing is becoming increasingly the weak link. This is alarming in sectors where CAD faces new challenges arising from fast point acquisition, big data, and mobile computing, but also in robotics, simulation, animation, fabrication and manufacturing, where CAD strives to address crucial societal and market needs. The challenge taken up by ARCADES is to invert the trend of CAD industry lagging behind mathematical breakthroughs and to build the next generation of CAD software based on strong foundations from algebraic geometry, differential geometry, scientific computing, and algorithm design. Our game-changing methods lead to real-time modelers for architectural geometry and visualisation, to isogeometric and design-through-analysis software for shape optimisation, and marine design & hydrodynamics, and to tools for motion design, robot kinematics, path planning, and control of machining tools.

## 7.3. International Initiatives

### *7.3.1. Inria International Partners*

NSFC collaboration project with Gang Xu, Hangzhou Dianzi University, China, “Research on theory and method of time-varying parameterization for dynamic isogeometric analysis”, 2018-2021.

## 7.4. International Research Visitors

### *7.4.1. Visits of International Scientists*

Aron Simis, University of Recife, Brazil, visited L. Busé for a week (October 8-12) to work on birationality of rational map by means of syzygy-based techniques.

Ibrahim Adamou, Univ. Dan Dicko Dankoulodo de Maradi, Niger, visited B. Mourrain (26 Nov.- 21 Dec.) to work on 3-dimensional VoronoïDiagrams of half-lines and medial axes of curve arcs.

#### *7.4.1.1. Internships*

Yairon Cid Ruiz, a PhD srudent at Barcelona in the Arcades network, visited L. Busé for 6 months (October 2017- March 2018) to work on birationality criteria for multi-graded rational maps with a view towards free form deformation problems.

Clément Laroche, a PhD student in Greece in the Arcades network, visited L. Busé and F. Yildirim for one month (October) for a collaboration on implicization matrices of rational curve in arbitrary dimension by means of quadratic relations.

Kim Perriguey, did a six months internship with L. Busé (December 2017-May 2018). She developed paramteric models for the human walk for the extraction of locomotive parameters. This work was done in collaboration with Pierre Alliez (EPI Titane) and the start-up Ekinnox (Sophia Antipolis).

### *7.4.2. Visits to International Teams*

*7.4.2.1. Research Stays Abroad*

F. Yildirim was on secondment at MISSLER Topsolid (France), for 3 months (Mai-July).

From October 25th to November 25th, E. Hubert visited the Institute for Computational and Experimental Research in Mathematics (Providence USA) during the program *Nonlinear Algebra*.

# 8. Dissemination

## 8.1. Promoting Scientific Activities

### *8.1.1. Scientific Events Organisation*

*8.1.1.1. General Chair, Scientific Chair*

Evelyne Hubert was the general and scientific chair for the conference *Symmetry and Computation* held at the Centre International de Recherche en Mathematiques (Marseille, France) April 3-7.

*8.1.1.2. Member of the Organizing Committees*

Laurent Busé organized the second "Learning Week" of the ARCADES Network : "Opportunity Recognition" at Inria Sophia Antipolis, March 19-23, 2018.

### *8.1.2. Scientific Events Selection*

*8.1.2.1. Reviewer*

Bernard Mourrain was reviewer for the conference ISSAC.

### *8.1.3. Journal*

*8.1.3.1. Member of the Editorial Boards*

Bernard Mourrain is associate editor of the Journal of Symbolic Computation (since 2007) and of the SIAM Journal on Applied Algebra and Geometry (since 2016).

Ioannis Emiris is associate editor of the Journal of Symbolic Computation (since 2003) and of Mathematics for Computer Science (since 2017).

Evelyne Hubert is associate editor of the Journal of Symbolic Computation (since 2007) and of Foundations of Computational Mathematics (since 2017).

*8.1.3.2. Reviewer - Reviewing Activities*

Laurent Busé reviewed for the journal *Linear Algebra and its Applications*, the journal *Computer Aided Geometric Design*, the journal of *Advances in Applied Mathematics*, the *Journal of Computational and Applied Mathematics*, the journal *Applicable Algebra in Engineering, Communication and Computing*, the journal *Computer Aided Design*, the *SIAM Journal on Applied Algebra and Geometry*.

Ioannis Emiris reviewed for the *SIAM Journal on Applied Algebra and Geometry*, the *Symposium of Computational Geometry*.

Bernard Mourrain reviewed for the *Journal of Algebra and its Application*, the journal *Computer Methods in Applied Mechanics and Engineering*, the journal *Computer Aided Geometric Design*, the *Journal of Computational and Applied Mathematics*, the *Journal of Symbolic Computation*, the journal *Mathematics of Computation*. He is also guest editor of the Special Issue of the *Journal Of Symbolic Computation* after MEGA 2017 [24].

Evelyne Hubert reviewed for the *Journal of Symbolic Computation*, the journal *Foundations of Comptutational Mathematics*, and the *Journal of Algebra*.

### 8.1.4. Invited Talks

Laurent Busé was an invited speaker at the conference "Applied and Computational Geometry" that took place at Loughborough University, Centre for Geometry and Applications, September 12-14, 2018.

Ioannis Emiris was an invited speaker at JRC Ispra, Italy, February 2018, at JK University (and gave a course), Linz, Austria, April-May 2018, at the "2nd Intern. Workshop on Geometry and Machine Learning" , within Computational Geometry Week, Budapest, Hungary, June 2018, at the "Symposium on Discrete Mathematics", of the German Mathematical Society, Graz, Austria, June 2018, at CHIPSET Training School on Large-Scale Data Mining and Machine Learning for Big Data Analytics (and gave a course), Thessaloniki, 19 September 2018.

Bernard Mourrain was an invited speaker at the Workshop "Structured Matrix Days", Lyon, 14-15 May, at the International conference on Approximation and Matrix Functions, Lille, May 31 - June 1, at the Workshop "Tensors" (and gave a course), Torino, 10-14 September, at the Workshop "A two-day journey in Computational Algebra and Algebraic Geometry" dedicated to Margherita Roggero, Torino, 27-28 Sep. He was invited at Univ. of Texas, Austin, for a collaboration with Pr. Chandajit Bajaj (29 January - 9 February), an invited participant of the semester on Nonlinear Algebra at ICERM, Providence, RI, USA from 1 to 19 Oct.

Evelyne Hubert gave a keynote lecture at the conference *Symmetry & Computation*, CIRM (Marseille, France) and was invited to give talks at the conference *Algebraic and Geometric Aspects of Numerical Methods for Differential Equations* held at the Mittag-Leffler Institute (Stockholm, Sweden), the *Séminaire différentiel*, jointly organized by Université Versailles St Quentin and Inria SIF, and the conference *Nonlinear Algebra in Applications* held at the Institute for Computational and Experimental Mathematics (Providence, USA).

### 8.1.5. Scientific Expertise

Evelyne Hubert was part of the hiring committees for the positions of Directeur de Recherche 2ème classe at Inria and for the position of Chargé de Recherche at Inria NGE. As part of the Commission d'Evaluation, she was also part of the promotion committee of Inria researchers (CRHC, DR1, DR0).

Evelyne Hubert was the external reviewer for the promotion of Wei Li to the rank of associate professor at the Chinese Academy of Science (Beijing).

### 8.1.6. Research Administration

Laurent Busé is a board member of the (national) labex AMIES (CRI-SAM representative) and a member of the steering committee of the MSI, *Maison de la Modélisation, de la Simulation et des Interactions* of the University Côte d'Azur. He is also an elected member of the CPRH (Commission Permanente de Ressources Humaines) of the math laboratory of the university of Nice, and is the Inria Sophia Antipolis centre representative at the "Academic Council" and the "Research Commission" of the University of Nice Sophia Antipolis. He participated to the hiring jury of junior researchers in Inria Sophia Antipolis.

Evelyne Hubert is a member of the Comission d'Evaluation, the national Inria evaluation committee. She is nominated to represent Inria at the Academic Council of the Université Côte d'Azur.

Bernard Mourrain is member of the BCEP (Bureau du Comité des Equipes Projet) of the center Inria- Sophia Antipolis.

## 8.2. Teaching - Supervision - Juries

### 8.2.1. Teaching

- Master : Laurent Busé, Geometric Modeling, 27h ETD, M2, EPU of the University of Nice-Sophia Antipolis.
- Master 2: Bernard Mourrain, Symbolic-Numeric Computation, 6h, Master ACSYON, Limoges.

### *8.2.2. Supervision*

PhD in progress: Evangelos Anagnostopoulos, Geometric algorithms for massive data, LAMBDA Marie Skłodowska-Curie RISE Network, started in September 2016, supervised by Ioannis Emiris.

PhD in progress: Ahmed Blidia, New geometric models for the design and computation of complex shapes. ARCADES Marie Skłodowska-Curie ITN, started in September 2016, supervised by Bernard Mourrain.

PhD in progress: Apostolos Chalkis, Sampling in high-dimensional convex regions, started in June 2018, supervised by Ioannis Emiris.

PhD in progress: Emmanouil Christoforou, Geometric approximation algorithms for clustering, Bioinformatics scholarship, started in January 2018, supervised by Ioannis Emiris.

PhD in progress: Alvaro-Javier Fuentes-Suàrez, Skeleton-based modeling of smooth shapes. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Evelyne Hubert.

PhD: Jouhayna Harmouch, Low rank structured matrix decomposition and applications. Cotutelle Univ. Liban, cosupervised by Houssam Khalil, Mustapha Jazar and Bernard Mourrain. Defended in December.

PhD in progress: Rima Khouja, Tensor decomposition, best approximations, algorithms and applications. Cotutelle Univ. Liban, started in November 2018, cosupervised by Houssam Khalil and Bernard Mourrain.

PhD in progress: Evangelos Bartzos, Algebraic elimination and Distance graphs. ARCADES Marie Skłodowska-Curie ITN, started in June 2016, supervised by Ioannis Emiris.

PhD in progress: Clément Laroche, Algebraic representations of geometric objects. ARCADES Marie Skłodowska-Curie ITN, started in November 2016, supervised by Ioannis Emiris.

PhD in progress: Ioannis Psarros, Dimensionality reduction and Geometric search, Greek scholarship foundation, started in Sep. 2015, supervised by Ioannis Emiris.

PhD in progress: Erick David Rodriguez Bazan, Symmetry preserving algebraic computation. CORDI Inria SAM, started in November 2017, supervised by Evelyne Hubert.

PhD in progress: Fatmanur Yildirim, Distances between points, rational Bézier curves and surfaces by means of matrix-based implicit representations. ARCADES Marie Skłodowska-Curie ITN, started in October 2016, supervised by Laurent Busé.

### *8.2.3. Juries*

L. Busé was a member of the committee of the PhD of Rémi Bignalet-Cazalet entitled *Géométrie de la projectivisation des idéaux et applications aux problèmes de birationalité*, University Bourgogne Franche-Comté, Dijon, France, October 24th.

I. Emiris was a member of two 3-person supervisory committees of PhD students Anuj Sharma and Emmanouil Kamarianakis, who defended their theses in December 2018, at NK University of Athens, and University of Crete, Greece, respectively.

E. Hubert was a member of the PhD committee of Timothé Pecatte from Ecole Normale Supérieure de Lyon, section informatique : *Bornes Inférieures et Algorithmes de Reconstruction pour des Sommes de Puissances Affines*.

## 8.3. Popularization

### *8.3.1. Interventions*

- Ioannis Emiris was an invited speaker at “Open Science Days”, Athens, 29 November 2018, and at “Mathematics Education Forum” within the Greek Mathematical Society annual meeting, Athens, December 2018.

# 9. Bibliography

## Publications of the year

### Articles in International Peer-Reviewed Journals


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