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Project-Team **CAGE**

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

RESEARCH CENTER
Paris

THEME
**Optimization and control of dynamic
systems**

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Project-Team CAGE

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- A6.1.1. - Continuous Modeling (PDE, ODE)
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- A6.4.3. - Observability and Controlability
- A6.4.4. - Stability and Stabilization
- A6.4.5. - Control of distributed parameter systems
- A6.4.6. - Optimal control

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- B1.2. - Neuroscience and cognitive science
- B2.6. - Biological and medical imaging
- B5.11. - Quantum systems
- B7.1.3. - Air traffic

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2. Overall Objectives

2.1. Overall Objectives

CAGE's activities take place in the field of mathematical control theory, with applications in three main directions: geometric models for vision, control of quantum mechanical systems, and control of systems with uncertain dynamics.

The relations between control theory and geometry of vision rely on the notion of sub-Riemannian structure, a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful control theoretical interpretation. We recall that nonholonomicity refers to the property of a velocity constraint that cannot be recast as a state constraint. In the language of differential geometry, a sub-Riemannian structure is a (possibly rank-varying) Lie bracket generating distribution endowed with a smoothly varying norm.

Sub-Riemannian geometry, and in particular the theory of associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the recent literature (including by members of our team). Our contributions to this field are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

Control theory is one of the components of the forthcoming quantum revolution ¹, since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance). The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. Time scales analysis is important for evaluation approaches based on adiabatic approximation theory, which is well-known to improve the robustness of the control strategy. CAGE works for the improvement of evaluation and design tools for efficient quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces.

Simultaneous control of a continuum of systems with slightly different dynamics is a typical problem in quantum mechanics and also a special case of the third applicative axis to which CAGE is contributing: control of systems with uncertain dynamics. The slightly different dynamics can indeed be seen as uncertainties in the system to be controlled, and simultaneous control rephrased in terms of a robustness task. Robustification, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. Our contributions to this research field concern both stabilization (either asymptotic or in finite time) and optimal control, where redundancies and probabilistic tools can be introduced to offset uncertainties.

¹As anticipated by the recent launch of the FET Flagship on Quantum Technologies

3. Research Program

3.1. Research domain

The activities of CAGE are part of the research in the wide area of control theory. This nowadays mature discipline is still the subject of intensive research because of its crucial role in a vast array of applications.

More specifically, our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [156]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [119]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [159], [118], [147]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [143], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [137] and models for neural activity [105]. Therapy analysis from the point of view of optimal control has also attracted a great attention [140].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [153].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [106].

3.2. Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, observability, optimal control... [68], [110]. The emphasis of such a geometric approach to control theory is put on intrinsic properties of the systems and it is particularly well adapted to study nonlinear and nonholonomic phenomena.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [91]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting from 2009 [94] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [152], [139]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [132], [121], those based on the differentiation of nonlinear flows such as the **return method** [99], [98], and those exploiting the **differential flatness** of the system [103].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [66] or shape optimization [74]. Examples of the second type are inactivation principles in human motricity [77] or neurogeometrical models for image representation of the primary visual cortex in mammals [88].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [129], geometric measure theory [70] and hypoelliptic operator theory [80].

The geometric control approach has historically been related to the development of finite-dimensional control theory. However, its impact in the analysis of distributed parameter control systems and in particular systems of controlled partial differential equations has been growing in the last decades, complementing analytical and numerical approaches, providing dynamical, qualitative and intrinsic insight [97]. CAGE's ambition is to be at the core of this development in the years to come.

4. Application Domains

4.1. First axis: Geometry of vision

A suggestive application of sub-Riemannian geometry and in particular of hypoelliptic diffusion comes from a model of geometry of vision describing the functional architecture of the primary visual cortex V1. In 1958, Hubel and Wiesel (Nobel in 1981) observed that the visual cortex V1 is endowed with the so-called **pinwheel structure**, characterized by neurons grouped into orientation columns, that are sensible both to positions and directions [109]. The mathematical rephrasing of this discovery is that the visual cortex lifts an image from \mathbf{R}^2 into the bundle of directions of the plane [95], [136], [138], [108].

A simplified version of the model can be described as follows: neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli at a given point of the retina and for a given direction on it. The retina is modeled by the real plane, i.e., each point is represented by a pair $(x, y) \in \mathbf{R}^2$, while the directions at a given point are modeled by the projective line, i.e. an element θ of the projective line P^1 . Hence, the primary visual cortex V1 is modeled by the so called projective tangent bundle $\text{PTR}^2 = \mathbf{R}^2 \times \mathbf{P}^1$. From a neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them being sensitive to stimuli at a given point (x, y) with any direction.

Orientation columns are connected between them in two different ways. The first kind of connections are the vertical (inhibitory) ones, which connect orientation columns belonging to the same hypercolumn and sensible to similar directions. The second kind of connections are the horizontal (excitatory) connections, which connect neurons belonging to different (but not too far) hypercolumns and sensible to the same directions. The resulting metric structure is sub-Riemannian and the model obtained in this way provides a convincing explanation in terms of sub-Riemannian geodesics of gestalt phenomena such as Kanizsa illusory contours.

The sub-Riemannian model for image representation of V1 has a great potential of yielding powerful bio-inspired image processing algorithms [102], [88]. Image inpainting, for instance, can be implemented by reconstructing an incomplete image by activating orientation columns in the missing regions in accordance with sub-Riemannian non-isotropic constraints. The process intrinsically defines an hypoelliptic heat equation on PTR^2 which can be integrated numerically using non-commutative Fourier analysis on a suitable semidiscretization of the group of roto-translations of the plane [86].

We have been working on the model and its software implementation since 2012. This work has been supported by several project, as the ERC starting grant GeCoMethods and the ERC Proof of Concept ARTIV1 of U. Boscain, and the ANR GCM.

A parallel approach that we will pursue and combine with this first one is based on **pattern matching in the group of diffeomorphisms**. We want to extend this approach, already explored in the Riemannian setting [151], [126], to the general sub-Riemannian framework. The paradigm of the approach is the following: consider a distortable object, more or less rigid, discretized into a certain number of points. One may track its distortion by considering the paths drawn by these points. One would however like to know how the object itself (and not its discretized version) has been distorted. The study in [151], [126] shed light on the importance of Riemannian geometry in this kind of problem. In particular, they study the Riemannian submersion obtained by making the group of diffeomorphisms act transitively on the manifold formed by the points of the discretization, minimizing a certain energy so as to take into account the whole object. Settled as such, the problem is Riemannian, but if one considers objects involving connections, or submitted to nonholonomic constraints, like in medical imaging where one tracks the motions of organs, then one comes up with a sub-Riemannian problem. The transitive group is then far bigger, and the aim is to lift curves submitted to these nonholonomic constraints into curves in the set of diffeomorphisms satisfying the corresponding constraints, in a unique way and minimizing an energy (giving rise to a sub-Riemannian structure).

4.2. Second axis: Quantum control

The goal of quantum control is to design efficient protocols for tuning the occupation probabilities of the energy levels of a system. This task is crucial in atomic and molecular physics, with applications ranging from photochemistry to nuclear magnetic resonance and quantum computing. A quantum system may be controlled by exciting it with one or several external fields, such as magnetic or electric fields. The goal of quantum control theory is to adapt the tools originally developed by control theory and to develop new specific strategies that tackle and exploit the features of quantum dynamics (probabilistic nature of wavefunctions and density operators, measure and wavefunction collapse, decoherence, ...). A rich variety of relevant models for controlled quantum dynamics exist, encompassing low-dimensional models (e.g., single-spin systems) and PDEs alike, with deterministic and stochastic components, making it a rich and exciting area of research in control theory.

The controllability of quantum system is a well-established topic when the state space is finite-dimensional [100], thanks to general controllability methods for left-invariant control systems on compact Lie groups [90], [111]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [154]. Nevertheless, weaker controllability properties, such as approximate controllability or controllability between eigenstates of the internal Hamiltonian (which are the most relevant physical states), may hold. In certain cases, when the state space is a function space on a 1D manifold, some rather precise description of the set of reachable states has been provided [75]. A similar description for higher-dimensional manifolds seems intractable and at the moment only approximate controllability results are available [127], [134], [112]. The most widely applicable tests for controllability of quantum systems in infinite-dimensional Hilbert spaces are based on the **Lie–Galerkin technique** [94], [83], [84]. They allow, in particular, to show that the controllability property is generic among this class of systems [124].

A family of algorithms which are specific to quantum systems are those based on adiabatic evolution [158], [157], [115]. The basic principle of adiabatic control is that the flow of a slowly varying Hamiltonian can be approximated (up to a phase factor) by a quasi-static evolution, with a precision proportional to the velocity of variation of the Hamiltonian. The advantage of the **adiabatic approach** is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty. The paradigm is based on the adiabatic perturbation theory developed in mathematical physics [81], [133], [150], where it plays an important role for understanding molecular dynamics. Approximation theory by adiabatic perturbation can be used to describe the evolution of the occupation probabilities of the energy levels of a slowly varying Hamiltonian. Results from the last 15 years, including those by members of our team [62], [87], have highlighted the effectiveness of control techniques based on adiabatic path following.

4.3. Third axis: Stability and uncertain dynamics

Switched and hybrid systems constitute a broad framework for the description of the heterogeneous aspects of systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [142], energy management [135] and congestion control [125].

Even if both controllability [146] and observability [113] of switched and hybrid systems have attracted much research efforts, the central role in their study is played by the problem of stability and stabilizability. The goal is to determine whether a dynamical or a control system whose evolution is influenced by a time-dependent signal is uniformly stable or can be uniformly stabilized [118], [147]. Uniformity is considered with respect to all signals in a given class. Stability of switched systems lead to several interesting phenomena. For example, even when all the subsystems corresponding to a constant switching law are exponentially stable, the switched systems may have divergent trajectories for certain switching signals [117]. This fact illustrates the fact that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the class of switching signals which is considered.

The most common class of switching signals which has been considered in the literature is made of all piecewise constant signals. In this case uniform stability of the system is equivalent to the existence of a common quadratic Lyapunov function [128]. Moreover, provided that the system has finitely many modes, the Lyapunov function can be taken polyhedral or polynomial [78], [79], [101]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see the surveys [120], [141] and the references therein). It is known, however, that the existence of a common quadratic Lyapunov function is not necessary for the global uniform exponential stability of a linear switched system with finitely many modes. Moreover, there exists no uniform upper bound on the minimal degree of a common polynomial Lyapunov function [123]. More refined tools rely on multiple and non-monotone Lyapunov functions [89]. Let us also mention linear switched systems techniques based on the analysis of the Lie algebra generated by the matrices corresponding to the modes of the system [65].

For systems evolving in the plane, more geometrical tests apply, and yield a complete characterization of the stability [82], [71]. Such a geometric approach also yields sufficient conditions for uniform stability in the linear planar case [85].

In many situations, it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Switching rules can also be imposed, for instance, by a timed automata. When constraints apply, the common Lyapunov function approach becomes conservative and new tools have to be developed to give more detailed characterizations of stable and unstable systems.

Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. One then looks at the corresponding Lyapunov exponents, whose existence is established by the multiplicative ergodic theorem. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [76], [96], [155].

4.4. Joint theoretical core

The theoretical questions raised by the different applicative area will be pooled in a research axis on the transversal aspects of geometric control theory and sub-Riemannian structures.

We recall that sub-Riemannian geometry is a generalization of Riemannian geometry, whose birth dates back to Carathéodory's seminal paper on the foundations of Carnot thermodynamics [92], followed by E. Cartan's address at the International Congress of Mathematicians in Bologna [93]. In the last twenty years,

sub-Riemannian geometry has emerged as an independent research domain, with a variety of motivations and ramifications in several parts of pure and applied mathematics. Let us mention geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics and optimal control (motion planning, robotics, nonholonomic mechanics, quantum control) [60], [61].

One of the main open problems in sub-Riemannian geometry concerns the regularity of length-minimizers [63], [130]. Length-minimizers are solutions to a variational problem with constraints and satisfy a first-order necessary condition resulting from the Pontryagin Maximum Principle (PMP). Solutions of the PMP are either *normal* or *abnormal*. Normal length-minimizers are well-known to be smooth, i.e., C^∞ , as it follows by the Hamiltonian nature of the PMP. The question of regularity is then reduced to abnormal length-minimizers. If the sub-Riemannian structure has step 2, then abnormal length-minimizers can be excluded and thus every length-minimizer is smooth. For step 3 structures, the situation is already more complicated and smoothness of length-minimizers is known only for Carnot groups [114], [149]. The question of regularity of length-minimizers is not restricted to the smoothness in the C^∞ sense. A recent result proves that length-minimizers, for sub-Riemannian structures of any step, cannot have corner-like singularities [107]. When the sub-Riemannian structure is analytic, more is known on the size of the set of points where a length-minimizer can lose analyticity [148], regardless of the rank and of the step of the distribution.

An interesting set of recent results in sub-Riemannian geometry concerns the extension to such a setting of the Riemannian notion of sectional curvature. The curvature operator can be introduced in terms of the symplectic invariants of the Jacobi curve [67], [116], [64], a curve in the Lagrange Grassmannian related to the linearization of the Hamiltonian flow. Alternative approaches to curvatures in metric spaces are based either on the associated heat equation and the generalization of the curvature-dimension inequality [72], [73] or on optimal transport and the generalization of Ricci curvature [145], [144], [122], [69].

5. Highlights of the Year

5.1. Highlights of the Year

Emmanuel Trélat has been invited speaker at the International Congress of Mathematicians (ICM2018) in Rio, Brazil, in the session “Control theory and optimization”.

5.1.1. Awards

- The poster “Adaptive Stimulation Strategy for Selective Brain Oscillations Disruption in a Neuronal Population Model with Delays” by **Jakub Orłowski**, Antoine Chaillet, **Mario Sigalotti**, and Alain Destexhe, has received the CPHS 2018 Best Poster Prize at the 2nd IFAC Conference on Cyber-Physical & Human Systems.

6. New Results

6.1. Geometry of vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In [7] we present a new image inpainting algorithm, the Averaging and Hypoelliptic Evolution (AHE) algorithm, inspired by the one presented in [86] and based upon a (semi-discrete) variation of the Citti–Petitot–Sarti model of the primary visual cortex V1. In particular, we focus on reconstructing highly corrupted images (i.e. where more than the 80% of the image is missing).
- In [6] we deal with a severe ill posed problem, namely the reconstruction process of an image during tomography acquisition with (very) few views. We present different methods that we have been investigated during the past decade. They are based on variational analysis.

- [13] is the first paper of a series in which we plan to study spectral asymptotics for sub-Riemannian Laplacians and to extend results that are classical in the Riemannian case concerning Weyl measures, quantum limits, quantum ergodicity, quasi-modes, trace formulae. Even if hypoelliptic operators have been well studied from the point of view of PDEs, global geometrical and dynamical aspects have not been the subject of much attention. As we will see, already in the simplest case, the statements of the results in the sub-Riemannian setting are quite different from those in the Riemannian one. Let us consider a sub-Riemannian (sR) metric on a closed three-dimensional manifold with an oriented contact distribution. There exists a privileged choice of the contact form, with an associated Reeb vector field and a canonical volume form that coincides with the Popp measure. We establish a Quantum Ergodicity (QE) theorem for the eigenfunctions of any associated sR Laplacian under the assumption that the Reeb flow is ergodic. The limit measure is given by the normalized Popp measure. This is the first time that such a result is established for a hypoelliptic operator, whereas the usual Shnirelman theorem yields QE for the Laplace-Beltrami operator on a closed Riemannian manifold with ergodic geodesic flow. To prove our theorem, we first establish a microlocal Weyl law, which allows us to identify the limit measure and to prove the microlocal concentration of the eigenfunctions on the characteristic manifold of the sR Laplacian. Then, we derive a Birkhoff normal form along this characteristic manifold, thus showing that, in some sense, all 3D contact structures are microlocally equivalent. The quantum version of this normal form provides a useful microlocal factorization of the sR Laplacian. Using the normal form, the factorization and the ergodicity assumption, we finally establish a variance estimate, from which QE follows. We also obtain a second result, which is valid without any ergodicity assumption: every Quantum Limit (QL) can be decomposed in a sum of two mutually singular measures: the first measure is supported on the unit cotangent bundle and is invariant under the sR geodesic flow, and the second measure is supported on the characteristic manifold of the sR Laplacian and is invariant under the lift of the Reeb flow. Moreover, we prove that the first measure is zero for most QLs.
- In [22] we study the validity of the Whitney C^1 extension property for horizontal curves in sub-Riemannian manifolds endowed with 1-jets that satisfy a first-order Taylor expansion compatibility condition. We first consider the equiregular case, where we show that the extension property holds true whenever a suitable non-singularity property holds for the input-output maps on the Carnot groups obtained by nilpotent approximation. We then discuss the case of sub-Riemannian manifolds with singular points and we show that all step-2 manifolds satisfy the C^1 extension property. We conclude by showing that the C^1 extension property implies a Lusin-like approximation theorem for horizontal curves on sub-Riemannian manifolds.
- In [34] we prove the C^1 regularity for a class of abnormal length-minimizers in rank 2 sub-Riemannian structures. As a consequence of our result, all length-minimizers for rank 2 sub-Riemannian structures of step up to 4 are of class C^1 .
- In [45] we address the double bubble problem for the anisotropic Grushin perimeter P_α , $\alpha \geq 0$, and the Lebesgue measure in \mathbb{R}^2 , in the case of two equal volumes. We assume that the contact interface between the bubbles lays on either the vertical or the horizontal axis. Since no regularity theory is available in this setting, in both cases we first prove existence of minimizers via the direct method by symmetrization arguments and then characterize them in terms of the given area by first variation techniques. Angles at which minimal boundaries intersect satisfy the standard 120-degree rule up to a suitable change of coordinates. While for $\alpha = 0$ the Grushin perimeter reduces to the Euclidean one and both minimizers coincide with the symmetric double bubble found in [104], for $\alpha = 1$ vertical interface minimizers have Grushin perimeter strictly greater than horizontal interface minimizers. As the latter ones are obtained by translating and dilating the Grushin isoperimetric set found in [131], we conjecture that they solve the double bubble problem with no assumptions on the contact interface.
- In [51] we study the notion of geodesic curvature of smooth horizontal curves parametrized by arc-length in the Heisenberg group, that is the simplest sub-Riemannian structure. Our goal is to give a metric interpretation of this notion of geodesic curvature as the first corrective term in the Taylor

expansion of the distance between two close points of the curve.

We would also like to mention the defense of the PhD thesis of Ludovic Sacchelli [3] on the subject.

6.2. Quantum control: new results

Let us list here our new results in quantum control theory.

- In [5] we consider a quantum particle in a potential $V(x)$ ($x \in \mathbb{R}^N$) in a time-dependent electric field $E(t)$ (the control). Boscain, Caponigro, Chambrion and Sigalotti proved in [83] that, under generic assumptions on V , this system is approximately controllable on the $L^2(\mathbb{R}^N, \mathbb{C})$ -sphere, in sufficiently large time T . In the present article we show that approximate controllability does not hold in arbitrarily small time, no matter what the initial state is. This generalizes our previous result for Gaussian initial conditions. Moreover, we prove that the minimal time can in fact be arbitrarily large.
- In [11] we consider the bilinear Schrödinger equation with discrete-spectrum drift. We show, for $n \in \mathbb{N}$ arbitrary, exact controllability in projections on the first n given eigenstates. The controllability result relies on a generic controllability hypothesis on some associated finite-dimensional approximations. The method is based on Lie-algebraic control techniques applied to the finite-dimensional approximations coupled with classical topological arguments issuing from degree theory.
- In [14] we consider the one dimensional Schrödinger equation with a bilinear control and prove the rapid stabilization of the linearized equation around the ground state. The feedback law ensuring the rapid stabilization is obtained using a transformation mapping the solution to the linearized equation on the solution to an exponentially stable target linear equation. A suitable condition is imposed on the transformation in order to cancel the non-local terms arising in the kernel system. This conditions also insures the uniqueness of the transformation. The continuity and invertibility of the transformation follows from exact controllability of the linearized system.
- In [33] we discuss how to control a parameter-dependent family of quantum systems. Our technique is based on adiabatic approximation theory and on the presence of curves of conical eigenvalue intersections of the controlled Hamiltonian. As particular cases, we recover chirped pulses for two-level quantum systems and counter-intuitive solutions for three-level stimulated Raman adiabatic passage (STIRAP). The proposed technique works for systems evolving both in finite-dimensional and infinite-dimensional Hilbert spaces. We show that the assumptions guaranteeing ensemble controllability are structurally stable with respect to perturbations of the parametrized family of systems.

6.3. Stability and uncertain dynamics: new results

Let us list here our new results about stability and stabilization of control systems, on the properties of systems with uncertain dynamics.

- In [8] we consider a one-dimensional controlled reaction-diffusion equation, where the control acts on the boundary and is subject to a constant delay. Such a model is a paradigm for more general parabolic systems coupled with a transport equation. We prove that it is possible to stabilize (in H^1 norm) this process by means of an explicit predictor-based feedback control that is designed from a finite-dimensional subsystem. The implementation is very simple and efficient and is based on standard tools of pole-shifting. Our feedback acts on the system as a finite-dimensional predictor. We compare our approach with the backstepping method.
- In [14] we consider the one dimensional Schrödinger equation with a bilinear control and prove the rapid stabilization of the linearized equation around the ground state. The feedback law ensuring the rapid stabilization is obtained using a transformation mapping the solution of the linearized equation to the solution of an exponentially stable target linear equation. A suitable condition is imposed on the transformation in order to cancel the non-local terms arising in the kernel system. This conditions also insures the uniqueness of the transformation. The continuity and invertibility of the transformation follows from exact controllability of the linearized system.

- Based on the notion of generalized homogeneity, we develop in [17] a new algorithm of feedback control design for a plant modeled by a linear evolution equation in a Hilbert space with a possibly unbounded operator. The designed control law steers any solution of the closed-loop system to zero in a finite time. Method of homogeneous extension is presented in order to make the developed control design principles to be applicable for evolution systems with non-homogeneous operators. The design scheme is demonstrated for heat equation with the control input distributed on the segment $[0, 1]$.
- In [19] we analyse the asymptotic behaviour of integro-differential equations modeling N populations in interaction, all structured by different traits. Interactions are modeled by non-local terms involving linear combinations of the total number of individuals in each population. These models have already been shown to be suitable for the modeling of drug resistance in cancer, and they generalise the usual Lotka–Volterra ordinary differential equations. Our aim is to give conditions under which there is persistence of all species. Through the analysis of a Lyapunov function, our first main result gives a simple and general condition on the matrix of interactions, together with a convergence rate. The second main result establishes another type of condition in the specific case of mutualistic interactions. When either of these conditions is met, we describe which traits are asymptotically selected.
- The goal of [20] is to compute a boundary control of reaction-diffusion partial differential equation. The boundary control is subject to a constant delay, whereas the equation may be unstable without any control. For this system equivalent to a parabolic equation coupled with a transport equation, a prediction-based control is explicitly computed. To do that we decompose the infinite-dimensional system into two parts: one finite-dimensional unstable part, and one stable infinite-dimensional part. A finite-dimensional delay controller is computed for the unstable part, and it is shown that this controller succeeds in stabilizing the whole partial differential equation. The proof is based on an explicit form of the classical Artstein transformation, and an appropriate Lyapunov function. A numerical simulation illustrate the constructive design method.
- [27] focuses on the (local) small-time stabilization of a Korteweg-de Vries equation on bounded interval, thanks to a time-varying Dirichlet feedback law on the left boundary. Recently, backstepping approach has been successfully used to prove the null controllability of the corresponding linearized system, instead of Carleman inequalities. We use the “adding an integrator” technique to gain regularity on boundary control term which clears the difficulty from getting stabilization in small-time.
- Motivated by improved ways to disrupt brain oscillations linked to Parkinson’s disease, we propose in [29] an adaptive output feedback strategy for the stabilization of nonlinear time-delay systems evolving on a bounded set. To that aim, using the formalism of input-to-output stability (IOS), we first show that, for such systems, internal stability guarantees robustness to exogenous disturbances. We then use this feature to establish a general result on scalar adaptive output feedback of time-delay systems inspired by the “ σ -modification” strategy. We finally apply this result to a delayed neuronal population model and assess numerically the performance of the adaptive stimulation.
- In [35] we consider open channels represented by Saint-Venant equations that are monitored and controlled at the downstream boundary and subject to unmeasured flow disturbances at the upstream boundary. We address the issue of feedback stabilization and disturbance rejection under Proportional-Integral (PI) boundary control. For channels with uniform steady states, the analysis has been carried out previously in the literature with spectral methods as well as with Lyapunov functions in Riemann coordinates. In [35], our main contribution is to show how the analysis can be extended to channels with non-uniform steady states with a Lyapunov function in physical coordinates.
- In [37], we study the exponential stabilization of a shock steady state for the inviscid Burgers equation on a bounded interval. Our analysis relies on the construction of an explicit strict control Lyapunov function. We prove that by appropriately choosing the feedback boundary conditions, we can stabilize the state as well as the shock location to the desired steady state in H^2 -norm, with an arbitrary decay rate.

- Given a discrete-time linear switched system $\Sigma(A)$ associated with a finite set A of matrices, we consider in [40] the measures of its asymptotic behavior given by, on the one hand, its deterministic joint spectral radius $\rho_d(A)$ and, on the other hand, its probabilistic joint spectral radii $\rho_p(v, P, A)$ for Markov random switching signals with transition matrix P and a corresponding invariant probability v . Note that $\rho_d(A)$ is larger than or equal to $\rho_p(v, P, A)$ for every pair (v, P) . In this paper, we investigate the cases of equality of $\rho_d(A)$ with either a single $\rho_p(v, P, A)$ or with the supremum of $\rho_p(v, P, A)$ over (v, P) and we aim at characterizing the sets A for which such equalities may occur.
- In [41], we introduce a method to get necessary and sufficient stability conditions for systems governed by 1-D nonlinear hyperbolic partial-differential equations with closed-loop integral controllers, when the linear frequency analysis cannot be used anymore. We study the stability of a general nonlinear transport equation where the control input and the measured output are both located on the boundaries. The principle of the method is to extract the limiting part of the stability from the solution using a projector on a finite-dimensional space and then use a Lyapunov approach. We improve a result of Trinh, Andrieu and Xu, and give an optimal condition for the design of the controller. The results are illustrated with numerical simulations where the predicted stable and unstable regions can be clearly identified.
- In [44] we construct explicit time-varying feedback laws leading to the global (null) stabilization in small time of the viscous Burgers equation with three scalar controls. Our feedback laws use first the quadratic transport term to achieve the small-time global approximate stabilization and then the linear viscous term to get the small-time local stabilization.
- In [46] we address the question of the exponential stability for the C^1 norm of general 1-D quasilinear systems with source terms under boundary conditions. To reach this aim, we introduce the notion of basic C^1 Lyapunov functions, a generic kind of exponentially decreasing function whose existence ensures the exponential stability of the system for the C^1 norm. We show that the existence of a basic C^1 Lyapunov function is subject to two conditions: an interior condition, intrinsic to the system, and a condition on the boundary controls. We give explicit sufficient interior and boundary conditions such that the system is exponentially stable for the C^1 norm and we show that the interior condition is also necessary to the existence of a basic C^1 Lyapunov function. Finally, we show that the results conducted in this article are also true under the same conditions for the exponential stability in the C^p norm, for any $p \geq 1$.
- In [47] we study the exponential stability for the C^1 norm of general 2×2 1-D quasilinear hyperbolic systems with source terms and boundary controls. When the eigenvalues of the system have the same sign, any nonuniform steady-state can be stabilized using boundary feedbacks that only depend on measurements at the boundaries and we give explicit conditions on the gain of the feedback. In other cases, we exhibit a simple numerical criterion for the existence of basic C^1 Lyapunov function, a natural candidate for a Lyapunov function to ensure exponential stability for the C^1 norm. We show that, under a simple condition on the source term, the existence of a basic C^1 (or C^p , for any $p \geq 1$) Lyapunov function is equivalent to the existence of a basic H^2 (or H^q , for any $q \geq 2$) Lyapunov function, its analogue for the H^2 norm. Finally, we apply these results to the nonlinear Saint-Venant equations. We show in particular that in the subcritical regime, when the slope is larger than the friction, the system can always be stabilized in the C^1 norm using static boundary feedbacks depending only on measurements of at the boundaries, which has a large practical interest in hydraulic and engineering applications.
- In [48] we study the exponential stability in the H^2 norm of the nonlinear Saint-Venant (or shallow water) equations with arbitrary friction and slope using a single Proportional-Integral (PI) control at one end of the channel. Using a local dissipative entropy we find a simple and explicit condition on the gain the PI control to ensure the exponential stability of any steady-states. This condition is independent of the slope, the friction, the length of the river, the inflow disturbance and, more surprisingly, the steady-state considered. When the inflow disturbance is time-dependent and no steady-state exist, we still have the Input-to-State stability of the system, and we show that changing slightly the PI control enables to recover the exponential stability of slowly varying trajectories.

- The exponential stability problem of the nonlinear Saint-Venant equations is addressed in [49]. We consider the general case where an arbitrary friction and space-varying slope are both included in the system, which lead to non-uniform steady-states. An explicit quadratic Lyapunov function as a weighted function of a small perturbation of the steady-states is constructed. Then we show that by a suitable choice of boundary feedback controls, that we give explicitly, the local exponential stability of the nonlinear Saint-Venant equations for the H^2 -norm is guaranteed.
- [53] elaborates control strategies to prevent clustering effects in opinion formation models. This is the exact opposite of numerous situations encountered in the literature where, on the contrary, one seeks controls promoting consensus. In order to promote declustering, instead of using the classical variance that does not capture well the phenomenon of dispersion, we introduce an entropy-type functional that is adapted to measuring pairwise distances between agents. We then focus on a Hegselmann-Krause-type system and design declustering sparse controls both in finite-dimensional and kinetic models. We provide general conditions characterizing whether clustering can be avoided as function of the initial data. Such results include the description of black holes (where complete collapse to consensus is not avoidable), safety zones (where the control can keep the system far from clustering), basins of attraction (attractive zones around the clustering set) and collapse prevention (when convergence to the clustering set can be avoided).
- In [54] we consider the problem of controlling parabolic semilinear equations arising in population dynamics, either in finite time or infinite time. These are the monostable and bistable equations on $(0, L)$ for a density of individuals $0 \leq y(t, x) \leq 1$, with Dirichlet controls taking their values in $[0, 1]$. We prove that the system can never be steered to extinction (steady state 0) or invasion (steady state 1) in finite time, but is asymptotically controllable to 1 independently of the size L , and to 0 if the length L of the interval domain is less than some threshold value L^* , which can be computed from transcendental integrals. In the bistable case, controlling to the other homogeneous steady state $0 < \theta < 1$ is much more intricate. We rely on a staircase control strategy to prove that θ can be reached in finite time if and only if $L < L^2$. The phase plane analysis of those equations is instrumental in the whole process. It allows us to read obstacles to controllability, compute the threshold value for domain size as well as design the path of steady states for the control strategy.
- Given a linear control system in a Hilbert space with a bounded control operator, we establish in [56] a characterization of exponential stabilizability in terms of an observability inequality. Such dual characterizations are well known for exact (null) controllability. Our approach exploits classical Fenchel duality arguments and, in turn, leads to characterizations in terms of observability inequalities of approximately null controllability and of α -null controllability. We comment on the relationships between those various concepts, at the light of the observability inequalities that characterize them.
- In [58] we use the backstepping method to study the stabilization of a 1-D linear transport equation on the interval $(0, L)$, by controlling the scalar amplitude of a piecewise regular function of the space variable in the source term. We prove that if the system is controllable in a periodic Sobolev space of order greater than 1, then the system can be stabilized exponentially in that space and, for any given decay rate, we give an explicit feedback law that achieves that decay rate.

Let us also mention the lecture notes [31] on stabilization of semilinear PDE's, which have been published this year.

6.4. Optimal control: new results

Let us list here our new results in optimal control theory beyond the sub-Riemannian framework.

- In [4] we focus on regional deterministic optimal control problems, i.e., problems where the dynamics and the cost functional may be different in several regions of the state space and present discontinuities at their interface. Under the assumption that optimal trajectories have a locally finite number of switchings (no Zeno phenomenon), we use the duplication technique to show that the value function of the regional optimal control problem is the minimum over all possible structures

of trajectories of value functions associated with classical optimal control problems settled over fixed structures, each of them being the restriction to some submanifold of the value function of a classical optimal control problem in higher dimension. The lifting duplication technique is thus seen as a kind of desingularization of the value function of the regional optimal control problem. In turn, we extend to regional optimal control problems the classical sensitivity relations and we prove that the regularity of this value function is the same (i.e., is not more degenerate) than the one of the higher-dimensional classical optimal control problem that lifts the problem.

- The goal of [9] is to show how non-parametric statistics can be used to solve some chance constrained optimization and optimal control problems. We use the Kernel Density Estimation method to approximate the probability density function of a random variable with unknown distribution, from a relatively small sample. We then show how this technique can be applied and implemented for a class of problems including the Goddard problem and the trajectory optimization of an Ariane 5-like launcher.
- In control theory the term chattering is used to refer to fast oscillations of controls, such as an infinite number of switchings over a finite time interval. In [10] we focus on three typical instances of chattering: the Fuller phenomenon, referring to situations where an optimal control features an accumulation of switchings in finite time; the Robbins phenomenon, concerning optimal control problems with state constraints, where the optimal trajectory touches the boundary of the constraint set an infinite number of times over a finite time interval; and the Zeno phenomenon, for hybrid systems, referring to a trajectory that depicts an infinite number of location switchings in finite time. From the practical point of view, when trying to compute an optimal trajectory, for instance, by means of a shooting method, chattering may be a serious obstacle to convergence. In [10] we propose a general regularization procedure, by adding an appropriate penalization of the total variation. This produces a family of quasi-optimal controls whose associated cost converge to the optimal cost of the initial problem as the penalization tends to zero. Under additional assumptions, we also quantify quasi-optimality by determining a speed of convergence of the costs.
- In [12], a new robust and fast method is developed to perform transfers that minimize fuel consumption between two invariant manifolds of periodic orbits in the circular restricted three-body problem. The method starts with an impulse transfer between two invariant manifolds to build an optimal control problem. This allows to choose an adequate fixed transfer time. Using the Pontryagin maximum principle, the resolution of the problem is formulated as that of finding the zero of a shooting function (indirect method). The algorithm couples different kinds of continuations (on cost, final state, and thrust) to improve robustness and to initialize the solver. The efficiency of the method is illustrated with numerical examples. Finally, the influence of the transfer time is studied numerically thanks to a continuation on this parameter, and it checks that, when transfer duration goes to zero, the control converges to the impulse transfer that it started with. It shows the robustness of the method and establishes a mathematical link between the two problems.
- In [15] we consider the controllability problem for finite-dimensional linear autonomous control systems, under state constraints but without imposing any control constraint. It is well known that, under the classical Kalman condition, in the absence of constraints on the state and the control, one can drive the system from any initial state to any final one in an arbitrarily small time. Furthermore, it is also well known that there is a positive minimal time in the presence of compact control constraints. We prove that, surprisingly, a positive minimal time may be required as well under state constraints, even if one does not impose any restriction on the control. This may even occur when the state constraints are unilateral, like the nonnegativity of some components of the state, for instance. Using the Brunovsky normal forms of controllable systems, we analyze this phenomenon in detail, that we illustrate by several examples. We discuss some extensions to nonlinear control systems and formulate some challenging open problems.
- In [18] we consider a system of two coupled integro-differential equations modeling populations of healthy and cancer cells under therapy. Both populations are structured by a phenotypic variable, representing their level of resistance to the treatment. We analyse the asymptotic behaviour of

the model under constant infusion of drugs. By designing an appropriate Lyapunov function, we prove that both densities converge to Dirac masses. We then define an optimal control problem, by considering all possible infusion protocols and minimising the number of cancer cells over a prescribed time frame. We provide a quasi-optimal strategy and prove that it solves this problem for large final times. For this modeling framework, we illustrate our results with numerical simulations, and compare our optimal strategy with periodic treatment schedules.

- In [21] we use conductance based neuron models and the mathematical modeling of Optogenetics to define controlled neuron models and we address the minimal time control of these affine systems for the first spike from equilibrium. We apply tools of geometric optimal control theory to study singular extremals and we implement a direct method to compute optimal controls. When the system is too large to theoretically investigate the existence of singular optimal controls, we observe numerically the optimal bang-bang controls.
- In [23] we first derive a general integral-turnpike property around a set for infinite-dimensional non-autonomous optimal control problems with any possible terminal state constraints, under some appropriate assumptions. Roughly speaking, the integral-turnpike property means that the time average of the distance from any optimal trajectory to the turnpike set converges to zero, as the time horizon tends to infinity. Then, we establish the measure-turnpike property for strictly dissipative optimal control systems, with state and control constraints. The measure-turnpike property, which is slightly stronger than the integral-turnpike property, means that any optimal (state and control) solution remains essentially, along the time frame, close to an optimal solution of an associated static optimal control problem, except along a subset of times that is of small relative Lebesgue measure as the time horizon is large. Next, we prove that strict strong duality, which is a classical notion in optimization, implies strict dissipativity, and measure-turnpike. Finally, we conclude the paper with several comments and open problems.
- In [24], we investigate the asymptotic behavior of optimal designs for the shape optimization of 2D heat equations in long time horizons. The control is the shape of the domain on which heat diffuses. The class of 2D admissible shapes is the one introduced by Sverák, of all open subsets of a given bounded open set, whose complementary sets have a uniformly bounded number of connected components. Using a Γ -convergence approach, we establish that the parabolic optimal designs converge as the length of the time horizon tends to infinity, in the complementary Hausdorff topology, to an optimal design for the corresponding stationary elliptic equation.
- In [25], we study the steady-state (or periodic) exponential turnpike property of optimal control problems in Hilbert spaces. The turnpike property, which is essentially due to the hyperbolic feature of the Hamiltonian system resulting from the Pontryagin maximum principle, reflects the fact that, in large time, the optimal state, control and adjoint vector remain most of the time close to an optimal steady-state. A similar statement holds true as well when replacing an optimal steady-state by an optimal periodic trajectory. To establish the result, we design an appropriate dichotomy transformation, based on solutions of the algebraic Riccati and Lyapunov equations. We illustrate our results with examples including linear heat and wave equations with periodic tracking terms.
- The Allee threshold of an ecological system distinguishes the sign of population growth either towards extinction or to carrying capacity. In practice human interventions can tune the Allee threshold for instance thanks to the sterile male technique and the mating disruption. In [26] we address various control objectives for a system described by a diffusion-reaction equation regulating the Allee threshold, viewed as a real parameter determining the unstable equilibrium of the bistable nonlinear reaction term. We prove that this system is the mean field limit of an interacting system of particles in which individual behaviours are driven by stochastic laws. Numerical simulations of the stochastic process show that population propagations are governed by wave-like solutions corresponding to traveling solutions of the macroscopic reaction-diffusion system. An optimal control problem for the macroscopic model is then introduced with the objective of steering the system to a target traveling wave. The relevance of this problem is motivated by the fact that traveling wave solutions model the fact that bounded space domains reach asymptotically an equilibrium

configuration. Using well known analytical results and stability properties of traveling waves, we show that well-chosen piecewise constant controls allow to reach the target approximately in sufficiently long time. We then develop a direct computational method and show its efficiency for computing such controls in various numerical simulations. Finally we show the efficiency of the obtained macroscopic optimal controls in the microscopic system of interacting particles and we discuss their advantage when addressing situations that are out of reach for the analytical methods. We conclude the article with some open problems and directions for future research.

- Consider a general nonlinear optimal control problem in finite dimension, with constant state and/or control delays. By the Pontryagin Maximum Principle, any optimal trajectory is the projection of a Pontryagin extremal. In [39] we establish that, under appropriate assumptions, Pontryagin extremals depend continuously on the parameter delays, for adequate topologies. The proof of the continuity of the trajectory and of the control is quite easy, however, for the adjoint vector, the proof requires a much finer analysis. The continuity property of the adjoint with respect to the parameter delay opens a new perspective for the numerical implementation of indirect methods, such as the shooting method. We also discuss the sharpness of our assumptions.
- In [43] we are concerned about the controllability of a general linear hyperbolic system of the form $\partial_t w(t, x) = \Sigma(x)\partial_x w(t, x) + \gamma C(x)w(t, x)$ ($\gamma \in \mathbb{R}$) in one space dimension using boundary controls on one side. More precisely, we establish the optimal time for the null and exact controllability of the hyperbolic system for generic γ . We also present examples which yield that the generic requirement is necessary. In the case of constant Σ and of two positive directions, we prove that the null-controllability is attained for any time greater than the optimal time for all $\gamma \in \mathbb{R}$ and for all C which is analytic if the slowest negative direction can be alerted by both positive directions. We also show that the null-controllability is attained at the optimal time by a feedback law when $C \equiv 0$. Our approach is based on the backstepping method paying a special attention on the construction of the kernel and the selection of controls.
- In [52] we consider a state-constrained optimal control problem of a system of two non-local partial-differential equations, which is an extension of the one introduced in a previous work in mathematical oncology. The aim is to minimize the tumor size through chemotherapy while avoiding the emergence of resistance to the drugs. The numerical approach to solve the problem was the combination of direct methods and continuation on discretization parameters, which happen to be insufficient for the more complicated model, where diffusion is added to account for mutations. In [52], we propose an approach relying on changing the problem so that it can theoretically be solved thanks to a Pontryagin Maximum Principle in infinite dimension. This provides an excellent starting point for a much more reliable and efficient algorithm combining direct methods and continuations. The global idea is new and can be thought of as an alternative to other numerical optimal control techniques.

We would also like to mention the defense of the PhD theses of Riccardo Bonalli [1] and Antoine Olivier [2] on the subject.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

A bilateral contract with CNES funded the PhD thesis of Antoine Olivier, who defended in October 2018.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

- ANR SRGI, for *Sub-Riemannian Geometry and Interactions*, coordinated by **Emmanuel Trélat**, started in 2015 and runs until 2020. Other partners: Toulon University and Grenoble University. SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.
- ANR Finite4SoS, for *Commande et estimation en temps fini pour les Systèmes de Systèmes*, coordinated by Wilfrid Perruquetti, started in 2015 and runs until 2019. Other partners: Inria Lille, CAOR - ARMINES. Finite4SoS aims at developing a new promising framework to address control and estimation issues of Systems of Systems subject to model diversity, while achieving robustness as well as severe time response constraints.
- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and runs until 2021. Other partners: Lorraine University. QUACO aims at contributing to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improve the efficiency of control algorithms for MRI.

8.2. European Initiatives

8.2.1. H2020 Projects

Program: ERC Proof of Concept

Project acronym: ARTIV1

Project title: An artificial visual cortex for image processing

Duration: From April 2017 to September 2018.

Coordinator: Ugo Boscain

Abstract: The ERC starting grant GECOMETHODS, on which this POC is based, tackled problems of diffusion equations via geometric control methods. One of the most striking achievements of the project has been the development of an algorithm of image reconstruction based mainly on non-isotropic diffusion. This algorithm is bio-mimetic in the sense that it replicates the way in which the primary visual cortex V1 of mammals processes the signals arriving from the eyes. It has performances that are at the state of the art in image processing. These results together with others obtained in the ERC project show that image processing algorithms based on the functional architecture of V1 can go very far. However, the exceptional performances of the primary visual cortex V1 rely not only on the particular algorithm used, but also on the fact that such algorithm 'runs' on a dedicated hardware having the following features: 1. an exceptional level of parallelism; 2. connections that are well adapted to transmit information in a non-isotropic way as it is required by the algorithms of image reconstruction and recognition. The idea of this POC is to create a dedicated hardware (called ARTIV1) emulating the functional architecture of V1 and hence having on one hand a huge degree of parallelism and on the other hand connections among the CPUs that reflect the non-isotropic structure of the visual cortex V1.

8.3. International Research Visitors

8.3.1. Research Stays Abroad

Jean-Michel Coron was at EPFL (Switzerland) from January to June 2018.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events Organisation

9.1.1.1. Member of the Organizing Committees

Ugo Boscain and Mario Sigalotti were Members of the Organizing Committee of the Workshop "Sub-Riemannian Geometry and Topolò(gy)", Topolò/Topolove, Italy, June 2018

9.1.2. Scientific Events Selection

9.1.2.1. Member of the Conference Program Committees

- Emmanuel Trélat was Member of the Program Committee of the 18th French-German-Italian Conference on Optimization (FGI'2018).
- Emmanuel Trélat was Member of the Scientific Committee of the 23rd International Symposium on Mathematical Programming (ISMP 2018).

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

- Ugo Boscain is Associate editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Member of the editorial board of Journal of Evolution Equations
- Jean-Michel Coron is Member of the editorial board of Asymptotic Analysis
- Jean-Michel Coron is Member of the editorial board of ESAIM : Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Member of the editorial board of Applied Mathematics Research Express
- Jean-Michel Coron is Member of the editorial board of Advances in Differential Equations
- Jean-Michel Coron is Member of the editorial board of Math. Control Signals Systems
- Jean-Michel Coron is Member of the editorial board of Annales de l'IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of ESAIM : Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM : Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of Syst. Cont. Letters
- Emmanuel Trélat is Associate editor of J. Dynam. Cont. Syst.
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM Math. Modelling Num. Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of J. Optim. Theory Appl.
- Emmanuel Trélat is Associate editor of Math. Control Related fields

9.1.4. Invited Talks

- Ugo Boscain was invited speaker at the International Conference "Optimal Control and Differential Games", dedicated to the 110th anniversary of L.S. Pontryagin, Dec. 2018.
- Ugo Boscain was invited speaker at the conference "Dynamics, Control, and Geometry", Banach Center, Warsaw, Sept. 2018.
- Ugo Boscain was invited speaker at Linköping University, Department of Electrical Engineering, Nov. 2018.
- Ugo Boscain was invited speaker at the conference "Analysis, Control and Inverse Problems for PDEs", Napoli (Italy), Nov. 2018.
- Mario Sigalotti was invited speaker at the Workshop Quantum control and feedback: foundations and applications, Paris, Jun. 2018.
- Emmanuel Trélat was invited speaker at ICM 2018, Rio, section "Control Theory and Optimization", Aug. 2018.

- Emmanuel Trélat was invited speaker at Analysis, Control and Inverse Problems for PDEs, Naples, Nov. 2018.
- Emmanuel Trélat was invited speaker at Dynamics Control and Geometry, Varsovie, Sept. 2018.
- Emmanuel Trélat was invited speaker at 14th Viennese Conference on Optimal Control and Dynamic Games, Vienna, July 2018.
- Emmanuel Trélat was invited speaker at Portuguese Meeting on Optimal Control 2018, Coimbra (Portugal), June 2018.
- Emmanuel Trélat was invited speaker at International Symposium on Mathematical Control Theory, Shanghai, June 2018.
- Emmanuel Trélat was invited speaker at GAMM Munich, March 2018.

9.1.5. Leadership within the Scientific Community

Emmanuel Trélat is director of the Fondation Sciences Mathématiques de Paris (FSMP).

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Ugo Boscain taught “Sub-elliptic diffusion” to PhD students at SISSA, Trieste Italy
- Ugo Boscain taught “Automatic Control” (with Mazyar Mirrahimi) at Ecole Polytechnique
- Ugo Boscain taught “MODAL of applied mathematics. Contrôle de modèles dynamiques” at Ecole Polytechnique
- Emmanuel Trélat taught “Control in finite and infinite dimension” at Master 2, Sorbonne Université

9.2.2. Supervision

PhD: Riccardo Bonalli, Optimal Control of Aerospace Systems with Control-State Constraints and Delays, Sorbonne Université, July 2018, supervised by Emmanuel Trélat.

PhD: Antoine Olivier, Optimal and robust attitude control of a launcher, Sorbonne Université, October 2018, supervised by Emmanuel Trélat and co-supervised by Thomas Haberkorn, Éric Bourgeois, David-Alexis Handschuh.

PhD: Ludovic Sacchelli, Singularities in sub-Riemannian geometry, Université Paris-Saclay, September 2018, supervised by Ugo Boscain and Mario Sigalotti.

PhD in progress: Nicolas Augier, “Contrôle adiabatique des systèmes quantiques”, started in September 2016, supervisors: Ugo Boscain, Mario Sigalotti.

PhD in progress: Amaury Hayat, “Contrôle et stabilisation en mécanique des fluides”, started in October 2016, supervisors: Jean-Michel Coron and Sébastien Boyaval

PhD in progress: Mathieu Kohli, “Volume and curvature in sub-Riemannian geometry”, started in September 2016, supervisors: Davide Barilari, Ugo Boscain.

PhD in progress: Gontran Lance, started in September 2018, supervisors: Emmanuel Trélat and Enrique Zuazua.

PhD in progress: Cyril Letrouit, “Équation des ondes sous-riemanniennes”, started in September 2019, supervisor Emmanuel Trélat.

PhD in progress: Jakub Orłowski, “Modeling and steering brain oscillations based on in vivo optogenetics data”, started in September 2016, supervisors: Antoine Chaillet, Alain Destexhe, and Mario Sigalotti.

PhD in progress: Eugenio Pozzoli, “Adiabatic Control of Open Quantum Systems”, started in September 2018, supervisors: Ugo Boscain and Mario Sigalotti.

PhD in progress: Shengquan Xiang, Stabilisation des fluides par feedbacks non-linéaires, September 2016, supervisor: Jean-Michel Coron.

PhD in progress: Christophe Zhang, started in October 2016, supervisor: Jean-Michel Coron

9.2.3. *Juries*

- Ugo Boscain was referee and member of the jury of the HDR of Jean-Marie Mirebeau, Université Paris-Sud.
- Mario Sigalotti was member of the jury of the PhD thesis of Abdelkrim Bahloul, Univ. Paris-Saclay.
- Emmanuel Trélat was co-supervisor and member of the jury of the PhD thesis of Camille Pouchol, Sorbonne Université.
- Emmanuel Trélat was member of the jury of the PhD thesis of F. Omnès, Sorbonne Université.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of S. Mitra, Univ. Toulouse.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of T. Weisser, Univ. Toulouse.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of S. Maslovskaya, Univ. Paris-Saclay.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of A. Vieira, Grenoble University.
- Emmanuel Trélat was member of the jury of the HDR of F. Chittaro, Univ. Toulon.

9.3. Popularization

Emmanuel Trélat is member of the Comité d'Honneur du Salon des Jeux et Culture Mathématique since November 2018

9.3.1. *Articles and contents*

- Nicolas Augier, Ugo Boscain, and Mario Sigalotti are authors of the popularization article [32] explaining how broken adiabatic paths can be used to enhance the control of a quantum systems.

9.3.2. *Education*

- Ugo Boscain and Jean-Michel Coron gave a lecture at journée ENS-UPS, ENS Paris
- Emmanuel Trélat gave a lecture at ENS Ulm to first-year students
- Emmanuel Trélat gave a lecture at Université Paris-Diderot to first- and second-year students

9.3.3. *Interventions*

- Ugo Boscain gave a lecture at Alfaclass, Saint-Barthélemy, Aosta, Italy
- Emmanuel Trélat gave a lecture at Salon des Jeux et Culture Mathématique

10. Bibliography

Publications of the year

Doctoral Dissertations and Habilitation Theses

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- [2] A. OLIVIER. *Optimal and robust attitude control of a launcher*, Sorbonne Université, October 2018, <https://tel.archives-ouvertes.fr/tel-01962542>
- [3] L. SACCHELLI. *Singularities in sub-Riemannian geometry*, Université Paris-Saclay, September 2018, <https://pastel.archives-ouvertes.fr/tel-01893068>

Articles in International Peer-Reviewed Journals

- [4] G. BARLES, A. BRIANI, E. TRÉLAT. *Value function for regional control problems via dynamic programming and Pontryagin maximum principle*, in "Mathematical Control and Related Fields", 2018, vol. 8, n^o 3-4, pp. 509–533, <https://arxiv.org/abs/1605.04079> , <https://hal.archives-ouvertes.fr/hal-01313559>
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