Activity Report 2018

Project-Team COMMANDS

Control, Optimization, Models, Methods and Applications for Nonlinear Dynamical Systems

IN COLLABORATION WITH: Centre de Mathématiques Appliquées (CMAP)

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team COMMANDS

Creation of the Project-Team: 2009 January 01

Keywords:

Computer Science and Digital Science:

A6.2.1. - Numerical analysis of PDE and ODE
A6.2.6. - Optimization
A6.2.7. - High performance computing
A6.3.2. - Data assimilation
A6.4.1. - Deterministic control
A6.4.2. - Stochastic control

Other Research Topics and Application Domains:

B4.4. - Energy delivery
B4.4.1. - Smart grids
B7.1.2. - Road traffic
B7.1.3. - Air traffic
B7.2.1. - Smart vehicles

1. Team, Visitors, External Collaborators

Research Scientists

Joseph Frederic Bonnans [Team leader, Inria, Senior Researcher, HDR]
Pierre Martinon [Inria, Researcher]

Post-Doctoral Fellows

Justina Gianatti [Inria, from May 2018]
Saeed Hadikhanloo [Inria, from Feb 2018]

PhD Students

Guillaume Bonnet [Univ. Paris Saclay, from Oct 2018]
Saeed Hadikhanloo [Inria, until Jan 2018]
Pierre Lavigne [École Nationale Supérieure de Techniques Avancées, from Oct 2018]
Arthur Le Rhun [Ifpen, from Sep 2016]
Cédric Rommel [Safety Line, until Oct 2018]

Technical staff

Liu Jinyan [Inria, until Jan 2018]

Interns

Guillaume Bonnet [Inria, from Apr 2018 until Sep 2018]
Pierre Lavigne [Inria, from Apr 2018 until Sep 2018]

Administrative Assistants

Hanadi Dib [Inria, until Oct 2018]
Ines Dumontier [Inria, from Nov 2018]
Hélèna Kutniak [Inria, from Oct 2018]

Visiting Scientist

Axel Kröner [Univ. Humboldt]

2. Overall Objectives

2.1. Scientific directions

Commands is a team devoted to dynamic optimization, both for deterministic and stochastic systems. This includes the following approaches: trajectory optimization, deterministic and stochastic optimal control, stochastic programming, dynamic programming and Hamilton-Jacobi-Bellman equation.

Our aim is to derive new and powerful algorithms for solving numerically these problems, with applications in several industrial fields. While the numerical aspects are the core of our approach it happens that the study of convergence of these algorithms and the verification of their well-posedness and accuracy raises interesting and difficult theoretical questions, such as, for trajectory optimization: qualification conditions and second-order optimality condition, well-posedness of the shooting algorithm, estimates for discretization errors; for the Hamilton-Jacobi-Bellman approach: accuracy estimates, strong uniqueness principles when state constraints are present, for stochastic programming problems: sensitivity analysis.

2.2. Industrial impact

For many years the team members have been deeply involved in various industrial applications, often in the framework of PhD theses. The Commands team itself has dealt since its foundation in 2009 with several types of applications:

- Space vehicle trajectories, in collaboration with CNES, the French space agency.
- Aeronautics, in collaboration with the startup Safety Line.
- Production, management, storage and trading of energy resources, in collaboration with Edf, ex-Gdf and Total.
- Energy management for hybrid vehicles, in collaboration with Renault and Ifpen.

We give more details in the Bilateral contracts section.

3. Research Program

3.1. Historical aspects

The roots of deterministic optimal control are the “classical” theory of the calculus of variations, illustrated by the work of Newton, Bernoulli, Euler, and Lagrange (whose famous multipliers were introduced in [28]), with improvements due to the “Chicago school”, Bliss [20] during the first part of the 20th century, and by the notion of relaxed problem and generalized solution (Young [33]).

Trajectory optimization really started with the spectacular achievement done by Pontryagin’s group [32] during the fifties, by stating, for general optimal control problems, nonlocal optimality conditions generalizing those of Weierstrass. This motivated the application to many industrial problems (see the classical books by Bryson and Ho [24], Leitmann [30], Lee and Markus [29], Ioffe and Tihomirov [27]).

Dynamic programming was introduced and systematically studied by R. Bellman during the fifties. The HJB equation, whose solution is the value function of the (parameterized) optimal control problem, is a variant of the classical Hamilton-Jacobi equation of mechanics for the case of dynamics parameterized by a control variable. It may be viewed as a differential form of the dynamic programming principle. This nonlinear first-order PDE appears to be well-posed in the framework of *viscosity solutions* introduced by Crandall and Lions [25]. The theoretical contributions in this direction did not cease growing, see the books by Barles [18] and Bardi and Capuzzo-Dolcetta [17].

3.2. Trajectory optimization

The so-called *direct methods* consist in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure. So the two main problems are the choice of the discretization and the nonlinear programming algorithm. A third problem is the possibility of refinement of the discretization once after solving on a coarser grid.

In the *full discretization approach*, general Runge-Kutta schemes with different values of control for each inner step are used. This allows to obtain and control high orders of precision, see Hager [26], Bonnans [21]. In the *indirect* approach, the control is eliminated thanks to Pontryagin's maximum principle. One has then to solve the two-points boundary value problem (with differential variables state and costate) by a single or multiple shooting method. The questions are here the choice of a discretization scheme for the integration of the boundary value problem, of a (possibly globalized) Newton type algorithm for solving the resulting finite dimensional problem in IR^n (n is the number of state variables), and a methodology for finding an initial point.

3.3. Hamilton-Jacobi-Bellman approach

This approach consists in calculating the value function associated with the optimal control problem, and then synthesizing the feedback control and the optimal trajectory using Pontryagin's principle. The method has the great particular advantage of reaching directly the global optimum, which can be very interesting when the problem is not convex.

Optimal stochastic control problems occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky and non-risky assets, super-replication with uncertain volatility, management of power resources (dams, gas). Air traffic control is another example of such problems.

For solving stochastic control problems, we studied the so-called Generalized Finite Differences (GFD), that allow to choose at any node, the stencil approximating the diffusion matrix up to a certain threshold [23]. Determining the stencil and the associated coefficients boils down to a quadratic program to be solved at each point of the grid, and for each control. This is definitely expensive, with the exception of special structures where the coefficients can be computed at low cost. For two dimensional systems, we designed a (very) fast algorithm for computing the coefficients of the GFD scheme, based on the Stern-Brocot tree [22].

4. Application Domains

4.1. Fuel saving by optimizing airplanes trajectories

We have a collaboration with the startup Safety Line on the optimization of trajectories for civil aircrafts. Key points include the reliable identification of the plane parameters (aerodynamic and thrust models) using data from the flight recorders, and the robust trajectory optimization of the climbing and cruise phases. We use both local (quasi-Newton interior-point algorithms) and global optimization tools (dynamic programming). The local method for the climb phase is in production and has been used for several hundreds of actual plane flights.

4.2. Hybrid vehicles

We have a collaboration with IFPEN on the energy management for hybrid vehicles. A significant direction is the analysis and classification of traffic data. More specifically, we focus on the traffic probability distribution in the (speed,torque) plane, with a time / space subdivision (road segments and timeframes).

4.3. Biological systems

2018 was the last year of IPL Algae in Silico in which we tackled the optimization of photobioreactors in turbid conditions. Our participation to IPL Cosy was strenghtened through the co-supervision of the PhD of E. Weill with team Lifeware (Inria and Pasteur), starting from september 2018, and focused on the oprimization of heterogenous populations of micro-organisms.

5. Highlights of the Year

5.1. Highlights of the Year

Variational analysis for options with stochastic volatility and multiple factors

Publication of the paper [3] in the SIAM J. finance. This paper clarifies the issue of well-posedness of some PDEs arising in finance.

A stochastic data-based traffic model applied to vehicles energy consumption estimation

Publication [10] of a new method for the analysis of road traffic, in relation with energy consumption.

6. New Software and Platforms

6.1. BOCOP

Boite à Outils pour le Contrôle OPtimal
KEYWORDS: Dynamic Optimization - Identification - Biology - Numerical optimization - Energy management - Transportation
FUNCTIONAL DESCRIPTION: Bocop is an open-source toolbox for solving optimal control problems, with collaborations with industrial and academic partners. Optimal control (optimization of dynamical systems governed by differential equations) has numerous applications in transportation, energy, process optimization, energy and biology. Bocop includes a module for parameter identification and a graphical interface, and runs under Linux / Windows / Mac.

RELEASE FUNCTIONAL DESCRIPTION: Handling of delay systems Alternate automatic differentiation tool: CppAD Update for CMake and MinGW (windows version)

- Participants: Benjamin Heymann, Virgile Andreani, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Pierre Martinon
- URL: http://bocop.org

6.2. Bocop HJB

KEYWORDS: Optimal control - Stochastic optimization - Global optimization
FUNCTIONAL DESCRIPTION: Toolbox for stochastic or deterministic optimal control, dynamic programming / HJB approach.

RELEASE FUNCTIONAL DESCRIPTION: User interface State jumps for switched systems Explicit handling of final conditions Computation of state probability density (fiste step to mean field games)

- Participants: Benjamin Heymann, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Joseph Frédéric Bonnans
- URL: http://bocop.org

6.3. Bocop Avion

KEYWORDS: Optimization - Aeronautics
FUNCTIONAL DESCRIPTION: Optimize the climb speeds and associated fuel consumption for the flight planning of civil airplanes.
NEWS OF THE YEAR: Improved atmosphere model 2D interpolations for temperature and wind data

- Participants: Gregorutti Baptiste, Cindie Andrieu, Anamaria Lupu, Joseph Frédéric Bonnans, Karim Tekkal, Pierre Jouniaux and Pierre Martinon
- Partner: Safety Line
- Contact: Pierre Martinon
- URL: http://www.safety-line.fr

6.4. Bocop HJB Avion

KEYWORDS: Optimization - Aeronautics
FUNCTIONAL DESCRIPTION: Optimize the climb and cruising trajectory of flight by a HJB approach.
NEWS OF THE YEAR: First demonstrator for cruise flight deployed at Safety Line

- Participants: Pierre Martinon, Joseph Frédéric Bonnans, Jinyan Liu, Gregorutti Baptiste and Anamaria Lupu
- Partner: Safety Line
- Contact: Pierre Martinon
- URL: http://www.safety-line.fr

7. New Results

7.1. Optimal control of ODEs

7.1.1. Optimal health insurance design

In [7] we analyze the design of optimal medical insurance under ex post moral hazard, i.e., when illness severity cannot be observed by insurers and policyholders decide for themselves on their health expenditures. The trade-off between ex ante risk sharing and ex post incentive compatibility is analyzed in an optimal revelation mechanism under hidden information and risk aversion. The optimal contract provides partial insurance at the margin, with a deductible when insurers' rates are affected by a positive loading, and it may also include an upper limit on coverage. The potential to audit the health state leads to an upper limit on out-of-pocket expenses.

Health insurance with audit. The 'out-of-pocket' expense $(m - I)$ remains bounded.

7.1.2. Optimal Battery Aging: an Adaptive Weights Dynamic Programming Algorithm

In [5] we present an algorithm to handle the optimization over a long horizon of an electric microgrid including a battery energy storage system. While the battery is an important and costly component of the microgrid, its aging process is often not taken into account by the Energy Management System, mostly because of modeling and computing challenges. We address the computing aspect by a new approach combining dynamic programming, decomposition and relaxation techniques. We illustrate this 'adaptive weight' method with numerical simulations for a toy microgrid model. Compared to a straightforward resolution by dynamic programming, our algorithm decreases the computing time by more than one order of magnitude, can be parallelized, and allows for online implementations. We believe that this approach can be used for other applications presenting fast and slow variables.

Optimal battery aging. Comparison of brute-force and adaptive weights algorithm.

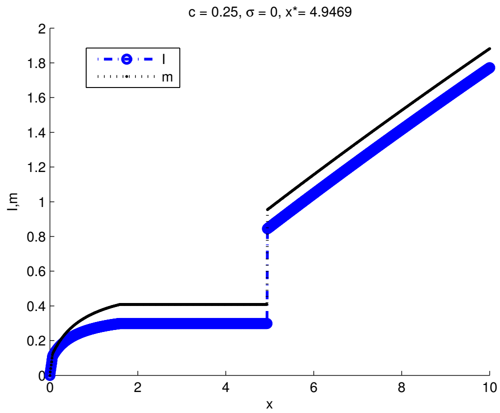

Figure 1.

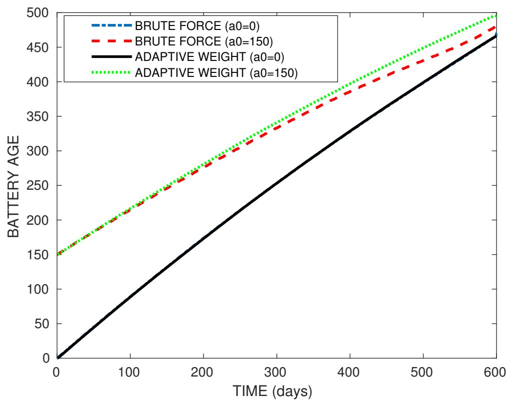

Figure 2.

7.1.3. Aircraft model identification and trajectory optimization

During the PhD of C. Rommel co-supervised with startup Safety Line, we investigated several formulations and methods for identifying an aircraft dynamics from recorded flight data. In particular, in [14] we introduce a block-sparse Bolasso approach for variable selection. In [12] we study how to quantify the closedness of a trajectory to a set of reference ones, based on the meean marginal likelihood. These works are combined with a gaussian mixture model in [15], allowing for a trade-off between optimality and acceptability of the aircraft trajectories.

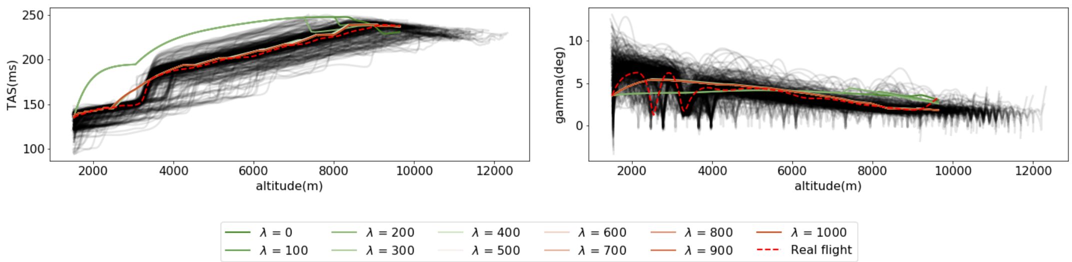

Figure 3.

Aircraft trajectory optimization. Illustration of the trade-off between performance (consumption) and acceptability (weighted by λ).

7.1.4. Microalgae cultivation in a turbid medium

In the context of IPL Algae in Silico, we study in [11] the cultivation of microalgae in a turbid medium. Microalgae cultivation with wastewater is a promising way of reducing the energetic needs for wastewater treatment and the costs of biofuel production. However, the very turbid medium is not favorable for the development of microalgae. Indeed, light, the key element for photosynthesis, rapidly vanishes along depth due to absorption and scattering. Therefore it is crucial to understand the effects of the depth on turbid cultures. In this work, we study theoretically the long-term behavior of a continuous culture of microalgae exposed to a periodic source of light. By allowing periodic variations of the depth and the hydraulic retention time, we show that the microalgae population is forced to a periodic regime. Finally, we address numerically the problem of determining the optimal variations of the depth and the hydraulic retention time for maximizing the productivity of the culture in the periodic regime.

7.1.5. Optimizing running a race on a curved track

Following on a previous study of optimal running strategies [16], we investigate in [9] the case of a curved track. In order to determine the optimal strategy to run a race on a curved track according to the lane number, we introduce a model based on differential equations for the velocity, the propulsive force and the anaerobic energy which takes into account the centrifugal force. This allows us to analyze numerically the different strategies according to the different types of track since the straight line is not always of the same length. In particular, we find that the tracks with shorter straight lines lead to better performances, while the double bend track with the longest straight line leads to the worst performances and the biggest difference between lanes. Then for a race with two runners, we introduce a psychological attraction to follow someone just ahead and the delay to benefit again from this interaction after being overtaken. We provide numerical simulations in different cases. Results are overall consistent with the IAAF rules for lanes drawing, indicating that middle lanes are the best, followed by the exterior lanes, interior lanes being the worst.

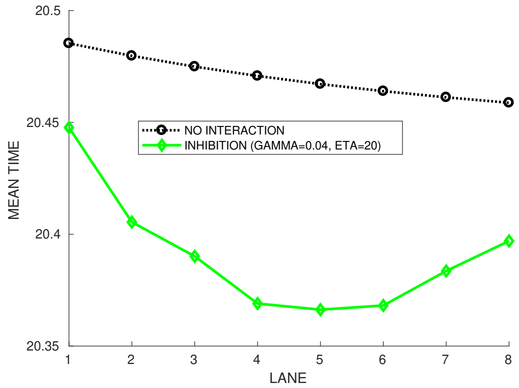

Figure 4.

Running on a curved track. Mean race times per lane, taking into account centrifugal force and psychological interaction.

7.2. Optimal control of PDEs and stochastic control

7.2.1. Sufficient optimality conditions for bilinear optimal control of the linear damped wave equation

In [8] we discuss sufficient optimality conditions for an optimal control problem for the linear damped wave equation with the damping parameter as the control. We address the case that the control enters quadratic in the cost function as well as the singular case that the control enters affine. For the non-singular case we consider strong and weak local minima , in the singular case we derive sufficient optimality conditions for weak local minima. Thereby, we take advantage of the Goh transformation applying techniques recently established in Aronna, Bonnans, and Kröner [Math. Program. 168(1):717–757, 2018]. Moreover, a numerical example for the singular case is presented.

7.2.2. Variational analysis for options with stochastic volatility and multiple factors

In [3] we perform a variational analysis for a class of European or American options with stochastic volatility models, including those of Heston and Achdou-Tchou. Taking into account partial correlations and the presence of multiple factors, we obtain the well-posedness of the related partial differential equations, in some weigthed Sobolev spaces. This involves a generalization of the commutator analysis introduced by Achdou and Tchou.

7.2.3. Infinite Horizon Stochastic Optimal Control Problems with Running Maximum Cost

In [6] we analyze an infinite horizon stochastic optimal control problem with running maximum cost. The value function is characterized as the viscosity solution of a second-order Hamilton-Jacobi-Bellman (HJB) equation with mixed boundary condition. A general numerical scheme is proposed and convergence is established under the assumptions of consistency, monotonicity and stability of the scheme. These properties are verified for a specific semi-Lagrangian scheme.

7.2.4. A stochastic data-based traffic model applied to vehicles energy consumption estimation

In the framework of the PhD of A. Le Rhun, we present in [10] a new approach to estimate traffic energy consumption via traffic data aggregation in (speed,acceleration) probability distributions. The aggregation is done on each segment composing the road network. In order to reduce data occupancy, clustering techniques are used to obtain meaningful classes of traffic conditions. Different times of the day with similar speed patterns and traffic behavior are thus grouped together in a single cluster. Different energy consumption models based on the aggregated data are proposed to estimate the energy consumption of the vehicles in the road network. For validation purposes, a microscopic traffic simulator is used to generate the data and compare the estimated energy consumption to the reference one. A thorough sensitivity analysis with respect to the parameters of the proposed method (i.e. number of clusters, size of the distributions support, etc.) is also conducted in simulation. Finally, a real-life scenario using floating car data is analyzed to evaluate the applicability and the robustness of the proposed method.

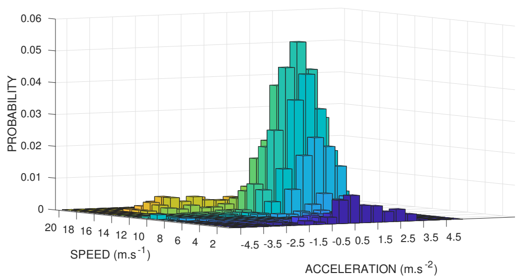

Figure 5.

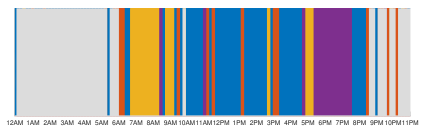

Figure 6.

Traffic modeling. Example of (speed,acceleration) distribution and illustration of clustering results with respect to day time. Slow traffic for yellow and purple clusters clearly corresponds to peak hours.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

Contract with Safety Line: support of an Ilab and of a Cifre PhD. Toolbox Bocop is a component of the commercial service OptiClimb used by several airplane companies.

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. IPL

9.1.1.1. Algae in Silico

Inria Project Lab ALGAE IN SILICO (2014-2018) was dedicated to provide an integrated platform for numerical simulation of microalgae “from genes to industrial process“. Commands joined the project in 2017 to tackle the optimization aspects. Our previous collaborations with teams Modemic and Biocore on bioreactors [31], [19] have been renewed in this framework, see [11]

9.1.1.2. Cosy

Inria Project Lab COSY (started in 2017) aims at exploiting the potential of state-of-art biological modelling, control techniques, synthetic biology and experimental equipment to achieve a paradigm shift in control of microbial communities. More precisely, we plan to determine and implement control strategies to make heterogeneous communities diversify and interact in the most profitable manner. Study of yeast cells has started in collaboration with team Lifeware (G. Batt) in the framework of the PhD of V. Andreani, and is pursued in the PhD of E. Weill (started 2018).

9.2. International Research Visitors

9.2.1. Visits of International Scientists

Several visits by L. Pfeiffer, U. Graz, and A. Kröner, U. Humboldt.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Selection

10.1.1.1. Member of the Conference Program Committees

- F. Bonnans: PGMO Days 2018.

10.1.2. Journal

10.1.2.1. Member of the Editorial Boards

- F. Bonnans: Associate Editor of “Applied Mathematics and Optimization” and of “Series on Mathematics and its Applications, Annals of The Academy of Romanian Scientists”.

10.1.3. Invited Talks

- F. Bonnans: CAESAR workshop, Palaiseau, Sept. 5-7, 2018.

10.1.4. Leadership within the Scientific Community

- F. Bonnans: French representative to the IFIP-TC7 committee (International Federation of Information Processing; TC7 devoted to System Modeling and Optimization).
- F. Bonnans: member of the PGMO board and Steering Committee (Gaspard Monge Program for Optimization and Operations Research, EDF-FMJH).

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Master :
F. Bonnans: *Numerical analysis of partial differential equations arising in finance and stochastic control*, 18h, M2, Ecole Polytechnique and U. Paris 6, France.
F. Bonnans: *Optimal control*, 15h, M2, Optimization master (U. Paris-Saclay) and Ensta, France.
A. Kröner : Optimal control of partial differential equations, 20h, M2, Optimization master (U. Paris-Saclay), France.

10.2.2. Supervision

Finished PhD : C. Rommel, Data exploration for the optimization of aircraft trajectories. Started November 2015, finished Oct 2018 (CIFRE fellowship with Safety Line), F. Bonnans and P. Martinon.

PhD in progress : A. Le Rhun, Optimal and robust control of hybrid vehicles. Started September 2016 (IFPEN fellowship), F. Bonnans and P. Martinon.

PhD in progress : G. Bonnet, Efficient schemes for the Hamilton-Jacobi-Bellman equation. Started Oct. 2018. F. Bonnans and J.-M. Mirebeau, LMO, U. Orsay.

PhD in progress : P. Lavigne, Mathematical study of economic equilibria for renewable energy sources. Started Oct. 2018. F. Bonnans.

PhD in progress : E. Weill, Optimal control of partial differential equation systems: Application to heterogeneous cell populations. Started Oct. 2018. F. Bonnans and G. Batt, Inria and Institut Pasteur.

10.3. Popularization

10.3.1. Internal or external Inria responsibilities

- F. Bonnans: codirection of a joint Allistene-Ancre commission (contribution to the national strategy for research), Numerics and Energy committee (2017-2018).
- F. Bonnans: Dimitrie Pompeiu Prize Committee (Academy of Romanian Scientists).
- P. Martinon is member of the CDT.

10.3.2. Articles and contents

- The work on races on curved tracks was covered in the Sciences page of Le Figaro.

10.3.3. Internal action

- The collaboration with startup Safety Line was presented to the “Journée Nationale des Nouveaux Arrivants”

11. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

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[7] P. MARTINON, P. PICARD, A. RAJ. *On the Design of Optimal Health Insurance Contracts under Ex Post Moral Hazard*, in "Geneva Risk and Insurance Review", July 2018, vol. 43, n^{o} 2, pp. 137-185 [*DOI :* 10.1057/S10713-018-0034-Y], https://hal-polytechnique.archives-ouvertes.fr/hal-01348551

Research Reports

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Other Publications

[9] A. AFTALION, P. MARTINON. *Optimizing running a race on a curved track*, November 2018, https://arxiv.org/abs/1811.12321 - working paper or preprint, https://hal.inria.fr/hal-01936993

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[11] C. MARTÍNEZ, F. MAIRET, P. MARTINON, O. BERNARD. *Dynamics and control of a periodically forced microalgae culture*, October 2018, working paper or preprint, https://hal.inria.fr/hal-01891648

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