Activity Report 2018

Team FACTAS

Functional Analysis for the ConcepTion and Assessment of Systems

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions).

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Creation of the Team: 2018 January 01

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1. Team, Visitors, External Collaborators

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2. Overall Objectives

2.1. Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modeling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviors. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider various inter-polation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE), in connection with inverse potential problems. A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),
- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro-encephalography), and paleomagnetism (determining the magnetization of rock samples).

In each case, the endeavor is to develop algorithms resulting in dedicated software.

3. Research Program

3.1. Introduction

Within the extensive field of inverse problems, much of the research by Factas deals with reconstructing solutions of classical elliptic PDEs from their boundary behavior. Perhaps the simplest example lies with harmonic identification of a stable linear dynamical system: the transfer-function f can be evaluated at a point $i\omega$ of the imaginary axis from the response to a periodic input at frequency ω. Since f is holomorphic in the right half-plane, it satisfies there the Cauchy-Riemann equation $\overline{\partial} f = 0$, and recovering f amounts to solve a Dirichlet problem which can be done in principle using, *e.g.* the Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [67]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n, then $\overline{\partial} f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles and δ_{z_j} is a Dirac unit mass at z_j. Thus, to find the domain of holomorphy (*i.e.* to locate the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address such questions, the team has developed a two-step approach as follows.

Step 1: To determine a complete model, that is, one which is defined at every frequency, in a sufficiently versatile function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behavior at non-measured frequencies.

Step 2: To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. We emphasize that deriving a complete model in step 1 is crucial to achieve stability of the reduced model in step 2.

Step 1 relates to extremal problems and analytic operator theory, see Section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and outputs, see Section 3.3.2.2. It also makes contact with the topology of rational functions, in particular to count critical points and to derive bounds, see Section 3.3.2. Step 2 raises further issues in approximation theory regarding the rate of convergence and the extent to which singularities of the approximant (*i.e.* its poles) tend to singularities of the approximated function; this is where logarithmic potential theory becomes instrumental, see Section 3.3.3.

Applying a realization procedure to the result of step 2 yields an identification procedure from incomplete frequency data which was first demonstrated in [75] to tune resonant microwave filters. Harmonic identification of nonlinear systems around a stable equilibrium can also be envisaged by combining the previous steps with exact linearization techniques from [31].

A similar path can be taken to approach design problems in the frequency domain, replacing the measured behavior by some desired behavior. However, describing achievable responses in terms of the design parameters is often cumbersome, and most constructive techniques rely on specific criteria adapted to the physics of the problem. This is especially true of filters, the design of which traditionally appeals to polynomial extremal problems [69], [54]. Apics contributed to this area the use of Zolotarev-like problems for multi-band synthesis, although we presently favor interpolation techniques in which parameters arise in a more transparent manner, as well as convex relaxation of hyperbolic approximation problems, see Sections 3.2.2 and 6.2.2.

The previous example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying $\mathbb{C}$ with $\mathbb{R}^2$, holomorphic functions become conjugate-gradients of harmonic functions, so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; when the portion of boundary where data are not available is itself unknown, we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [59] and subsequently received considerable attention. It makes clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations [1] [29], [79]. Such questions are particular instances of the so-called inverse potential problem, where a measure μ has to be recovered from the knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ. For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. Nevertheless it is a useful concept bringing perspective on how problems could be raised and solved, using tools from harmonic analysis.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set produce the same field outside this set; this phenomenon is called *balayage* [66]. In the two steps approach previously described, we implicitly removed this indeterminacy by requiring in step 1 that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half-space), and by requiring in step 2 that the measure be discrete in the left half-plane (in fact: a sum of point masses $\sum_1^n a_j \delta_{z_j}$). The discreteness assumption also prevails in 3-D inverse source problems, see Section 4.3. Conditions that ensure uniqueness of the solution to the inverse potential problem are part of the so-called regularizing assumptions which are needed in each case to derive efficient algorithms.

To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. This differs from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

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[1]There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere. Such a "bad" subset, however, cannot have interior points on the sphere.

Along these lines, Factas advocates the use of steps 1 and 2 above, along with some singularity analysis, to approach issues of nondestructive control in 2-D and 3-D [2], [39], [42]. The team is currently engaged in the generalization to inverse source problems for the Laplace equation in 3-D, to be described further in Section 3.2.1. There, holomorphic functions are replaced by harmonic gradients; applications are to inverse source problems in neurosciences (in particular in EEG/MEG) and inverse magnetization problems in geosciences, see Section 4.3.

The approximation-theoretic tools developed by Apics and now by Factas to handle issues mentioned so far are outlined in Section 3.3. In Section 3.2 to come, we describe in more detail which problems are considered and which applications are targeted.

Note that the Inria project-team Apics reached the end of its life cycle by the end of last year. The proposal for our new team Factas is since then being built and examined by the CEP (Comité des Équipes-Projets) of the Research Center, on its way to become a project-team.

3.2. Range of inverse problems

3.2.1. *Elliptic partial differential equations (PDE)*

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Konstantinos Mavreas.

By standard properties of conjugate differentials, reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain, when these conditions are already known on a subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E. This is the problem raised on the half-plane in step 1 of Section 3.1. It makes good sense in holomorphic Hardy spaces where functions are entirely determined by their values on boundary subsets of positive linear measure, which is the framework for Problem (P) that we set up in Section 3.3.1. Such issues naturally arise in nondestructive testing of 2-D (or 3-D cylindrical) materials from partial electrical measurements on the boundary. For instance, the ratio between the tangential and the normal currents (the so-called Robin coefficient) tells one about corrosion of the material. Thus, solving Problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape yields non destructive testing of a potentially corroded piece of material, part of which is inaccessible to measurements. This was an initial application of holomorphic extremal problems to non-destructive control [52], [55].

Another application by the team deals with non-constant conductivity over a doubly connected domain, the set E being now the outer boundary. Measuring Dirichlet-Neumann data on E, one wants to recover level lines of the solution to a conductivity equation, which is a so-called free boundary inverse problem. For this, given a closed curve inside the domain, we first quantify how constant the solution on this curve. To this effect, we state and solve an analog of Problem (P), where the constraint bears on the real part of the function on the curve (it should be close to a constant there), in a Hardy space of a conjugate Beltrami equation, of which the considered conductivity equation is the compatibility condition (just like the Laplace equation is the compatibility condition of the Cauchy-Riemann system). Subsequently, a descent algorithm on the curve leads one to improve the initial guess. For example, when the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D situation), this method can be used to estimate the shape of a plasma subject to magnetic confinement.

This was actually carried out in collaboration with CEA (French nuclear agency) and the University of Nice (JAD Lab.), to data from *Tore Supra* in [58]. The procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation in terms of Bessel functions was found in this case. Generalizing this approach in a more systematic manner to free boundary problems of Bernoulli type, using descent algorithms based on shape-gradient for such approximation-theoretic criteria, is an interesting prospect to the team.

The piece of work we just mentioned requires defining and studying Hardy spaces of conjugate Beltrami equations, which is an interesting topic. For Sobolev-smooth coefficients of exponent greater than 2, they were investigated in [5], [32]. The case of the critical exponent 2 is treated in [28], which apparently provides the first example of well-posedness for the Dirichlet problem in the non-strictly elliptic case: the conductivity may be unbounded or zero on sets of zero capacity and, accordingly, solutions need not be locally bounded. More importantly perhaps, the exponent 2 is also the key to a corresponding theory on very general (still rectifiable) domains in the plane, as coefficients of pseudo-holomorphic functions obtained by conformal transformation onto a disk are merely of L^2-class in general, even if the initial problem deals with coefficients of L^r-class for some $r > 2$.

Generalized Hardy classes as above are used in [29] where we address the uniqueness issue in the classical Robin inverse problem on a Lipschitz domain of $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with uniformly bounded Robin coefficient, L^2 Neumann data and conductivity of Sobolev class $W^{1,r}(\Omega)$, $r > n$. We show that uniqueness of the Robin coefficient on a subset of the boundary, given Cauchy data on the complementary part, does hold in dimension $n = 2$, thanks to a unique continuation result, but needs not hold in higher dimension. In higher dimension, this raises an open issue on harmonic gradients, namely whether the positivity of the Robin coefficient is compatible with identical vanishing of the boundary gradient on a subset of positive measure.

The 3-D version of step 1 in Section 3.1 is another subject investigated by Factas: to recover a harmonic function (up to an additive constant) in a ball or a half-space from partial knowledge of its gradient. This prototypical inverse problem (*i.e.* inverse to the Cauchy problem for the Laplace equation) often recurs in electromagnetism. At present, Factas is involved with solving instances of this inverse problem arising in two fields, namely medical imaging *e.g.* for electroencephalography (EEG) or magneto-encephalography (MEG), and paleomagnetism (recovery of rocks magnetization) [2], [35], see Section 6.1. In this connection, we collaborate with two groups of partners: Athena Inria project-team and INS (Institut de Neurosciences des Systèmes, http://ins.univ-amu.fr/), hospital la Timone, Aix-Marseille Univ., on the one hand, Geosciences Lab. at MIT and Cerege CNRS Lab. on the other hand. The question is considerably more difficult than its 2-D counterpart, due mainly to the lack of multiplicative structure for harmonic gradients. Still, substantial progress has been made over the last years using methods of harmonic analysis and operator theory.

The team is further concerned with 3-D generalizations and applications to non-destructive control of step 2 in Section 3.1. A typical problem is here to localize inhomogeneities or defaults such as cracks, sources or occlusions in a planar or 3-dimensional object, knowing thermal, electrical, or magnetic measurements on the boundary. These defaults can be expressed as a lack of harmonicity of the solution to the associated Dirichlet-Neumann problem, thereby posing an inverse potential problem in order to recover them. In 2-D, finding an optimal discretization of the potential in Sobolev norm amounts to solve a best rational approximation problem, and the question arises as to how the location of the singularities of the approximant (*i.e.* its poles) reflects the location of the singularities of the potential (*i.e.* the defaults we seek). This is a fairly deep issue in approximation theory, to which Apics contributed convergence results for certain classes of fields expressed as Cauchy integrals over extremal contours for the logarithmic potential [7], [36], [49]. Initial schemes to locate cracks or sources *via* rational approximation on planar domains were obtained this way [39], [42], [52]. It is remarkable that finite inverse source problems in 3-D balls, or more general algebraic surfaces, can be approached using these 2-D techniques upon slicing the domain into planar sections [8], [40]. More precisely, each section cuts out a planar domain, the boundary of which carries data which can be proved to match an algebraic function. The singularities of this algebraic function are not located at the 3-D sources, but are related to them: the section contains a source if and only if some function of the singularities in that section meets a relative extremum. Using bisection it is thus possible to determine an extremal place along all sections parallel to a given plane direction, up to some threshold which has to be chosen small enough that one does not miss a source. This way, we reduce the original source problem in 3-D to a sequence of inverse poles and branchpoints problems in 2-D. This bottom line generates a steady research activity within Factas, and again applications are sought to medical imaging and geosciences, see Sections 4.3, 4.2 and 6.1.

Conjectures may be raised on the behavior of optimal potential discretization in 3-D, but answering them is an ambitious program still in its infancy.

3.2.2. Systems, transfer and scattering

Participants: Laurent Baratchart, Sylvain Chevillard, Adam Cooman, Martine Olivi, Fabien Seyfert.

Through contacts with CNES (French space agency), members of the team became involved in identification and tuning of microwave electromagnetic filters used in space telecommunications, see Section 4.4. The initial problem was to recover, from band-limited frequency measurements, physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant cavities with negligible loss, hence its scattering matrix is modeled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory comes into play, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces provide a framework to transform this ill-posed issue into a series of regularized analytic and meromorphic approximation problems. More precisely, the procedure sketched in Section 3.1 goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving a problem analogous to (P) in Section 3.3.1, while taking into account prior knowledge on the decay of the response outside the bandwidth, see [11] for details.

2. A stable rational approximation of appropriate degree to the model obtained in the previous step is performed. For this, a descent method on the compact manifold of inner matrices of given size and degree is used, based on an original parametrization of stable transfer functions developed within the team [24], [11].

3. Realizations of this rational approximant are computed. To be useful, they must satisfy certain constraints imposed by the geometry of the device. These constraints typically come from the coupling topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled according to some specific graph. This realization step can be recast, under appropriate compatibility conditions [53], as solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Gröbner basis techniques and continuation methods which team up in the Dedale-HF software (see Section 3.4.1).

Let us mention that extensions of classical coupling matrix theory to frequency-dependent (reactive) couplings have been carried-out in recent years [1] for wide-band design applications.

Factas also investigates issues pertaining to design rather than identification. Given the topology of the filter, a basic problem in this connection is to find the optimal response subject to specifications that bear on rejection, transmission and group delay of the scattering parameters. Generalizing the classical approach based on Chebyshev polynomials for single band filters, we recast the problem of multi-band response synthesis as a generalization of the classical Zolotarev min-max problem for rational functions [23] [10]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These were implemented in the software easy-FF (see easy-FF). Currently, the team is engaged in the synthesis of more complex microwave devices like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role here, because scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [74], who devised a recursive test to check for contractivity of a holomorphic function in the disk. The so-called Schur parameters of a function may be viewed as Taylor coefficients for the hyperbolic metric of the disk, and the fact that Schur functions are contractions for that metric lies at the root of Schur's test. Generalizations thereof turn out to be efficient to parametrize solutions to contractive interpolation problems [25]. Dwelling on this, Factas contributed differential parametrizations (atlases of charts) of lossless matrix functions [24], [68], [63]

which are fundamental to our rational approximation software RARL2 (see Section 3.4.4). Schur analysis is also instrumental to approach de-embedding issues, and provides one with considerable insight into the so-called matching problem. The latter consists in maximizing the power a multiport can pass to a given load, and for reasons of efficiency it is all-pervasive in microwave and electric network design, *e.g.* of antennas, multiplexers, wifi cards and more. It can be viewed as a rational approximation problem in the hyperbolic metric, and the team presently deals with this hot topic using contractive interpolation with constraints on boundary peak points, within the framework of the (defense funded) ANR Cocoram, see Sections 6.2 and 8.2.2.

In recent years, our attention was driven by CNES and UPV (Bilbao) to questions about stability of high-frequency amplifiers. Contrary to previously discussed devices, these are *active* components. The response of an amplifier can be linearized around a set of primary current and voltages, and then admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The initial goal is to check for stability of the linearized model, so as to ascertain existence of a well-defined working state. The network is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive reactors, transmission lines, and controlled current sources. Our research so far has focused on describing the algebraic structure of admittance functions, so as to set up a function-theoretic framework where the two-steps approach outlined in Section 3.1 can be put to work. The main discovery is that the unstable part of each partial transfer function is rational and can be computed by analytic projection, see Section 6.3. We now start investigating the linearized harmonic transfer-function around a periodic cycle, to check for stability under non necessarily small inputs. This generalization generates both doctoral and postdoctoral work by new students in the team.

3.3. Approximation

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Fabien Seyfert.

3.3.1. *Best analytic approximation*

In dimension 2, the prototypical problem to be solved in step 1 of Section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, to recover a holomorphic function from its values on a subset K of the boundary of D. For the discussion it is convenient to normalize D, which can be done by conformal mapping. So, in the simply connected case, we fix D to be the unit disk with boundary unit circle T. We denote by H^p the Hardy space of exponent p, which is the closure of polynomials in $L^p(T)$-norm if $1 \le p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function g in D matching some measured values f approximately on a sub-arc K of T, we formulate a constrained best approximation problem as follows.

> (P) Let $1 \le p \le \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \smallsetminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \smallsetminus K)} \le M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capturing *a priori* assumptions on the behavior of the model off K, while M is some admissible deviation thereof. The value of p reflects the type of stability which is sought and how much one wants to smooth out the data. The choice of L^p classes is suited to handle pointwise measurements.

To fix terminology, we refer to (P) as a *bounded extremal problem*. As shown in [38], [41], [46], the solution to this convex infinite-dimensional optimization problem can be obtained when $p \ne 1$ upon iterating with respect to a Lagrange parameter the solution to spectral equations for appropriate Hankel and Toeplitz operators. These spectral equations involve the solution to the special case $K = T$ of (P), which is a standard extremal problem [61]:

> (P_0) Let $1 \le p \le \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

The case $p = 1$ is more or less open.

Various modifications of (P) can be tailored to meet specific needs. For instance when dealing with lossless transfer functions (see Section 4.4), one may want to express the constraint on $T \smallsetminus K$ in a pointwise manner: $|g - \psi| \leq M$ a.e. on $T \smallsetminus K$, see [13]. In this form, the problem comes close to (but still is different from) H^∞ frequency optimization used in control [64], [73]. One can also impose bounds on the real or imaginary part of $g - \psi$ on $T \smallsetminus K$, which is useful when considering Dirichlet-Neumann problems.

The analog of Problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see Sections 3.2.1, 4.3, 6.1.3). It may serve as a tool to approach Bernoulli type problems, where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the solution. In this case, the Lagrange parameter indicates how to deform the inner contour in order to improve data fitting. Similar topics are discussed in Section 3.2.1 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\operatorname{div}(\sigma \nabla u) = 0$ where σ is no longer constant. Then, the Hardy spaces in Problem (P) are those of a so-called conjugate Beltrami equation: $\overline{\partial} f = \nu \overline{\partial f}$ [65], which are studied for $1 < p < \infty$ in [5], [28], [32] and [56]. Expansions of solutions needed to constructively handle such issues in the specific case of linear fractional conductivities (occurring for instance in plasma shaping) have been expounded in [58].

Though originally considered in dimension 2, Problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and some $\mathbb{R}^n$-valued vector field V on an open subset O of the boundary of Ω, we seek a harmonic function in Ω whose gradient is close to V on O.

When Ω is a ball or a half-space, a substitute for holomorphic Hardy spaces is provided by the Stein-Weiss Hardy spaces of harmonic gradients [77]. Conformal maps are no longer available when $n > 2$, so that Ω can no longer be normalized. More general geometries than spheres and half-spaces have not been much studied so far.

On the ball, the analog of Problem (P) is

> (P_1) Let $1 \leq p \leq \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \smallsetminus O)$ be $\mathbb{R}^n$-valued vector fields. Given $M > 0$, find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \smallsetminus O)} \leq M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, Problem (P_1) was solved in [2] as well as its analog on a shell, when the tangent component of V is a gradient (when O is Lipschitz the general case follows easily from this). The solution extends the work in [38] to the 3-D case, using a generalization of Toeplitz operators. The case of the shell was motivated by applications to the processing of EEG data. An important ingredient is a refinement of the Hodge decomposition, that we call the *Hardy-Hodge* decomposition, allowing us to express a $\mathbb{R}^n$-valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H^p(B)$, a vector field in $H^p(\mathbb{R}^n \smallsetminus \overline{B})$, and a tangential divergence free vector field on S; the space of such divergence-free fields is denoted by $D(S)$. If $p = 1$ or $p = \infty$, L^p must be replaced by the real Hardy space or the space of functions with bounded mean oscillation. More generally this decomposition, which is valid on any sufficiently smooth surface (see Section 6.1), seems to play a fundamental role in inverse potential problems. In fact, it was first introduced formally on the plane to describe silent magnetizations supported in $\mathbb{R}^2$ (*i.e.* those generating no field in the upper half space) [35].

Just like solving problem (P) appeals to the solution of problem (P_0), our ability to solve problem (P_1) will depend on the possibility to tackle the special case where $O = S$:

> (P_2) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find a harmonic gradient $G \in H^p(B)$ such that $\|G - V\|_{L^p(S)}$ is minimum.

Problem (P_2) is simple when $p = 2$ by virtue of the Hardy-Hodge decomposition together with orthogonality of $H^2(B)$ and $H^2(\mathbb{R}^n \smallsetminus \overline{B})$, which is the reason why we were able to solve (P_1) in this case. Other values of p cannot be treated as easily and are still under investigation, especially the case $p = \infty$ which is of particular interest and presents itself as a 3-D analog to the Nehari problem [71].

Companion to problem (P_2) is problem (P_3) below.

> (P_3) Let $1 \leq p \leq \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find $G \in H^p(B)$ and $D \in D(S)$ such that $\|G + D - V\|_{L^p(S)}$ is minimum.

Note that (P_2) and (P_3) are identical in 2-D, since no non-constant tangential divergence-free vector field exists on T. It is no longer so in higher dimension, where both (P_2) and (P_3) arise in connection with inverse potential problems in divergence form, like source recovery in electro/magneto encephalography and paleomagnetism, see Sections 3.2.1 and 4.3.

3.3.2. *Best meromorphic and rational approximation*

The techniques set forth in this section are used to solve step 2 in Section 3.2 and they are instrumental to approach inverse boundary value problems for the Poisson equation $\Delta u = \mu$, where μ is some (unknown) measure.

3.3.2.1. *Scalar meromorphic and rational approximation*

We put R_N for the set of rational functions with at most N poles in D. By definition, meromorphic functions in $L^p(T)$ are (traces of) functions in $H^p + R_N$.

A natural generalization of problem (P_0) is:

> (P_N) Let $1 \leq p \leq \infty$, $N \geq 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and f continuous is it known how to solve (P_N) in semi-closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), which connects the spectral decomposition of Hankel operators with best approximation [71].

The case where $p = 2$ is of special importance for it reduces to rational approximation. Indeed, if we write the Hardy decomposition $f = f^+ + f^-$ where $f^+ \in H^2$ and $f^- \in H^2(\mathbb{C} \setminus \overline{D})$, then $g_N = f^+ + r_N$ where r_N is a best approximant to f^- from R_N in $L^2(T)$. Moreover, r_N has no pole outside D, hence it is a *stable* rational approximant to f^-. However, in contrast to the case where $p = \infty$, this best approximant may *not* be unique.

The former Miaou project (predecessor of Apics) designed a dedicated steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; until now it seems to be the only procedure meeting this property. This gradient algorithm proceeds recursively with respect to N on a compactification of the parameter space [30]. Although it has proved to be effective in all applications carried out so far (see Sections 4.3, 4.4), it is still unknown whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, Section 3.4.4).

In order to establish global convergence results, Apics has undertaken a deeper study of the number and nature of critical points (local minima, saddle points, ...), in which tools from differential topology and operator theory team up with classical interpolation theory [43], [45]. Based on this work, uniqueness or asymptotic uniqueness of the approximant was proved for certain classes of functions like transfer functions of relaxation systems (*i.e.* Markov functions) [47] and more generally Cauchy integrals over hyperbolic geodesic arcs [50]. These are the only results of this kind. Research by Apics on this topic remained dormant for a while by reasons of opportunity, but revisiting the work [26] in higher dimension is a worthy and timely endeavor today. Meanwhile, an analog to AAK theory was carried out for $2 \leq p < \infty$ in [46]. Although not as effective computationally, it was recently used to derive lower bounds [4]. When $1 \leq p < 2$, problem (P_N) is still quite open.

A common feature to the above-mentioned problems is that critical point equations yield non-Hermitian orthogonality relations for the denominator of the approximant. This stresses connections with interpolation, which is a standard way to build approximants, and in many respects best or near-best rational approximation may be regarded as a clever manner to pick interpolation points. This was exploited in [51], [48], and is used in an essential manner to assess the behavior of poles of best approximants to functions with branched singularities, which is of particular interest for inverse source problems (*cf.* Sections 3.4.2 and 6.1).

In higher dimensions, the analog of Problem (P_N) is best approximation of a vector field by gradients of discrete potentials generated by N point masses. This basic issue is by no means fully understood, and it is an exciting field of research. It is connected with certain generalizations of Toeplitz or Hankel operators, and with constructive approaches to so-called weak factorizations for real Hardy functions [57].

Besides, certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus (*i.e.* a Schur function). In particular, Schur interpolation lately received renewed attention from the team, in connection with matching problems. There, interpolation data are subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix), and the main difficulty is to put interpolation points on the boundary of D while controlling both the degree and the extremal points (peak points for the modulus) of the interpolant. Results obtained by Apics in this direction generalize a variant of contractive interpolation with degree constraint as studied in [62]. We mention that contractive interpolation with nodes approaching the boundary has been a subsidiary research topic by the team in the past, which plays an interesting role in the spectral representation of certain non-stationary stochastic processes [34], [37].

3.3.2.2. Matrix-valued rational approximation

Matrix-valued approximation is necessary to handle systems with several inputs and outputs but it generates additional difficulties as compared to scalar-valued approximation, both theoretically and algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the usual notion of degree for rational functions. For instance when poles are simple, the McMillan degree is the sum of the ranks of the residues.

The basic problem that we consider now goes as follows: *let* $\mathcal{F} \in (H^2)^{m \times l}$ *and* n *an integer; find a rational matrix of size* $m \times l$ *without poles in the unit disk and of McMillan degree at most* n *which is nearest possible to* $\mathcal{F}$ *in* $(H^2)^{m \times l}$. Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm derived in [30] and mentioned in Section 3.3.2.1 generalizes to the matrix-valued situation [60]. The first difficulty here is to parametrize inner matrices (*i.e.* matrix-valued functions analytic in the unit disk and unitary on the unit circle) of given McMillan degree degree n. Indeed, inner matrices play the role of denominators in fractional representations of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree is a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (local parametrizations) and to handle changes of charts in the course of the algorithm. Such parametrization can be obtained using interpolation theory and Schur-type algorithms, the parameters of which are vectors or matrices ([24], [63], [68]). Some of these parametrizations are also interesting to compute realizations and achieve filter synthesis ([63], [68]). The rational approximation software "RARL2" developed by the team is described in Section 3.4.4.

Difficulties relative to multiple local minima of course arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of degree n or small perturbations thereof (the consistency problem) was solved in [44]. Matrix-valued Markov functions are the only known example beyond this one [27].

Let us stress that RARL2 seems the only algorithm handling rational approximation in the matrix case that demonstrably converges to a local minimum while meeting stability constraints on the approximant. It is still a working pin of many developments by Factas on frequency optimization and design.

3.3.3. Behavior of poles of meromorphic approximants

Participant: Laurent Baratchart.

We refer here to the behavior of poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. Normalizing the contour to be the unit circle T, we are back to Problem (P_N) in Section 3.3.2.1; invariance of the latter under conformal mapping was established in [42]. Research so far has focused on functions whose singular

set inside the contour is polar, meaning that the function can be continued analytically (possibly in a multiple-valued manner) except over a set of logarithmic capacity zero.

Generally speaking in approximation theory, assessing the behavior of poles of rational approximants is essential to obtain error rates as the degree goes large, and to tackle constructive issues like uniqueness. However, as explained in Section 3.2.1, the original twist by Apics, now Factas, is to consider this issue also as a means to extract information on singularities of the solution to a Dirichlet-Neumann problem. The general theme is thus: *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see Section 4.3). It can be used as a computationally cheap initial condition for more precise but much heavier numerical optimizations which often do not even converge unless properly initialized. As regards crack detection or source recovery, this approach boils down to analyzing the behavior of best meromorphic approximants of given pole cardinality to a function with branch points, which is the prototype of a polar singular set. For piecewise analytic cracks, or in the case of sources, we were able to prove ([7], [42], [36]), that the poles of the approximants accumulate, when the degree goes large, to some extremal cut of minimum weighted logarithmic capacity connecting the singular points of the crack, or the sources [39]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution on this cut in D, therefore it charges the singular points if one is able to approximate in sufficiently high degree (this is where the method could fail, because high-order approximation requires rather precise data).

The case of two-dimensional singularities is still an outstanding open problem.

It is remarkable that inverse source problems inside a sphere or an ellipsoid in 3-D can be approached with such 2-D techniques, as applied to planar sections, see Section 6.1. The technique is implemented in the software FindSources3D, see Section 3.4.2.

3.4. Software tools of the team

In addition to the above-mentioned research activities, Factas develops and maintains a number of long-term software tools that either implement and illustrate effectiveness of the algorithms theoretically developed by the team or serve as tools to help further research by team members. We present briefly the most important of them.

3.4.1. *DEDALE-HF*

Scientific Description

Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the coupling matrix synthesis problem (C.M. problem for short) associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. corresponding to the user-specified filter characteristics. The reference files are computed off-line using Gröbner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such continuation techniques, combined with an efficient implementation of the integrator, drastically reduces the computational time.

Dedale-HF has been licensed to, and is currently used by TAS-Espana

Functional Description

Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the filtering community. Given a coupling topology, the coupling matrix synthesis problem consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics. Solving the latter is crucial during the design step of a filter in order to derive its physical dimensions, as well as during the tuning process where coupling values need to be extracted from frequency measurements.

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- URL: http://www-sop.inria.fr/apics/Dedale/

3.4.2. *FindSources3D*

KEYWORDS: Health - Neuroimaging - Visualization - Compilers - Medical - Image - Processing
FindSources3D is a software program dedicated to the resolution of inverse source problems in electroencephalography (EEG). From pointwise measurements of the electrical potential taken by electrodes on the scalp, FindSources3D estimates pointwise dipolar current sources within the brain in a spherical model.

After a first data transmission "cortical mapping" step, it makes use of best rational approximation on 2-D planar cross-sections and of the software RARL2 in order to locate singularities. From those planar singularities, the 3-D sources are estimated in a last step, see [8].

The present version of FindSources3D (called FindSources3D-bolis) provides a modular, ergonomic, accessible and interactive platform, with a convenient graphical interface and a tool that can be distributed and used, for EEG medical imaging. Modularity is now granted (using the tools dtk, Qt, with compiled Matlab libraries). It offers a detailed and nice visualization of data and tuning parameters, processing steps, and of the computed results (using VTK).

A new version is being developed that will incorporate a first Singular Value Decomposition (SVD) step in order to be able to handle time dependent data and to find the corresponding principal static components.

- Participants: Juliette Leblond, Maureen Clerc (team Athena, Inria Sophia), Jean-Paul Marmorat, Théodore Papadopoulo (team Athena).
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- URL: http://www-sop.inria.fr/apics/FindSources3D/en/index.html

3.4.3. *PRESTO-HF*

SCIENTIFIC DESCRIPTION

For the matrix-valued rational approximation step, Presto-HF relies on RARL2. Constrained realizations are computed using the Dedale-HF software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following assumption: far off the pass-band, one can reasonably expect a good approximation of the rational components of S11 and S22 by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox has been licensed to, and is currently used by Thales Alenia Space in Toulouse and Madrid, Thales airborne systems and Flextronics (two licenses). XLIM (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements have been granted to the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

FUNCTIONAL DESCRIPTION

Presto-HF is a toolbox dedicated to low-pass parameter identification for microwave filters. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single stroke:

• Determination of delay components caused by the access devices (automatic reference plane adjustment),

• Automatic determination of an analytic completion, bounded in modulus for each channel,

• Rational approximation of fixed McMillan degree,

• Determination of a constrained realization.

- Participants: Fabien Seyfert, Jean-Paul Marmorat and Martine Olivi
- Contact: Fabien Seyfert
- URL: https://project.inria.fr/presto-hf/

3.4.4. *RARL2*

Réalisation interne et Approximation Rationnelle L2

Scientific Description

The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in Matlab, is based on state-space representations.

RARL2 performs the rational approximation step in the software tools PRESTO-HF and FindSources3D. It is distributed under a particular license, allowing unlimited usage for academic research purposes. It was released to the universities of Delft and Maastricht (the Netherlands), Cork (Ireland), Brussels (Belgium), Macao (China) and BITS-Pilani Hyderabad Campus (India).

Functional Description

RARL2 is a software for rational approximation. It computes a stable rational L2-approximation of specified order to a given L2-stable (L2 on the unit circle, analytic in the complement of the unit disk) matrix-valued function. This can be the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

• its internal realization,

• its first N Fourier coefficients,

• discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L2 norm.

It thus performs model reduction in the first or the second case, and leans on frequency data identification in the third. For band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation.

An appropriate Möbius transformation allows to use the software for continuous-time systems as well.

- Participants: Jean-Paul Marmorat and Martine Olivi
- Contact: Martine Olivi
- URL: http://www-sop.inria.fr/apics/RARL2/rarl2.html

3.4.5. *Sollya*

Keywords: Numerical algorithm - Supremum norm - Curve plotting - Remez algorithm - Code generator - Proof synthesis

Functional Description

Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, i.e. the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

Among other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. As well, it provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license.

- Participants: Sylvain Chevillard, Christoph Lauter, Mioara Joldes and Nicolas Jourdan
- Partners: CNRS - ENS Lyon - UCBL Lyon 1
- Contact: Sylvain Chevillard
- URL: http://sollya.gforge.inria.fr/

4. Application Domains

4.1. Introduction

Application domains are naturally linked to the problems described in Sections 3.2.1 and 3.2.2. By and large, they split into a systems-and-circuits part and an inverse-source-and-boundary-problems part, united under a common umbrella of function-theoretic techniques as described in Section 3.3.

4.2. Inverse magnetization problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Konstantinos Mavreas.

Generally speaking, inverse potential problems, similar to the one appearing in Section 4.3, occur naturally in connection with systems governed by Maxwell's equation in the quasi-static approximation regime. In particular, they arise in magnetic reconstruction issues. A specific application is to geophysics, which led us to form the Inria Associate Team IMPINGE (Inverse Magnetization Problems IN GEosciences) together with MIT and Vanderbilt University. A recent collaboration with Cerege (CNRS, Aix-en-Provence), in the framework of the ANR-project MagLune, completes this picture, see Sections 6.1.2, 8.2.1.

To set up the context, recall that the Earth's geomagnetic field is generated by convection of the liquid metallic core (geodynamo) and that rocks become magnetized by the ambient field as they are formed or after subsequent alteration. Their remanent magnetization provides records of past variations of the geodynamo, which is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

For a long time, paleomagnetic techniques were only capable of analyzing bulk samples and compute their net magnetic moment. The development of SQUID microscopes has recently extended the spatial resolution to sub-millimeter scales, raising new physical and algorithmic challenges. The associate team IMPINGE aims at tackling them, experimenting with the SQUID microscope set up in the Paleomagnetism Laboratory of the department of Earth, Atmospheric and Planetary Sciences at MIT. Typically, pieces of rock are sanded down to a thin slab, and the magnetization has to be recovered from the field measured on a planar region at small distance from the slab.

Mathematically speaking, both inverse source problems for EEG from Section 4.3 and inverse magnetization problems described presently amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the potential:

$$\int_\Omega \frac{\operatorname{div} m(x')\, dx'}{|x-x'|}\,, \tag{1}$$

outside the volume Ω of the object. The difference is that the distribution m is located in a volume in the case of EEG, and on a plane in the case of rock magnetization. This results in quite different identifiability properties, see [35] and Section 6.1.1, but the two situations share a substantial mathematical common core.

Another timely instance of inverse magnetization problems lies with geomagnetism. Satellites orbiting around the Earth measure the magnetic field at many points, and nowadays it is a challenge to extract global information from those measurements. In collaboration with C. Gerhards (since September with the Geomathematics and Geoinformatics Group, Technische Universität Bergakademie Freiberg, Germany; until then, Univ. of Vienna, Austria), we started to work on the problem of separating the magnetic field due to the magnetization of the globe's crust from the magnetic field due to convection in the liquid metallic core. The techniques involved are variants, in a spherical context, from those developed within the IMPINGE associate team for paleomagnetism, see Section 6.1.1.

4.3. Inverse source problems in EEG

Participants: Laurent Baratchart, Juliette Leblond, Jean-Paul Marmorat.

This work is conducted in collaboration with Maureen Clerc and Théo Papadopoulo from the team Athena (Inria Sophia).

Solving overdetermined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see Section 3.2.1) is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG, see [8]. Indeed, the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary shell down to the center of the domain where the singularities (*i.e.* the sources) lie. Once propagated to the innermost sphere, it turns out that traces of the boundary data on 2-D cross sections coincide with analytic functions with branched singularities in the slicing plane [7], [40]. The singularities are related to the actual location of the sources, namely their moduli reach in turn a maximum when the plane contains one of the sources. Hence we are back to the 2-D framework of Section 3.3.3, and recovering these singularities can be performed *via* best rational approximation. The goal is to produce a fast and sufficiently accurate initial guess on the number and location of the sources in order to run heavier descent algorithms on the direct problem, which are more precise but computationally costly and often fail to converge if not properly initialized. Our belief is that such a localization process can add a geometric, valuable piece of information to the standard temporal analysis of EEG signal records.

Numerical experiments obtained with our software FindSources3D give very good results on simulated data and we are now engaged in the process of handling real experimental data (see Sections 3.4.2 and 6.1), in collaboration with our partners at INS, hospital la Timone, Marseille.

4.4. Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Jean-Paul Marmorat, Martine Olivi, Fabien Seyfert.

This is joint work with Stéphane Bila (XLIM, Limoges).

One of the best training grounds for function-theoretic applications by the team is the identification and design of physical systems whose performance is assessed frequency-wise. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study of the Helmholtz equation states that an essentially discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected).

Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since screws are conductors, they behave as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of an iris is opposite to that of a screw: no condition is imposed on a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued once a satisfactory response has been obtained. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 GHz.

Figure 1. Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

Near the resonance frequency, a good approximation to the Helmholtz equations is given by a second order differential equation. Thus, one obtains an electrical model of the filter as a sequence of electrically-coupled resonant circuits, each circuit being modeled by two resonators, one per mode, the resonance frequency of which represents the frequency of a mode, and whose resistance accounts for electric losses (surface currents) in the cavities.

This way, the filter can be seen as a quadripole, with two ports, when plugged onto a resistor at one end and fed with some potential at the other end. One is now interested in the power which is transmitted and reflected. This leads one to define a scattering matrix S, which may be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The matrix S is approximately rational of order 4 times the number of cavities (that is 16 in the example on Figure 2), and the key step consists in expressing the components of the equivalent electrical circuit as functions of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the quality of a design, in particular the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may no longer have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the strategy for identification is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80MHz in the example).
- Solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. Section 3.3.1) in order to get a models for the scattering matrix as an analytic matrix-valued function. This provides

us with a scattering matrix known to be close to a rational matrix of order roughly 1/4 of the number of data points.

- Approximating this scattering matrix by a true rational transfer-function of appropriate degree (8 in this example) via the Endymion or RARL2 software (cf. Section 3.3.2.2).

- A state space realization of S, viewed as a transfer function, can then be obtained, where additional symmetry constraints coming from the reciprocity law and possibly other physical features of the device have to be imposed.

- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).
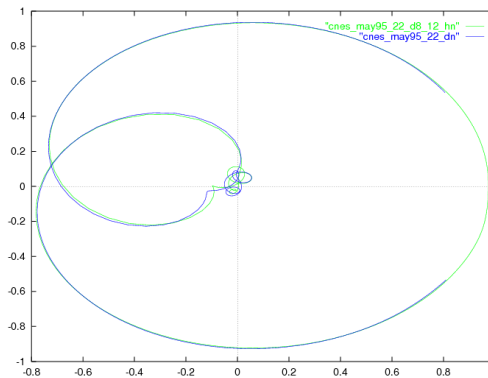

Figure 2. Nyquist Diagram. Rational approximation (degree 8) and data - S_{22}.

The final approximation is of high quality. This can be interpreted as a confirmation of the linearity assumption on the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2). Non-physical couplings are less than 10^{-2}.

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, which are useful for the synthesis of repeating devices.

The team further investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was proposed, in order to derive guaranteed optimal multi-band filter responses subject to modulus constraints [10]. This generalizes the classical single band design techniques based on Chebyshev polynomials and elliptic functions. The approach relies on the fact that the modulus of the scattering parameter $|S_{1,2}|$ admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$, namely

$$|S_{1,2}|^2 = \frac{1}{1 + D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2, the numerator of the reflection and transmission scattering factors, that may be chosen freely. The denominator q is then obtained as the unique stable unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1 p_1^* + p_2 p_2^*.$$

The relative simplicity of the derivation of a filter's response, under modulus constraints, owes much to the possibility of forgetting about Feldtkeller's equation and express all design constraints in terms of the filtering function. This no longer the case when considering the synthesis N-port devices for $N > 3$, like multiplexers, routers and power dividers, or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is the subject of recent investigation by Factas, using techniques based on constrained Nevanlinna-Pick interpolation (see Section 6.2).

Through contacts with CNES (Toulouse) and UPV (Bilbao), Apics got additionally involved in the design of amplifiers which, unlike filters, are active devices. A prominent issue here is stability. A twenty years back, it was not possible to simulate unstable responses, and only after building a device could one detect instability. The advent of so-called *harmonic balance* techniques, which compute steady state responses of linear elements in the frequency domain and look for a periodic state in the time domain of a network connecting these linear elements *via* static non-linearities made it possible to compute the harmonic response of a (possibly nonlinear and unstable) device [78]. This has had tremendous impact on design, and there is a growing demand for software analyzers. The team is also becoming active in this area.

In this connection, there are two types of stability involved. The first is stability of a fixed point around which the linearized transfer function accounts for small signal amplification. The second is stability of a limit cycle which is reached when the input signal is no longer small and truly nonlinear amplification is attained (*e.g.* because of saturation). Work by the team so far has been concerned with the first type of stability, and emphasis is put on defining and extracting the "unstable part" of the response, see Section 6.3. The stability check for limit cycles is now under investigation.

5. New Software and Platforms

5.1. pisa

KEYWORDS: Electrical circuit - Stability
FUNCTIONAL DESCRIPTION: To minimise prototyping costs, the design of analog circuits is performed using computer-aided design tools which simulate the circuit's response as accurately as possible.

Some commonly used simulation tools do not impose stability, which can result in costly errors when the prototype turns out to be unstable. A thorough stability analysis is therefore a very important step in circuit design. This is where pisa is used.

pisa is a Matlab toolbox that allows designers of analog electronic circuits to determine the stability of their circuits in the simulator. It analyses the impedance presented by a circuit to determine the circuit's stability. When an instability is detected, pisa can estimate location of the unstable poles to help designers fix their stability issue.

RELEASE FUNCTIONAL DESCRIPTION: First version

- Authors: Adam Cooman, David Martinez Martinez, Fabien Seyfert and Martine Olivi
- Contact: Fabien Seyfert
- Publications: Model-Free Closed-Loop Stability Analysis: A Linear Functional Approach - On Transfer Functions Realizable with Active Electronic Components
- URL: https://project.inria.fr/pisa

5.2. PUMA-HF

PUMA-HF: Passive Uniform Matching
KEYWORD: Transfer functions

FUNCTIONAL DESCRIPTION: PUMA computes a passive rational 2-port filtering function presenting a reference impedance (i.e. 50 Ohm) at the first port, and the conjugate of the given load impedance within a frequency band at the second port.

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- Publication: Synthesis Method for Matching Filters
- URL: https://project.inria.fr/puma/

6. New Results

6.1. Inverse problems for Poisson-Laplace equations

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Jean-Paul Marmorat, Konstantinos Mavreas.

6.1.1. Inverse magnetization issues from planar data

This work has been carried out in the framework of the Inria Associate Team IMPINGE, comprising Cauê Borlina, Eduardo Andrade Lima and Benjamin Weiss from the Earth Sciences department at MIT (Boston, USA) and Douglas Hardin, Edward Saff and Cristobal Villalobos from the Mathematics department at Vanderbilt University (Nashville, USA).

The overall goal of IMPINGE was to determine magnetic properties of rock samples (*e.g.* meteorites or stalactites), from weak field measurements close to the sample that can nowadays be obtained using SQUIDs (superconducting quantum interference devices). Depending on the geometry of the rock sample, the magnetization distribution can either be considered to lie in a plane or in a parallelepiped of thickness r. Some of our results apply to both frameworks (the former appears as a limiting case when r goes to 0), while others concern the 2-D case and have no 3-D counterpart as yet.
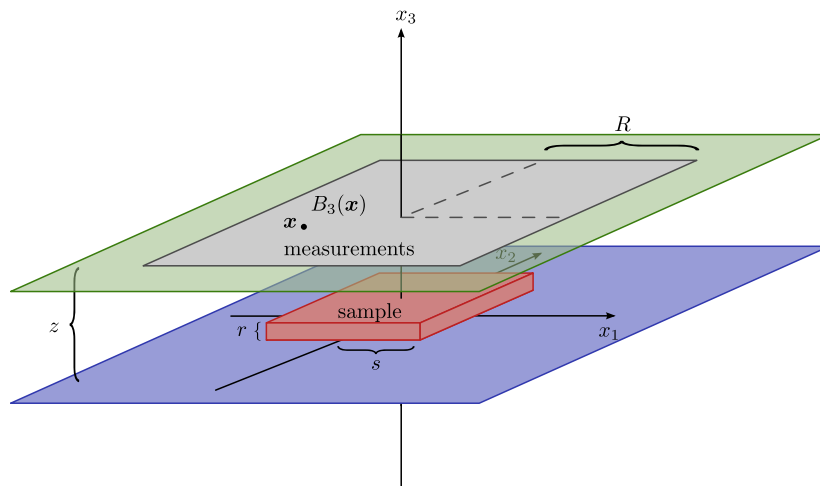

Figure 3. Schematic view of the experimental setup

Figure 3 presents a schematic view of the experimental setup: the sample lies on a horizontal plane at height 0 and its support is included in a parallelepiped. The vertical component B_3 of the field produced by the sample is measured in points of a horizontal square at height z.

We pursued our research efforts towards designing algorithms for net moment recovery. The net moment is the integral of the magnetization over its support, and it is a valuable piece of information to physicists which has the advantage of being determined solely by the field: whereas two different magnetizations can generate the same field, the net moment depends only on the field and not on which magnetization produced it. Hence the goal may be described as to build a numerical magnetometer, capable of analyzing data close to the sample. This is in contrast to classical magnetometers which regard the latter as a single dipole, an approximation which is only valid away from the sample and is not suitable to handle weak fields which get quickly blurred by ambient magnetic sources if one measures the field at a distance from the sample.

The first approach consists in using the fact that the integral of B_3 against polynomials of order less or equal to 1 on some domains symmetric with respect to the origin provides an estimate of the net moment, asymptotically when R grows large [20], [72]. This year, on the one hand, we conducted with our colleagues at MIT a campaign of fairly systematic tests (with various sensitivity parameters for the sensor, step size between measurement points, overall size of the measurement rectangle, etc.) to observe how our method behaves on true data. The results are overall very good when the signal-to-noise ratio is not poor; however, this revealed a few situations where the observed asymptotics does not fit the theoretical one, and we have currently no clue of the reason of this phenomenon (is it a manipulation error on some of the samples of our campaign? a bug in our implementation? a noise on the data with a surprisingly large effect on the asymptotic? the fact that the asymptotic regime was not yet reached?) Understanding this phenomenon is still an on-going work. On the other hand, we spotted in the literature approaches that are somehow similar to ours: they compute the same integral, but on the whole plane, and try to account for the finiteness of the measurement rectangle by more or less heuristic methods. As the finiteness of the rectangle is built-in in our approach (we exploit the fact the nature of the asymptotic behavior with respect to R is analytically known), we have good hope that our method should compare favorably against its competitors, but we did not conduct systematic tests yet.

The second approach attempts to generalize the previous expansions in the case when R is moderately large (but only in the thin slab framework, modeling the sample as a rectangle). For this purpose, we setup a bounded extremal problem (BEP, see Section 3.3.1) and submitted an article last year on the subject. It has been accepted for publication this year, see [12].

A third approach developed during the previous years was to design an alternate procedure to compute a good linear estimator, dwelling on expansions on a family of piecewise affine functions, with a restricted number of pieces. A key point here is that it is possible to derive explicit formulas for the adjoint operator B_3^* (in appropriate L^2 spaces) to the operator B_3 mapping a magnetization to the vertical component of the field, when applied to polynomials. We derived this year explicit recurrence formulas that allow one to efficiently compute B_3^* of a polynomial of any degree in linear time with respect to the number of monomials. We currently only have draft notes of this research.

Concerning full inversion of thin samples, after preliminary experiments on regularization with L^1 constraints (a heavy trend in linear inverse problems today to favor sparse solutions), we started studying magnetizations modeled by signed measures. A loop decomposition of silent sources was obtained, which sharpens in 2-D setting the structure theorem of [76]. Moreover, a characterization of equivalent sources having minimal total variation has been obtained when the support of the magnetization is very scattered (namely: purely 1-unrectifiable, which holds in particular for dipolar models) and also for certain magnetizations of physical interest like unidirectional ones. Thus, it seems that constraining the total variation to regularize the recovery process is appropriate in some important cases. The theoretical analysis has shown that the optimum is then always sparse, in that it has Hausdorff dimension at most 1. This stems from the real analyticity of the operators relating the magnetization to the field, which prevents them from assuming constant level on large sets. Moreover, we proved that the argument of the minimum of the regularized criterion $\|f - B_3\mu\|_2^2 + \lambda\|\mu\|_{TV}$ is unique; here, μ is the measure representing the magnetization with respect to which the criterion gets optimized, f is the data and $\lambda > 0$ a regularization parameter, while $\|\mu\|_{TV}$ is the total variation of μ. An

implementation is currently being set up with promising results, discretization beforehand on a fixed grid. Yet, a deeper understanding on how to adjust the parameters of the method is required. This topic is studied in collaboration with D. Hardin and C. Villalobos from Vanderbilt University. [21].

Besides, we considered a simplified 2-D setup for magnetizations and magnetic potentials (of which the magnetic field is the gradient). When both the sample and the measurement set are parallel intervals, some best approximation issues related to inverse recovery and relevant BEP problems in Hardy classes of holomorphic functions (see Section 3.3.1). Note that, in the present case, the criterion no longer acts on the boundary of the holomorphy domain (namely, the upper half-plane), but on a strict subset thereof, while the constraint acts on the support of the approximating function. Both involve functions in the Hilbert Hardy space of the upper half-plane. This is the subject of ongoing work with E. Pozzi (Department of Mathematics and Statistics, St Louis Univ., St Louis, Missouri, USA).

For magnetizations supported in a volume Ω with boundary $\partial\Omega$, there is a greater variety of silent sources, since they have much more space to live in. Now, to each magnetization m supported in Ω there is a unique magnetization supported on $\partial\Omega$ (the balayage of m) and producing the same field outside Ω. Thus, describing silent sources supported on $\partial\Omega$ is a way to factor out some of the complexity of the situation. When m is located in the plane, the Hardy-Hodge decomposition introduced in [35] (see Section 3.3.1) was used there to characterize all silent magnetizations from above (resp. below) as being those having no harmonic gradient from below (resp. above) in their decomposition. When m is supported on a closed compact surface, a similar decomposition exists for $\mathbb{R}^3$-valued vector fields on $\partial\Omega$, (see Section 6.4), that allows us to characterize all magnetizations on $\partial\Omega$ which are silent from outside as being those whose harmonic components satisfy a certain spectral relation for the double layer potential on $\partial\Omega$. The significance and the algorithmic implications of that equation are under study.

Other types of inverse magnetization problems can be tackled using such techniques, in particular global Geomagnetic issues which arise in spherical geometry. In collaboration with C. Gerhards from the Technische Universität Bergakademie Freiberg (Germany), we developed a method to separate the crustal component of the Earth's magnetic field from its core component, if an estimate of the field is known on a subregion of the globe [33]. This assumption is not unrealistic: parts of Australia and of northern Europe are considered as fairly well understood from the magnetostatic view point. We are currently working to test the algorithm against real data, in collaboration with Geophysicists.

6.1.2. *Inverse magnetization issues from sparse cylindrical data*

The team Factas is a partner of the ANR project MagLune on Lunar magnetism, headed by the Geophysics and Planetology Department of Cerege, CNRS, Aix-en-Provence (see Section 8.2.1). Recent studies let geoscientists think that the Moon used to have a magnetic dynamo for a while. However, the exact process that triggered and fed this dynamo is still not understood, much less why it stopped. The overall goal of the project is to devise models to explain how this dynamo phenomenon was possible on the Moon.

The geophysicists from Cerege went a couple of times to NASA to perform measurements on a few hundreds of samples brought back from the Moon by Apollo missions. The samples are kept inside bags with a protective atmosphere, and geophysicists are not allowed to open the bags, nor to take out samples from NASA facilities. Moreover, the process must be carried out efficiently as a fee is due to NASA by the time when handling these moon samples. Therefore, measurements were performed with some specific magnetometer designed by our colleagues from Cerege. This device measures the components of the magnetic field produced by the sample, at some discrete set of points located on circles belonging to three cylinders (see Figure 4). The objective of Factas is to enhance the numerical efficiency of post-processing data obtained with this magnetometer.

Under the hypothesis that the field can be well explained by a single magnetic pointwise dipole, and using ideas similar to those underlying the FindSources3D tool (see Sections 3.4.2 and 6.1.3), we try to recover the position and the moment of the dipole using the available measurements. This is still on-going work which constitutes the main topic of the PhD thesis of K. Mavreas.
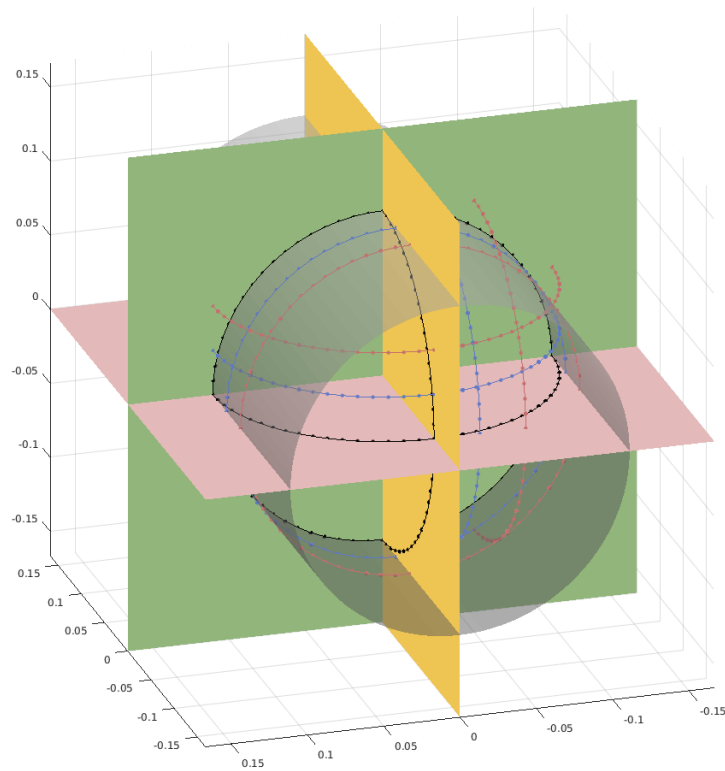
Figure 4. Typical measurements obtained with the instrument of Cerege. Measurements of the field are performed on nine circles, given as sections of three cylinders. On each circle, only one component of the field is measured: the component B_h along the axis of the corresponding cylinder (blue points), the component B_n radial with respect to the circle (black points), or the component B_τ tangential to the circle (red points).

In a given cylinder, using the associated cylindrical system of coordinates, recovering the position of the dipole boils down to determine its height z, its radial distance ρ and its azimuth ϕ. We use a rational approximation technique which, for the circle of measurements at height h, gives us an estimate of the complex number $u_h = \frac{1+\rho^2+(h-z)^2}{\rho} e^{i\phi}$, from which ϕ is directly obtained. Besides, from the relation $\rho|u_h| = 1+\rho^2+(h-z)^2$, we see that the point (ρ, z) lies on a circle C_h. Therefore, with measurements at three different heights h_i $(i = 1, 2, 3)$, we can in principle recover (ρ, z) as the intersection of the three circles C_{h_1}, C_{h_2} and C_{h_3}.

In practice, due to the many sources of imprecision (the first of all being that the field is not truly generated by a single dipole), the circles do not all truly intersect. This year, we studied three different strategies to estimate the pseudo-intersection point of the circles. In the plane, for a point P and a circle C of center O and radius R, we define $d(P, C) = |\|PO\| - R|$ where $\| \cdot \|$ denotes the Euclidean distance, and $h(P, C) = \|PO\|^2 - R^2$ and we formulate the problem of finding a pseudo-intersection point between C_{h_1}, C_{h_2} and C_{h_3} as either:

1. finding a point P that minimizes $d(P, C_{h_1}) + d(P, C_{h_2}) + d(P, C_{h_3})$;
2. finding a point P for which $h(P, C_{h_1}) = h(P, C_{h_2}) = h(P, C_{h_3})$;
3. finding a point P that minimizes $h(P, C_{h_1})^2 + h(P, C_{h_2})^2 + h(P, C_{h_3})^2$.

The first case turns out to actually be a generalization of two classical concepts: the Fermat point (or Torricelli point) of a triangle, and the Alhazen optical problem. The second case corresponds to a classical notion called the *radical center* of three circles (intersection of the three corresponding radical axes). Finally, the third case does not seem to have a documented solution. We solve it by writing the algebraic system of two equations corresponding to the critical points of the function, after an appropriate change of coordinates in order to reduce the degree. Finally, we get a superset of the solutions by estimating the roots of the resultant of both polynomials. First experiments showed that the third formulation led to the most satisfying estimate of the pseudo-intersection. We also implemented a heuristic numerical procedure (without theoretical formulas for its solution) to estimate the point P that minimizes $d(P, C_{h_1})^2 + d(P, C_{h_2})^2 + d(P, C_{h_3})^2$, and it also gives fairly acceptable estimates. This work has not yet been submitted for publication.

Another important part of our work this year has been to extensively test our implementation of the rational approximation procedure which is at the heart of our method (and which is also used for the problem described in Section 6.1.3). These tests allowed us to detect situations in which the algorithm was falling into an infinite loop or was converging towards a local minimum that was not really the best approximation. It also revealed that all initialization strategies for the iterative optimization algorithm were not equally sensitive to the noise. This led us to redesign our implementation.

Finally, the article that we submitted last year, with a rudimentary approach to recover ρ and z from the data obtained at several heights, has been accepted and will be published soon, see [14].

6.1.3. Inverse problems in medical imaging

In 3-D, functional or clinically active regions in the cortex are often modeled by pointwise sources that have to be localized from measurements, taken by electrodes on the scalp, of an electrical potential satisfying a Laplace equation (EEG, electroencephalography). In the works [7], [40] on the behavior of poles in best rational approximants of fixed degree to functions with branch points, it was shown how to proceed via best rational approximation on a sequence of 2-D disks cut along the inner sphere, for the case where there are finitely many sources (see Section 4.3).

In this connection, a dedicated software FindSources3D (FS3D, see Section 3.4.2) is being developed, in collaboration with the Inria team Athena and the CMA - Mines ParisTech. In addition to the Matlab version of FS3D, a new (C++) version of the software that automatically performs the estimation of the quantity of sources is being built, specifically this year in the framework of the AMDT Bolis2 ("Action Mutualisée de Développement Technologique", "Boîte à Outils Logiciels pour l'Identification de Sources"), together with engineers from the SED (Service d'Expérimentation et de Développement) of the Research Center. This new version, still under development, is modular, portable and possesses a nice GUI (using Qt5, dtk, vtk), while non regression (continuous integration) is ensured.

It appears that, in the rational approximation step, *multiple* poles possess a nice behavior with respect to branched singularities. This is due to the very physical assumptions on the model from dipolar current sources: for EEG data that correspond to measurements of the electrical potential, one should consider *triple* poles; this will also be the case for MEG – magneto-encephalography – data. However, for (magnetic) field data produced by magnetic dipolar sources, like in Section 6.1.2, one should consider poles of order five. Though numerically observed in [8], there is no mathematical justification so far why multiple poles generate such strong accumulation of the poles of the approximants. This intriguing property, however, is definitely helping source recovery and will be the topic of further study. It is used in order to automatically estimate the "most plausible" number of sources (numerically: up to 3, at the moment). Last but not least, the version of the software currently under development takes as inputs actual EEG measurements, like time signals, and performs a suitable singular value decomposition in order to separate independent sources.

Magnetic data from MEG recently became available along with EEG data, by our medical partners at INS in Marseille; indeed, it is now possible to use simultaneously both measurement devices, in order to measure both the electrical potential and a component of the magnetic field (its normal component on the MEG helmet, that can be assumed to be spherical). This should enhance the accuracy of our source recovery algorithms. We will add the treatment of MEG data as another functionality of the software FS3D.

Concerning dipolar source estimation from EEG, joint work with Marion Darbas (Univ. Picardie Jules Verne, Laboratoire Amiénois de Mathématique Fondamentale et Appliquée, LAMFA) is in progress for neonates data and models. Their specificity is that the skull does not have a constant conductivity (at the fontanels location, the bone is spongy). We pursue together a study of the influence of the skull conductivity on the inverse EEG problem, using in particular FS3D, see also [70].

We also consider non quasi-static models in order to more precisely analyze the time influence on the behavior of the solutions to the inverse source problems in EEG and MEG. This is current work with Iannis Stratis and Atanasios Yannacopoulos (National and Kapodistrian University of Athens, Greece, Department of Mathematics).

6.2. Matching problems and their applications

Participants: Laurent Baratchart, Martine Olivi, Gibin Bose, David Martinez Martinez, Fabien Seyfert.

Filter synthesis is usually performed under the hypothesis that both ports of the filter are loaded on a constant resistive load (usually 50 Ohm). In complex systems, filters are however cascaded with other devices, and end up being loaded, at least at one port, on a non purely resistive frequency varying load. For example, in an emitter-receiver, the antenna is followed by a filter. Whereas the antenna can usually be regarded as a resistive load at some frequencies, this is far from being true on the whole pass-band. A mismatch between the antenna and the filter, however, causes irremediable power losses, both in emission and transmission. Antennas are not the only systems where matching is of importance: in multiplexer design, one of the most complicated problems among microwave device synthesis, each filter is plugged at one of its accesses on a load made of the common manifold and all other channel filters. This load is far from being matched, and leads to a complex simultaneous matching problem, of all filters connected via the common manifold junction. Our goal is therefore to develop a method for filter synthesis that allows us to match varying loads on specific frequency bands, while enforcing some rejection properties away from the pass-band.

Figure 5 shows a filter with scattering matrix S, plugged at its right port on a frequency varying load with reflection parameter $L_{1,1}$. If the filter is lossless, simple algebraic manipulations show that on the frequency axis the reflex-ion parameter satisfies:

$$|G_{1,1}| = \left| \frac{S_{2,2} - \overline{L_{1,1}}}{1 - S_{2,2}L_{1,1}} \right| = \delta(\overline{L_{1,1}}, S_{2,2}).$$
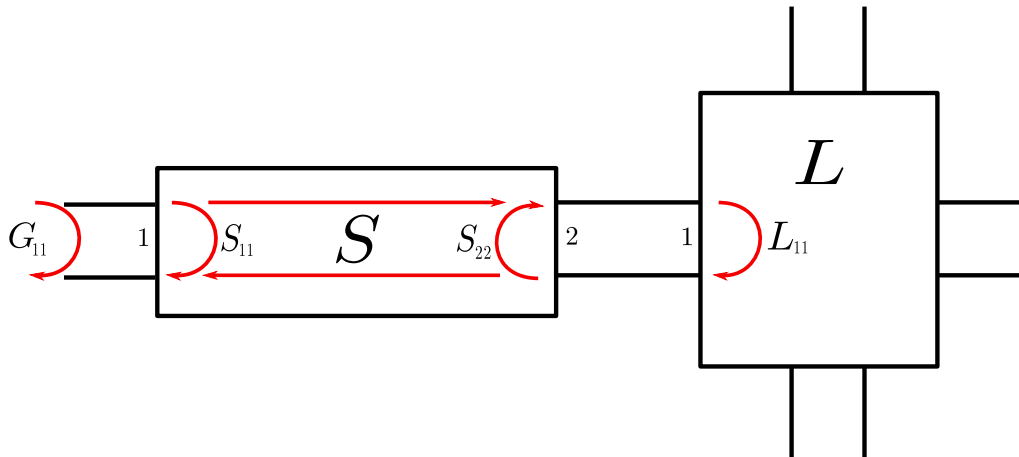
Figure 5. Filter plugged on a system with reflection coefficient L_{11}

The matching problem of minimizing $|G_{1,1}|$ amounts therefore to minimize the pseudo-hyperbolic distance δ between the filter's reflection parameter $S_{2,2}$ and the conjugate of the load's reflection $\overline{L_{1,1}}$, on a given frequency band. On the contrary enforcing a rejection level on a stop band, amounts to maintaining the value of $\delta(L_{1,1}, S_{2,2})$ above a certain threshold on this frequency band. For a broad class of filters, namely those that can be modeled by a circuit of n coupled resonators, the scattering matrix S is a rational function of McMillan degree n in the frequency variable. The matching problem thus appears to be a rational approximation problem in the hyperbolic metric.

6.2.1. *Multiplexer synthesis via interpolation and common junction design*

Based on our work linking Nevanlinna-Pick interpolation and point-wise matching techniques [6] we tackled the synthesis of a multiplexer on the basis of frequency specifications relative to a triplexer furnished by CNES and deemed to be problematic. Theoretical results guaranty the existence in this case, under a strict contractivity hypothesis on the common junction, of a simultaneous pointwise matching solution of all three channel filters. This result is however not constructive as it relies on Brouwer's fixed point theorem, a purely topological argument. In the context of the PhD of D. Martinez, we developed a continuation algorithm, starting from a completely decoupling junction and ending up with the manifold junction of interest. When the junction decouples all the channels, the overall matching problem results in 3 independent Nevanlinna-Pick interpolation problems as described in [6]: this yield the starting point of our continuation approach. The manifold peaks have also been identified as crucial here: these are resonances taking place in the common junction of the multiplexer, at which transmission of energy becomes impossible between a given channel and the common port. These peaks have been mathematically characterized, and a systematic combinatorial algorithm designed for the synthesis of a "peak free" manifold junction. In particular it was proven, that an extremely compact junction is needed to fit the specifications furnished by CNES: when dispersive effects increase (and they do with the size of the junction) the appearance of manifold peaks has been proven inevitable. Using a continuation approach to compute the channel filters responses and a combinatorial procedure to design a peak-free junction, a complete triplexer has been synthesized that fits the CNES specifications. The synthesis was first considered in terms of circuits and eventually transformed in a real hardware realized in waveguide technology (see Figure.6). The latter is currently being manufactured at XLIM laboratories, where our colleague S. Bila is co-advising D. Martinez' PhD.

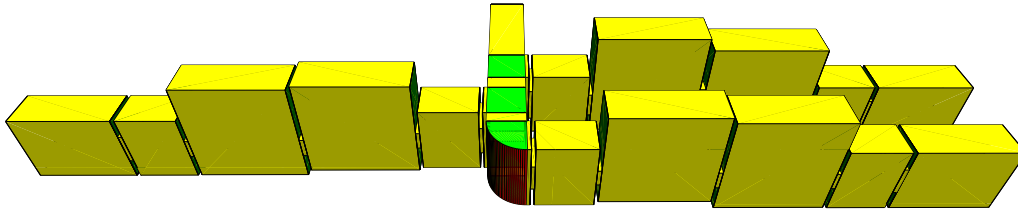### 6.2.2. *Uniform matching and global optimality considerations*

Figure 6. Full wave simulation of the realized triplexer, constituted of a very compact fishbone junction and three channel filters, each of order 6

The interpolation procedure of [6] provides us with matching/rejecting filtering characteristics at a discrete set of frequencies. It can serve as a starting point for heavier optimization procedures, where the matching and rejection specifications are expressed uniformly over the bandwidth. Although the practical results thus obtained are quite convincing, we have no proof of their global optimality. This has led us to seek alternative approaches allowing us to assess, at least in simple cases, global optimality of the obtained response. By optimality of a response we mean, as in classical filtering, a best match of the response in the uniform norm on a given pass-band, while meeting given rejection constraints on a stop-band, and this for a given maximal degree n_c of the matching network. Following the approach of Fano and Youla, we considered the problem of designing a 2×2 loss-less frequency response, under the condition that a specified load can be "unchained" from one of its port. This classically amounts to set interpolation conditions on the response at the transmission zeros of the Darlington extension of the load. When the load admits a rational representation of degree $n_a = 1$, and if the transmission zeros of the overall system are fixed, we were able to show that the uniform matching problem over an interval, together with rejection constraints at other frequency locations, reduces to a convex minimization problem with convex constraints over the set of non-negative polynomials of given degree. When the load is of degree greater than 1, our approach is a convex relaxation of the original problem as the computed matching circuit has in general degree $n_c + n_a - 1$. This delivers lower bounds for the original matching problem, the first that are known of this problem under degree constraint.

The internal representation of our problem, relies on the use of a Pick matrix $\mathcal{P}$ depending on a positive polynomial p, that needs to be positive in terms of matrices, that is $\mathcal{P}(p) \succeq 0$. Although we know for the convexity of our global formulation [17], the concavity of the matrix valued map $\mathcal{P}$ is not evident at all. Using arguments of matrix valued Nevanlinna-Pick interpolation we were able to establish it this years, that is: for all pair of positive polynomials (p_1, p_2) the following holds,

$$\mathcal{P}(\alpha p_1 + (1 - \alpha)p_2) \succeq \alpha \mathcal{P}(p_1) + (1 - \alpha)\mathcal{P}(p_2).$$

This is an important result, as it justifies the use of Lagrangian relaxation for our problem that appears to be a non-linear, convex, semi-definite optimization program (NLSDP), the hardest class among current convex optimization. We plan to build on this result, in order to obtain a critical point equation to our problem, in terms of extremal points of the involved polynomials.

The software implementation developed within the PhD of D. Martinez, combining logarithmic barrier functions and Lagrangian relaxation techniques has been made available for practitioners as Matlab library called Puma 5.2. Results obtained thank to this tool, have been presented in [15], [18] and on a dedicated workshop at the conference IMS2018. Design of use cases, for antenna matching problems relevant in 5G and IOT applications are currently being analyzed with LEAT, within the context of G. Bose's PhD.

6.3. Stability assessment of microwave amplifiers and design of oscillators

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert, Sébastien Fueyo, Adam Cooman.

The goal is here to help design amplifiers, in particular to detect instability at an early stage of the design. This activity has gained momentum with the doctoral work of (S. Fueyo), co-advised with J.-B. Pomet (from the McTao Inria project-team) and the postdoctoral stay of (A. Cooman) that eventually resulted in substantial software developments. Application of our work to oscillator design methodologies started recently in collaboration with Smain Amari from the Royal Military College of Canada (Kingston, Canada).

As opposed to Filters and Antennas, Amplifiers and Oscillators are active components that intrinsically entail a non-linear functioning. The latter is due to the use of transistors governed by electric laws exhibiting saturation effects, and therefore inducing input/output characteristics that are no longer proportional to the magnitude of the input signal. Hence, they typically produce non-linear distortions. A central question arising in the design of amplifiers is to assess stability. The latter may be understood around a functioning point when no input but noise is considered, or else around a periodic trajectory when an input signal at a specified frequency is applied. For oscillators, a precise estimation of their oscillating frequency is crucial during the design process. As regards devices devised to operate at relative low frequencies, time domain simulations, based on the integration of the underlying non-linear dynamical system, answers these questions satisfactorily. For complex microwave amplifiers and oscillators, the situation is however drastically different: the time step necessary to integrate the transmission line's dynamical equations (which behave like simple electrical wire at low frequency) becomes so small that simulations are intractable in reasonable time. In addition to this problem, most linear components of these circuits are known through their frequency response, and require therefore a preliminary, numerically unstable step to obtain their impulse response, prior to any time domain simulation.

For all these reasons it is widely preferred to perform the analysis of such systems in the frequency domain. In the case of stability issues around a functioning point, where only small input signals are considered, the stability of the linearized system obtained by a first order approximation of each non-linear dynamic is considered. This is done by means of the analysis of transfer impedance functions computed at some ports of the circuit. We have shown, that under some realistic hypothesis on the building blocks of the circuit, these transfer functions are meromorphic functions of the frequency variable s, with at most a finite number of unstable poles in the right half-plane [19].

Dwelling on the unstable/stable decomposition in Hardy Spaces, we developed a procedure to assess the stability or instability of the transfer functions at hand, from their evaluation on a finite frequency grid [9], that we further improved in [16] to address the design of oscillators, in collaboration with Smain Amari. The evaluation of the admittance function of interest is furnished, on a finite frequency band, by a circuit simulator. Progress were made on the interpolation procedure and the determination of a filtering function that are used to obtain a functional representation of high order of the unstable part of the admittance function to be analyzed. The latter was tested on a time-delayed Chua oscillator circuit: the analytical model of this circuit is known in closed form, and using continuation techniques on the involved delay components it is possible to compute the exact unstable poles of this circuit: two being exactly at the oscillators frequency, while two others spurious poles at DC frequency are present and are usually hard to detect with classical methods. Our approximation based procedure, which was fed with incomplete frequency data estimation of the admittance, was able to recover all poles within a relative error of less than 0.01%. A real world example of an MMIC oscillator was also analyzed and confirmed the procedure's effectiveness. A complete software library called pisa (see Section 5.1) have been developed to render these techniques accessible for practitioners. Although these results are very satisfying in practice, we are aiming for a result that would link together, the width of the measurement band, the density of the measurement points with the precision with which a pole, located within a certain depth into the complex plan can be identified. Extensions of our procedure to the strong signal case, where linearization is considered around a periodic trajectory, yielding harmonic transfer functions is also being worked on.

When stability is studied around a periodic trajectory, which is determined in practice by Harmonic Balance algorithms, linearization yields a linear time varying dynamical system with periodic coefficients and a periodic trajectory thereof. While in finite dimension the stability of such systems is well understood via the Floquet theory, this is no longer the case in the infinite dimensional setting when delays are considered. Dwelling on the theory of retarded systems, S. Fueyo's PhD work showed in previous years that, for certain simple circuits with properly positioned resistors, the monodromy operator is a compact perturbation of a stable operator, and that only finitely many unstable point of its spectrum can occur. This year, we proved a similar result for general circuits, provided that they are passive at very high frequency. For this, we use Lyapunov functions for the transmission lines to establish exponential L^2-stability, and then rely on counting techniques and impulse response estimates to obtain L^∞ stability from the exponential L^2-stability. We are currently developing the link between the monodromy operators of a general circuit and the so-called Harmonic Transfer Function of the circuit. A practical application of this result will be to generalize the previously described techniques of stability assessment around a functioning point into a stability assessment technique around periodic trajectories. This can be recast in terms of the finiteness of the number of abscissas of unstable poles of the Harmonic Transfer functions of the circuit. It will be of great importance to generalize such considerations to more complex circuits, whose structure is less well understood at present.

6.4. The Hardy-Hodge decomposition

Participant: Laurent Baratchart.

(This is joint work with T. Qian and P. Dang from the university of Macao.) It was proven in previous year that on a smooth compact hypersurface Σ embedded in $\mathbb{R}^n$, a $\mathbb{R}^n$-valued vector field of L^p class decomposes as the sum of a harmonic gradient from inside Σ, a harmonic gradient from outside Σ, and a tangent divergence-free field. This year we extended this result to Lipschitz surfaces for $2 - \varepsilon < p < 2 + \varepsilon'$, where ε and ε' depend on the Lipschitz constant of the surface. We also proved that the decomposition is valid for $1 < p < \infty$ when Σ is VMO-smooth (*i.e.* Σ is locally the graph of Lipschitz function with derivatives in VMO). By projection onto the tangent space, this gives a Helmholtz-Hodge decomposition for vector fields on a Lipschitz hypersurface, which is apparently new since existing results deal with smooth surfaces. In fact, the Helmholtz-Hodge decomposition is valid on surfaces (not just hypersurfaces), and an article is currently being written on this topic. The Hardy-Hodge decomposition generalizes the classical Plemelj formulas from complex analysis.

6.5. Imaging and modeling ancient materials

Participants: Vanna Lisa Coli, Juliette Leblond.

This is a very recent activity of the team, linked to images classification in archaeology in the framework of the project ToMaT, "Multiscale Tomography: imaging and modeling ancient materials, technical traditions and transfers" (see Section 8.1), and to the post-doctoral stay of V. L. Coli ; it is pursued in collaboration with L. Blanc-Féraud (project-team Morpheme, I3S-CNRS/Inria Sophia/iBV), D. Binder (CEPAM-CNRS, Nice), in particular.

The pottery style is classically used as the main cultural marker within Neolithic studies. Archaeological analyses focus on pottery technology, and particularly on the first stages of pottery manufacturing processes. These stages are the most demonstrative for identifying the technical traditions, as they are considered as crucial in apprenticeship processes. Until now, the identification of pottery manufacturing methods was based on macro-traces analysis, i.e. surface topography, breaks and discontinuities indicating the type of elements (coils, slabs...) and the way they were put together for building the pots. Overcoming the limitations inherent to the macroscopic pottery examination requires a complete access to the internal structure of the pots. Micro-computed tomography (μCT) has recently been used for exploring ancient materials microstructure. This non-invasive method provides quantitative data for a big set of proxies and is perfectly adapted to the analysis of Cultural heritage materials.

The main challenge of our current analyses aims to overcome the lack of existing protocols to apply in order to quantify observations. In order to characterize the manufacturing sequences, the mapping of the paste variability (distribution and composition of temper) and the discontinuities linked to different classes of pores, fabrics and/or organic inclusions appears promising. The totality of the acquired data composes a set of 2-D and 3-D surface and volume data at different resolutions and with specific physical characteristics related to each acquisition modality (multimodal and multi-scale data). Specific shape recognition methods need to be developed by application of robust imaging techniques and 3-D-shapes recognition algorithms.

In a first step, we devised a method to isolate pores from the 3-D data volumes; we are currently focusing our investigation on 2-D slices displaying pores locations and we considering several data processing treatments, such as multiresolution processing and Hough transform (derived from Radon transform), in order to evaluate their outcome when applied to these very particular images. Different possibilities of investigation will be analyzed as well, such as "a contrario" analysis and deep learning techniques.

6.6. Behavior of poles in meromorphic approximation

Participant: Laurent Baratchart.

We proved this year that if a function is holomorphic outside a disk of radius $r < 1$ in the complex plane, then its best approximant on the unit circle, in the uniform norm, by a meromorphic function having at most n poles in the unit disk has at most m poles of modulus greater than r, where m is independent of n. This is the first result on the behavior of singularities in meromorphic approximation to a function with 2-D singular set. We are currently working on analogs in a non-circular geometry and in rational rather than meromorphic approximation.

6.7. Tools for numerically guaranteed computations

Participant: Sylvain Chevillard.

The overall and long-term goal is to enhance the quality of numerical computations. The software tool Sollya (see Section 3.4.5), developed together with C. Lauter (Université Pierre et Marie Curie) intends to provide an interactive environment for performing numerically rigorous computations. It comes as a standalone tool and also as a C library that allows one to benefit from all the features of the tool in C programs.

In September 2018, we released version 7.0. Among other things, this release fixes some bugs and improves the way base functions are internally handled. Another important novelty of 2018 is that Sollya now comes with the companion `pythonsollya` [2] proposed by third-party developers to provide all the features of Sollya within Python. Some of the novelties in the API of the library version of Sollya 7.0, were made to ease the development of `pythonsollya`.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

7.1.1. Contract CNES-Inria-XLIM

This contract (reference Inria: 11282) accompanies the PhD of David Martinez and focuses on the development of efficient techniques for the design of matching network tailored for frequency varying loads. Applications of the latter to the design output multiplexers occurring in space applications will be considered.

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[2] https://gitlab.com/metalibm-dev/pythonsollya/

7.1.2. Contract Inria-SKAVENJI

This is a scientific consulting activity for the start-up company SKAVENJI. The latter develops an innovative and communicative device to facilitate the production and home consumption of small amounts of energy, produced by one or more local sources of renewable energy. Our contribution consisted in designing a simple controller improving the energy efficiency of the energy production while minimizing the number of charge and discharge cycles of the associated battery. The retained control strategy was based on consumption and production profiles.

8. Partnerships and Cooperations

8.1. Regional Initiatives

- The team participates in the project WIMAG (Wave IMAGing) funded by the Idex UCA[Jedi]. It aims at identifying and gathering the research and development by partners of UCA involved in wave imaging systems. Other partners are UNS and CNRS (GéoAzur, I3S, LEAT, LJAD), together with Orange Labs.

- The team co-advises a PhD (G. Bose) with the CMA team of LEAT (http://leat.unice.fr/pages/activites/cma.html) funded by the Labex UCN@Sophia on the co-conception of Antennas and Filters.

- The team participates in the transverse action C4PO funded by the Idex UCA[Jedi]. This "Center for Planetary Origin" brings together scientists from various fields to advance and organize Planetary Science at the the University of Nice, and supports research and teaching initiatives within its framework, among which the workshop "Inverse problems and approximation techniques in planetary sciences" organized by members of Factas in May, about inverse problems in harmonic electromagnetism and approximation, with applications mainly dedicated to geomagnetism and paleomagnetism, see Section 9.1.1.

- The team also participates in the project ToMaT, "Multiscale Tomography: imaging and modeling ancient materials, technical traditions and transfers", funded by the Idex UCA[Jedi] ("programme structurant Matière, Lumière, Interactions"). This project brings together researchers in archaeological, physical, and mathematical sciences, with the purpose of modeling and detecting low level signals in 3-D images of ancient potteries. The other concerned scientists are from CEPAM-CNRS-UCA (project coordinator: Didier Binder), Nice http://www.cepam.cnrs.fr, the team Morpheme, CNRS-I3S-Inria http://www.inria.fr/equipes/morpheme, and IPANEMA, CNRS, Ministère de la Culture et de la Communication, Université Versailles Saint Quentin http://ipanema.cnrs.fr/. Since March 2018, they co-advise together the post-doctoral research of Vanna Lisa Coli, see Section 6.5.

8.2. National Initiatives

8.2.1. ANR MagLune

The ANR project MagLune (Magnétisme de la Lune) has been approved July 2014. It involves the Cerege (Centre de Recherche et d'Enseignement de Géosciences de l'Environnement, joint laboratory between Université Aix-Marseille, CNRS and IRD), the IPGP (Institut de Physique du Globe de Paris) and ISTerre (Institut des Sciences de la Terre). Associated with Cerege are Inria (Apics, then Factas team) and Irphe (Institut de Recherche sur les Phénomènes Hors Équilibre, joint laboratory between Université Aix-Marseille, CNRS and École Centrale de Marseille). The goal of this project (led by geologists) is to understand the past magnetic activity of the Moon, especially to answer the question whether it had a dynamo in the past and which mechanisms were at work to generate it. Factas participates in the project by providing mathematical tools and algorithms to recover the remanent magnetization of rock samples from the moon on the basis of measurements of the magnetic field it generates. The techniques described in Section 6.1 are instrumental for this purpose.

8.2.2. ANR Cocoram

The ANR (Astrid) project Cocoram (Co-design et co-intégration de réseaux d'antennes actives multi-bandes pour systèmes de radionavigation par satellite) started January 2014 and ended October 2018. We were associated with three other teams from XLIM (Limoges University), geared respectively towards filters, antennas and amplifiers design. The core idea of the project was to realize dual band reception an emission chains by co-conceiving the antenna, the filters, and the amplifier. A complete chain has been synthesized using matching filters placed after each of the accesses of a network of 4 bi-polarized antennas, resulting in a substantial gain in energy effectiveness of 30%. This communication chain has been manufactured by our partner XLIM, and presented to the final meeting of the ANR in October.

8.3. European Initiatives

8.3.1. Collaborations with Major European Organizations

Apics is part of the European Research Network on System Identification (ERNSI) since 1992.

System identification deals with the derivation, estimation and validation of mathematical models of dynamical phenomena from experimental data.

8.4. International Initiatives

8.4.1. Inria Associate Teams Not Involved in an Inria International Labs

8.4.1.1. IMPINGE

Title: Inverse Magnetization Problems IN GEosciences.

International Partner (Institution - Laboratory - Researcher):

Massachusetts Institute of Technology (United States) - Department of Earth, Atmospheric and Planetary Sciences - Benjamin P. Weiss

Start year: 2016

See also: http://www-sop.inria.fr/apics/IMPINGE/

This proposal is concerned with the inverse problem of recovering a magnetization distribution from measurements of the magnetic field in a portion of space nearby. The application domain is to Earth and planetary sciences. Indeed, the remanent magnetization of rocks provides valuable information on their history. The proposal aims at renewing the existing "Équipe Associée" Impinge ending 2015, between Apics (now Factas) team at Inria and the Department of Earth, Atmospheric and Planetary Sciences at MIT (Cambridge, MA, USA), with the Department of Mathematics at Vanderbilt University (Nashville, TN, USA) as a secondary partner. Several research paths were broken towards magnetization recovery and promising numerical experiments have been conducted. This initial effort must be continued to achieve a reasonably complete methodology for reconstructing magnetizations and checking for hypotheses by geophysicists (e.g., unidirectionality of magnetization distributions).

8.4.2. Inria International Partners

8.4.2.1. Informal International Partners

MIT-France seed funding is a competitive collaborative research program ran by the Massachusetts Institute of Technology (Cambridge, Ma, USA). Together with E. Lima and B. Weiss from the Earth and Planetary Sciences dept. at MIT, Apics obtained two-years support from the above-mentioned program to run a project entitled: "Development of Ultra-high Sensitivity Magnetometry for Analyzing Ancient Rock Magnetism"

NSF Grant L. Baratchart, S. Chevillard and J. Leblond are external investigators in the NSF Grant 2015-2018, "Collaborative Research: Computational methods for ultra-high sensitivity magnetometry of geological samples" led by E. B. Saff (Vanderbilt Univ.) and B. Weiss (MIT).

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Axel Ringh (KTH Royal Institute of Technology, Stockholm, Sweden), June 11 -July 10.
- Elodie Pozzi (St Louis Univ., St. Louis, Missouri, USA), Brett Wick (Washington University, St. Louis, Missouri, USA), July 25.
- Cristobal Villalobos, Douglas Hardin, Edward Saff (Vanderbilt University, Nashville, Tennessee, USA), December, 6-19, 13-19, and 13-20 respectively.
- M. Olivi proposed with B. Hanzon and R. Peeters a Research in Pair event at CIRM « A state-space approach to parametrization of lossless and stable systems ». which was accepted and took place on August 27-31 https://conferences.cirm-math.fr/2126.html

8.6. List of international and industrial partners

- Collaboration under contract with Thales Alenia Space (Toulouse, Cannes, and Paris), CNES (Toulouse), XLIM (Limoges), LEAT (Sophia Antipolis), University of Bilbao (Universidad del País Vasco / Euskal Herriko Unibertsitatea, Spain), Flextronics.
- Regular contacts with research groups at UST (Villeneuve d'Asq), Universities of Bordeaux-I (Talence), Orléans (MAPMO), Aix-Marseille (CMI-LATP), Nice Sophia Antipolis (Lab. JAD), Grenoble (IJF and LJK), Paris 6 (P. et M. Curie, Lab. JLL), Inria Saclay (Lab. Poems, ENSTA), IMT Atlantique (Institut Mines-Télécom., Brest), Cerege-CNRS (Aix-en-Provence), CWI (the Netherlands), MIT (Boston, USA), Vanderbilt University (Nashville USA), Steklov Institute (Moscow), Michigan State University (East-Lansing, USA), Texas A&M University (College Station USA), Indiana University-Purdue University at Indianapolis, St Louis University and Washington University (St Louis, Missouri, USA), Cinvestav (Queretaro, Mexico), Politecnico di Milano (Milan, Italy), University of Trieste (Italy), RMC (Kingston, Canada), University of Leeds (UK), of Maastricht (the Netherlands), of Cork (Ireland), Vrije Universiteit Brussel (Belgium), TU-Wien and Universität Wien (Austria), TFH-Berlin and Technische Universität Bergakademie Freiberg (Germany), ENIT (Tunis), KTH (Stockholm), University of Cyprus (Nicosia, Cyprus), University of Macau (Macau, China), SIAE Microelettronica (Milano), Université Picardie Jules Verne (France), National and Kapodistrian University of Athens (Greece).
- The project is involved in the GDR-project AFHP (CNRS), in the ANR (Astrid program) project Cocoram (with XLIM, Limoges, and DGA), in the ANR (Défis de tous les savoirs program) project MagLune (with Cerege, IPGP, ISTerre, Irphe), in a MIT-France collaborative seed funding, in the Associate Inria Team IMPINGE (with MIT, Boston), and in a NSF grant (with Vanderbilt University and MIT).

9. Dissemination

9.1. Promoting Scientific Activities

- V. L. Coli presented oral communications at the "Journées Idex UCA^{JEDI} Matière, Lumière, Interactions", October 4-5, Fréjus, France, http://univ-cotedazur.fr/events/MLI/fr, and at the Journée d'échanges "Applications de la tomographie par rayons X", November 23, Paris, France, https://jea-tomo3d-2018.sciencesconf.org/.
- J. Leblond and K. Mavreas presented oral communications at the 5th International Conference "Modern Mathematical Methods in Science and Technology 2018 (M3ST '18)", September 2-4, Kalamata, Greece.
- K. Mavreas gave a talk at the PhD Seminar at Inria Sophia, France, November 13, https://phd-seminars-sam.inria.fr.

- G. Bose, S. Fueyo and D. Martinez Martinez presented posters at the Workshop ERNSI 2018, http://mlg.eng.cam.ac.uk/ernsi2018/.

- G. Bose gave a talk at the 23rd International Symposium on Mathematical Theory of Networks and Systems http://mtns2018.ust.hk/, July 16-20 2018, Hong-Kong.

- D. Martinez Martinez gave a talk at the 7th International Workshop on Microwave Filters https://artes.esa.int/news/7th-international-workshop-microwave-filters, 17-19 Apr 2018, Noord-wijk, Netherlands, and a talk at the 2nd URSI AT-RASC (Atlantic Radio Science) http://www.ursi.org/events.php?atrasc=on, May 28 - June 1, 2018, Gran Canaria, Spain, and a talk at IMS 2018 https://ims2018.org/, June 10-15 2018., Philadelphia, USA

- A. Cooman gave a talk at IMS 2018 https://ims2018.org/, June 10-15, 2018, Philadelphia, USA.

- F. Seyfert gave a talk at IMS 2018 https://ims2018.org/, June 10-15 2018, Philadelphia, USA and at the KTH's seminar May 15, Stockholm, Sweden and at XLIM seminar, October 3, Limoges, France

9.1.1. Scientific Events Organisation

9.1.1.1. General Chair, Scientific Chair

- L. Baratchart, S. Chevillard, and J. Leblond organized (scientifically and practically) the Spring School/Workshop "Inverse problems and approximation techniques in planetary sciences" [3], at Inria Sophia Antipolis, May 16-18. It was financially supported by UCA$^{\text{Jedi}}$ and gathered around 20 participants.

9.1.1.2. Member of the Organizing Committees

- L. Baratchart and J. Leblond organized a mini-symposium [4] on "Inverse source problems with applications to planetary sciences and medical imaging" at the 9th international conference "Inverse problems: modeling and simulation (IPMS 2018)", Malta, May 21-25.

- Fabien Seyfert organized a workshop at IMS 2018, on "Advanced Synthesis Techniques for reduced size filtering techniques", https://ims2018.org/technical-program/workshops-and-short-courses#2018-06-15, Philadelphia, USA.

9.1.2. Scientific Events Selection

9.1.2.1. Member of the Conference Program Committees

L. Baratchart sits on the Program Committee of *Applied Inverse Problems 2019*, to be held in Grenoble, France.

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

L. Baratchart sits on the editorial boards of "Constructive Methods in Function Theory" and "Complex Analysis and Operator Theory".

9.1.3.2. Reviewer - Reviewing Activities

S. Chevillard was a reviewer for the journal *Transactions on Mathematical Software*.

J. Leblond was a reviewer for the journals *Numerical Algorithms, DMV Jahresberichte*.

M. Olivi was a reviewer for the journal *Automatica* and for the *18th IFAC Symposium on System Identification* (SYSID 2018).

F. Seyfert is a reviewer of *IEEE Transactions on Microwave Theory and Techniques*.

L. Baratchart was a reviewer for the seed fund program at MIT.

9.1.4. Invited Talks

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[3]http://www-sop.inria.fr/apics/IPAPS18/
[4]http://www.ipms-conference.org/ipms2018/index.php/m20

- L. Baratchart was an invited speaker at the SIGMA day organized by SMAI at Université Pierre et Marie Curie, November 30.
- L. Baratchart and J. Leblond were invited to give talks at the IPMS 2018 conference, Malta, May (see Section 9.1.1). They were invited speakers at the Conference "Advances in Operator Theory with Applications to Mathematical Physics", http://www1.chapman.edu/~alpay/conf2018/conf2018.html, Chapman University, Orange, CA, USA, November.
- L. Baratchart and K. Mavreas gave talks at the workshop "Inverse problems and approximation techniques in planetary sciences", Sophia Antipolis, France, May.

9.1.5. *Scientific Expertise*

L. Baratchart is sitting on committee 40 of the ANR.

J. Leblond was a reviewer for the Strategic Research Programmes of the Research Council of the Vrije Universiteit Brussel (VUB, Brussel, Belgium).

F. Seyfert is a member of the IEEE MTT-8 committee for filters and passive components.

9.1.6. *Research Administration*

L. Baratchart sits on the committee "Mathématiques et Informatique" of the French Agency for research (ANR).

J. Leblond is an elected member of the "Conseil Scientifique" and of the "Commission Administrative Paritaire" of Inria.

9.2. Teaching - Supervision - Juries

9.2.1. *Teaching*

Colles: S. Chevillard is giving "Colles" at Centre International de Valbonne (CIV) (2 hours per week).

9.2.2. *Supervision*

PhD in progress: K. Mavreas, *Inverse source problems in planetary sciences: dipole localization in Moon rocks from sparse magnetic data*, since October 2015, advisors: S. Chevillard, J. Leblond.

PhD in progress: D. Martinez Martinez, *Méthodologie et Outils de Synthèse pour des Fonctions de Filtrage Chargées par des Impédances complexes*, since October 2015, advisors: L. Baratchart, F. Seyfert, M. Olivi.

PhD in progress: G. Bose, *Filter Design to Match Antennas*, since December 2016, advisors: F. Ferrero, F. Seyfert and M. Olivi.

PhD in progress: S. Fueyo, *Cycles limites et stabilité dans les circuits*, since October 2016, advisors: L. Baratchart and J.-B. Pomet (Inria Sophia, McTao).

Post-doc. in progress: V. L. Coli, *Multiscale Tomography: imaging and modeling ancient materials*, since March 2018, advisors: J. Leblond, L. Blanc-Féraud (project-team Morpheme, I3S-CNRS/Inria Sophia/iBV), D. Binder (CEPAM-CNRS, Nice).

Post-doc.: A. Cooman, from January 2017 until June 2018, advisors: F. Seyfert and M. Olivi.

9.2.3. *Juries*

J. Leblond was a member of the "Comités de Suivi Doctoraux" (preliminary evaluation committees, after one year of PhD) for the ongoing PhDs of Kostiantyn Maksymenko (project-team Athena, doctoral school STIC, UCA), and for Arne Bechensteen (project-team Morpheme, I3S-CNRS), May. She was a member of a "Comité de Sélection" for professors at the University Paris Descartes (MAP5).

M. Olivi was a member of the "jury d'admission du concours CR" of the Inria Research Center.

9.3. Popularization

9.3.1. Internal or external Inria responsibilities

M. Olivi is responsible for Scientific Mediation and president of the Committee MASTIC (Commission d'Animation et de Médiation Scientifique) https://project.inria.fr/mastic/. Her main contributions related with this mission were:

- submit applications to get founding for 2019: "cordées de la réussite" (accepted) et "APOCS région".
- co-organize 10 robotic sessions for 2 classes of middle school students (device "MEDITES" http://medites.fr, founded by ANRU, the "Agence Nationale de Rénovation Urbaine"),
- co-organize the "stage MathC2+" https://project.inria.fr/mastic/mediation/stage-mathc2/, a four-day internship for 50 high school students ("secondes", about 16 years old) organized by the Committee MASTIC and its partners (June 19-22),
- co-organize Inria participation to the event "Le Village des Sciences et de l'Innovation" in Antibes (October 20-21, 10000 people), and the event "Mouans-Sartoux fête les sciences du quotidien" http://univ-cotedazur.fr/events/fetedelascience.
- co-organize about 10 "cafés scientifiques" (c@fé-in's and cafés Techno, 30 to 80 participants each) https://project.inria.fr/mastic/category/cafein/.

9.3.2. Education

- M. Olivi animated a half-day workshop session "activités débranchées" at "l'ESPE de Nice" for primary school students (April 10).

9.3.3. Interventions

- National events: M. Olivi participated to the event "Le Village des Sciences et de l'Innovation" in Antibes (October 20-21, 8000 people) and to the event "Mouans-Sartoux fête les sciences du quotidien" (October 13, 1000 people).
- In educational institutions: M. Olivi participated to the event "bilan Medium's" at the "collège Émile Roux, Le Cannet".
- Welcoming of schoolchildren or the general public in an Inria center: M. Olivi animated a workshop session during the "stage MathC2+", a four-day internship for high-school students organized by the Committee MASTIC and its partners (June 19-22).

9.3.4. Internal action

- M. Olivi presented the scientific objects produced during the year at the CaféIn of the Research Center, https://project.inria.fr/mastic/cafe-in-12-fevrier-manipulons-et-echangeons/, February.
- J. Leblond together with members of the SED gave a communication at the CaféIn of the Research Center, http://project.inria.fr/mastic/category/cafein/, October.

9.3.5. Creation of media or tools for science outreach

M. Olivi participated in the creation of the web pages https://pixees.fr/jouons-avec-des-experiences-scientifiques/.

10. Bibliography

Major publications by the team in recent years

[1] S. AMARI, F. SEYFERT, M. BEKHEIT. *Theory of Coupled Resonator Microwave Bandpass Filters of Arbitrary Bandwidth*, in "Microwave Theory and Techniques, IEEE Transactions on", August 2010, vol. 58, n⁰ 8, pp. 2188 -2203

[2] B. ATFEH, L. BARATCHART, J. LEBLOND, J. R. PARTINGTON. *Bounded extremal and Cauchy-Laplace problems on the sphere and shell*, in "J. Fourier Anal. Appl.", 2010, vol. 16, n° 2, pp. 177–203, Published online Nov. 2009, http://dx.doi.org/10.1007/s00041-009-9110-0

[3] L. BARATCHART, A. BORICHEV, S. CHAABI. *Pseudo-holomorphic functions at the critical exponent*, in "Journal of European Math. Soc.", 2016, vol. 18, n° 9, pp. 1919–1960

[4] L. BARATCHART, S. CHEVILLARD, T. QIAN. *Minimax principle and lower bounds in H^2-rational approximation*, in "Journal of Approximation Theory", 2015, vol. 206, pp. 17–47

[5] L. BARATCHART, J. LEBLOND, S. RIGAT, E. RUSS. *Hardy spaces of the conjugate Beltrami equation*, in "Journal of Functional Analysis", 2010, vol. 259, n° 2, pp. 384-427, http://dx.doi.org/10.1016/j.jfa.2010.04.004

[6] L. BARATCHART, M. OLIVI, F. SEYFERT. *Boundary nevanlinna-pick interpolation with prescribed peak points. Application to impedance matching*, in "SIAM Journal on Mathematical Analysis", 2017 [*DOI :* 10.1137/16M1085577], https://hal.inria.fr/hal-01377782

[7] L. BARATCHART, H. STAHL, M. YATTSELEV. *Weighted Extremal Domains and Best Rational Approximation*, in "Advances in Mathematics", 2012, vol. 229, pp. 357-407, http://hal.inria.fr/hal-00665834

[8] M. CLERC, J. LEBLOND, J.-P. MARMORAT, T. PAPADOPOULO. *Source localization using rational approximation on plane sections*, in "Inverse Problems", May 2012, vol. 28, n° 5, 24 p. , http://hal.inria.fr/inria-00613644

[9] A. COOMAN, F. SEYFERT, M. OLIVI, S. CHEVILLARD, L. BARATCHART. *Model-Free Closed-Loop Stability Analysis: A Linear Functional Approach*, in "IEEE Transactions on Microwave Theory and Techniques", 2017, https://arxiv.org/abs/1610.03235 , https://hal.inria.fr/hal-01381731

[10] V. LUNOT, F. SEYFERT, S. BILA, A. NASSER. *Certified Computation of Optimal Multiband Filtering Functions*, in "IEEE Transactions on Microwave Theory and Techniques", 2008, vol. 56, n° 1, pp. 105-112, http://dx.doi.org/10.1109/TMTT.2007.912234

[11] M. OLIVI, F. SEYFERT, J.-P. MARMORAT. *Identification of microwave filters by analytic and rational H2 approximation*, in "Automatica", January 2013, vol. 49, n° 2, pp. 317-325 [*DOI :* 10.1016/J.AUTOMATICA.2012.10.005], http://hal.inria.fr/hal-00753824

Publications of the year

Articles in International Peer-Reviewed Journals

[12] L. BARATCHART, S. CHEVILLARD, D. P. HARDIN, J. LEBLOND, E. A. LIMA, J.-P. MARMORAT. *Magnetic moment estimation and bounded extremal problems*, in "Inverse Problems and Imaging ", February 2019, vol. 13, n° 1, 29 p. , https://hal.inria.fr/hal-01623991

[13] L. BARATCHART, J. LEBLOND, F. SEYFERT. *Constrained extremal problems in H2 and Carleman's formulas*, in "Matematicheskii Sbornik", 2018, vol. 209, n° 7, 36 p. , https://hal.archives-ouvertes.fr/hal-01625840

[14] S. Chevillard, J. Leblond, K. Mavreas. *Dipole recovery from sparse measurements of its field on a cylindrical geometry*, in "International Journal of Applied Electromagnetics and Mechanics", 2018, 7 p. , https://hal.inria.fr/hal-01618885

Invited Conferences

[15] D. Martínez Martínez, G. Bose, F. Seyfert, M. Olivi. *Convex optimisation method for matching filters synthesis*, in "2nd URSI AT-RASC (Atlantic Radio Science)", Gran Canaria, Spain, May 2018, https://hal.inria.fr/hal-01909586

International Conferences with Proceedings

[16] A. Cooman, F. Seyfert, S. Amari. *Estimating unstable poles in simulations of microwave circuits*, in "IMS 2018", Philadelphia, United States, June 2018, https://hal.inria.fr/hal-01958795

Conferences without Proceedings

[17] G. Bose, D. Martínez Martínez, F. Seyfert, M. Olivi. *A Convex Approach to the Finite Dimensional Matching Problem in Communication Systems*, in "The 23rd International Symposium on Mathematical Theory of Networks and Systems", Hong Kong, China, July 2018, https://hal.inria.fr/hal-01936784

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Other Publications

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[22] D. Martínez Martínez, G. Bose, F. Seyfert, M. Olivi, L. Baratchart, S. Bila, F. Ferrero. *A convex optimisation approach to Youla's broadband matching theory*, September 2018, 27th ERNSI Workshop in System Identiifcation, Poster, https://hal.inria.fr/hal-01909618

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