Activity Report 2018

Team GAIA

Geometry, Algebra, Informatics, Applications

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions).

RESEARCH CENTER
Lille - Nord Europe

THEME
Algorithmics, Computer Algebra and Cryptology

Table of contents

Team GAIA

Creation of the Team: 2018 January 01, end of the Team: 2018 December 31

Keywords:

Computer Science and Digital Science:

A6.4. - Automatic control
A8.3. - Geometry, Topology
A8.4. - Computer Algebra

Other Research Topics and Application Domains:

B6.6. - Embedded systems
B9.5.1. - Computer science
B9.5.2. - Mathematics

1. Team, Visitors, External Collaborators

Research Scientists

Alban Quadrat [Team leader, Inria, Senior Researcher, HDR]
Rosane Ushirobira [Inria, Researcher, HDR]

Faculty Members

Francois Boulier [Univ des sciences et technologies de Lille, Professor, from Mar 2018]
Francois Lemaire [Univ des sciences et technologies de Lille, Associate Professor, from Mar 2018]
Adrien Poteaux [Univ des sciences et technologies de Lille, Associate Professor]
Michele Triestino [Univ de Bourgogne, Associate Professor, from Nov 2018]

Post-Doctoral Fellows

Cyrille Chenavier [Inria, from Oct 2018]
Yacine Bouzidi [Inria]

PhD Students

Guillaume Rance [Safran Electronics & Defense, until Sep 2018]
Grace Younes [Inria, from Oct 2018]

Administrative Assistant

Corinne Jamroz [Inria]

Visiting Scientists

Marc Moreno Maza [Univ Western Ontario, from Sep 2018 until Oct 2018]
Fredrik Olsson [Univ Uppsala, Nov 2018]

2. Overall Objectives

2.1. Algebraic and geometric studies of functional systems

Systems of functional equations or simply *functional systems* are systems whose unknowns are functions, such as systems of ordinary (OD) or partial differential (PD) equations, of differential time-delay equations, of difference equations, of integro-differential equations, etc. [34], [35]. Functional systems play a fundamental role in the mathematical modeling of physical phenomena studied in natural science such as physics, or in engineering sciences such as mathematical systems theory control theory, signal processing, etc. [34], [35]. Numerical aspects of functional systems, especially OD and PD systems, have largely been studied in applied mathematics due to the importance of numerical simulation issues.

Complementary approaches, based on algebraic and differential or algebraic geometric methods, are usually upstream or help the numerical simulation of systems of functional systems. These methods tackle questions and problems such as algebraic preconditioning, elimination and simplification, completion to formal integrability or involution, computation of integrability conditions or compatibility conditions, index reduction, reduction of variables, choice of adapted coordinate systems based on symmetries, computation of first integrals of motion, conservation laws, and Lax pairs, study of Liouville integrability or of the asymptotic behavior of solutions at a singularity, etc. For more details, see [36], [41], [51], [67], [75], [76], [81], [85], [101], [104], [109] and the references therein.

Let us state a few interests of an algebraic approach for the study of functional systems:

- Algebraic methods are clearly suitable for an algorithmic study, and thus for the development of efficient algorithms implementable in computer algebra systems.
- It can be used to finely study the behavior of the solutions of a system with respect to unfixed model parameters – which is usually a difficult numerical issue. Moreover, the boundaries of the zones in the parameter space over which the behavior of the solution changes can be algorithmically characterized by means of algebraic methods, which yields a safe use of numeric methods in each regular zones (symbolic-numeric methods).
- The existence of closed-form solutions can highly simplify certain problems studied in applications by avoiding the use of time-consuming optimization problems, and thus fits well with nowadays real-time applications.

The GAIA team aims to develop algebraic and geometric methods for the study of functional systems.

2.2. Effective algebraic theories and their implementations in computer algebra systems

Although not yet very popular in applied mathematics, algebraic and differential geometric approaches of functional systems have lengthly been studied in foundamental mathematics. We can state a few names of mathematical theories such as (differential) Galois theory, Lie groups, exterior differential systems, differential algebra, algebraic analysis, etc. [36], [41], [51], [67], [75], [76], [81], [85], [101], [104], [109].

Over the past years, some of these algebraic theories for the study of functional systems have been investigated in the computer algebra community within an algorithmic viewpoint, mostly driven by applications to engineering sciences such as mathematical systems theory and control theory.

Gröbner or Janet bases [40], [102] for noncommutative polynomial rings of functional operators or differential elimination techniques for differential systems [45], [46], [72], based on differential algebra [101], [76], are remarkable examples of those effective algebraic methods. They are nowadays implemented in standard computer algebra systems (e.g. `Maple`, `Mathematica`, `Magma`).

These effective algebraic approaches also form the algorithmic "engines" at the basis of the first developments of effective versions of modern algebraic theories (algebraic geometry, differential algebra, module theory and homological algebra over certain noncommutative polynomial rings of functional operators, algebraic analysis, etc.).

The above-mentioned results are just the tip of the iceberg and much more effort must be made in the future for making effective larger parts of fundamental mathematics and making them largely available in standard computer algebra systems. This "democratization" process towards the accessibility of fundamental mathematics is important, for instance, for educational issues where computers can be used to teach them to students, scientists of other communities and engineerings, and for learning by doing and computing. Further developing effective mathematics, making them accessible to a larger audience through dedicated software, and demonstrating them through interesting engineering problems are at the core of the GAIA team. The latter engineering problems are major sources of motivation for the development of effective algebraic theories and their implementations in computer algebra systems as explained in the next section.

2.3. A rich interplay between computer algebra and control theory

A major source of motivation for the development of the effective study of algebraic theories is represented by control theory issues. Indeed, certain problems studied in control theory can be better understood and finely studied by means of algebraic or geometric structures and techniques. The rich interplay between algebra, computer algebra, and control theory has a long history.

The first main paper on Gröbner bases [50] written by their creators, Buchberger, was published in Bose's book [42] on control theory of multidimensional systems since they play a fundamental role in this theory. They were the first main applications of Gröbner bases outside the field of algebraic geometry and they still play a fundamental role in multidimensional systems theory [43].

The differential algebra approach to nonlinear control theory [58], [59], [63], [84] was a major motivation for the effective study of differential algebra [101], [76] (differential elimination theory, triangular sets, regular chains, etc.) [45], [46], [72] and its implementations in `Maple`. Within this effective differential algebra approach to nonlinear control systems, observability, identifiability, parameter estimation, invertibility, differential flatness, etc., have received appealing and checkable algebraic characterizations.

Linear control theory [73] and multidimensional systems theory [42], [43] have recently been profoundly developed due to the so-called behavior approach [80], [83] and the module approach [64], [84]. Based on ideas of *algebraic analysis* [75], system properties of those systems (e.g. controllability, parametrizability, differential flatness) are intrinsically characterized by means of properties of certain algebraic structures (namely finitely presented left modules over noncommutative polynomial rings of functional operators). To effectively check the latter properties, the development of effective versions of two important algebraic theories, namely module theory [79] and homological algebra [103], had to be iniated based on functional elimination techniques (i.e. Gröbner or Janet basis techniques for noncommutative polynomial rings) [7], [8] (see also [91]). Dedicated packages, written in `Maple`, `Mathematica` and `GAP`, are now available.

The GAIA team wants to further develop its expertise in this direction by considering new classes of functional systems (e.g. differential varying/distributed delay systems, ordinary integro-differential systems) interesting in control theory and in signal processing.

2.4. Main objectives of the GAIA team

The first goal of the GAIA team is to study classes of *functional systems* which are interesting in practice (e.g. differential systems, differential constant/varying/distributed delay systems, ordinary integro-differential systems) by means of *algebraic* and *geometric* methods (algebraic analysis, algebraic/differential/noncommutative geometry, etc.), *computer algebra* (e.g. algorithmic, symbolic and symbolic-numeric methods, librairies), and *mathematical systems theory*. The systems to be investigated can be linear, nonlinear, continuous, discrete, or originated from real life applications.

The second goal of the GAIA team is to study important problems coming from:

- *control theory* (e.g. parametric robust control, stability and stabilization of multidimensional systems or of differential constant/distributed/time-varying delay systems)
- *signal processing* (e.g. parameter estimation problem, metric multidimensional unfolding, autocalibration)
- *multidisciplinary domains* (e.g. marine bivalves behavior, human-machine interaction, chemical reaction networks, ionic activities in neuroscience)

The third goal of the GAIA team is to develop (`Maple`, `Mathematica`, `C/C++`) *packages and librairies* dedicated to functional systems and to their applications, and in parallel, eventual *industry transfer* (e.g. *Safran Electronics & Defense*, *Safran Tech*, *Maplesoft*).

3. Research Program

3.1. Effective algebra

To develop a computational study of problems coming from control theory, signal processing, and multidisciplinary domains, parts of algebraic theories must be studied within an effective approach: methods and theoretical results must be made algorithmic based on computer algebra techniques appropriated for efficient implementations in computer algebra systems.

3.1.1. Polydisc Nullstellensatz & effective version of a theorem of Deligne

The works on stability and stabilization problems of multidimensional systems, developed in the former ANR MSDOS (2014–2018), have shown the importance for developing an effective version of the module theory over the ring of rational functions without poles in the closed unit polydisc of $\mathbb{C}^n$ [90], [47]. The stabilizability (resp. the existence of a doubly coprime factorization) of a multidimensional system is related to a module-theoretical property (projectivity, resp. freeness) that has to be algorithmically verified prior to compute stabilizing controllers (resp. the standard Youla-Kučera parametrization of all stabilizing controllers). Based on the works [89], [90], in [47], we have recently proved that the stabilizability condition is related to the development of an algorithmic proof of the so-called *Polydisc Nullstellensatz* [49], a natural extension of Hilbert's *Nullstellensatz* for the above-mentioned ring (see e.g. [40]). In addition, the existence of a doubly coprime factorization is related to a theorem obtained by (the Fields medalist) Deligne with a non-constructive proof [90]. This theorem can be seen as an extension of the famous Quillen-Suslin theorem (Serre's conjecture) [77]. Based on our experience of the first implementation of the Quillen-Suslin theorem in the computer algebra system (`Maple`) [62], we aim to develop this effective framework as well as a dedicated `Maple` package.

3.1.2. Effective version of Spencer's theory of formal integrability of PD systems

A differential geometric counterpart of differential algebra and differential elimination theory [101], [76] is the so-called Spencer's theory of formal integrability and involutive PD systems [85], [104]. For linear PD systems, this theory can be seen as an intrinsic approach to Janet or Gröbner bases for noncommutatibe polynomial rings of PD operators. No complete algorithmic study of Spencer's theory has been developed yet. We aim to develop it as well as to implement it. The understanding of the connections between the different differential elimination theories (Janet [102] or Gröbner bases [40], Thomas decomposition [102], differential algebra [101], [76], Spencer's theory [85], exterior differential systems [51], etc.) will also be investigated. On a longer term, applications of Spencer's theory to Lie pseudogroups and their applications in mathematical physics (e.g. variational formulations based on Lie (pseudo)groups) [86] will be investigated and implemented.

3.1.3. Rings of integro-differential operators & integro-differential algebra

The main contribution of this axis is the development of effective elimination theories for both linear and nonlinear systems of integro-differential equations.

3.1.3.1. Linear systems of integro-differential equations

The rings of integro-differential operators are more complex than the purely differential case [96], [97] due to the existence of zero-divisors or the fact of having a *coherent ring* instead of a *Noetherian ring* [39]. We want to develop an algorithmic study of these rings. Following the direction initiated in [95] for the computation of zero divisors, we first want to develop algorithms for the computation of left/right kernels and left/right/generalized inverses of matrices with entries in such rings, and to use them to develop a module-theoretic approach to linear systems of integro-differential equations. Following [95], standard questions addressed within the computer algebra community such as the computation of rational/exponential/hyperexponential/etc. solutions will also be addressed. Moreover, famous Stafford's results [105], algorithmically studied in [96], [97] for rings of PD operators, are known to still hold for rings of integro-differential operators [39]. Their algorithmic extensions will be investigated and our corresponding implementation will be extended accordingly. Finally, following [93], [95], an algorithmically study of rings of integro-differential-delay operators will be further developed as well as their applications to the equivalence problem of differential constant/varying/distributed delay systems (e.g. Artstein's reduction, Fiagbedzi-Pearson's transformation) and their applications to control theory.

3.1.3.2. Nonlinear systems of integro-differential equations

Integro-differential algebra is an extension of Ritt-Kolchin's *differential algebra* [101], [76] that also includes integral operators. This extension is now attracting more attention in mathematics, computer algebras, and control theory. This new type of algebras will be algorithmically studied for integro-differential nonlinear systems. To do that, concepts such as integro-differential ideals and varieties have to be introduced and studied for developing an integro-differential elimination theory which extends the current differential elimination theory [45], [46], [72]. A `Maple` prototype will first be developed and then a `C` library when experience will be gained.

3.2. Computer algebra

We aim to further reinforce our expertise in the computer algebra aspects of functional systems and algebraic curves by attacking remaining technical obstacles and by considering new classes of functional systems, notably those coming from interesting applications in engineering sciences and particularly in control theory.

3.2.1. Efficient algorithms for the study of singularities of algebraic curve and its applications

On an algorithmic viewpoint, there are mainly four different approaches for the study of the singularities of plane algebraic curves. Let us shortly list them:

- The well-known *Newton-Puiseux* algorithm, initiated by Cramer and Puiseux, which follows an idea due to Newton. This approach has successively been improved in [61], [88], [9], [10].
- The *Extended Hensel Construction*, developed in [74] and recently improved in [82].
- The work [68] concentrates on the factorisation of polynomials defined over valued fields (the local study of a plane algebraic curve enters in this approach).
- The work [33] introduces the concept of an*approximate root*.

The first two methods are based on Puiseux series computations. They use techniques which are equivalent to the standard blowing-up of a singularity of an algebraic curve, which has the drawback to be bottlenecks in terms of complexity and practical efficiency. Nevertheless, the recent work [88] provides the best complexity currently known. The last two methods study singularities without computing Puiseux series. They both use the concept of an *extended evaluation*.

A recent very efficient algorithm for the factorisation of a univariate polynomial based on its Newton polygon has recently been obtained in [52]. The key ingredient of this algorithm is to work on the given polynomial and not after changes of variables as usually done in the literature.

To improve the complexity results of [68], [38], we want to combine the above different approaches using approximate roots and a generalization of the results of [52] to the context of [68].

The method proposed above is important in practice since it is based on well-known and efficiently implemented algorithms (mainly Newton iteration and gcd computations). Nevertheless, these algorithms involve technical difficulties on the computer science side: the main one is the need to improve the accuracy of computations due to truncations. These issues, including also run-time compilation, are well-studied in the `BPAS` library [1] based on specific data structures to deal with power series computation (a power series is represented by terms that have been computed and a program that enables to compute more terms when required).

Another part of the code development concerns issues on certified numerical computations for univariate polynomials (with algebraic coefficients) that will be used for the development of certified symbolic-numeric algorithms making effective the strategy proposed in [87].

[1] http://www.bpaslib.org/index.html

3.2.2. Differential algebra

A major bottleneck of computational differential algebra methods is the computation of greatest common divisors of multivariate commutative polynomials. Any algorithmic progress in this direction would highly improve the efficiency of differential algebra software such as, for instance, the `C` library `BLAD` [44]. Moreover, numerous computer algebra problems and related implementations could also highly profit from any success in this direction.

A major application of the effective differential algebra approach developed by GAIA's members [45], [46] is the possibility to reduce a nonlinear (implicit) differential system, particularly differential algebraic equations, to so-called regular chains of differentiation index 0, i.e. to systems which do not need differentiation of their equations to be rewritten as pure differential systems [69]. Based on our expertise on differential techniques, we want to study the consistent initialization problem and develop numerical integrators for nonlinear differential algebraic systems, and used them in the study of coupled algebraic and differential systems, interconnected systems, or networks [69].

3.2.3. A Maple package and a C/C++ open source library for integro-differential algebra

A package dedicated to nonlinear integro-differential equations will be developed in a `Maple` prototype and then in a standalone `C/C++` open source library (as it was already done for the `diffalg` and `DifferentialAlgebra` packages). General purpose solvers such as `Maple` `dsolve` or `pdesolve` may call differential elimination methods for computing essential singular solutions of differential equations, for computing systems of polynomial differential equations admitting a given function as a solution, etc. On the long run, one may foresee enhanced general purpose solvers able to handle integro-differential equations processed through integro-differential elimination methods. It will rely on a sub-package dedicated to the problem of effectively handling integro-differential expressions.

These packages will rely on existing software such as `BLAD`, `DifferentialAlgebra`, and `MABSys`. It is worth pointing out that proof-of-concept methods are already available. See [29] and [2]. The collaboration with modelers will also enhance the software user-interface for a better usability.

The study of numerical integration of integro-differential equations (a necessary component of software dedicated to the parameter estimation problem) will also be further studied following the direction initiated in [29], leading to the `Maple/C` library `BLINEIDE`. This software has currently no widely available challenger.

The `Maple` prototype software dedicated to nonlinear integro-differential equations will also be implemented in a standalone `C/C++` open source library, leading to software easier to integrate in modeling platforms such as `OpenModelica`. The GAIA team has quite some expertise in releasing software satisfying industrial standards: its `C` open source `BLAD` libraries, dedicated to differential elimination, are currently integrated in `Maple` and called through the `Maple` package `DifferentialAlgebra`. The `Modelica` programming language, which emphasizes programming with equations and permits to call external code, can integrate software dedicated to integro-differential equations developed in the GAIA team.

3.3. Applications to control theory and signal processing

3.3.1. Robust stability analysis and stabilization problems for functional systems

Our expertise in the computer algebra aspects to stability and stabilization problems for multidimensional systems and for differential constant/distributed/varying delay systems [5], [6], [48], [47] will further be developed.

3.3.1.1. Computation of Lyapunov functions for homogeneous dynamical systems

We shall investigate the possibility to develop a computer algebra package for the design of Lyapunov functions for homogeneous dynamical systems based on an effective study of the differential algebra of *generalized forms*, i.e. of Puiseux polynomials in signed powers [106], [107].

3.3.1.2. Robust stability analysis of differential time-delay systems

The symbolic-numeric study of the robust stability of a differential constant time-delay system with respect to the delay h, via the variation of the zero locus of the associated quasipolynomial $p(s, e^{-h\,s})$ [70] in the stability (resp. unstability) region $\mathbb{C}_- = \{s \in \mathbb{C} \mid \Re(s) < 0\}$ (resp. $\overline{\mathbb{C}_-} = \mathbb{C} \setminus \mathbb{C}_+$) of $\mathbb{C}$, initiated in [5], will be further developed. This problem is another motivation for the development of a fast numerical algorithm for the computation of Netwon-Puiseux series and its implementation in a `C` library. They will be used to study how the different branches of a quasipolynomial at a critical pair (namely $(h_{\star}, \omega_{\star}) \in \mathbb{R}_{>0} \times \mathbb{R}$ such that $p(i\,\omega_{\star}, e^{-i\,h\,\omega_{\star}}) = 0$) vary in $\mathbb{C}_-$ and in $\mathbb{C}_+$ with respect to h. See [5] and the references therein.

Moreover, in collaboration with Mouze (Centrale Lille, France), we want to develop an effective study of the ring $\mathcal{E} = \mathbb{R}(s)[e^{-h\,s}] \cap E$, where E is the ring of entire functions. The ring $\mathcal{E}$ plays an important role for differential time-delay systems [66], [78]. Effective computation of Smith normal forms for matrices with entries in $\mathcal{E}$ and its implementation in a symbolic-numeric package will have many applications for synthesis problems of differential time-delay systems.

3.3.1.3. Stabilization problems for functional systems

We want to use the results developed on the module-theoretic aspects of the ring of multivariate rational functions without poles in the closed unit polydisc of $\mathbb{C}^n$ to effectively compute stabilizing controllers of multidimensional systems, as well as the Youla-Kučera parametrization of all the stabilizing controllers [90]. This last parametrization can be used to transform standard H_∞-optimal control problems, which are nonlinear by nature, into affine, and thus convex optimal problems. See [37], [90] and the references therein. Applications addressed in the former ANR MSDOS (2014–2018) will be developed in collaboration with Bachelier (U. Poitiers). The algorithms obtained in this direction will be unified in a unique `Maple` package.

Finally, the *noncommutative geometric approach* to robust problems for infinite-dimensional linear systems (e.g. differential time-delay or PD systems) [54], initiated in [92], will be further studied based on the mathematical concepts and methods introduced by (the Fields medalist) Connes [53]. We particularly want to investigate generalizations of Nyquist's theorem to infinite-dimensional systems based on index theory (pairing of K-theory and K-homology), model reduction based on Connes' interpretation of infinitesimal operators, robustness metrics particularly the ν-gap metric, etc. The quantized differential calculus [53], based on Hankel operators, as well as the connections and curvatures on stabilizable systems will be further studied [92]. We aim to exploit these noncommutative differential geometric structures on the systems to get new inside in both the topology and geometry aspects of the H_∞-control theory for infinite-dimensional systems [54].

3.3.2. Parameter estimation for linear & nonlinear functional systems

3.3.2.1. Linear functional systems

Our expertise on algebraic parameter estimation problem, coming from the former NON-A project-team, will be further developed. Following [65], this problem consists in estimating a set θ of parameters of a signal $x(\theta, t)$ – which satisfies a certain dynamics – when the signal $y(t) = x(\theta, t) + \gamma(t) + \varpi(t)$ is observed, where γ denotes a structured perturbation and ϖ a noise. For instance, x can be a multi-sinusoidal waveform signal and θ phases, frequencies, or amplitudes [13]. Based on a combination of algebraic analysis techniques (rings of differential operators), differential elimination theory (computation of annihilators), and *operational calculus* (Laplace transform, convolution), [65] shows how θ can sometimes be explicitly determined by means of closed-form expressions using iterated integrals of y. These integrals usually help to filter the effect of the noise ϖ on the estimation of the parameters θ.

A first aim in this direction is to develop to a greater extent our recent work [108] that shows how the above approach can cover wider classes of signals such as *holonomic signals* (e.g. signals decomposed into orthogonal polynomial bases, special functions, possibly wavelets).

Moreover, [94] explains how larger classes of structured perturbations γ can be considered when the approach developed in [65], [108], based on computation of *annihilators*, is replaced by a new approach based on the more general algebraic concept of *syzygies* [103]. This general approach to the algebraic parameter estimation problem will be developed. Following the ideas of [94], an effective version of this general approach will also be done based on differential elimination techniques, i.e. Gröbner basis techniques for rings of differential operators. It will be implemented in a dedicated `Maple` package which will extend the current prototype `NonA` package [94].

Furthermore, as an alternative to passing forth and backwards from the time domain to the operational (Laplace/frequency) domain by means of Laplace transform and its inverse as done in the standard algebraic parameter estimation method [65], [108], we aim to develop a direct time domain approach based on calculus on rings of integro-differential operators as described by the following picture (L denotes the Laplace transform):

$$\begin{array}{ccc} \text{temporal domain} & & \text{frequency domain } L(z)=\widehat{z} \\ z(t)=x(t,\theta)+\gamma(t) & \Longrightarrow & \widehat{z}(s)=\widehat{x}(s,\theta)+\widehat{\gamma}(s) \\ \textbf{integro}-\textbf{diff}.\ \textbf{calculus}\ \Downarrow & & \Downarrow \\ \text{closed-form expressions} & & \text{differential algebraic calculus} \\ \theta=g\left(\int^i z(t)\right) & \Longleftarrow & \theta=f(s^{-i}\,\widehat{z}(s)) \end{array}$$

The direct computation will be handled by means of the effective methods of rings of integro-differential operators described in the above sections.

3.3.2.2. Nonlinear functional systems

For nonlinear control systems, the approach to the parameter estimation problem, recently proposed in [3] and based on the computation of integro-differential input-output equations, will be further developed based on the integration of fractions [2]. Such a representation better suits a numerical estimation of the parameters as shown in [3].

In [29], we have recently initiated an extension of the results developed in [3] to handle integro-differential equations such as Volterra-Kostitzin's equation. This general approach, based on an extension of the input-output ideal method for ordinary differential equations to the integro-differential ones, will be further developed based on the effective elimination theory for systems of integro-differential equations. An important advantage of this approach is that not only it solves the identifiability theoretical question but it also prepares a further parameter estimation step [57].

4. Application Domains

4.1. Adaptive & parametric robust control – collaboration with Safran Electronics & Defense

We have developed a collaboration with *Safran Electronics & Defense* (Massy Palaiseau) and Rouillier (OURAGAN, Inria Paris) on a *parametric robust control theory* based on computer algebra methods (symbolic-numeric methods), as well as its applications to the robust stabilization of certain mechanical systems (e.g. gyrostabilized systems, two mass-spring-damper system, stabilized mirrors).

For low-dimensional systems of ODEs, this approach aims to determine closed-form solutions for robust controllers and for the robustness margins in terms of the model parameters (e.g. mass, length, inertia, mode) [12], [98], [100]. The main applications of these results are twofold: the feasibility of an industrial project can be simplified by speeding up the computation of robust controllers and robust margins for systems with rapidly changing architecture parameters, and avoiding usual time-consuming optimization techniques. Secondly, adaptive and embeddable schemes for robust controllers can be proposed and tested while coupling our approach with real-time parameter estimation methods such as the ones developed in the GAIA team. For more details, see [12].

Preliminary works in the direction have opened a great variety of questions such as the explicit search for positive definite solutions of algebraic or differential Riccati equations (i.e. polynomial or differential systems) with model parameters, the reduction of these equations, and of the parameters based on symmetries, the development, of efficient tools for plotting high degree curves and surfaces showing the robustness margins in terms of the model parameters (collaboration with Moroz (GAMBLE, Inria Nancy)), the use of a certified numeric Newton-Puiseux algorithm for the design of robust controllers, etc. [12], [98], [100]. These results require the use of a large spectrum of computer algebra methods such as linear algebra with parameters, polynomial systems with parameters, ordinary differential systems with parameters, symmetries and reduction, rational parametrizations, discriminant varieties, semi-algebraic sets, critical point methods, real root isolation methods, etc. We shall further develop the parametric robust control in collaboration with *Safran Electronics & Defense*.

In connection with the above results, parameter estimation methods will be studied to develop *adaptive robust controllers* for gyrostabilized systems. Indeed, combining explicit characterizations of robust controllers in terms of the model parameters with time-to-time estimations of these model parameters (which can change with the system production, the heat, the wear, etc.), the robust controllers can then be automatically tuned to conserve their robustness performances [12], [99].

Finally, as explained in [11], [99], constant and distributed delays naturally appear in *Safran E & D* systems (e.g. gyrostabilized systems using visual trackers, stabilized mirror models). Extensions of the above problems and results will be studied for differential time-delay systems based on robust control techniques for infinite-dimensional systems (see, e.g., [54] and the references therein) and its algebraic extension to include model parameters.

4.2. Self calibration problem & Gear fault diagnosis – collaboration with Safran Tech

4.2.1. Self calibration problem

Due to numerous applications (e.g. sensor network, mobile robots), sources and sensors localization has intensively been studied in the literature of signal processing. The *anchor position self calibration problem*, a well-known problem in signal processing, consists in estimating the positions of both the moving sources and a set of fixed sensors (anchors) when only the distance information between the points from the two different sets is available. The position self-calibration problem is a particular case of the *Multidimensional Unfolding* (MDU) problem for the Euclidean space of dimension 3.

Based on computer algebra methods for polynomial systems, we have recently proposed a new approach for the MDU problem which yields closed-form solutions and an efficient algorithm for the estimation of the positions [56] only based on linear algebra techniques. This first result, obtained in collaboration with Dagher (Research Engineer, Inria Chile) and Zheng (DEFROST, Inria Lille - Nord Europe), yields a recent *patent* [55]. Real tests are now carried out. Our first results will be further developed, improved, tested, and demonstrated.

The MDU problem is just one instance of localization problems: more problems can be addressed for which a computer algebra expertise can brought new interesting results, especially in finding closed-form solutions, yielding new estimation techniques which avoid the use of optimization algorithms as commonly done in the signal processing literature. The main differences between these localization problems can essentially be read on a certain matrix of distance called the *Euclidean distance matrix* [56].

4.2.2. Gear fault diagnosis

We have a collaboration with Barau (*Safran Tech*) and Hubert (*Safran Tech*), and Dagher (Research Engineer, Inria Chile) on the symbolic-numeric study of the new multi-carrier demodulation method developed in [71]. *Gear fault diagnosis* is an important issue in aeronautics industry since a damage in a gearbox, which is not detected in time, can have dramatic effects on the safety of a plane.

Since the vibrations of a spur gear can be modeled as a product of two periodic functions related to the gearbox kinematic, [71] has proposed to recover each function from the global signal by means of an optimal reconstruction problem which, by means of Fourier analysis, can be rewritten as

$$\mathrm{argmin}_{u \in \mathbb{C}^n, v_1, v_2 \in \mathbb{C}^m} \|M - u\, v_1^{\star} - D\, u\, v_2^{\star}\|_F,$$

where $M \in \mathbb{C}^{n \times m}$ (resp. $D \in \mathbb{C}^{n \times n}$) is a given (resp. diagonal) matrix with a special shape, $\| \cdot \|_F$ denotes the Frobenius norm, and $v^{\star}$ the Hermitian transpose of v. Based on closed-form solutions of the exact problem $-$ which are defined by a system of polynomial equations in the unknowns $-$ we have recently proposed efficient numerical algorithms to numerically solve the problem. The first results are interesting and they will be further developed and tested on different data sets. Finally, we shall continue to study the extremal solutions of the corresponding polynomial problem by means of symbolic and numeric methods, etc.

4.3. Applications of the parameter estimation problem to multidisciplinary domains $-$ collaboration with an INSERM team (Rouen University)

For linear systems, the closed-form expressions of the parameters obtained by means of the algebraic parameter estimation problem will continue to provide robust estimates in our multidisciplinary collaborations, in marine biology and human-machine interactions, as it is already the case of existing NON-A results (in collaboration with LOKI, Inria Lille - Nord Europe).

For nonlinear systems, a collaboration with biologists and modelers has been developed for a few years already [29]. Our partners are a team from the Applied Mathematical Department of Le Havre University (modelers) and an INSERM team at Rouen University (neurobiologists). The targeted biological problem is the cortical spreading depression, a brain disease likely to occur after cerebrovascular accidents [60]. We seek $-$ ultimately $-$ a mathematical integro-differential model permitting to predict the triggering of this disease for patients arising in the emergency services of hospitals. The key phenomenon to reproduce is a slow depolarization wave of neurons. Our approach is original because it focuses on the role calcium fluxes in neurons and astrocytes.

5. Highlights of the Year

5.1. Highlights of the Year

Computer Algebra in Scientific Computing

The GAIA team organized the conference *Computer Algebra in Scientific Computing* (CASC), University of Lille, 17–21 September 2018.

6. New Software and Platforms

6.1. ADHOMFI

Adaptive Homogeneous Filtering
KEYWORDS: Automatic differentiation - Filtering

FUNCTIONAL DESCRIPTION: allows to reconstruct a signal based on derivatives estimation and to filter high amplitude and wide frequencies spectrum perturbations.

- Contact: Denis Efimov

6.2. Platforms

6.2.1. BLINEIDE library

We have released the first version of the BLINEIDE library. BLINEIDE stands for Bibliothèques Lilloises dédiées à l'Intégration Numérique des Équations Intégro-Différentielles. It is an open source C library dedicated to the numerical integration of systems of integro-differential equations. for more details, see https://pro.univ-lille.fr/francois-boulier/logiciels/blineide.

7. New Results

7.1. Regular (differential) chains

[17] provides new equivalence theorems for regular chains and regular differential chains, which are generalizations of Ritt's characteristic sets. These theorems focus on regularity properties of elements of residue class rings defined by these chains, which are revealed by resultant computations. New corollaries to these theorems have quite simple formulations.

[30] contains a description of the management of the parameters in the`Maple DifferentialAlgebra package` and, in particular, in the RosenfeldGroebner function.

7.2. Systems of integro-differential equations

[28], [29] present a proof of concept for symbolic and numeric methods dedicated to the parameter estimation problem for models formulated by means of nonlinear integro-differential equations. In particular, we address the computation of the model input-output equation and the numerical integration of integro-differential systems (the BLINEIDE library).

7.3. Certified non-conservative tests for the structural stability of discrete multidimensional systems

In collaboration with Fabrice Rouillier (Inria Paris, Ouragan), in [18], we propose a new approach for testing the stability of nD systems. We first show that the standard characterization of the structural stability of a multivariate rational transfer function (namely, the denominator of the transfer function does not have solutions in the unit polydisc of $\mathbb{C}^n$) is equivalent to the fact that a certain system of polynomials does not have real solutions. We then use state-of-the-art computer algebra algorithms to check this last condition, and thus the structural stability of multidimensional systems. Our results have been implemented in a `Maple` prototype.

7.4. Using symbolic computation to solve algebraic Riccati equations arising in invariant filtering

In this joint work with Axel Barrau from Safran Tech [23], we propose a new step in the development of invariant observers. In the past, this theory led to impressive simplifications of the error equations encountered in estimation problems, especially those related to navigation. This was used to reduce computation load or derive new theoretical properties. Here, we leverage this advantage to obtain closed-form solutions of the underlying algebraic Riccati equations through advanced symbolic computation methods.

7.5. Parametric sub-optimal H_∞ controllers for an optro-mechanical system

In collaboration with *Safran Electronics & Defense*, in [15], we studied the robust stabilization of the line of sight of a stabilized mirror system. This system can be modeled by a single-input single- output time-delay system. Due to large model uncertainties, non-parametric methods are usually too conservative. Hence, we consider here unfixed model parameters. Using an additive decomposition, we show how to compute parametric H_∞ controllers of the time-delay model. Such a symbolic approach is interesting in the context of adaptive control and is illustrated throughout a simulation with an ideal parameter estimator.

7.6. A symbolic approach for signal demodulation and application to gearbox vibration analysis

This work is made in collaboration with Axel Barrau and Elisa Hubert (Safran Tech), and Roudy Dagher (Research Engineer, Inria Chile). The problem under study, which reduces to a certain signal factorization problem, was shown by Barrau et. al. to be equivalent to a Frobenius norm minimization problem. Starting from this optimization problem, we investigate the use of computer algebra methods to compute explicit solutions for the original problem. Along the way, we exhibit interesting algebraic and geometric properties of the underlying polynomial system. A paper is currently in development to summarize these results.

7.7. Curve analysis for the stability of time-delay systems

This work aims to design a new symbolic-numerical Puiseux-free approach for the study of the stability of differential time-delay systems. The idea behind is to replace the costly computations of Puiseux developpements around the *critical pairs* of the characteristic function by the numerical analysis of the branches of a well chosen 3D curve. The preliminary results show that this approach is easier to implement and turns out to be more efficient in practice. This ongoing work will be the subject of a future publication.

8. Bilateral Contracts and Grants with Industry

8.1. Safran Electronics & Defense

Within the CIFRE PhD thesis (2014-2018) [15], we have studied new robust stabilization techniques for gyrostabilized systems with unfixed model parameters (e.g. modes, masses, stiffness of springs, damper magnitudes). Parameters of their models indeed slowly change with the temperature, fatigue, etc., yielding time-consuming re-computations of robust controllers. Moreover, the possibility to quickly know robustness indicators (e.g. margins) and explicit robust controllers in terms of the model parameters can highly speed up the design of a project. Finally, closed-form solutions for robust controllers in terms of the model parameters are the first steps towards the development of adaptive robust controllers which can be embedded in gyrostabilized platforms since no optimization algorithms are then required for a real-time implementation and only the parameters have to be estimated from time to time to re-compute the robust controller (based on a basic arithmetic). To do that, we have introduced algebraic methods and computer algebra techniques to initiate a new approach entitled *parametric robust control*. For mor details, see [15] and [98], [100], [99]. This new approach will be further developed in the future since it opens both theoretical and practical interesting questions. In particular, the new PhD thesis of Grace in GAIA aims to study the underlying mathematical problem from both a theoretical and an implementation perspectives.

8.2. Ellcie Healthy

A new collaboration with Ellcie Healthy, a company based in Nice began in October 2017. It involves the analyze of signals coming from optical sensors installed in glasses. With Denis Efimov, the first studies obtained were very promising. This collaboration was formalized with the signature of a first contract in March 2018. The first objective of this project was to design algorithms for intelligent filtering of data coming from infrared sensors, especially for light-related disturbances. Discussions are currently underway for the submission of new joint projects.

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR

- *ANR project MSDOS* (Multidimensional System: Digression on Stability, coordinator: Nima Yeganefar (Poitiers University), 2014-2018) aimed at studying stability and stabilization problems for multidimensional systems by means of both analytic and algebraic methods. For more information, see https://www.lias-lab.fr/msdos/doku.php.
- *ANR TurboTouch* (High-performance touch interactions, coordinator: G. Casiez (MJOLNIR team, Inria), 2014–2019) develops methods and tools on transfer functions to allow high performance tactile interactions (e.g. high precision and low latency) adapted to the user and to the task. This research project is developed in collaboration with the Loki team, Inria Lille – Nord Europe (project leader). For more information, seehttp://mjolnir.lille.inria.fr/turbotouch/.
- *ANR WaQMoS* (Coastal waters Quality surveillance using bivalve Mollusk-based Sensors, coordinator: D. Efimov (Non-A Post, Inria), 2015–2020) develops a biosensor, based on measurements and interpretation of bi- valves mollusks behavior, for remote online detection of coastal water pollution and cli- mate change consequences. This research project is developed in collaboration with the Valse team, Inria Lille – Nord Europe (project leader). For more information, see https://team.inria.fr/non-a/anr-waqmos/.

9.2. European Initiatives

9.2.1. Collaborations with Major European Organizations

Mohamed Barakat: University of Siegen (Germany)

Effective module theory, effective homological algebra, algebraic analysis, computer algebra, implementation.

Georg Regensburger: Institute for Algebra, Johannes Kepler University Linz (Austria)

Rings of integro-differential-delay operators, computer algebra, implementation.

Daniel Robertz: University of Plymouth (United Kingdom)

Effective algebraic analysis, mathematical systems theory, computer algebra, implementation.

9.3. International Initiatives

9.3.1. Inria Associate Teams Not Involved in an Inria International Labs

WeCare, Inria Northern European Associate Team with the team of A. Medvedev from Uppsala University on effective algorithms for estimation and control in wearable devices for health and care, 2018–2020.

We participate in *HoTSMoCE*, an Inria Associated team with Non-A Post and the team of L. Fridman (UNAM, Mexico), on the development of algebraic and homogeneous tools for sliding mode control and estimation.

9.4. International Research Visitors

9.4.1. Visits of International Scientists

- Thomas Cluzeau, XLIM, University of Limoges, May 2018.
- Marc Moreno Maza, University of Western Ontario, London, Ontario, Canada, September 2018.
- Alexander Medvedev, University of Uppsala (03–05/10/2018).
- Fredrik Olsson, University of Uppsala (26–30/11/2018).
- Elisa Hubert (Safran Tech) visited us twice (23–24/07/2018, 12–13/09/2018) to work on the problem of gear fault diagnostic based on algebraic and symbolic approaches.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

The GAIA team organized the 2018 edition of the *Computer Algebra in Scientific Computing* (CASC) international workshops series (17–21 September 2018). For more details, see http://www.casc.cs.uni-bonn.de/2018/.

10.1.1.1. General Chair, Scientific Chair

- Since 2018, François Boulier is a *General Chair* of CASC. See http://www.casc-conference.org/.
- A. Quadrat is a member of the *IFAC Technical Committee* “Linear Control Systems”, International Federation of Automatic Control, TC2.2.

10.1.1.2. Member of the Organizing Committees

The GAIA team organized the *Computer Algebra in Scientific Computing* (CASC), University of Lille, 17–21 September 2018.

A. Quadrat is a member of the organization committee of the *Journées Nationales de Calcul Formel* (JNCF), Luminy, France, 22–26/01/2018.

10.1.2. Scientific Events Selection

10.1.2.1. Member of the Conference Program Committees

F. Lemaire was Poster Chair of the *International Symposium on Symbolic and Algebraic Computation* (ISSAC’2018) Poster Session, New York, 16–19/05/2018.

10.1.2.2. Reviewer

- Y. Bouzidi reviewed a publication for ISSAC 2018 and for TDS 2018.
- F. Lemaire reviewed a publication for CASC 2018.
- A. Quadrat reviewed two publications for ISSAC 2018.

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

A. Quadrat is associate editor of *Multidimensional Systems and Signal Processing*.

10.1.3.2. Reviewer - Reviewing Activities

A. Quadrat reviewed papers for the journal *Multidimensional Systems and Signal Processing*.

10.1.4. Invited Talks

Y. Bouzidi gave the following talks:

- A Symbolic Approach for Solving Algebraic Riccati Equations, *Journées Nationales de Calcul Formel* (JNCF), CIRM, Marseille - January 22th 2018.
- Parametric study of the critical pairs of linear differential systems with commensurate delays, *Inria DISCO team-project*, Paris - April 30th 2018.
- Using symbolic computation to solve algebraic Riccati equations arising in invariant filtering, *European Control Conference*, Limassol - June 15th 2018.
- A symbolic approach for a parametric H_∞ control problem, *visualization group*, ISCD, Paris - Novembre 30th 2018.

10.1.5. Leadership within the Scientific Community

A. Quadrat co-organized with N. Yeganefar (University of Poitiers) the invited session *New Results in Multidimensional Systems Theory* at the *2018 European Control Conference*, Cyprus, June 12-15, 2018.

R. Ushirobira was co-responsible with Denis Efimov (Non-A Post, Inria) and Gilberto Pin (Electrolux Italia) of a special issue Finite-time estimation, diagnosis and synchronization of uncertain systems for the *European Journal of Control*.

10.1.6. Scientific Expertise

A. Quadrat is a member of the *Bureau du Comité des Equipes-Projets* (BCEP) and of the *Commission des Emplois de Recherche*, Inria Lille - Nord Europe.

A. Quadrat wrote a comparative report for the position “Algorithmische Algebra” (W2), University of Siegen, Germany.

10.1.7. Research Administration

- F. Boulier has been Head of the *Spécialité GIS at Polytech Lille* since June 2018.
- A. Quadrat is in charge with C. Jamroz of the *RaWeb 2018* for Inria Lille – Nord Europe.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Rosane Ushirobira taught around 90h at Polytech Lille and University of Lille (Linear algebra, analysis and logic).

10.2.2. Supervision

PhD: Guillaume Rance, “Commande H_∞ paramétrique et application aux viseurs gyrostabilisés”, University Paris - Saclay, 09/07/2018, Alban Quadrat & Arnaud Quadrat & Hugues Mounier.

PhD in progress: Grace Younes, “Calcul de multiplicités de racines de polynoômes et de quasi-polynoômes”, 01 /10/2018, Alban Quadrat & Yacine Bouzidi & Adrien Poteaux.

Master: Ambroise Fleury (Licence 3 ème année training), “Computation of gcd using AVX”, F. Lemaire.

Master: Guillaume Maitrot (Master 2 training), “Improving the BLINEIDE library with OpenMP”, F. Lemaire.

10.2.3. Juries

F. Boulier was referee of the PhD thesis of G. Rance, University Paris Orsay, 09/07/2018.

A. Quadrat was a jury member of the CRCN Inria 2018 competition for the Lille – Nord Europe center.

10.3. Popularization

10.3.1. Internal or external Inria responsibilities

Since December 2103, R. Ushirobira organize the cycle “30 minutes of science”, a rotating monthly seminar for all researchers at Inria LIlle. On average, 40 people participate in this seminar.

10.3.2. Education

In 2017/2018, Rosane Ushirobira was a referent researcher for *the Math en Jean program* at Arthur Rimbaud College (Villeneuve d’Ascq).

10.3.3. Internal action

Y. Bouzidi, *Symbolic-numeric method for a parametric control problem*, presentation in the *30 minutes of science Inria event*, Lille - January 10th 2018.

11. Bibliography

Major publications by the team in recent years

[1] F. BOULIER, C. CHEN, F. LEMAIRE, M. MORENO-MAZA. *Real root isolation of regular chains*, in "Computer Mathematics", 2014, pp. 33–38

[2] F. BOULIER, F. LEMAIRE, J. LALLEMAND, G. REGENSBURGER, M. ROSENKRANZ. *Additive normal forms and integration of differential fractions*, in "Journal of Symbolic Computation", 2016, vol. 77, pp. 16–38

[3] F. BOULIER, F. LEMAIRE, M. ROSENKRANZ, R. USHIROBIRA, N. VERDIÈRE. *On symbolic approaches to integro-differential equations*, Springer, 2019, pp. 165–184

[4] Y. BOUZIDI, S. LAZARD, G. MOROZ, M. POUGET, F. ROUILLIER. *Separating linear forms and rational univariate representations of bivariate systems*, in "Journal of Symbolic Computation", 2015, vol. 68, pp. 84–119

[5] Y. BOUZIDI, A. POTEAUX, A. QUADRAT. *A symbolic computation approach towards the asymptotic stability analysis of differential systems with commensurate delays*, in "Delays and Interconnections: Methodology, Algorithms and Applications, Advances in Delays and Dynamics (ADD)", Springer, 2018

[6] Y. BOUZIDI, A. QUADRAT, F. ROUILLIER. *Certified Non-conservative Tests for the Structural Stability of Discrete Multidimensional Systems*, in "Multidimensional Systems and Signal Processing", June 2018, https://hal.inria.fr/hal-01951765

[7] F. CHYZAK, A. QUADRAT, D. ROBERTZ. *Effective algorithms for parametrizing linear control systems over Ore algebras*, in "Applicable Algebra in Engineering, Communications and Computing", 2005, vol. 16, pp. 319–376

[8] T. CLUZEAU, A. QUADRAT. *Factoring and decomposing a class of linear functional systems*, in "Linear Algebra and Its Applications", 2008, vol. 428, pp. 324–381

[9] A. POTEAUX, M. RYBOWICZ. *Good reduction of Puiseux series and applications*, in "Journal of Symbolic Computation", 2012, vol. 47, pp. 32–63

[10] A. POTEAUX, M. RYBOWICZ. *Improving complexity bounds for the computation of Puiseux series over finite fields*, in "ISSAC", 2015, pp. 299–306

[11] A. QUADRAT, A. QUADRAT. *Delay effects in visual tracking problems for an optronic sighting system*, in "Low-Complexity Controllers for Time-Delay Systems, Advances in Delays and Dynamics (ADD)", Springer, 2014, vol. 2, pp. 77–92

[12] G. RANCE, Y. BOUZIDI, A. QUADRAT, A. QUADRAT. *H_∞ parametric control via algebraic Riccati equations*, in "22nd International Symposium on Mathematical Theory of Networks and Systems (MTNS", 2016

[13] R. USHIROBIRA, W. PERRUQUETTI, M. MBOUP. *An algebraic continuous time parameter estimation for a sum of sinusoidal waveform signals*, in "International Journal of Adaptive Control and Signal Processing", 2016, vol. 30, pp. 1689–1713

[14] R. USHIROBIRA, A. QUADRAT. *Algebraic estimation of a biased and noisy continuous signal via orthogonal polynomials*, in "55th IEEE Conference on Decision and Control (CDC)", 2016

Publications of the year

Doctoral Dissertations and Habilitation Theses

[15] G. RANCE. *Parametric H∞ control and its application to gyrostabilized sights*, Université Paris-Saclay, July 2018, https://tel.archives-ouvertes.fr/tel-01904086

Articles in International Peer-Reviewed Journals

[16] H. AHMED, R. USHIROBIRA, D. EFIMOV. *Robust global synchronization of Brockett oscillators*, in "IEEE Transactions on Control of Network Systems", 2018, pp. 1-10 [*DOI :* 10.1109/TCNS.2018.2813927], https://hal.inria.fr/hal-01965539

[17] F. BOULIER, F. LEMAIRE, A. POTEAUX, M. MORENO MAZA. *An Equivalence Theorem For Regular Differential Chains*, in "Journal of Symbolic Computation", 2018, pp. 1-27, https://hal.archives-ouvertes.fr/hal-01391768

[18] Y. M. BOUZIDI, A. QUADRAT, F. ROUILLIER. *Certified Non-conservative Tests for the Structural Stability of Discrete Multidimensional Systems*, in "Multidimensional Systems and Signal Processing", June 2018, https://hal.inria.fr/hal-01951765

[19] X. DERU, F. LEMAIRE, D. NICOLAS. *From Etaples to Pompei. Discussion about Metal Vessel in Roman Household*, in "Herom", 2018, Définition d'un modèle à partir des données écrites et archéologiques du camp napoléonien d'Etaples, confrontation des données céramologiques romaines en Gaule et en Italie, https://hal.univ-lille3.fr/hal-01737593

[20] D. EFIMOV, R. USHIROBIRA, J. A. MORENO, W. PERRUQUETTI. *Homogeneous Lyapunov functions: from converse design to numerical implementation*, in "SIAM Journal on Control and Optimization", December 2018, https://hal.inria.fr/hal-01851761

[21] J. G. RUEDA-ESCOBEDO, R. USHIROBIRA, D. EFIMOV, J. MORENO. *Gramian-based uniform convergent observer for stable LTV systems with delayed measurements*, in "International Journal of Control", 2018, https://hal.inria.fr/hal-01889193

[22] H. SILM, R. USHIROBIRA, D. EFIMOV, J.-P. RICHARD, W. MICHIELS. *A note on distributed finite-time observers*, in "IEEE Transactions on Automatic Control", May 2018 [*DOI :* 10.1109/TAC.2018.2838042], https://hal.inria.fr/hal-01827041

International Conferences with Proceedings

[23] A. BARRAU, G. RANCE, Y. BOUZIDI, A. QUADRAT, A. QUADRAT. *Using symbolic computation to solve algebraic Riccati equations arising in invariant filtering*, in "ECC2018 - European Control Conference", Limassol, Cyprus, June 2018, https://hal.inria.fr/hal-01667341

[24] P.-A. BLIMAN, D. EFIMOV, R. USHIROBIRA. *A class of nonlinear adaptive observers for SIR epidemic model*, in "ECC 2018 - European Control Conference", Limassol, Cyprus, June 2018, 6 p. , https://hal.inria.fr/hal-01724989

[25] M. NANCEL, S. ARANOVSKIY, R. USHIROBIRA, D. EFIMOV, S. POULMANE, N. ROUSSEL, G. CASIEZ. *Next-Point Prediction for Direct Touch Using Finite-Time Derivative Estimation*, in "Proceedings of the ACM Symposium on User Interface Software and Technology (UIST 2018)", Berlin, Germany, October 2018 [*DOI :* 10.1145/3242587.3242646], https://hal.inria.fr/hal-01893310

[26] J. G. RUEDA-ESCOBEDO, R. USHIROBIRA, D. EFIMOV, J. MORENO. *A Gramian-based observer with uniform convergence rate for delayed measurements*, in "ECC 2018 - 16th annual European Control Conference", Limassol, Cyprus, June 2018, https://hal.inria.fr/hal-01895389

[27] H. SILM, R. USHIROBIRA, D. EFIMOV, W. MICHIELS, J.-P. RICHARD, E. FRIDMAN. *Comparison of the Time-Delay Margin of a Distributed and Centralized Observer*, in "ECC 2018 - European Control Conference", Limassol, Cyprus, June 2018, https://hal.inria.fr/hal-01761390

Conferences without Proceedings

[28] F. BOULIER. *On the Parameter Estimation Problem for Integro-Differential Models*, in "AADIOS @ ACA 2018", Santiago de Compostela, Spain, June 2018, https://hal.archives-ouvertes.fr/hal-01780909

[29] F. BOULIER, H. CASTEL, N. CORSON, V. LANZA, F. LEMAIRE, A. POTEAUX, A. QUADRAT, N. VERDIÈRE. *Symbolic-Numeric Methods for Nonlinear Integro-Differential Modeling*, in "CASC 2018 - The 20th International Workshop on Computer Algebra in Scientific Computing", Lille, France, September 2018, https://hal.archives-ouvertes.fr/hal-01765409

Other Publications

[30] F. BOULIER. *The Management of Parameters in the MAPLE DifferentialAlgebra Package*, June 2018, working paper or preprint, https://hal.archives-ouvertes.fr/hal-01825191

[31] F. BOULIER, F. FAGES, O. RADULESCU, S. S. SAMAL, A. SCHUPPERT, W. M. SEILER, T. STURM, S. WALCHER, A. WEBER. *The SYMBIONT Project: Symbolic Methods for Biological Networks*, August 2018, F1000Research 7:1341 (poster), https://hal.inria.fr/hal-01889825

[32] G. PEROZZI. *Optimization of the control effort for quadrotors using a varying-gain quasi-continuous sliding mode control influenced by a wind estimator*, April 2018, working paper or preprint, https://hal.inria.fr/hal-01758112

References in notes

[33] S. S. ABHYANKAR. *Irreducibility criterion for germs of analytic functions of two complex variable*, in "Advances in Mathematics", 1989, vol. 74, pp. 190–257

[34] J. ACZÉL. *Lectures on Functional Equations and Their Applications*, Academic Press, 1966

[35] J. ACZÉL, J. DHOMBRES. *Functional Equations in Several Variables*, Cambridge University Press, 1989

[36] M. AUDIN. *Hamiltonian Systems and Their Integrability*, American Mathematical Society, 2008

[37] J. A. BALL, S. TER HORST. *Robust control, multidimensional systems and multivariable Nevanlinna-Pick interpolation*, Topics in operator theory. Volume 2. Systems and Mathematical Physics, Birkhäuser, 2010, pp. 13–88

[38] J.-D. BAUCH, E. NART, H. STAINSBY. *Complexity of the OM factorizations of polynomials over local fields*, in "LMS Journal of Computation and Mathematics", 2013, vol. 16, pp. 139–171

[39] V. V. BAVULA. *The algebra of integro-differential operators on an affine line and its modules*, in "J. Pure Appl. Algebra", 2013, vol. 217, pp. 495–529

[40] T. BECKER, V. WEISPFENNING. *Gröbner Bases. A Computational Approach to Commutative Algebra*, Springer, 1993

[41] J. E. BJÖRK. *Rings of Differential Operators*, North Holland, 1979

[42] N. K. BOSE. *Multidimensional Systems Theory - Progress, Directions and Open Problems in Multidimensional Systems*, Reidel Publishing Company, 1985

[43] N. K. BOSE. *Multidimensional Systems Theory and Applications*, Springer, 1995

[44] F. BOULIER. *BLAD: Bibliothèques Lilloises d'Algèbre Différentielle*, http://cristal.univ-lille.fr/~boulier/pmwiki/pmwiki.php/Main/BLAD

[45] F. BOULIER. *An optimization of Seidenberg's elimination algorithm in differential algebra*, in "Mathematics and Computers in Simulation", 1996, vol. 42, pp. 439–448

[46] F. BOULIER, D. LAZARD, F. OLLIVIER, M. PETITOT. *Computing representations for radicals of finitely generated differential ideals*, in "Appl. Algebra Engrg. Comm. Comput.", 2009, vol. 20, pp. 73–121

[47] Y. BOUZIDI, T. CLUZEAU, G. MOROZ, A. QUADRAT. *Computing effectively stabilizing controllers for a class of nD systems*, in "Proc. of the 20th World Congress of the International Federation of Automatic Control (IFAC'17), 9-14 July 2017", 2017

[48] Y. BOUZIDI, A. QUADRAT, F. ROUILLIER. *Computer algebra methods for testing the structural stability of multidimensional systems*, in "IEEE 9th International Workshop on Multidimensional Systems (NDS)", 2015

[49] D. BRIDGES, R. MINES, F. RICHMAN, P. SCHUSTER. *The polydisk Nullstellensatz*, in "Proceedings of the American Mathematical Society", 2003, vol. 132, pp. 2133–2140

[50] B. BUCHBERGER. *Gröbner bases: An algorithmic method in polynomial ideal theory*, in "Multidimensional Systems Theory: Progress, Directions and Open Problems in Multidimensional Systems", N. K. BOSE (editor), Reidel, 1985, pp. 184–232

[51] E. CARTAN. *Les systèmes différentiels extérieurs et leurs applications géométriques*, Hermann, 1971

[52] X. CARUSO, D. ROE, T. VACCON. *Division and slope factorization of p-adic polynomials*, in "International Symposium on Symbolic and Algebraic Computation (ISSAC)", 2016

[53] A. CONNES. *Noncommutative Geometry*, Academic Press, 1994

[54] R. F. CURTAIN, H. J. ZWART. *An Introduction to Infinite-Dimensional Linear Systems Theory*, Springer, 1995

[55] R. DAGHER, A. QUADRAT, G. ZHENG. *Auto-localisation par mesure de distances*, Pattern FR1853553, Inria France, 2018

[56] R. DAGHER, A. QUADRAT, G. ZHENG. *Algebraic solutions to the metric multidimensional unfolding. Application to the position self-calibration problem*, in "Pattern under development", 2017

[57] L. DENIS–VIDAL, G. JOLY–BLANCHARD, C. NOIRET. *System identifiability (symbolic computation) and parameter estimation*, in "Numerical Algorithms", 2003, vol. 34, pp. 282—292

[58] S. DIOP. *Elimination in control theory*, in "Math. Control Signals Systems", 1991, vol. 4, pp. 17–32

[59] S. DIOP. *Differential-algebraic decision methods and some applications to system theory*, in "Theoret. Comput. Sci.", 1992, vol. 98, pp. 137–161

[60] J. P. DREIER, S. MAJOR, A. MANNING, J. WOITZIK, C. DRENCKHAHN, J. STEINBRINK, C. TOLIAS, A. I. OLIVEIRA-FERREIRA, M. FABRICIUS, J. A. HARTINGS, P. VAJKOCZY, M. LAURITZEN, U. DIRNAGL, G. BOHNER, A. J. STRONG. *Cortical spreading ischaemia is a novel process involved in ischaemic damage in patients with aneurysmal subarachnoid haemorrhage*, in "Brain", 2009, vol. 132, pp. 1866–1881

[61] D. DUVAL. *Rational Puiseux expansions*, in "Compositio Mathematica", 1989, vol. 70, pp. 119–154

[62] A. FABIAŃSKA, A. QUADRAT. *Applications of the Quillen-Suslin theorem to multidimensional systems theory*, in "Gröbner Bases in Control Theory and Signal Processing, Radon Series on Computation and Applied Mathematics", D. GRUYTER (editor), H. Park, G. Regensburger, 2007, vol. 3, pp. 23–106, http://chercheurs.lille.inria.fr/aquadrat/QuillenSuslin.html

[63] M. FLIESS, S. T. GLAD. *An algebraic approach to linear and nonlinear control*, in "Essays on Control: Perspectives in the Theory and its Applications", BIRKHAÜSER (editor), 1993, pp. 223–267

[64] M. FLIESS, H. MOUNIER. *Controllability and observability of linear delay systems: an algebraic approach*, in "ESAIM Control Optim. Calc. Var.", 1998, vol. 3, pp. 301–314

[65] M. FLIESS, H. SIRA-RAMÍREZ. *An algebraic framework for linear identification*, in "ESAIM Control Optim. Calc. Variat.", 2003, vol. 9, pp. 151–168

[66] H. GLUESING-LUERSSEN. *Linear Delay-Differential Systems with Commensurate Delays: An Algebraic Approach*, Lecture Notes in Mathematics, Springer, 1995, vol. 1770

[67] M. GROMOV. *Partial Differential Relations*, Springer, 1986

[68] J. GUÀRDIA, J. MONTES, E. NART. *Newton polygons of higher order in algebraic number theory*, in "Transactions of the American Mathematical Society", 2012, vol. 364, pp. 361–416

[69] E. HAIRER, G. WANNER. *Solving Ordinary Differential Equations II: Stiff and Differential-Algebraic Problems*, Springer Series in Computational Mathematics, Springer, 2010, vol. 14

[70] J. K. HALE, S. M. V. LUNEL. *Introduction to Functional Differential Equations*, Applied Mathematical Sciences, Springer, 1993, vol. 99

[71] E. HUBERT, A. BARRAU, M. E. BADAOUI. *New multi-carrier demodulation method applied to gearbox vibration analysis*, in "2018 IEEE International Conference on Acoustics, Speech, and Signal, Calgary, 15-20/04/2018", 2018

[72] E. HUBERT. *Notes on triangular sets and triangulation-decomposition algorithms II: Differential Systems*, in "Symbolic and Numerical Scientific Computations, LNCS", SPRINGER (editor), Springer, 2003, vol. 2630, pp. 40–87

[73] T. KAILATH. *Linear Systems*, Prentice-Hall, 1980

[74] F. KAKO, T. SASAKI. *Solving multivariate algebraic equations by Hensel construction*, in "Japan J. of Industrial and Applied Math.", 1999, vol. 16, pp. 257–285

[75] M. KASHIWARA. *Algebraic Study of Systems of Partial Differential Equation*, Mémoires de la Société Mathématiques de France, 1995, vol. 63, Transl. from Japanese of his 1970 Master's Thesis

[76] E. R. KOLCHIN. *Differential Algebra and Algebraic Groups*, Academic Press, 1973

[77] T. Y. LAM. *Serre's Problem on Projective Modules*, Monograph in Mathematics, Springer, 2006

[78] J. J. LOISEAU. *Algebraic tools for the control and stabilization of time-delay systems*, in "IFAC Reviews, Annual Reviews in Control", 2000, vol. 24, pp. 135–149

[79] J. C. MCCONNELL, J. C. ROBSON. *Noncommutative Noetherian Rings*, American Mathematical Society, 2000.

[80] U. OBERST. *Multidimensional constant linear systems*, in "Acta Appl. Math.", 1990, vol. 20, pp. 1–175

[81] P. OLVER. *Applications of Lie Groups to Differential Equations*, Springer, 1993

[82] A. PARISA, A. MASOUD, M. M. MAZA. *On the extended Hensel construction and its application to the computation of limit points*, in "International Symposium on Symbolic and Algebraic Computation (ISSAC)", 2017

[83] J. W. POLDERMAN, J. C. WILLEMS. *Introduction to Mathematical Systems Theory. A Behavioral Approach*, TAM, Springer, 1998, vol. 26

[84] J.-F. POMMARET. *Partial Differential Control Theory. Volume II: Control Systems*, Kluwer, 2001

[85] J.-F. POMMARET. *Systems of Partial Differential Equations and Lie Pseudogroups*, Gordon and Breach, 1978

[86] J.-F. POMMARET. *Partial Differential Equations and Group Theory: New Perspectives for Applications*, Kluwer, 1994

[87] A. POTEAUX. *Calcul de développements de Puiseux et application au calcul du groupe de monodrmie d'une courbe algébrique plane*, University of Limoges, 2008

[88] A. POTEAUX, M. WEIMANN. *A dichotomic Newton-Puiseux algorithm using dynamic evaluation*, in "Preprint available at https://hal.inria.fr/hal-01578214", 2017

[89] A. QUADRAT. *The fractional representation approach to synthesis problems: an algebraic analysis viewpoint. Part I: (weakly) doubly coprime factorizations, Part II: internal stabilization*, in "SIAM J. Control & Optimization", 2003, vol. 42, pp. 266–299, 300–320

[90] A. QUADRAT. *A lattice approach to analysis and synthesis problems. On a generalization of the Youla-Kučera parametrization. Part II: The lattice approach to MIMO systems*, in "Mathematics of Control, Signals, and Systems", 2006, vol. 18, pp. 147–186, 199–235

[91] A. QUADRAT. *An introduction to constructive algebraic analysis and its applications*, in "Les cours du CIRM, Journées Nationales de Calcul Formel", CIRM, 2010, vol. 1, pp. 281–471

[92] A. QUADRAT. *Noncommutative geometric structures on stabilizable infinite-dimensional linear systems*, in "European Control Conference, Strasbourg (France)", 2014

[93] A. QUADRAT. *A constructive algebraic analysis approach to Artstein's reduction of linear time-delay systems*, in "Proceedings of 12th IFAC Workshop on Time Delay Systems (TDS)", 2015

[94] A. QUADRAT. *Towards an effective study of the algebraic parameter estimation problem*, in "Proceedings of 20th World Congress of the International Federation of Automatic Control (IFAC'17)", 2017

[95] A. QUADRAT, G. REGENSBURGER. *Computing polynomial solutions and annihilators of integro-differential operators with polynomial coefficients*, in "Algebraic and Symbolic Computation Methods in Dynamical Systems", Springer, 2018, vol. 9, pp. 87–111

[96] A. QUADRAT, D. ROBERTZ. *Computation of bases of free modules over the Weyl algebras*, in "Journal of Symbolic Computation", 2007, vol. 42, pp. 1113–1141

[97] A. QUADRAT, D. ROBERTZ. *A constructive study of the module structure of rings of partial differential operators*, in "Acta Applicandæ Mathematicæ", 2014, vol. 133, pp. 187–243

[98] G. RANCE, Y. BOUZIDI, A. QUADRAT, A. QUADRAT. *Explicit H_∞ controllers for 1st to 3rd order single-input single-output systems with parameters*, in "Proceedings of IFAC 2017 Workshop Congress", 2017

[99] G. RANCE, Y. BOUZIDI, A. QUADRAT, A. QUADRAT. *Parametric sub-optimal H_∞ controllers for an optro-mechanical system modeled by a time-delay 4th order system*, in "8th International Symposium on Optronics in Defence and Security, OPTRO2018, accepted", 2018

[100] G. RANCE, Y. BOUZIDI, A. QUADRAT, A. QUADRAT, F. ROUILLIER. *Explicit H_∞ controllers for 4th order single-input single-output systems with parameters and their applications to the two mass-spring system with damping*, in "Proceedings of IFAC 2017 Workshop Congress", 2017

[101] J. F. RITT. *Differential Algebra*, RI: Amer. Math. Soc., 1950

[102] D. ROBERTZ. *Formal Algorithmic Elimination for PDEs*, Lecture Notes in Mathematics, Springer, 2014, vol. 2121

[103] J. J. ROTMAN. *An Introduction to Homological Algebra*, Academic Press, 1979

[104] W. M. SEILER. *Involution. The Formal Theory of Differential Equations and its Applications in Computer Algebra*, Springer, 2010

[105] J. T. STAFFORD. *Module structure of Weyl algebras*, in "J. London Math. Soc.", 1978, vol. 18, pp. 429–442

[106] T. SÁNCHEZ, J. A. MORENO. *A constructive Lyapunov function design method for a class of homogeneous systems*, in "53rd IEEE Conference on Decision and Control, Los Angeles", 2014, pp. 5500–5505

[107] T. SÁNCHEZ, J. A. MORENO. *An SOS method for the design of continuous and discontinuous differentiators*, in "International Journal of Control", 2017, vol. https://doi.org/10.1080/00207179.2017.1393564, pp. 1–18

[108] R. USHIROBIRA, A. QUADRAT. *Algebraic estimation of a biased and noisy continuous signal via orthogonal polynomials*, in "55th IEEE Conference on Decision and Control", 2016

[109] M. V. DER PUT, M. SINGER. *Galois Theory of Linear Differential Equations*, Springer, 2002