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**Université Nice - Sophia  
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**Université de Bourgogne**

Activity Report 2018

## **Project-Team MCTAO**

# Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Institut Mathématique de Bourgogne, Laboratoire Jean-Alexandre Dieudonné (JAD)

RESEARCH CENTER  
**Sophia Antipolis - Méditerranée**

THEME  
**Optimization and control of dynamic  
systems**



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## Project-Team MCTAO

*Creation of the Team: 2012 January 01, updated into Project-Team: 2013 January 01*

### Keywords:

#### Computer Science and Digital Science:

- A5.10.3. - Planning
- A5.10.4. - Robot control
- A6.1.1. - Continuous Modeling (PDE, ODE)
- A6.1.5. - Multiphysics modeling
- A6.2.1. - Numerical analysis of PDE and ODE
- A6.2.6. - Optimization
- A6.4. - Automatic control
  - A6.4.1. - Deterministic control
  - A6.4.3. - Observability and Controlability
  - A6.4.4. - Stability and Stabilization
  - A6.4.6. - Optimal control
- A6.5. - Mathematical modeling for physical sciences
- A8.2.3. - Calculus of variations
- A8.12. - Optimal transport

#### Other Research Topics and Application Domains:

- B2.6. - Biological and medical imaging
  - B2.7.2. - Health monitoring systems
- B5.2.3. - Aviation
- B5.2.4. - Aerospace
- B5.6. - Robotic systems

## 1. Team, Visitors, External Collaborators

### Research Scientists

- Jean-Baptiste Pomet [Team leader, Inria, Senior Researcher, HDR]
- Lamberto Dell'Elce [Inria, Researcher, since Nov 2018]
- Laetitia Giraldi [Inria, Researcher]

### Faculty Members

- Bernard Bonnard [Université de Bourgogne, Professor, HDR]
- Jean-Baptiste Caillaud [Université Côte d'Azur, Professor, HDR]
- Pierre Lissy [Université Paris-Dauphine, Associate Professor, on leave at Inria until Jun 2018]
- Ludovic Rifford [Université de Nice - Sophia Antipolis, Professor, HDR]
- Gilles Lebeau [Université Côte d'Azur, Professor, on leave at Inria since Sep 2018, HDR]

### Post-Doctoral Fellows

- Lamberto Dell'Elce [Inria, until Aug 2018]
- Walid Djema [Inria]

### PhD Students

- Luca Berti [Université de Strasbourg, since Oct 2018]
- Yacine El Alaoui-Faris [Inria]

Sébastien Fueyo [Inria]  
Clément Moreau [Ecole Normale Supérieure Cachan]  
Michael Orioux [Université Paris-Dauphine]  
Agustin Yabo [Université Côte d’Azur, since Oct 2018]

#### **Administrative Assistant**

Claire Senica [Inria]

#### **Visiting Scientist**

Zeinab Badreddine [Université de Bourgogne, until Jul 2018]

#### **External Collaborators**

Thierry Dargent [Thales Alenia Space, Engineer]  
Joseph Gergaud [ENSEEIH Toulouse, Professor]  
Jérémy Rouot [EPF Troyes, Associate Professor]

## 2. Overall Objectives

### 2.1. Control, Transport and Dynamics

Our goal is to develop methods in geometric control theory for finite-dimensional nonlinear systems, as well as in optimal transport, and to transfer our expertise through real applications of these techniques.

Our primary domain of industrial applications in the past years is space engineering, namely designing trajectories in space mechanics using optimal control and stabilization techniques: transfer of a satellite between two Keplerian orbits, rendez-vous problem, transfer of a satellite from the Earth to the Moon or more complicated space missions. A second field of applications is quantum control with applications to Nuclear Magnetic Resonance and medical image processing. A third and more recent one is the control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number.

There is also a form of transfer to other mathematical fields: some problems in dynamical systems are being solved thanks to control theory techniques.

## 3. Research Program

### 3.1. Control Problems

McTAO’s major field of expertise is control theory in the large. Let us give an overview of this field.

**Modelling.** Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form  $dx/dt = f(x, u)$  where  $x$  is the *state*, ideally finite dimensional, and  $u$  the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes  $x$  as an input. Modelling amounts to deciding the nature and dimension of  $x$ , as well as the dynamics (roughly speaking the function  $f$ ). Connected to modeling is identification of parameters when a finite number of parameters are left free in “ $f$ ”.

**Controllability, path planning.** Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by “physical” first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open.

Path planning is the problem of constructing the control that actually steers one state to another.

**Optimal control.** In optimal control, one wants to find, among the controls that satisfy some constraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion.

This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [70], [47], [37], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions.

See more about our research program in optimal control in section 3.2.

**Feedback control.** In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems.

On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of “Control Lyapunov Functions”, see [61], [72].

**Classification of control systems** One may perform various classes of transformations acting on systems, or rather on models... The simpler ones come from point-to-point transformations (changes of variables) on the state and control, and more intricate ones consist in embedding an extraneous dynamical system into the model, these are dynamic feedback transformations, they change the dimension of the state.

In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomena, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves.

These topics are central in control theory; they are present in the team, see for instance the classification aspect in [52] or —although this research has not been active very recently— the study [69] of dynamic feedback and the so-called “flatness” property [64].

## 3.2. Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondence between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

**Cut and conjugate loci.** The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [74], [38], [39] and [53] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [46], followed by the *Hampath* [55] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number.

With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [56] and [43]).

Recently, computation of conjugate points was conducted in [6], [5] to determine the optimality status in swimming at low Reynolds number; because of symmetries, and of the periodicity constraint, a tailor-made notion of conjugate point had to be used, see more in [65]. In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [36]. This point of view withstands generalizations far beyond the smooth case: In  $L^1$ -minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [60]).

The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on “The cut locus: A bridge over differential geometry, optimal control, and transport”, co-organized by team members and Japanese colleagues, see section 10.1.1.

**Riemann and Finsler geometry.** Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some transformations, like averaging (see section 3.2) or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry.

On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [63], [62]. The analysis in the oblate geometry [44] was completed in [59] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci.

Averaging in systems coming from space mechanics control (see sections 3.2 and 4.1) with  $L^2$ -minimization yields a Riemannian metric, thoroughly computed in [42] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [45] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [41]. In [51], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a “drift”, *i.e.* the Finsler metric is no longer symmetric.

**Sub-Riemannian Geometry.** Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, like smoothness of minimisers, see the recent monograph [68] for an introduction. Let us detail one open question related to these singular trajectories: the Sard conjecture in sub-Riemannian geometry.

Given a totally non-holonomic distribution on a smooth manifold, the Sard Conjecture is concerned with the size of the set of points that can be reached by singular horizontal paths starting from a given point. In the setting of rank-two distributions in dimension three, the Sard conjecture is that this set should be a subset of the so-called Martinet surface, indeed small both in measure and in dimension. In [4], it has been proved that the conjecture holds in the case where the Martinet surface is smooth. Moreover, the case of singular real-analytic Martinet surfaces was also addressed. In this case, it was shown that the Sard Conjecture holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. It is, of course, very interesting to get rid of the remaining technical assumption, or to go to higher dimension. Note that any Sard-type result has strong consequences on the regularity of sub-Riemannian distance functions and in turn on optimal transport problems in the sub-Riemannian setting.

**Small controls and conservative systems, averaging.** Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [40], [73].



This line of research stemmed out of applications to space engineering (see section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including  $J_2$  effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure.

Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [41], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [58], standard averaging ([40], [73]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an “averaged Pontryagin Maximum Principle” certainly fails. In particular, one cannot hope for  $C^1$ -regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [57].)

**Optimality of periodic solutions/periodic controls.** When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [48], for the problem of optimizing the efficiency of the displacement of some micro-swimmers (see section 4.3) with periodic deformations, we used the sufficient optimality conditions established by R. Vinter’s group [78], [65] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P. Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [78], [65].

Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the “turnpike theorem” (classical, recently applied to nonlinear situations in [77]).

**Software.** These applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. The software *Hampath* (see section 6.1) is maintained by former members of the team in close collaboration with McTAO. We also use direct discretization approaches (such as the **Bocop** solver developed by COMMANDS) in parallel. Apart from this, we develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms (see sections 4.1 to 4.3) on one hand, and with optimal transport, and more recently Hamiltonian dynamics, on the other.

### 3.3. Optimal Transport

Given two measures, and calling transport maps the maps that transport the first measure into the second one, the Monge-Kantorovich problem of Optimal Transport is the search of the minimum of some cost on the set of transport maps. The cost of a map usually comes from some point to point cost and the transport measure. This topic attracted renewed attention in the last decade, and has ongoing applications of many types. Matching optimal transport with geometric control theory is one originality of our team.

Let us sketch an important class of open problems. In collaboration with R. McCann [67], we worked towards identifying the costs that admit unique optimizers in the Monge-Kantorovich problem of optimal transport between arbitrary probability densities. For smooth costs and densities on compact manifolds, the only known examples for which the optimal solution is always unique require at least one of the two underlying spaces to be homeomorphic to a sphere. We have introduced a multivalued dynamics induced by the transportation cost between the target and source space, for which the presence or absence of a sufficiently large set of periodic trajectories plays a role in determining whether or not optimal transport is necessarily unique. This insight allows us to construct smooth costs on a pair of compact manifolds with arbitrary topology, so that the optimal transport between any pair of probability densities is unique. We investigated further this problem of uniquely minimizing costs and obtained in collaboration with Abbas Moameni [12] a result of density of uniquely minimizing costs in the  $C^0$ -topology. The results in higher topology should be the subject of some further research.

## 4. Application Domains

### 4.1. Aerospace Engineering

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its “life”), which amounts to  $L^1$ -minimization. Both topics are studied by the team.

We have a steady relationships with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 3 PhDs and 2 post-docs in the Sophia location of the team in the decade and are a source of inspiration even at the methodological level. Team members also have close connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

### 4.2. Magnetic resonance imaging (MRI)

The starting point of our interest in optimal control for quantum systems was a collaboration with physicist from ICB, University of Burgundy (Dominique Sugny), motivated by an ANR project where we worked on the control of molecular orientation in a dissipative environment using a laser field, and developed optimal control tools, combined with numerical simulations, to analyze the problem for Qubits. This was related to quantum computing rather than MRI.

Using this expertise and under the impulse of Prof. S. Glaser and his group (Chemistry, TU München), we investigated Nuclear Magnetic Resonance (NMR) for medical imaging (MRI), where the model is the Bloch equation describing the evolution of the magnetization vector controlled by a magnetic field, but in fine is a specific Qubit model without decoherence. We worked on, and brought strong contributions to, the contrast problem: typically, given two chemical substances that have an importance in medicine, like oxygenated and de-oxygenated blood, find the (time-dependent) magnetic field that will produce the highest difference

in brightness between these two species on the image resulting from Nuclear Magnetic Resonance. This has immediate and important industrial applications in medical imaging. Our contacts are with the above mentioned physics academic labs, who are themselves in contact with major companies.

The team has produced and is producing important work on this problem. One may find a good overview in [50], a reference book has been published on the topic [54], a very complete numerical study comparing different optimization techniques was performed in [49]. We conduct this project in parallel with S. Glaser team, which validated experimentally the pertinence of the methods, the main achievement being the *in vivo* experiments realized at the Creatis team of Insa Lyon showing the interest to use optimal control methods implemented in modern softwares in MRI in order to produce a better image in a shorter time. A goal is to arrive to a cartography of the optimal contrast with respect to the relaxation parameters using LMI techniques and numerical simulations with the Hampath and Bocop code; note that the theoretical study is connected to the problem of understanding the behavior of the extremal solutions of a controlled pair of Bloch equations, and this is an ambitious task. Also, one of the difficulties to go from the obtained results, checkable on experiments, to practical control laws for production is to deal with magnetic field space inhomogeneities.

### 4.3. Swimming at low-Reynolds number

Following the historical reference for low Reynolds number locomotion [71], the study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. In this regime, it turns out that the infinite dimensional dynamics of the fluid do not have to be retained as state variables, so that the dynamics of a micro-swimmer can be expressed by ordinary differential equations if its shape has a finite number of degrees of freedom. Assuming this finite dimension, and if the control is the rate of deformation, one obtains a control system that is linear (affine without drift) with respect to the controls, *i.e.* the optimal control problem with a quadratic cost defines a sub-Riemannian structure (see section 3.2). This is the case where the shape is “fully actuated”, *i.e.* if all the variables describing the shape are angles, there is an actuator on each of these angles. For artificial micro-swimmers, this is usually unrealistic, hence (artificial) magneto-elastic micro-swimmers, that are magnetized in order to be deformed by an external magnetic field. In this case, the control functions are the external magnetic field.

In both cases, questions are controllability (straightforward in the fully actuated case), optimal control, possibly path planning. We collaborate with teams that have physical experiments for both.

- In collaboration with D. Takagi and M. Chyba (Univ of Hawaii), this approach is currently at the experimental level for copepod-like swimmer at the university of Hawaii: on the one hand, this zooplankton and its locomotion can be observed, and a robot micro swimmer mimicking a copepod has been constructed, but in fact large enough for direct actuation to be possible, and the low Reynolds number is achieved by using a more viscous fluid. This gives possibilities, through an inverse optimization problem, to determine what cost can be optimised by these crustaceans, see [5], [76], and to validate models on the robot.
- For magneto-elastic micro-robots, Y. El-Alaoui’s PhD is co-advised with Stéphane Régnier from the robotics lab ISIR, Univ. Paris 6. Magneto-elastic micro-robots and their magnetic actuation are actually built at ISIR and the aim of the collaboration is to validate models and improve the existing control laws both in performance and in energy; of course, the micro scale does make things difficult.

The questions about optimality of periodic controls raised in Section 3.2 are related to these applications for periodic deformations, or strokes, play an important role in locomotion.

### 4.4. Stability of high frequency amplifiers

Nonlinear hyper-frequency amplifiers are ubiquitous in cell phone relays and many other devices. They must be as compact as possible, yielding complicated design. Computer Assisted Design tools are extensively used;

for a given amplifier design, they provide frequency responses but fail to provide information of the stability of the response for each frequency. This stability is crucial for an unstable response will not be observed in practice; the actual device should not be built before stability is asserted. Predicting stability/instability from “simulations” in the Computer Assisted Design tool is of utmost importance (simulation between quotation marks because these simulations are in fact computations in the frequency domain).

Some techniques do exist (see a state of the art in [75]). They are pioneering but mildly reliable and not clearly mathematically grounded. Potential transfer is important.

This is the topic of an ongoing collaboration between McTAO and the project-team APICS. See results in section 7.15.

## 5. Highlights of the Year

### 5.1. Highlights of the Year

Let us mention two events

- Lamberto Dell’Elce was hired as a permanent researcher in 2018. This is not a scientific achievement in itself, but it is an important point in the life of a research team.
- Alessio Figalli **recieved a Fields Medal** at ICM 2018 in Rio. He is a close collaborator of Ludovic Rifford, member of the team.

## 6. New Software and Platforms

### 6.1. Hampath

**KEYWORDS:** Optimal control - Second order conditions - Differential homotopy - Ordinary differential equations

**FUNCTIONAL DESCRIPTION:** Hampath is a software developed to solve optimal control problems by a combination of Hamiltonian et path following methods. Hampath includes shooting and computation of conjugate points. It is an evolution of the software cotcot ([apo.enseeiht.fr/cotcot](http://apo.enseeiht.fr/cotcot)). It has a Fortran kernel, uses Tapenade ([www-sop.inria.fr/tropics/tapenade.html](http://www-sop.inria.fr/tropics/tapenade.html)) for automatic differentiation and has a Matlab interface.

- Participants: Jean-Baptiste Caillaud, Joseph Gergaud and Olivier Cots
- Contact: Jean-Baptiste Caillaud
- URL: <http://www.hampath.org>

## 7. New Results

### 7.1. Well posedness in Optimal Transport

**Participants:** Zeinab Badreddine, Ludovic Rifford, Robert McCann [Univ of Toronto, Canada], Abbas Moameni [Carleton Univ, Ottawa, Canada].

Concerning the Kantorovitch problem, in continuation of the work by McCann and Rifford [67], Moameni and Rifford have studied (see [12]) some conditions on the cost which are sufficient for the uniqueness of optimal plans (provided that the measures are absolutely continuous with respect to the Lebesgue measure). As a by-product of their results, the authors show that the costs which are uniquely minimizing for the Kantorovitch problem are dense in the  $C^0$ -topology. Many others applications and examples are investigated.

Concerning the Monge problem in the sub-Riemannian setting, Zeinab Badreddine [2] obtained the first result of well-posedness in cases where singular minimizing curves may be present. This study is related to the so-called measure contraction property. In collaboration with Rifford [22], Badreddine obtained new classes of sub-Riemannian structures satisfying measure contraction properties.

## 7.2. Strong Sard conjecture for sub-Riemannian structures

**Participants:** Ludovic Rifford, André Belotto Da Silva [Université Aix-Marseille, France], Alessio Figalli [ETH, Switzerland], Adam Parusinski [Université Côte d'Azur, France].

In [25], we address the strong Sard conjecture for sub-Riemannian structures on 3-dimensional analytic manifolds. More precisely, given a totally non-holonomic analytic distribution of rank 2 on a 3-dimensional analytic manifold, we investigate the size of the set of points that can be reached by singular horizontal paths starting from a given point and prove that it has Hausdorff dimension at most 1. In fact, this set is a semi-analytic curve, provided that the lengths of the singular curves under consideration are bounded with respect to a given complete Riemannian metric. As a consequence, combining these techniques with recent developments on the regularity of sub-Riemannian minimizing geodesics, we prove that minimizing sub-Riemannian geodesics in 3-dimensional analytic manifolds are always of class  $C^1$ , and actually are analytic outside of a finite set of points. This paper can be seen as a major step toward a proof of the Sard conjecture in any dimension.

This is a drastic improvement of the results published in [4] (appeared this year), that proved a slightly weaker property for a less general class of systems.

## 7.3. Optimal approximation of internal controls for a wave-type problem with fractional Laplacian using finite-difference method

**Participants:** Pierre Lissy, Ionel Roventa [University of Craiova, Romania].

In paper [30], a finite-difference semi-discrete scheme for the approximation of internal controls of a one-dimensional evolution problem of hyperbolic type involving the spectral fractional Laplacian is considered. The continuous problem is controllable in arbitrary small time. However, the high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh size) controllability property of the semi-discrete model in the natural setting. For all initial data in the natural energy space, if we filter the high frequencies of these initial data in an optimal way, the uniform controllability property in arbitrary small time is restored. The proof is mainly based on a (non-classic) moment method.

## 7.4. Singularities in minimum time control

**Participants:** Jean-Baptiste Caillaud, Michaël Orioux, Jacques Féjoz [Univ. Paris Dauphine], Robert Roussarie [Univ. Bourgogne-Franche Comté].

We analyze singularities arising in minimum time systems. Consider a control affine system in dimension four with control on the disc such that the controlled fields together with their first order Lie brackets with the drift have full rank. There is a natural stratification of the codimension two singular set in the cotangent bundle leading to a local classification of extremals in terms of singular and bang arcs. This analysis was done in [56] using the nilpotent model, and extended in [35] by interpreting the singularities of the extremal flow as equilibrium points of a regularized dynamics to prove the continuity of the flow. After a suitable blow-up, one can actually treat these singularities as connections of pairs of normally hyperbolic invariant manifolds in order to find a suitable stratification of the flow and prove finer regularity properties. Another issue is to be able to give global bounds on the number of these heteroclinic connections. This can be done by means of *à la Sturm* estimations. This work is part of the PhD thesis of Michaël Orioux [1] and is described in the preprint [28]. In an ongoing work, we also investigate second order sufficient conditions for extremals with such singularities.

## 7.5. Software advances

**Participants:** Jean-Baptiste Caillau, Olivier Cots [Univ. Toulouse], Lamberto Dell’Elce, Thibaud Kloczko, Pierre Martinon [COMMANDS team], Jean-Baptiste Pomet.

McTAO and COMMANDS have been awarded an AMDT (Action Mutualisée de Développement) funding of two years to develop a common interface for *Hampath* and BOCOP. This AMDT, coined *ct* for "Control tools" is to start in January 2019. Our midterm goal is to set the standard for the numerical resolution of optimal control problems. On the basis of the two well established codes BOCOP and *Hampath* from the optimal control community, thanks to this ADT we want to design a high-level modular architecture in order to:

- interoperate BOCOP and *Hampath*,
- offer a high-level common user interface for the two codes.

Another expected outcome of the ADT is to integrate state of the art processes into the development of the two solvers (collaborative dev tools, reliable repositories, continuous integration...)

## 7.6. Averaging optimal control problems with two frequencies

**Participants:** Jean-Baptiste Caillau, Lamberto Dell’Elce, Jean-Baptiste Pomet.

Averaging is a valuable technique to gain understanding in the long-term evolution of fast-oscillating dynamical systems. Recent contributions (pioneered by McTAO members in the framework of a long-standing project funded by CNES and Thales Alenia Space) proved that averaging can be applied to the extremal flow of optimal control problems. This study extends the aforementioned results by tackling averaging of time optimal systems with two fast variables with particular emphasis on the treatment of adjoint variables and on the understanding of resonance effects on their dynamics. The chapter [18] details part of this work, and a dedicated paper is in preparation [29].

## 7.7. Integrability properties of the controlled Kepler problem

**Participants:** Jean-Baptiste Caillau, Michaël Orioux, Jacques Féjóz [Univ. Paris Dauphine], Robert Roussarie [Univ. Bourgogne-Franche Comté].

We prove, using Moralès–Ramis theorem, that the minimum-time controlled Kepler problem is not meromorphically integrable in the Liouville sense on the Riemann surface of its Hamiltonian. The Kepler problem is a classical reduction of the two-body problem. We think of the Cartesian coordinate as being the position of a spacecraft, and of the attraction as the action of the Earth. We are interested in controlling the transfer of the spacecraft from one Keplerian orbit towards another one, in the plane. By virtue of Pontrjagin maximum principle, the minimum time dynamics is a Hamiltonian system. The controlled Kepler problem can be embedded in the two parameter family obtained when considering the control of the circular restricted three-body problem. In the uncontrolled model, it is well known that the Kepler case is integrable and geodesic (there exists a Riemannian metric such that Keplerian curves are geodesics of this metric), while there are obstructions to integrability for positive ratio of masses. In the controlled case, the Kepler problem for the energy cost has been shown to be integrable (and geodesic) when suitably averaged. The aim of this work is to study the integrability properties of the Kepler problem for time minimization. The pioneering work of Ziglin in the 80s, followed by the modern formulation of differential Galois theory in the late 90s by Moralès, Ramis and Simó, have led to a very diverse literature on the integrability of Hamiltonian systems. According to Pontrjagin Maximum principle, one can turn general optimization problems with dynamical constraints into Hamiltonian systems, which are generally not everywhere differentiable. Optimal control theory thus provides an abundant class of dynamical systems for which integrability is a central question. Yet, differential Galois theory has not so often been applied in this context, in part because of the difficulty brought by the singularities. Notwithstanding these singularities, we show how to apply these ideas to the Kepler system, and prove that it is not meromorphically integrable in the Liouville sense on the Riemann surface of its Hamiltonian. This work is also part of the PhD thesis of Michaël Orioux [1] and is described in the paper [11].

## 7.8. Quasi-satellite orbits in the proximity of Martian moons

**Participants:** Lamberto Dell’Elce, Nicola Baresi [JAXA, Japan], Josué Cardoso Dos Santos [Sao Paolo State University, Brasil], Yasuhiro Kawakatsu [JAXA, Japan].

The Martian Moons eXploration mission is currently under development at the Japan space agency (JAXA) and will be the first spacecraft mission to retrieve pristine samples from the surface of Phobos. In preparation for the sampling operations, MMX will collect observations of Phobos from stable retrograde relative trajectories, which are referred to as quasi-satellite orbits (QSOs). This study investigates the navigability of mid- and high-altitude QSOs in terms of relative orbit elements. After developing an analytical model for the long-term evolution of QSOs and a numerical map between mean and osculating (instantaneous) orbital elements, we use a Lyapunov control law for orbit maintenance purposes based on mean relative orbit element differences. These results were presented in [16] and they pave the way for a perspective collaboration between JAXA and McTAO.

## 7.9. The Copepod and Purcel swimmer

**Participants:** Bernard Bonnard, Jérémy Rouot, Piernicola Bettiol, Monique Chyba [U. Hawaii].

In the continuation to J. Rouot Phd thesis (2016), our results are presented in a series of papers [7], [5], [6]. The most efficient strokes are computed using geometric studies and numerical simulations, in relation with sub-Riemannian geometry and periodic optimal control algorithms. In the copepod case the model is validated by simulations and a copepod robot was constructed at Hawaii to mimic the copepod. The experiment was reproduced at EPF Troyes under the supervision of J. Rouot. The reference [21] gathers the results of MRI and swimmers in a unified setting combining geometric and numeric techniques developed in McTAO.

## 7.10. Multi-link vs flexible filament swimmers

**Participants:** Laetitia Giraldi, Clément Moreau, Jean-Baptiste Pomet, Hermes Gadhêla [Univ. of York, United Kingdom].

The inertialess fluid-structure interactions of active or passive inextensible filaments and slender-rods are ubiquitous in nature, from the dynamics of semi-flexible polymers and cytoskeletal filaments to cellular mechanics and flagella, or in artificial micro-swimmers (see Section 7.12).

For a microscopic inextensible elastic filament immersed into a fluid, even approximating the fluid-structure interaction by the Resistive Force Theory formulation, the system of PDEs resulting from elasto-hydrodynamical laws is structurally convoluted and demanding numerically. The  $N$ -link swimmer model, where the continuous filament is replaced by  $N$  segments with elasticity concentrated at the joints, can be seen as a coarse-graining formulation of the latter. In [10] (see also [31]), the  $N$ -link swimmer model is presented in this perspective and it is demonstrated numerically how this system can be used as an alternative. It can be solved numerically with any ODE solver and overcomes well-known numerical instabilities when solving numerically the full PDE for the filament. Computations can be as much as a hundred times faster. Generalisations for more complex interactions are demonstrated on four examples commonly found in biological systems, a Matlab code is provided as a basis for further generalisations.

More theoretical study of this approximation property are under investigation, also in the framework of Clément Moreau’s PhD.

## 7.11. Energy-optimal strokes for multi-link micro-swimmers

**Participants:** Laetitia Giraldi, François Alouges [École Polytechnique], Antonio Desimone [SISSA Trieste, Italy], Yshar Or [Technion, Haifa, Israel], Oren Wiezel [Technion, Haifa, Israel].

In a common work that is presented in [33], submitted to *New Journal of Physics*, we consider a slender planar multi-link micro-swimmer ( $N$  links, see Section 7.10), where the time derivatives of the angles defining the shape are taken as controls, and we are mostly interested in small-amplitude undulations about its straight configuration.

Based only on the leading order dynamics in that vicinity, the optimal stroke to achieve a given prescribed displacement in a given time period is then obtained as the largest eigenvalue solution of a constrained optimal control problem. Remarkably, the optimal stroke is an ellipse lying within a two-dimensional plane in the  $(N-1)$ -dimensional space of joint angles, where  $N$  can be arbitrarily large. For large  $N$ , the optimal stroke is a traveling wave of bending, modulo edge effects.

We also solved, numerically, the fully non-linear optimal control problem for the cases  $N = 3$  (Purcell's three-link swimmer) and  $N = 5$  showing that, as the prescribed displacement becomes small, the optimal solutions obtained using the small-amplitude assumption are recovered. We also show that, when the prescribed displacements become large, the picture is different. For  $N = 3$  we recover the non-convex planar loops already known from previous studies. For  $N = 5$  we obtain non-planar loops, raising the question of characterizing the geometry of complex high-dimensional loops.

## 7.12. Swimming magnetic micro-robots

**Participants:** Luca Berti, Yacine El Alaoui-Faris, Laetitia Giraldi, Jean-Baptiste Pomet, Christophe Prud'Homme [Université de Strasbourg], Stéphane Régnier [UPMC - Sorbonne Universités].

We are in a collaboration with the Parisian robotics laboratory ISIR (**Institut des Systèmes Intelligents et de Robotique**) to enhance the control of artificial micro-swimmers that are actually built and implemented there. This involves building models and using them for control design. These robots are "magnetic micro-swimmers": part of them is magnetized and the control is an ambient magnetic field.

Yacine El Alaoui-Faris's PhD, co-advised with Stéphane Régnier, started October, 2017. It is centered on finite-dimensional models. The robots under consideration are made of a magnetic head and a flexible tail; the model is a 3-D counterpart of the planar "multi-link micro-swimmers" discussed in Section 7.10.

The validation of this nonlinear ODE model, with or without magnetic actuation, has been achieved this year, both against continuous models present in the literature and against experimental data at ISIR. This model has a definite interest by itself, and in the case of magnetic actuation, it allowed the numerical computation of periodic controls (magnetic field) that optimize the longitudinal velocity with prescribed maximum amplitude of the magnetic field oscillations. This process is described in a manuscript under preparation, to be submitted to *Physical Review Letters*.

These controls have very recently been tested in lab and a very significant efficiency gain over classical sinusoidal oscillations has been evidenced. This is a very encouraging experimental result.

Luca Berti's PhD, co-advised with Christophe Prud'homme, started this fall. It is focused on PDE models that are closer to the real physics but more intricate. His master's thesis was mostly a numerical project in the framework of Cemracs 2018 (<http://smai.emath.fr/cemracs/cemracs18/>), where we modeled the displacement of a deformable swimmer using a coupling between Stokes equations and hyper-elasticity equations. The PDEs were solved using the Feel++ finite elements library. We validated the fluid model using an exact solution for a rotating rigid body. The motion of a one-hinged swimmer (which obeys to the scallop theorem) was successfully simulated. The physical robots from ISIR are now considered in his PhD.

## 7.13. Necessary conditions for local controllability, motivated by the Two-link Magneto-elastic Micro-swimmer

**Participants:** Laetitia Giraldi, Pierre Lissy [Univ. Paris Dauphine], Clément Moreau, Jean-Baptiste Pomet.

After proving in [66] a local controllability for the 2-link magneto-elastic swimmer around its straight configuration and noting that this property is weaker than Small-Time Locally Controllable (STLC), we investigated "full" STLC for this system, and were able to show that, except for very specific values of the lengths and magnetizations, this system is *not* STLC. This is published in [9]. This led us to necessary conditions for STLC for more general classes of systems. This is part of Clément Moreau's doctoral research and a publication is under preparation, for classes of control-affine systems with two controls that are not micro-swimmer models, but stem from the observations in [9] and [66].



## 7.14. Numerical and Symbolic computations in MRI

**Participants:** Bernard Bonnard, Jérémy Rouot, Thibaut Verron, Olivier Cots [ENSEEIH Toulouse].

Academic year 2016-17 with the two Postdoctoral position at Toulouse (J. Rouot at LAAS and T. Verron, Enseeth) was the opportunity to complete our investigations about the contrast and multi-saturation problem in MRI. This concerns numeric and symbolic computations using HamPath, Bocop, Gloptipoly and Maple software. The reference [27], to be published in MCRF, contains all the results and techniques about this project and is the final paper of this longstanding work in quantum theory.

## 7.15. Stability of nonlinear high frequency amplifiers

**Participants:** Laurent Baratchart [FACTAS project-team], Sébastien Fueyo, Jean-Baptiste Pomet, Gilles Lebeau.

Sébastien Fueyo's PhD is co-advised on this topic. The problem is presented in Section 4.4.

These amplifiers contain on the one hand nonlinear active components and on the other hand lines, that induce some sort of delays and make the system infinite-dimensional: they are, for each choice of a periodic input, a nonlinear infinite dimensional dynamical system. The Computer Aided Design tools mentioned in Section 4.4 provide a periodic solution under this periodic forcing and may also give the frequency response of the linearized system along this trajectory with some artificial "small" excitation. The goal is to deduce stability from these data.

It is an opportunity to build theoretical basis and justification to a stability analysis through harmonic identification; the latter is one of the specialties of FACTAS, we collaborate on the infinite-dimensional nonlinear stability analysis for periodic solutions and how it works with the results of harmonic identification.

On academic examples of simple circuits, we have given full justification (with some possible obstructions) to the prediction of stability through transfer function identification. The theoretical interest is that the spectrum of the operator that gives stability is not as elementary as predicted in the literature, but stability can be predicted nonetheless. This was presented at a local conference, [Université Côte d'Azur Complex Days](#) in January, and a more complete publication is under preparation.

On more general structures, new results are available too, publications are in progress.

## 7.16. Optimal sampled-control with applications to Muscular Control

**Participants:** Bernard Bonnard, Toufik Bakir [L2I, Univ. de Bourgogne Franche Comté], Jérémy Rouot.

The study was initialized two years ago under the impulse of Toufik Bakir (LE2I-UBFC). Based on preliminary experimental studies, the chosen model to muscular control integrates the fatigue variables and is known as *Ding et al force-fatigue model* in the literature. It is a refinement of the historical Hill model (Medecine Nobel Prize 1922). Preliminary results lead to construct a nonlinear observer and the optimized pulses trains are computed using MPC methods [15]. More recently in collaboration with L. Bourdin (Limoges), optimal control techniques were introduced in the framework of sampled-control problems. Pontryagin type necessary conditions were obtained with preliminary numerical simulations. On these topics, paper [23] has been submitted to *JOTA*, October 2018; [3] is accepted for publication in *Networks and Heterogeneous Media*, 2018.

## 7.17. An Optimal Control Strategy Separating Two Species of Microalgae in Photobioreactors

**Participants:** Olivier Bernard [BIOCORE project-team], Walid Djema, Laetitia Giraldi.

We investigate a minimum time control problem in a chemostat continuous photobioreactor model that describes the dynamics of two distinct microalgae populations. Our objective is to optimize the time of selection – or separation – between two species of microalgae. In [17], we focus on Droop’s model which takes into account the internal quota storage of each microalgae species. Using the Pontryagin’s principle, we find a dilution-based control strategy that steers the model trajectories to a suitable target in minimal time. Our study reveals that singular arcs play a key role in the optimal synthesis. Using numerical simulations, we show that the optimal control strategy is mainly of type bang-singular. A numerical optimal synthesis is performed throughout the paper, thereby confirming the optimality of the provided feedback-control law.

## 8. Bilateral Contracts and Grants with Industry

### 8.1. Bilateral Contract with Industry

A bilateral research contract between the company **CGG** and the team took place in 2018. Duration: 6 months.

### 8.2. Bilateral Grant with Industry

A grant “PEPS AMIES”, title: “Conception d’un électrostimulateur intelligent”, has been obtained, co-financed by **AMIES** and **SEGULA**. PI: Bernard Bonnard. Start: December 2018. Duration: two years.

A grant PEPS UCA MSI (Maison de la Simulation de l’Innovation) on “Effet des résonances sur la moyennisation en contrôle optimal appliqué à la mécanique spatiale” with Inria and Thales Alenia Space (Cannes). PI: J.-B. Caillaud Start: January 2018. Duration: six months

## 9. Partnerships and Cooperations

### 9.1. National Initiatives

#### 9.1.1. ANR

**Sub-Riemannian Geometry and Interactions (SRGI)**. Started 2015 (decision ANR-15-CE40-0018), duration: 4 years. L. Rifford is a member.

**Intéractions Systèmes Dynamiques Équations d’Évolution et Contrôle (ISDEEC)**. Started 2016 (decision ANR-16-CE40-0013), duration: 4 years. L. Rifford is a member.

**Maximic: optimal control of microbial cells by natural and synthetic strategies**. Started 2017, duration: 4 years. J.-B. Caillaud, L. Giraldu, J.-B. Pomet are members.

#### 9.1.2. Others

**Défi InfIniti CNRS project, Control and Optimality of Magnetic Microrobots**, (PI L. Giraldu). Started 2017, duration: 1 years. This project involves colleagues from Paris Sorbonne Université (around S. Régnyier’s team) and from University of Strasbourg (around C. Prud’Homme’s team).

**PGMO** grant (2016-2018) on “Metric approximation of minimizing trajectories and applications” (PI J.-B. Caillaud). This project involved colleagues from Université Paris Dauphine and has funding for two year (originally one, extended), including one intership (M2 level).

**PGMO** grant (2017-2019) on “Algebro-geometric techniques with applications to global optimal control for Magnetic Resonance Imaging (MRI)”. B. Bonnard, A. Nolot and J. Rouot participate in this project, the PI is O. Cots, from ENSEIHHT, Toulouse.

The McTAO team participates in the **GdR MOA**, a CNRS network on Mathematics of Optimization and Applications.

J.-B. Caillaud is associate researcher of the CNRS team **Parallel Algorithms & Optimization** at ENSEEIHT, Univ. Toulouse.

P. Lissy was the PI of a PEPS project JCJC (young researchers).

## 9.2. European Initiatives

### 9.2.1. *Bilateral program with Portugal*

Program: FCT (Fundação para a Ciência e a Tecnologia)

Grant no. : PTDC/MAT-CAL/4334/2014

Project title: "Extremal spectral quantities and related problems"

Duration: 05/2016-05/2019

Coordinator: P. Freitas (Univ. Lisbon)

Team member involved: J.-B. Caillaud

Other partners: Univ. Lisbon, Univ. Luxembourg, Czech Nuclear Physics Institute, Univ. Bern

Link: <https://team.inria.fr/mctao/fct-project-extremal-spectral-quantities-and-related-problems-2016-2019>

### 9.2.2. *Bilateral ANR-DFG program with Germany*

Program: Projets de recherche collaborative-internationale ANR-DFG (Germany)

Grant no. : ANR-14-CE35-0013-01; DFG-GI 203/9-1

Project title: "Exploring the physical limits of spin systems (Explosys)."

Duration: 11/2014-10/2018

Coordinator: D. Sugny (Univ. de Bourgogne) for France, Glaser (TU München) for Germany.

Team member involved: Bernard Bonnard.

Other partners: TU München, Univ. de Bourgogne (IMB and UCB).

This project involves specialists in physics and control theory in order to make important progresses in the use of spin dynamics, in particular for Magnetic Resonance Medical Imaging.

Link: <http://www.agence-nationale-recherche.fr/fileadmin/aap/2014/selection/pa-2014-selection-franco-allemand-dfg.pdf>

## 9.3. International Research Visitors

### 9.3.1. *Visits of International Scientists*

Zhen Chen, Technion. Two day visit in July, 2018. Gave a talk "Shortest Dubins Paths through Three Points" at McTAO seminar.

### 9.3.2. *Visits to International Teams*

Lamberto Dell'Elce visited Department of Aerospace Engineering at Technion (Haifa, Israel) for a week in July, 2018.

Pierre Lissy was invited one month at Fudan University (China) in March and June, 2018.

## 10. Dissemination

### 10.1. Promoting Scientific Activities

#### 10.1.1. Scientific Events Organisation

Members of the team have been involved in creating a series of conferences on “The cut locus: A bridge over differential geometry, optimal control, and transport”, together with Japanese colleagues, see motivations in Section 3.2. The first one took place in Bangkok, Thailand, in 2016 and the second one was organized this year, September 3-6 in Sapporo, Japan. There are plans to organise the third conference in Nice in 2020.

J.-B. Caillau and L. Rifford were members of the scientific and organising committee for this second edition.

J.-B. Caillau was member of the Scientific committee of the PGMO days 2018, hosted by EDF labs in Saclay, and supported by the Fondation Mathématique Jacques Hadamard (FMJH). The conference gathered about 280 scientists working in optimization and data science. Together with H. Zidani (ENSTA Paristech), J.-B. Caillau organized two sessions of invited talks on “Optimal control and applications” during the conference.

J.-B. Caillau is chair (together with D. Auroux, UCA) of the 19th French-German-Swiss conference on Optimization that will take place in Nice in September 2019. This conference is the main European biennial event in optimization in the broad sense.

#### 10.1.2. Journal

B. Bonnard is a member of the editorial board of the *Pacific Journal of Mathematics for Industry*.

#### 10.1.3. Invited Talks

B. Bonnard, J.-B. Pomet and J. Rouot gave three invited talks at the conference *Dynamics, Control, and Geometry*, Banach center, Warsaw (Poland): “Sub-Riemannian geometry and the Copepod Micro-swimmer”, “Geometric and numerical methods in optimal control for the time minimal saturation of a pair of spins”, “Dynamic equivalence and flatness of control systems: some results and open questions”.

Olivier Cots gave an invited talk at *PGMO Days*. Olivier Cots, Bernard Bonnard, Jérémy Rouot and Thibaut Verron, “Geometric and numerical methods for the saturation problem in Magnetic Resonance Imaging”.

J.-B. Pomet gave an invited talk at the 2<sup>nd</sup> conference on “The cut locus: A bridge over differential geometry, optimal control, and transport”, September 3-6, Sapporo, Japan.

P. Lissy gave two seminars at Fudan University, Shanghai, China (February).

P. Lissy gave two plenary talks at *PICOF conference*, Beyrouth, Libanon (June) and at *Workshop on Microlocal analysis, numerical analysis and kinetic equations*, Madrid, Spain (February).

J.-B. Caillau gave the following invited talks:

- “Optimal control of slow-fast mechanical systems” (in January), *UCA Complex days*, Nice
- “Smooth and broken Hamiltonian curves in optimal control” (in February), *Recent advances in Hamiltonian dynamics and symplectic topology*, Padova

L. Dell’Elce gave the following seminars:

22/2/2018 *Robust trajectory design using invariant manifolds. Application to the asteroid (65803) Didymos* at Astrogeo, Sophia Antipolis, France.

7/5/2018 *Two-phase averaging of optimal control systems with application to the Earth-Moon transfer* at JAXA, Sagamihara, Japan.

9/7/2018 *Two-phase averaging of optimal control systems* at Technion, Haifa, Israel.

### 10.1.4. Leadership within the Scientific Community

J.-B. Caillau is member of the following committees:

- Conseil scientifique du GdR Calcul
- Conseil scientifique de l’Institut de Mécanique Céleste et de Calcul des Éphémérides (Observatoire de Paris)
- Conseil scientifique PGMO, Fondation Mathématique Jacques Hadamard
- Jury du prix de thèse PGMO

J.-B. Caillau, L. Dell’Elce and J.-B. Pomet are members of the Centre Spatial Universitaire UCA (projet CubeSat).

### 10.1.5. Scientific Expertise

J.-B. Caillau and L. Giraldi were hired for a one day expertise for Smart’n Go (startup working in marine routing).

### 10.1.6. Research Administration

J.-B. Caillau is

- co-organizer of the Séminaire de géométrie hamiltonienne of Sorbonne Université
- member of the Conseil Scientifique 3IA (project on AI supported by Nice-Sophia Antipolis)

Laetitia Giraldi is

- a member of CSD (*Comité du Suivi Doctoral*) at Inria Sophia-Antipolis,
- a redactor of the meeting reports of the *Comité des Équipes-Projets* at Inria Sophia-Antipolis.

Jean-Baptiste Pomet is

- a member of the steering committee of the Center for Planetary Origin (C4PO),
- a member of the scientific council of Académie 2 “Complex system”, both for Université Côte d’Azur (UCA),
- an elected member of Commission d’Évaluation (Inria permanent evaluation committee).

Ludovic Rifford is the Executive Director of the CIMPA (Centre International de Mathématiques Pures et Appliquées).

Pierre Lissy is elected member of the “CCR” of the CEREMADE (committee in charge of the recruitment at Dauphine, notably for ATER, months of invited and composition of hiring committees).

Pierre Lissy is member of the team of the website “Opération Postes”.

## 10.2. Teaching - Supervision - Juries

### 10.2.1. Teaching

B. Bonnard taught 220 hours at undergraduate level (ESIREM engineering school)

J.-B. Caillau taught 200 hours at L3, M1 and M2 level (Polytech Nice Sophia, Université Nice Sophia).

L. Giraldi taught 10 hours in the Master 2 recherche, Cell Physics (Université de Strasbourg). She also teaches in Classe préparatoire aux grandes écoles d’ingénieurs (Centre International de Valbonne) as a interrogatrice in MP\* and MPSI (4 hours per week).

P. Lissy gave a six hour lecture on “Spectral Geometry” at the CIMPA spring school “PDEs and geometry” (May 2018, Jijel, Algeria).

J.-B. Caillau is director of the Applied Math. & Modelling department of Polytech Nice Sophia.

J.-B. Caillau and L. Giraldi are members of the jury of Agrégation externe de mathématiques.

P. Lissy was coordinator of a work group on the recast of the Teaching in Analysis for the initial education in Maths at Université Paris-Dauphine.

### 10.2.2. Supervision

PhD : Michaël Orioux, “Quelques propriétés et applications du contrôle en temps minimum” [1], Université Paris Dauphine, November 27, 2018, co-supervised by J.-B. Caillau and J. Féjoz (Univ. Paris-Dauphine).

PhD in progress : Sébastien Fueyo, “Testing stability of nonlinear amplifier by frequency-domain methods”, started October, 2016, co-supervised by J.-B. Pomet and L. Baratchart (FACTAS team).

PhD in progress : Yacine El alaoui-faris, “modeling magneto-elastic micro-robot from theory to experiment”, started October, 2017, co-supervised by L. Giraldi, J.-B. Pomet and Stephane Régnier (Sorbonne Université).

PhD in progress : Clément Moreau, “Contrôlabilité de systèmes en dimension finie ou infinie issus du vivant”, started September, 2017, co-supervised by L. Giraldi, P. Lissy (Univ. Paris-Dauphine) and J.-B. Pomet.

PhD in progress : Agustin Yabo, “Control of biological processes”, started September, 2018, co-supervised by J.-B. Pomet and J.-L. Gouzé (Biocore team).

PhD in progress : Luca Berti, “Modeling magneto-elastic micro-robot using PDE”, started October, 2018, co-supervised by L. Giraldi and C. Prud’Homme (Université de Strasbourg).

### 10.2.3. Juries

B. Bonnard was a reviewer of Sofya Maslovskaya’s PhD (ENSTA ParisTech), and he sat in the jury for Toufik Bakir’s Habilitation defense (Univ. de Bourgogne Franche Comté).

J.-B. Caillau was a reviewer of Ivan Beschastnyi PhD thesis (SISSA, September 2018), Francesca Chittaro Habilitation (Univ. Toulon, December 2018), Antoine Olivier PhD thesis (Sorbonne Université, October 2018), Cédric Rommel PhD thesis (École Polytechnique, October 2018), and he sat in the juries of Clément Gazzino PhD thesis (Univ. Toulouse, January 2018), Ricardo Bonalli PhD thesis (Sorbonne Université, July 2018).

J.-B. Pomet was a reviewer of Riccardo Bonalli’s PhD (Univ. Paris Sorbonne), and he sat in the jury for Sofya Maslovskaya’s PhD (ENSTA ParisTech) and for Toufik Bakir’s Habilitation defense (Univ. de Bourgogne Franche Comté).

L. Rifford was a reviewer of Vincenzo Basco’s PhD (Sorbonne Université) and sat in the jury for Nicolas Juillet’s Habilitation à Diriger des Recherches (Université de Strasbourg).

Pierre Lissy has been a member of hiring committees for “Maître de Conférences” positions at Université Paris-Dauphine (2 positions) and Université Sorbonne Université (1 position).

## 10.3. Popularization

### 10.3.1. Internal or external Inria responsibilities

Clément Moreau is a member of the team « équipe actualités » of [Images des Mathématiques](#) since October, 2018.

J.-B. Caillau is member of the MASTIC initiative at Inria Sophia (Médiation et animation scientifiques Inria) and delivers regular talks in high school or college.

### 10.3.2. Articles and contents

Clément Moreau participated in the radio show « [La méthode scientifique](#) » on March 7, 2018.

Clément Moreau participated in the national competition « [Ma thèse en 180 secondes](#) ».

### 10.3.3. Education

Lamberto Dell'Elce is involved in the PoBot challenge promoted by MEDITES. Specifically he supervises a class in the College Emile Roux in Cannes.

### 10.3.4. Interventions

Clément Moreau participated in the « journées portes ouvertes » at Inria Sophia Antipolis in the framework of the 2018 « Fête de la Science », October 7, 2018, and in a scientific discovery workshop for secondary school students at the concrete Art Museum, Mouans-Sartoux, October 11, 2018.

Lamberto Dell'Elce gave the talk "CubeSats: Concevoir et Realiser un Satellite à l'Université" at the *Cafe In* meeting held at Inria Sophia on June 6, 2018, and for high school students during the **stage MathC2+** in June, 2018.

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### Publications of the year

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