IN PARTNERSHIP WITH:
CNRS

Université Rennes 1

École normale supérieure de Rennes

Activity Report 2018

Project-Team MINGUS

MultI-scale Numerical Geometric Schemes

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

RESEARCH CENTER
Rennes - Bretagne-Atlantique

THEME
Numerical schemes and simulations

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Creation of the Team: 2018 January 01, updated into Project-Team: 2018 August 01

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1. Team, Visitors, External Collaborators

Research Scientists

Philippe Chartier [Inria, Senior Researcher, HDR]
Nicolas Crouseilles [Inria, Team leader, Researcher, HDR]
Erwan Faou [Inria, Senior Researcher, HDR]
Mohammed Lemou [CNRS, Senior Researcher, HDR]

Faculty Members

François Castella [Univ de Rennes I, Professor, HDR]
Anais Crestetto [Univ de Nantes, Associate Professor, until Aug 2018]
Arnaud Debussche [Ecole normale supérieure de Rennes, Professor, HDR]
Florian Méhats [Univ de Rennes I, Professor, HDR]

Post-Doctoral Fellows

Guillaume Morel [Inria, from Oct 2018]
Xiaofei Zhao [Inria, from Feb 2018]

PhD Students

Joackim Bernier [Univ de Rennes I]
Marine Fontaine [Ecole normale supérieure de Rennes, until Aug 2018]
Josselin Massot [Univ de Rennes I, from Oct 2018]
Angelo Rosello [Ecole normale supérieure de Rennes]
Leopold Tremant [Inria, from Oct 2018]

Technical staff

Pierre Navaro [CNRS, Technical staff]

Interns

Josselin Massot [IRSTEA, from Apr 2018 until Sep 2018]
Leopold Tremant [Inria, from Mar 2018 until Aug 2018]

Administrative Assistant

Stéphanie Lemaile [Inria]

Visiting Scientist

Fernando Casas [Universitat Jaume I, from Sep 2018]

2. Overall Objectives

2.1. Presentation

In applications involving complex physics, such as plasmas and nanotechnologies, numerical simulations serve as a prediction tool supplementing real experiments and are largely endorsed by engineers or researchers. Their performances rely not only on computational power, but also on the efficiency of the underlying numerical method and the complexity of the underlying models. The contribution of applied mathematics is then required, on the one hand for a better understanding of qualitative properties and a better identification of the different regimes present in the model, and on the other hand, for a more sounded construction of new models based on asymptotic analysis. This mathematical analysis is expected to greatly impact the design of *multiscale* numerical schemes.

The proposed research group MINGUS will be dedicated to the mathematical and numerical analysis of (possibly stochastic) partial differential equations (PDEs), originating from plasma physics and nanotechnologies, with emphasis on *multiscale* phenomena either of **highly-oscillatory**, of **dissipative** or **stochastic** types. These equations can be also encountered in applications to rarefied gas dynamics, radiative transfer, population dynamics or laser propagation, for which the multiscale character is modelled by a scale physical parameter ε.

Producing accurate solutions of multiscale equations is extremely challenging owing to severe restrictions to the numerical methods imposed by fast (or stiff) dynamics. *Ad-hoc* numerical methods should aim at capturing the slow dynamics solely, instead of resolving finely the stiff dynamics at a formidable computational cost. At the other end of the spectrum, the separation of scales -as required for numerical efficiency- is envisaged in asymptotic techniques, whose purpose is to describe the model in the limit where the small parameter ε tends to zero. MINGUS aspires to accommodate sophisticated tools of mathematical analysis and heuristic numerical methods in order to produce simultaneously rich asymptotic models and efficient numerical methods.
To be more specific, MINGUS aims at finding, implementing and analysing new multiscale numerical schemes for the following physically relevant multiscale problems:

Highly-oscillatory Schrödinger equation for nanoscale physics: In quantum mechanics, the Schrödinger equation describes how the quantum state of some physical system changes with time. Its mathematical and numerical study is of paramount importance to fundamental and applied physics in general. We wish to specifically contribute to the mathematical modeling and the numerical simulation of confined quantum mechanical systems (in one or more space dimensions) possibly involving stochastic terms. Such systems are involved in quantum semi-conductors or atom-chips, as well as in cold atom physics (Bose-Einstein condensates) or laser propagation in optical fibers.

The prototypical equation is written

$$\boxed{i\varepsilon\partial_t\psi^\varepsilon = \frac{\varepsilon^2}{\beta}\Delta\psi^\varepsilon + |\psi^\varepsilon|^2\psi^\varepsilon + \psi^\varepsilon\xi} \tag{1}$$

where the function $\psi^\varepsilon = \psi^\varepsilon(t,x) \in \mathbb{C}$ depends on time $t \geq 0$ and position $x \in \mathbb{R}^3$, $\xi = \xi(x,t)$ is a white noise and where the small parameter ε is the Planck's constant describing the microscopic/macroscopic ratio. The limit $\varepsilon \to 0$ is referred to as the semi-classical limit. The regime $\varepsilon = 1$ and $\beta \to 0$ (this can be for instance the relative length of the optical fiber) is highly-oscillatory. The noise ξ acts as a potential, it may represent several external perturbations. For instance temperature effects in Bose-Einstein condensation or amplification in optical fibers. The highly oscillatory regime combined with noise introduces new challenges in the design of efficient schemes.

Highly-oscillatory or highly-dissipative kinetic equations: Plasma is sometimes considered as the fourth state of matter, obtained for example by bringing a gas to a very high temperature. A globally neutral gas of neutral and charged particles, called plasma, is then obtained and is described by a kinetic equation as soon as collective effects dominate as compared to binary collisions. A situation of major importance is magnetic fusion in which collisions are not predominant. In order to confine such a plasma in devices like tokamaks (ITER project) or stellarators, a large magnetic field is used to endow the charged particles with a cyclotronic motion around field lines. Note that kinetic models are also widely used for modeling plasmas in earth magnetosphere or in rarefied gas dynamics.

Denoting $f^\varepsilon = f^\varepsilon(t,x,v) \in \mathbb{R}^+$ the distribution function of charged particles at time $t \geq 0$, position $x \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$, a typical kinetic equation for f^ε reads

$$\boxed{\partial_t f^\varepsilon + v \cdot \nabla_x f^\varepsilon + \left(E + \tfrac{1}{\varepsilon}(v \times B)\right) \cdot \nabla_v f^\varepsilon = \tfrac{1}{\beta} Q(f^\varepsilon) + f^\varepsilon m^\varepsilon} \tag{2}$$

where (E,B) is the electro-magnetic field (which may itself depend on f through Maxwell's equations), m^ε is a random process (which may describe absorption or creation of particles) and Q is a collision operator. The dimensionless parameters ε, β are related to the cyclotronic frequency and the mean free path. Limits $\varepsilon \to 0$ and $\beta \to 0$ do not share the same character (the former is oscillatory and the latter is dissipative) and lead respectively to gyrokinetic and hydrodynamic models. The noise term m^ε is correlated in space and time. At the limit $\varepsilon \to 0$, it converges formally to a white noise and stochastic PDEs are obtained.

MINGUS project is the follow-up of IPSO, ending in december in 2017. IPSO original aim was to extend the analysis of geometric schemes from ODEs to PDEs. During the last evaluation period, IPSO also considered the numerical analysis of geometric schemes for (S)PDEs, possibly including multiscale phenomena. Breakthrough results [36], [38], [39], [42] have been recently obtained which deserve to be deepened and extended. It thus appears quite natural to build the MINGUS team upon these foundations.

The objective of MINGUS is twofold: the construction and the analysis of numerical schemes (such as "Uniformly Accurate numerical schemes", introduced by members of the IPSO project) for multiscale (S)PDEs originating from physics. In turn, this requires (i) a deep mathematical understanding of the (S)PDEs under consideration and (ii) a strong involvement into increasingly realistic problems, possibly resorting to parallel computing. For this aspect, we intend to benefit from the Inria Selalib software library which turns out to be the ideal complement of our activities.

3. Research Program

3.1. Research Program

The MINGUS project is devoted to the mathematical and numerical analysis of models arising in plasma physics and nanotechnology. The main goal is to construct and analyze numerical methods for the approximation of PDEs containing multiscale phenomena. Specific multiscale numerical schemes will be proposed and analyzed in different regimes (namely highly-oscillatory and dissipative). The ultimate goal is to dissociate the physical parameters (generically denoted by ε) from the numerical parameters (generically denoted by h) with a uniform accuracy. Such a task requires mathematical prerequisite of the PDEs.

Then, for a given stiff (highly-oscillatory or dissipative) PDE, the methodology of the MINGUS team will be the following

- Mathematical study of the asymptotic behavior of multiscale models.
  This part involves averaging and asymptotic analysis theory to derive asymptotic models, but also long-time behavior of the considered models.
- Construction and analysis of multiscale numerical schemes.

This part is the core of the project and will be deeply inspired for the mathematical prerequisite. In particular, our ultimate goal is the design of *Uniformly Accurate* (UA) schemes, whose accuracy is independent of ε.

- Validation on physically relevant problems.
  The last goal of the MINGUS project is to validate the new numerical methods, not only on toy problems, but also on realistic models arising in physics of plasmas and nanotechnologies. We will benefit from the Selalib software library which will help us to scale-up our new numerical methods to complex physics.

3.1.1. Dissipative problems

In the dissipative context, the asymptotic analysis is quite well understood in the deterministic case and multiscale numerical methods have been developed in the last decades. Indeed, the so-called Asymptotic-Preserving schemes has retained a lot of attention all over the world, in particular in the context of collisional kinetic equations. But, there is still a lot of work to do if one is interesting in the derivation high order asymptotic models, which enable to capture the original solution for all time. Moreover, this analysis is still misunderstood when more complex systems are considered, involving non homogeneous relaxation rates or stochastic terms for instance. Following the methodology we aim at using, we first address the mathematical analysis before deriving multiscale efficient numerical methods.

A simple model of dissipative systems is governed by the following differential equations

$$\left\{ \begin{array}{rcll} \dfrac{dx^\varepsilon(t)}{dt} & = & \mathcal{G}(x^\varepsilon(t), y^\varepsilon(t)), & x^\varepsilon(0) = x_0, \\ \dfrac{dy^\varepsilon(t)}{dt} & = & -\dfrac{y^\varepsilon(t)}{\varepsilon} + \mathcal{H}(x^\varepsilon(t), y^\varepsilon(t)), & y^\varepsilon(0) = y_0, \end{array} \right. \tag{3}$$

for given initial condition $(x_0, y_0) \in \mathbb{R}^2$ and given smooth functions $\mathcal{G}, \mathcal{H}$ which possibly involve stochastic terms.

3.1.1.1. Asymptotic analysis of dissipative PDEs (F. Castella, P. Chartier, A. Debussche, E. Faou, M. Lemou)

Derivation of asymptotic problems
Our main goal is to analyze the asymptotic behavior of dissipative systems of the form (3) when ε goes to zero. The *center manifold theorem* [35] is of great interest but is largely unsatisfactory from the following points of view

- a constructive approach of h and x_0^ε is clearly important to identify the high-order asymptotic models: this would require expansions of the solution by means of B-series or word-series [37] allowing the derivation of error estimates between the original solution and the asymptotic one.
- a better approximation of the transient phase is strongly required to capture the solution for small time: extending the tools developed in averaging theory, the main goal is to construct a suitable change of variable which enables to approximate the original solution for all time.

Obviously, even at the ODE level, a deep mathematical analysis has to be performed to understand the asymptotic behavior of the solution of (3). But, the same questions arise at the PDE level. Indeed, one certainly expects that dissipative terms occurring in collisional kinetic equations (2) may be treated theoretically along this perspective. The key new point indeed is to see the center manifold theorem as a change of variable in the space on unknowns, while the standard point of view leads to considering the center manifold as an asymptotic object.

Stochastic PDEs

We aim at analyzing the asymptotic behavior of stochastic collisional kinetic problems, that is equation of the type (2). The noise can describe creation or absorption (as in (2)), but it may also be a forcing term or a random magnetic field. In the parabolic scaling, one expect to obtain parabolic SPDEs at the limit. More precisely, we want to understand the fluid limits of kinetic equations in the presence of noise. The noise is smooth and non delta correlated. It contains also a small parameter and after rescaling converges formally to white noise. Thus, this adds another scale in the multiscale analysis. Following the pioneering work [38], some substantial progresses have been done in this topic.

More realistic problems may be addressed such as high field limit describing sprays, or even hydrodynamic limit. The full Boltzmann equation is a very long term project and we wish to address simpler problems such as convergences of BGK models to a stochastic Stokes equation.

The main difficulty is that when the noise acts as a forcing term, which is a physically relevant situation, the equilibria are affected by the noise and we face difficulties similar to that of high field limit problems. Also, a good theory of averaging lemma in the presence of noise is lacking. The methods we use are generalization of the perturbed test function method to the infinite dimensional setting. We work at the level of the generator of the infinite dimensional process and prove convergence in the sense of the martingale problems. A further step is to analyse the speed of convergence. This is a prerequisite if one wants to design efficient schemes. This requires more refined tools and a good understanding of the Kolmogorov equation.

3.1.1.2. Numerical schemes for dissipative problems (All members)

The design of numerical schemes able to reproduce the transition from the microscopic to macroscopic scales largely matured with the emergence of the Asymptotic Preserving schemes which have been developed initially for collisional kinetic equations (actually, for solving (2) when $\beta \to 0$). Several techniques have flourished in the last decades. As said before, AP schemes entail limitations which we aim at overcoming by deriving

- AP numerical schemes whose numerical cost diminishes as $\beta \to 0$,
- Uniformly accurate numerical schemes, whose accuracy is independent of β.

Time diminishing methods
The main goal consists in merging Monte-Carlo techniques [33] with AP methods for handling *automatically* multiscale phenomena. As a result, we expect that the cost of the so-obtained method decreases when the asymptotic regime is approached; indeed, in the collisional (i.e. dissipative) regime, the deviational part becomes negligible so that a very few number of particles will be generated to sample it. A work in this direction has been done by members of the team.

We propose to build up a method which permits to realize the transition from the microscopic to the macroscopic description without domain decomposition strategies which normally oblige to fix and tune an interface in the physical space and some threshold parameters. Since it will permit to go over domain decomposition and AP techniques, this approach is a very promising research direction in the numerical approximation of multiscale kinetic problems arising in physics and engineering.

Uniformly accurate methods
To overcome the accuracy reduction observed in AP schemes for intermediate regimes, we intend to construct and analyse multiscale numerical schemes for (3) whose error is uniform with respect to ε. The construction of such a scheme requires a deep mathematical analysis as described above. Ideally one would like to develop schemes that preserve the center manifold (without computing the latter!) as well as schemes that resolve numerically the stiffness induced by the fast convergence to equilibrium (the so-called transient phase). First, our goal is to extend the strategy inspired by the central manifold theorem in the ODE case to the PDE context, in particular for collisional kinetic equations (2) when $\beta \to 0$. The design of Uniformly Accurate numerical schemes in this context would require to generalize two-scale techniques introduced in the framework of highly-oscillatory problems [36].

Multiscale numerical methods for stochastic PDEs

AP schemes have been developed recently for kinetic equations with noise in the context of Uncertainty Quantification UQ [41]. These two aspects (multiscale and UQ) are two domains which usually come within the competency of separate communities. UQ has drawn a lot of attention recently to control the propagation of data pollution; undoubtedly UQ has a lot of applications and one of our goals will be to study how sources of uncertainty are amplified or not by the multiscale character of the model. We also wish to go much further and by developing AP schemes when the noise is also rescaled and the limit is a white noise driven SPDE, as described in section (3.1.1.1). For simple nonlinear problem, this should not present much difficulties but new ideas will definitely be necessary for more complicated problems when noise deeply changes the asymptotic equation.

3.1.2. Highly-oscillatory problems

As a generic model for highly-oscillatory systems, we will consider the equation

$$\frac{du^\varepsilon(t)}{dt} = \mathcal{F}(t/\varepsilon, u^\varepsilon(t)), \quad u^\varepsilon(0) = u_0, \tag{4}$$

for a given u_0 and a given periodic function $\mathcal{F}$ (of period P w.r.t. its first variable) which possibly involves stochastic terms. Solution u^ε exhibits high-oscillations in time superimposed to a slow dynamics. Asymptotic techniques -resorting in the present context to *averaging* theory [45]- allow to decompose

$$u^\varepsilon(t) = \Phi_{t/\varepsilon} \circ \Psi_t \circ \Phi_0^{-1}(u_0), \tag{5}$$

into a fast solution component, the εP-periodic change of variable $\Phi_{t/\varepsilon}$, and a slow component, the flow Ψ_t of a non-stiff *averaged* differential equation. Although equation (5) can be satisfied only up to a small remainder, various methods have been recently introduced in situations where (4) is posed in $\mathbb{R}^n$ or for the Schrödinger equation (1).

In the asymptotic behavior $\varepsilon \to 0$, it can be advantageous to replace the original singularly perturbed model (for instance (1) or (2)) by an approximate model which does not contain stiffness any longer. Such reduced models can be derived using asymptotic analysis, namely averaging methods in the case of highly-oscillatory problems. In this project, we also plan to go beyond the mere derivation of limit models, by searching for better approximations of the original problem. This step is of mathematical interest *per se* but it also paves the way of the construction of multiscale numerical methods.

3.1.2.1. Asymptotic analysis of highly-oscillatory PDEs (All members)

Derivation of asymptotic problems
We intend to study the asymptotic behavior of highly-oscillatory evolution equations of the form (4) posed in an infinite dimensional Banach space.

Recently, the stroboscopic averaging has been extended to the PDE context, considering nonlinear Schrödinger equation (1) in the highly-oscillatory regime. A very exciting way would be to use this averaging strategy for highly-oscillatory kinetic problem (2) as those encountered in strongly magnetized plasmas. This turns out to be a very promising way to re-derive gyrokinetic models which are the basis of tokamak simulations in the physicists community. In contract with models derived in the literature (see [34]) which only capture the average with respect to the oscillations, this strategy allows for the complete recovery of the exact solution from the asymptotic (non stiff) model. This can be done by solving companion transport equation that stems naturally from the decomposition (5).

Long-time behavior of Hamiltonian systems

The study of long-time behavior of nonlinear Hamiltonian systems have received a lot of interest during the last decades. It enables to put in light some characteristic phenomena in complex situations, which are beyond the reach of numerical simulations. This kind of analysis is of great interest since it can provide very precise properties of the solution. In particular, we will focus on the dynamics of nonlinear PDEs when the initial condition is close to a stationary solution. Then, the long-time behavior of the solution is studied through mainly three axis

- *linear stability*: considering the linearized PDE, do we have stability of a stationary solution ? Do we have linear Landau damping around stable non homogeneous stationary states?
- *nonlinear stability*: under a criteria, do we have stability of a stationary solution in energy norm like in [42], and does this stability persist under numerical discretization? For example one of our goals is to address the question of the existence and stability of discrete travelling wave in space and time.
- do we have existence of damped solutions for the full nonlinear problem ? Around homogeneous stationary states, solutions scatter towards a modified stationary state (see [43], [39]). The question of existence of Landau damping effects around non homogeneous states is still open and is one of our main goal in the next future.

Asymptotic behavior of stochastic PDEs
The study of SPDEs has known a growing interest recently, in particular with the fields medal of M. Hairer in 2014. In many applications such as radiative transfer, molecular dynamics or simulation of optical fibers, part of the physical interactions are naturally modeled by adding supplementary random terms (the noise) to the initial deterministic equations. From the mathematical point of view, such terms change drastically the behavior of the system.

- In the presence of noise, highly-oscillatory dispersive equations presents new problems. In particular, to study stochastic averaging of the solution, the analysis of the long time behavior of stochastic dispersive equations is required, which is known to be a difficult problem in the general case. In some cases (for instance highly-oscillatory Schrödinger equation (1) with a time white noise in the regime $\varepsilon \ll 1$), it is however possible to perform the analysis and to obtain averaged stochastic equations. We plan to go further by considering more difficult problems, such as the convergence of a stochastic Klein-Gordon-Zakharov system to as stochastic nonlinear Schrödinger equation.
- The long-time behavior of stochastic Schrödinger equations is of great interest to analyze mathematically the validity of the Zakharov theory for wave turbulence (see [44]). The problem of wave turbulence can be viewed either as a deterministic Hamiltonian PDE with random initial data or a randomly forced PDEs where the stochastic forcing is concentrated in some part of the spectrum (in this sense it is expected to be a hypoelliptic problem). One of our goals is to test the validity the Zakharov equation, or at least to make rigorous the spectrum distribution spreading observed in the numerical experiments.

3.1.2.2. Numerical schemes for highly-oscillatory problems (All members)

This section proposes to explore numerical issues raised by highly-oscillatory nonlinear PDEs for which (4) is a prototype. Simulating a highly-oscillatory phenomenon usually requires to adapt the numerical parameters in order to solve the period of size ε so as to accurately simulate the solution over each period, resulting in a unacceptable execution cost. Then, it is highly desirable to derive numerical schemes able to advance the solution by a time step independent of ε. To do so, our goal is to construct *Uniformly Accurate* (UA) numerical schemes, for which the numerical error can be estimated by Ch^p (h being any numerical parameters) with C independent of ε and p the order of the numerical scheme.

Recently, such numerical methods have been proposed by members of the team in the highly-oscillatory context [36]. They are mainly based on a separation of the fast and slow variables, as suggested by the decomposition (5). An additional ingredient to prove the uniformly accuracy of the method for (4) relies on the search for an appropriate initial data which enables to make the problem smooth with respect to ε.

Such an approach is assuredly powerful since it provides a numerical method which enables to capture the high oscillations in time of the solution (and not only its average) even with a large time step. Moreover, in the asymptotic regime, the potential gain is of order $1/\varepsilon$ in comparison with standard methods, and finally averaged models are not able to capture the intermediate regime since they miss important information of the original problem. We are strongly convinced that this strategy should be further studied and extended to cope with some other problems. The ultimate goal being to construct a scheme for the original equation which degenerates automatically into a consistent approximation of the averaged model, without resolving it, the latter can be very difficult to solve.

- Space oscillations:
  When rapidly oscillating coefficients in **space** (*i.e.* terms of the form $a(x, x/\varepsilon)$) occur in elliptic or parabolic equations, homogenization theory and numerical homogenization are usually employed to handle the stiffness. However, these strategies are in general not accurate for all $\varepsilon \in]0, 1]$. Then, the construction of numerical schemes which are able to handle both regimes in an uniform way is of great interest. Separating fast and slow *spatial* scales merits to be explored in this context. The delicate issue is then to extend the choice suitable initial condition to an *appropriate choice of boundary conditions* of the augmented problem.
- Space-time oscillations:
  For more complex problems however, the recent proposed approaches fail since the main oscillations cannot be identified explicitly. This is the case for instance when the magnetic field B depends on t or x in (2) but also for many other physical problems. We then have to deal with the delicate issue of space-time oscillations, which is known to be a very difficult problem from a mathematical and a numerical point of view. To take into account the space-time mixing, a periodic motion has to be detected together with a phase S which possibly depends on the time and space variables. These techniques originate from **geometric optics** which is a very popular technique to handle highly-frequency waves.
- Geometrical properties:
  The questions related to the geometric aspects of multiscale numerical schemes are of crucial importance, in particular when long-time simulations are addressed (see [40]). Indeed, one of the main questions of geometric integration is whether intrinsic properties of the solution may be passed onto its numerical approximation. For instance, if the model under study is Hamiltonian, then the exact flow is symplectic, which motivates the design of symplectic numerical approximation. For practical simulations of Hamiltonian systems, symplectic methods are known to possess very nice properties (see [40]). It is important to combine multiscale techniques to geometric numerical integration. All the problems and equations we intend to deal with will be addressed with a view to preserve intrinsic geometric properties of the exact solutions and/or to approach the asymptotic limit of the system in presence of a small parameter. An example of a numerical method developed by members of the team is the multi-revolution method.
- Quasi-periodic case:
  So far, numerical methods have been proposed for the periodic case with single frequency. However, the quasi-periodic case [1] is still misunderstood although many complex problems involve multi-frequencies. Even if the quasi-periodic averaging is doable from a theoretical point of view in the ODE case (see [45]), it is unclear how it can be extended to PDEs. One of the main obstacle being the requirement, usual for ODEs like (4), for $\mathcal{F}$ to be analytic in the periodic variables, an assumption which is clearly impossible to meet in the PDE setting. An even more challenging problem is then the design of numerical methods for this problem.
- extension to stochastic PDEs:

[1] replacing t/ε by $t\omega/\varepsilon$ in (4), with $\omega \in \mathbb{R}^d$ a vector of non-resonant frequencies

All these questions will be revisited within the stochastic context. The mathematical study opens the way to the derivation of efficient multiscale numerical schemes for this kind of problems. We believe that the theory is now sufficiently well understood to address the derivation and numerical analysis of multiscale numerical schemes. Multi-revolution composition methods have been recently extended to highly-oscillatory stochastic differential equations The generalization of such multiscale numerical methods to SPDEs is of great interest. The analysis and simulation of numerical schemes for highly-oscillatory nonlinear stochastic Schrödinger equation under diffusion-approximation for instance will be one important objective for us. Finally, an important aspect concerns the quantification of uncertainties in highly-oscillatory kinetic or quantum models (due to an incomplete knowledge of coefficients or imprecise measurement of datas). The construction of efficient multiscale numerical methods which can handle multiple scales as well as random inputs have important engineering applications (see (4.1)).

4. Application Domains

4.1. Applications

The MINGuS project aims at applying the new numerical methods on realistic problems arising for instance in physics of nanotechnology and physics of plasmas. Therefore, in addition to efforts devoted to the design and the analysis of numerical methods, the inherent large size of the problems at hand requires advanced mathematical and computational methods which are hard to implement. Another application is concerned with population dynamics for which the main goal is to understand how the spatial propagation phenomena affect the demography of a population (plankton, parasite fungi, ...).

Our activity is mostly at an early stage in the process of transfer to industry. However, all the models we use are physically relevant and all have applications in many areas (ITER, Bose-Einstein condensate, wave turbulence, optical tomography, transport phenomena, population dynamics, $\cdots$). As a consequence, our research aims at reaching theoretical physicists or computational scientists in various fields who have strong links with industrial applications.

In order to tackle as realistic physical problems as possible, a fundamental aspect will consist in working on the realization of numerical methods and algorithms which are able to make an efficient use of a large number of processors. Then, it is essential for the numerical methods developed in the MINGuS project to be thought through this prism. We will benefit from the strong expertise of P. Navaro in scientific computing and more precisely on the Selalib software library (see description below).

Below, we detail our main applications: first, the modeling and numerical approximation of magnetized plasmas is our major application and will require important efforts in terms of software developments to scale-up our multiscale methods; second, the transport of charged particles in nanostructures has very interesting applications (like graphene material), for which our contributions will mainly focus on dedicated problems; lastly, applications on population dynamics will be dedicated to mathematical modeling and some numerical validations.

4.1.1. Kinetic problems

The Selalib (SEmi-LAgrangian LIBrary) software library [2] is a modular library for kinetic and gyrokinetic simulations of plasmas in fusion energy devices. Selalib is a collection of fortran modules aimed at facilitating the development of kinetic simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities and modules to help parallelization (both MPI and OpenMP), as well as pre-packaged simulations.

[2] SELALIB, http://selalib.gforge.inria.fr.

Its main objective is to develop a documented library implementing several numerical methods for the numerical approximation of kinetic models of the form (2). Another objective of the library is to provide physicists with easy-to-use gyrokinetic solvers. It has been originally developed by E. Sonnendrücker and his collaborators in the past CALVI Inria project, and has played an important role in the activities of the IPL FRATRES. P. Navaro is one of the main software engineer of this library and as such he played an important daily role in its development and its portability on supercomputers. Though Selalib has reached a certain maturity some additional works are needed to make available by the community. There are currently discussions for a possible evolution of Selalib, namely the writing of a new release which will be available for free download. Obviously, the team will be involved in this process.

At the scientific level, Selalib is of great interest for us since it provides a powerful tool with which we can test, validate and compare our new methods and algorithms (*users level*). Besides numerical algorithms the library provides low-level utilities, input-output modules as well as parallelization strategies dedicated to kinetic problems. Moreover, a collection of simulations for typical test cases (of increasing difficulties) with various discretization schemes supplements the library. This library turns out to be the ideal complement of our activities and it will help us to scale-up our numerical methods to high-dimensional kinetic problems. During the last years, several experiments have been successfully performed in this direction (especially with PhD students) and it is important for us that this approach remains throughout.
Then, we intend to integrate several of the numerical methods developed by the team within the Selalib library, with the strong help of P. Navaro (*contributors level*). This work has important advantages: (i) it will improve our research codes (in terms of efficiency but also of software maintenance point of view); (ii) it will help us to promote our research by making our methods available to the research community.

4.1.2. Quantum problems

Nowadays, a great challenge consists in the downscaling at the nanometer scale of electronic components in order to improve speed and efficiency of semiconductor materials. In this task, modeling and numerical simulations play an important role in the determination of the limit size of the nanotransistors At the nanoscale, quantum effects have to be considered and the Schrödinger equation (1) is prominent equation in this context. In the so-called semiclassical regime or when the transport is strongly confined, the solution endows space-time highly oscillations which are very difficult to capture numerically.

An important application is the modeling of charged particles transport in graphene. Graphene is a sheet of carbone made of a single layer of molecule, organised in a bidimensional honeycomb crystal. The transport of charged particles in this structure is usually performed by Dirac equation (which is the relativistic counterpart of the Schrödinger equation). Due to the unusual properties of graphene -at room temperature, electrons moving in graphene behave as massless relativistic particles- physicists and compagnies are nowadays actively studying this material. Here, predicting how the material properties are affected by the uncertainties in the hexagonal lattice structure or in external potentials, is a major issue. Then, one can wonder how to combine multiscale UA schemes with some well-known UQ numerical methods (as stochastic Galerkin (SG) approaches) ?

4.1.3. Population dynamics

The main goal is to characterize how spatial propagation phenomena (diffusion, transport, advection, ...) affect the time evolution of the demography of a population. In collaboration with Y. Lagadeuc (ECOBIO, Rennes), this question has been studied for plankton. In this context, mathematical models have been proposed and it has been shown that the spatial dynamic (in this context, due to the marine current) which is fast compared to demographic scales, can strongly modify the demographic evolution of the plankton.
In collaboration with Ecole d'Agronomie de Rennes, a mathematical study on the demography of a parasite fungi of plants has been performed. In this context, the demography is specific: the fungi can proliferate through sexual reproduction or through parthenogenesis. This two ways of reproduction give rise mathematically to quadratic and linear growth rates with respect to the population variable. The demography is then coupled with transport (transport of fungi spore by wind). Here, the goal is characterize the propagation of the

fungi population by finding travelling waves solutions which are well adapted to describe the evolution of invasive fronts. Moreover, this approach enables to recover with a good agreement realistic examples (infection of ash or banana tree) for which experimental data are available.
In these contexts, mathematical models are a powerful tool for biologists since measurements are very complicated to obtain and laboratory experiments hardly reproduce reality. The models derived are multiscale due to the nature of the underlying phenomena and the next step is to provide efficient numerical schemes.

5. Highlights of the Year

5.1. Highlights of the Year

5.1.1. Awards

Arnaud Debussche has been awarded the senior IUF.

5.1.2. Contracts

- New associated team ANTIpODE with the University of Wisconsin.
- New contract with the startup RAVEL.
- New contract with the startup CAILABS.

6. New Software and Platforms

6.1. Selalib

SEmi-LAgrangian LIBrary
KEYWORDS: Plasma physics - Semilagrangian method - Parallel computing - Plasma turbulence
SCIENTIFIC DESCRIPTION: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.
FUNCTIONAL DESCRIPTION: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

- Partners: Max Planck Insitute - Garching - Université de Strasbourg
- Contact: Philippe Helluy
- URL: http://selalib.gforge.inria.fr/

7. New Results

7.1. Highly-oscillatory problems

7.1.1. Highly-oscillatory problems with time-dependent vanishing frequency

In the analysis of highly-oscillatory evolution problems, it is commonly assumed that a single frequency is present and that it is either constant or, at least, bounded from below by a strictly positive constant uniformly in time. Allowing for the possibility that the frequency actually depends on time and vanishes at some instants introduces additional difficulties from both the asymptotic analysis and numerical simulation points of view. This work [27] is a first step towards the resolution of these difficulties. In particular, P. Chartier, M. Lemou, F. Méhats and G. Vilmart show that it is still possible in this situation to infer the asymptotic behaviour of the solution at the price of more intricate computations and we derive a second order uniformly accurate numerical method.

7.1.2. Uniformly accurate methods for Vlasov equations with non-homogeneous strong magnetic field

In this paper [26], the authors P. Chartier, N. Crouseilles, M. Lemou, F. Méhats and X. Zhao consider the numerical solution of highly-oscillatory Vlasov and Vlasov-Poisson equations with non-homogeneous magnetic field. Designed in the spirit of recent uniformly accurate methods, our schemes remain insensitive to the stiffness of the problem, in terms of both accuracy and computational cost. The specific difficulty (and the resulting novelty of our approach) stems from the presence of a non-periodic oscillation, which necessitates a careful ad-hoc reformulation of the equations. Our results are illustrated numerically on several examples.

7.1.3. Uniformly accurate time-splitting methods for the semiclassical linear Schrödinger equation

The paper [8] is devoted to the construction of numerical methods which remain insensitive to the smallness of the semiclassical parameter for the linear Schrödinger equation in the semiclassical limit. We specifically analyse the convergence behavior of the first-order splitting. Our main result is a proof of uniform accuracy. The authors illustrate the properties of our methods with simulations. Philippe Chartier, Loïc Le Treust, Florian Méhats then illustrate the properties of the methods with simulations.

7.1.4. Numerical methods for the two-dimensional Vlasov-Poisson equation in the finite Larmor radius approximation regime

In this paper [7], the authors P. Chartier, N. Crouseilles and X. Zhao consider the numerical methods for solving the two-dimensional Vlasov-Poisson equation in the finite Larmor radius approximation regime. The model describes the behaviour of charged particles under a strong external magnetic field and the finite Larmor radius approximation. We discretise the equation under Particle-in-Cell method, where the characteristics equations are highly oscillatory system in the limit regime. We apply popular numerical integrators including splitting methods, multi-revolution composition methods, two-scale formulation method and limit solver to integrate the characteristics. Dissuasions are made to highlight the strength and drawback of each method. Numerical experiments are done, and comparisons on the accuracy, efficiency and long-time behaviour of the methods are made, aiming to suggest the method with the best performance for the problem.

7.1.5. A new class of uniformly accurate numerical schemes for highly oscillatory evolution equations

In [9], we introduce a new methodology to design uniformly accurate methods for oscillatory evolution equations. The targeted models are envisaged in a wide spectrum of regimes, from non stiff to highly oscillatory. Thanks to an averaging transformation, the stiffness of the problem is softened, allowing for standard schemes to retain their usual orders of convergence. Overall, high order numerical approximations are obtained with errors and at a cost independent of the regime.

7.1.6. Uniformly accurate exponential-type integrators for Klein-Gordon equations with asymptotic convergence to classical splitting schemes in the nonlinear Schrödinger limit

In [2], we introduce efficient and robust exponential-type integrators for Klein-Gordon equations which resolve the solution in the relativistic regime as well as in the highly-oscillatory non-relativistic regime without any step-size restriction, and under the same regularity assumptions on the initial data required for the integration of the corresponding limit system. In contrast to previous works we do not employ any asymptotic/multiscale expansion of the solution. This allows us derive uniform convergent schemes under far weaker regularity assumptions on the exact solution. In particular, the newly derived exponential-type integrators of first-, respectively, second-order converge in the non-relativistic limit to the classical Lie, respectively, Strang splitting in the nonlinear Schrödinger limit.

7.1.7. A micro-macro method for a kinetic graphene model in one-space dimension

In [29], for the one space dimensional semiclassical kinetic graphene model recently introduced in the literature, we propose a micro-macro decomposition based numerical approach, which reduces the computational dimension of the nonlinear geometric optics method based numerical method for highly oscillatory transport equation developed in a previous work. The method solves the highly oscillatory model in the original coordinate, yet can capture numerically the oscillatory space-time quantum solution pointwisely even without numerically resolving the frequency. We prove that the underlying micro-macro equations have smooth (up to certain order of derivatives) solutions with respect to the frequency, and then prove the uniform accuracy of the numerical discretization for a scalar model equation exhibiting the same oscillatory behavior. Numerical experiments verify the theory.

7.1.8. Multiscale Particle-in-Cell methods and comparisons for the long-time two-dimensional Vlasov-Poisson equation with strong magnetic field

In [12], we applied different kinds of multiscale methods to numerically study the long-time Vlasov-Poisson equation with a strong magnetic field. The multiscale methods include an asymptotic preserving Runge-Kutta scheme, an exponential time differencing scheme, stroboscopic averaging method and a uniformly accurate two-scale formulation. We briefly review these methods and then adapt them to solve the Vlasov-Poisson equation under a Particle-in-Cell discretization. Extensive numerical experiments are conducted to investigate and compare the accuracy, efficiency, and long-time behavior of all the methods. The methods with the best performance under different parameter regimes are identified.

7.1.9. Symmetric high order Gautschi-type exponential wave integrators pseudospectral method for the nonlinear Klein-Gordon equation in the nonrelativistic limit regime

In [19], a group of high order Gautschi-type exponential wave integrators (EWIs) Fourier pseudospectral method are proposed and analyzed for solving the nonlinear Klein-Gordon equation (KGE) in the nonrelativistic limit regime, where a parameter which is inversely proportional to the speed of light, makes the solution propagate waves with wavelength in time and in space. With the Fourier pseudospectral method to discretize the KGE in space, we propose a group of EWIs with designed Gautschi's type quadratures for the temporal integrations, which can offer any intended even order of accuracy provided that the solution is smooth enough, while all the current existing EWIs offer at most second order accuracy. The scheme is explicit, time symmetric and rigorous error estimates show the meshing strategy of the proposed method is time step and mesh size as, which is optimal among all classical numerical methods towards solving the KGE directly in the limit regime, and which also distinguish our methods from other high order approaches such as Runge-Kutta methods which require . Numerical experiments with comparisons are done to confirm the error bound and show the superiority of the proposed methods over existing classical numerical methods.

7.1.10. On the rotating nonlinear Klein-Gordon equation: non-relativistic limit and numerical methods

In [32], we consider both numerics and asymptotics aspects for the rotating nonlinear Klein Gordon (RKG) equation, an important PDE in relativistic quantum physics that can model a rotating galaxy in Minkowski metric and serves also as a model e.g. for a "cosmic superfluid". Firstly, we formally show that in the non-relativistic limit RKG converges to coupled rotating nonlinear Schrödinger equations (RNLS), which is used to describe the particle-antiparticle pair dynamics. Investigations of the vortex state of RNLS are carried out. Secondly, we propose three different numerical methods to solve RKG from relativistic regimes to non-relativistic regimes in polar and Cartesian coordinates. In relativistic regimes, a semi-implicit finite difference Fourier spectral method is proposed in polar coordinates where both rotation terms are diagonalized simultaneously. While in non relativistic regimes, to overcome the fast temporal oscillations, we adopt the rotating Lagrangian coordinates and introduce two efficient multiscale methods with uniform accuracy, i.e., the multi revolution composition method and the exponential integrator. Various numerical results confirm (uniform) accuracy of our methods. Simulations of vortices dynamics are presented.

7.2. Numerical schemes for Hamiltonian PDEs

7.2.1. On numerical Landau damping for splitting methods applied to the Vlasov-HMF model

In [14], we consider time discretizations of the Vlasov-HMF (Hamiltonian Mean-Field) equation based on splitting methods between the linear and non-linear parts. We consider solutions starting in a small Sobolev neighborhood of a spatially homogeneous state satisfying a linearized stability criterion (Penrose criterion). We prove that the numerical solutions exhibit a scattering behavior to a modified state, which implies a nonlinear Landau damping effect with polynomial rate of damping. Moreover, we prove that the modified state is close to the continuous one and provide error estimates with respect to the time stepsize.

7.2.2. Unconditional and optimal H^2-error estimates of two linear and conservative finite difference schemes for the Klein-Gordon-Schrödinger equation in high dimensions

In [17], The focus of this paper is on the optimal error bounds of two finite difference schemes for solving the d-dimensional ($d = 2, 3$) nonlinear Klein-Gordon-Schrödinger (KGS) equations. The proposed finite difference schemes not only conserve the mass and energy in the discrete level but also are efficient in practical computation because only two linear systems need to be solved at each time step. Besides the standard energy method, an induction argument as well as a lifting technique are introduced to establish rigorously the optimal H^2-error estimates without any restrictions on the grid ratios, while the previous works either are not rigorous enough or often require certain restriction on the grid ratios. The convergence rates of the proposed schemes are proved to be at $O(h^2 + \tau^2)$ with mesh-size h and time step τ in the discrete H^2-norm. The analysis method can be directly extended to other linear finite difference schemes for solving the KGS equations in high dimensions. Numerical results are reported to confirm the theoretical analysis for the proposed finite difference schemes.

7.2.3. Modulation equations approach for solving vortex and radiation in nonlinear Schrödinger equation

In [16], we apply the modulation theory to study the vortex and radiation solution in the 2D nonlinear Schrödinger equation. The full modulation equations which describe the dynamics of the vortex and radiation separately are derived. A general algorithm is proposed to efficiently and accurately find vortices with prescribed values of energy and spin index. The modulation equations are solved by accurate numerical method. Numerical tests and simulations of radiation are given.

7.2.4. Unconditional L^∞-convergence of two compact conservative finite difference schemes for the nonlinear Schrödinger equation in multi-dimensions

In [18], we are concerned with the unconditional and optimal L^∞-error estimates of two fourth-order (in space) compact conservative finite difference time domain schemes for solving the nonlinear Schrödinger equation in two or three space dimensions. The fact of high space dimension and the approximation via compact finite difference discretization bring difficulties in the convergence analysis. The two proposed schemes preserve the total mass and energy in the discrete sense. To establish the optimal convergence results without any constraint on the time step, besides the standard energy method, the cut-off function technique as well as a lifting technique are introduced. On the contrast, previous works in the literature often require certain restriction on the time step. The convergence rate of the proposed schemes are proved to be of $O(h^4 + \tau^2)$ with time step τ and mesh size h in the discrete L^∞-norm. The analysis method can be directly extended to other finite difference schemes for solving the nonlinear Schrödinger-type equations. Numerical results are reported to support our theoretical analysis, and investigate the effect of the nonlinear term and initial data on the blow-up solution.

7.2.5. Verification of $2D$x$2D$ and two-species Vlasov-Poisson solvers

Recently $1D$x$1D$ two-species Vlasov-Poisson simulations have been performed by the semi-Lagrangian method. Thanks to a classical first order dispersion analysis, we are able to check in [1] the validity of their simulations; the extension to second order is performed and shown to be relevant for explaining further details.

In order to validate multi-dimensional effects, we propose a $2D$x$2D$ single species test problem that has true $2D$ effects coming from the sole second order dispersion analysis. Finally, we perform, in the same code, full $2D$x$2D$ non linear two-species simulations with mass ratio $\sqrt{0.01}$, and consider the mixing of semi-Lagrangian and Particle-in-Cell methods.

7.2.6. An exponential integrator for the drift-kinetic model

In [11], we propose an exponential integrator for the drift-kinetic equations in polar geometry. This approach removes the CFL condition from the linear part of the system (which is often the most stringent requirement in practice) and treats the remainder explicitly using Arakawa's finite difference scheme. The present approach is mass conservative, up to machine precision, and significantly reduces the computational effort per time step. In addition, we demonstrate the efficiency of our method by performing numerical simulations in the context of the ion temperature gradient instability. In particular, we find that our numerical method can take time steps comparable to what has been reported in the literature for the (predominantly used) splitting approach. In addition, the proposed numerical method has significant advantages with respect to conservation of energy and efficient higher order methods can be obtained easily. We demonstrate this by investigating the performance of a fourth order implementation.

7.2.7. Convergence of a normalized gradient algorithm for computing ground states

In [15], we consider the approximation of the ground state of the one-dimensional cubic nonlinear Schrödinger equation by a normalized gradient algorithm combined with linearly implicit time integrator, and finite difference space approximation. We show that this method, also called imaginary time evolution method in the physics literature, is con-vergent, and we provide error estimates: the algorithm converges exponentially towards a modified solitons that is a space discretization of the exact soliton, with error estimates depending on the discretization parameters.

7.3. Analysis of PDE

7.3.1. Bounds on the growth of high discrete Sobolev norms for the cubic discrete nonlinear Schrödinger equations on $h\mathbb{Z}$

In [22], we consider the discrete nonlinear Schrödinger equations on a one dimensional lattice of mesh h, with a cubic focusing or defocusing nonlinearity. We prove a polynomial bound on the growth of the discrete Sobolev norms, uniformly with respect to the stepsize of the grid. This bound is based on a construction of higher modified energies.

7.3.2. Existence and stability of traveling waves for discrete nonlinear Schrödinger equations over long times

In [23], we consider the problem of existence and stability of solitary traveling waves for the one dimensional discrete non linear Schrödinger equation (DNLS) with cubic nonlinearity, near the continuous limit. We construct a family of solutions close to the continuous traveling waves and prove their stability over long times. Applying a modulation method, we also show that we can describe the dynamics near these discrete traveling waves over long times.

7.3.3. Smoothing properties of fractional Ornstein-Uhlenbeck semigroups and null-controllability

In [20], we study fractional hypoelliptic Ornstein-Uhlenbeck operators acting on $L^2(\mathbb{R}^n)$ satisfying the Kalman rank condition. We prove that the semigroups generated by these operators enjoy Gevrey regularizing effects. Two byproducts are derived from this smoothing property. On the one hand, we prove the null-controllability in any positive time from thick control subsets of the associated parabolic equations posed on the whole space. On the other hand, by using the interpolation theory, we get global L^2 subelliptic estimates for the these operators.

7.3.4. Stable ground states for the HMF Poisson model

In [31], we prove the nonlinear orbital stability of a large class of steady states solutions to the Hamiltonian Mean Field (HMF) system with a Poisson interaction potential. These steady states are obtained as minimizers of an energy functional under one, two or infinitely many constraints. The singularity of the Poisson potential prevents from a direct run of the general strategy which was based on generalized rearrangement techniques, and which has been recently extended to the case of the usual (smooth) cosine potential. Our strategy is rather based on variational techniques. However, due to the boundedness of the space domain, our variational problems do not enjoy the usual scaling invariances which are, in general, very important in the analysis of variational problems. To replace these scaling arguments, we introduce new transformations which, although specific to our context, remain somehow in the same spirit of rearrangements tools introduced in the references above. In particular, these transformations allow for the incorporation of an arbitrary number of constraints, and yield a stability result for a large class of steady states.

7.4. Dissipative problems

7.4.1. A formal series approach to the center manifold theorem

In [4], the author considers near-equilibrium systems of ordinary differential equations with explicit separation of the slow and stable manifolds. Formal B-series like those previously used to analyze highly-oscillatory systems or to construct modified equations are employed here to construct expansions of the change of variables, the center invariant manifold and the reduced model. The new approach may be seen as a process of reduction to a normal form, with the main advantage, as compared to the standard view conveyed by the celebrated center manifold theorem, that it is possible to recover the complete solution at any time through an explicit change of variables.

7.4.2. Analysis of an asymptotic preserving scheme for stochastic linear kinetic equations in the diffusion limit

In [21], we present an asymptotic preserving scheme based on a micro-macro decomposition for stochastic linear transport equations in kinetic and diffusive regimes. We perform a mathematical analysis and prove that the scheme is uniformly stable with respect to the mean free path of the particles in the simple telegraph model and in the general case. We present several numerical tests which validate our scheme.

7.4.3. A particle micro-macro decomposition based numerical scheme for collisional kinetic equations in the diffusion scaling

In [28], we derive particle schemes, based on micro-macro decomposition, for linear kinetic equations in the diffusion limit. Due to the particle approximation of the micro part, a splitting between the transport and the collision part has to be performed, and the stiffness of both these two parts prevent from uniform stability. To overcome this difficulty, the micro-macro system is reformulated into a continuous PDE whose coefficients are no longer stiff, and depend on the time step Δt in a consistent way. This non-stiff reformulation of the micro-macro system allows the use of standard particle approximations for the transport part, and extends a previous work of the authors where a particle approximation has been applied using a micro-macro decomposition on kinetic equations in the fluid scaling. Beyond the so-called asymptotic-preserving property which is satisfied by our schemes, they significantly reduce the inherent noise of traditional particle methods, and they have a computational cost which decreases as the system approaches the diffusion limit.

7.4.4. Time diminishing schemes (TDS) for kinetic equations in the diffusive scaling

In [28], we develop a new class of numerical schemes for collisional kinetic equations in the diffusive regime. The first step consists in reformulating the problem by decomposing the solution in the time evolution of an equilibrium state plus a perturbation. Then, the scheme combines a Monte Carlo solver for the perturbation with a Eulerian method for the equilibrium part, and is designed in such a way to be uniformly stable with respect to the diffusive scaling and to be consistent with the asymptotic diffusion equation. Moreover, since

particles are only used to describe the perturbation part of the solution, the scheme becomes computationally less expensive - and is thus time diminishing (TDS) - as the solution approaches the equilibrium state due to the fact that the number of particles diminishes accordingly. This contrasts with standard methods for kinetic equations where the computational cost increases (or at least does not decrease) with the number of interactions. At the same time, the statistical error due to the Monte Carlo part of the solution decreases as the system approaches the equilibrium state: the method automatically degenerates to a solution of the macroscopic diffusion equation in the limit of infinite number of interactions. After a detailed description of the method, we perform several numerical tests and compare this new approach with classical numerical methods on various problems up to the full three dimensional case.

7.5. Stochastic PDE

7.5.1. Linearized wave turbulence convergence results for three-wave systems

In [30], E. Faou considers stochastic and deterministic three-wave semi-linear systems with bounded and almost continuous set of frequencies. Such systems can be obtained by considering nonlinear lattice dynamics or truncated partial differential equations on large periodic domains. We assume that the nonlinearity is small and that the noise is small or void and acting only in the angles of the Fourier modes (random phase forcing). We consider random initial data and assume that these systems possess natural invariant distributions corresponding to some Rayleigh-Jeans stationary solutions of the wave kinetic equation appearing in wave turbulence theory. We consider random initial modes drawn with probability laws that are perturbations of theses invariant distributions. In the stochastic case, we prove that in the asymptotic limit (small nonlinearity, continuous set of frequency and small noise), the renormalized fluctuations of the amplitudes of the Fourier modes converge in a weak sense towards the solution of the linearized wave kinetic equation around these Rayleigh-Jeans spectra. Moreover, we show that in absence of noise, the deterministic equation with the same random initial condition satisfies a generic Birkhoff reduction in a probabilistic sense, without kinetic description at least in some regime of parameters.

7.5.2. Large deviations for the dynamic Φ_d^{2n} model

In [5], we are dealing with the validity of a large deviation principle for a class of reaction-diffusion equations with polynomial non-linearity, perturbed by a Gaussian random forcing. We are here interested in the regime where both the strength of the noise and its correlation are vanishing, on a length scale ρ and $\delta(\rho)$, respectively, with $0 < \rho, \delta(\rho) \ll 1$. We prove that, under the assumption that ρ and $\delta(\rho)$ satisfy a suitable scaling limit, a large deviation principle holds in the space of continuous trajectories with values both in the space of square-integrable functions and in Sobolev spaces of negative exponent. Our result is valid, without any restriction on the degree of the polynomial nor on the space dimension.

7.5.3. Kolmogorov equations and weak order analysis for SPDES with nonlinear diffusion coefficient

In [3], we provide new regularity results for the solutions of the Kolmogorov equation associated to a SPDE with nonlinear diffusion coefficients and a Burgers type nonlinearity. This generalizes previous results in the simpler cases of additive or affine noise. The basic tool is a discrete version of a two sided stochastic integral which allows a new formulation for the derivatives of these solutions. We show that this can be used to generalize the weak order analysis performed previously. The tools we develop are very general and can be used to study many other examples of applications.

7.5.4. Large deviations for the two-dimensional stochastic Navier-Stokes equation with vanishing noise correlation

In [6], we are dealing with the validity of a large deviation principle for the two-dimensional Navier-Stokes equation, with periodic boundary conditions, perturbed by a Gaussian random forcing. We are here interested in the regime where both the strength of the noise and its correlation are vanishing, on a length scale ε and $\delta(\varepsilon)$, respectively, with $0 < \varepsilon, \delta(\varepsilon) \ll 1$. Depending on the relationship between ε and $\delta(\varepsilon)$ we will prove the validity of the large deviation principle in different functional spaces.

7.5.5. The Schrödinger equation with spatial white noise potential

In [13], we consider the linear and nonlinear Schrödinger equation with a spatial white noise as a potential in dimension 2. We prove existence and uniqueness of solutions thanks to a change of unknown originally used in a paper by Hairer and Labbé (2015) and conserved quantities.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

Contract with RAVEL (01/09/2018-31/08/2019, budget 15000 euros) : P. Chartier, M. Lemou and F. Méhats initiated a collaboration with the startup RAVEL on a one-year basis (with possible renewal at the end of the year). The objective is to study the mathematical fondations of artificial intelligence and in particular machine learning algorithms for data anonymized though homomorphic encryption.

Contract with CaiLabs (20/12/2018-20/06/2019, budget 8774 euros) : E. Faou initiated a collaboration with the startup CaiLabs on a six-months basis (with possible renewal at the end of the contract). The collaboration between CaiLabs and MINGuS aims at modelling optical devices allowing the recognition of simple objects. The structure of the devices combines quantum propagation phenomena, reflection mirrors and frequency absorbers and possesses a deep neural networks structure.

9. Partnerships and Cooperations

9.1. Regional Initiatives

- A. Crestetto is a member of the regional initiative "Pari Scientifique Exprodil".
- M. Lemou is the head of the project "MUNIQ" of ENS Rennes. This two-years project (2018-2019) intends to gather multiscale numerical methods and uncertainty quantification techniques. The MINGuS members are P. Chartier, N. Crouseilles, M. Lemou and F. Méhats and colleagues from university of Madison-Wisconsin also belongs to this project.

9.2. National Initiatives

9.2.1. ANR

Participants: François Castella, Philippe Chartier, Nicolas Crouseilles, Mohammed Lemou, Florian Méhats, Arnaud Debussche, Anais Crestetto.

ANR MOONRISE: 2015-2019

The project *Moonrise* submitted by Florian Méhats has been funded by the ANR for 4 years, for the period 2015-2019. This project aims at exploring modeling, mathematical and numerical issues originating from the presence of high-oscillations in nonlinear PDEs from the physics of nanotechnologies (quantum transport) and from the physics of plasmas (magnetized transport in tokamaks). The partners of the project are the IRMAR (Rennes), the IMT (Toulouse) and the CEA Cadarache. In the MINGuS team, François Castella, Philippe Chartier, Nicolas Crouseilles and Mohammed Lemou are members of the project Moonrise.

Postdocs

- Loïc Le Treust has been hired as a Postdoc, under the supervision of Philippe Chartier and Florian Méhats. His contract started in september 2015 and ended in august 2016. Loïc Le Treust is now assistant professor at the university of Marseille.
- Yong Zhang has been hired as a Postdoc, under the supervision of Philippe Chartier and Florian Méhats. His contract started in september 2015 and ended in august 2016. Yong Zhang is now professor at the Tianjin university.
- Xiaofei Zhao has been hired as a Postdoc from september 2015 to september 2016 under the supervision of Florian Méhats. Xiaofei Zhao is now postdoc in the MINGuS team.

ANR MFG: 2016-2020

Mean Field Games (MFG) theory is a new and challenging mathematical topic which analyzes the dynamics of a very large number of interacting rational agents. Introduced ten years ago, the MFG models have been used in many areas such as, e.g., economics (heterogeneous agent models, growth modeling,...), finance (formation of volatility, models of bank runs,...), social sciences (crowd models, models of segregation) and engineering (data networks, energy systems...). Their importance comes from the fact that they are the simplest ("stochastic control"-type) models taking into account interactions between rational agents (thus getting beyond optimization), yet without entering into the issues of strategic interactions. MFG theory lies at the intersection of mean field theories (it studies systems with a very large number of agents), game theory, optimal control and stochastic analysis (the agents optimize a payoff in a possibly noisy setting), calculus of variations (MFG equilibria may arise as minima of suitable functionals) and partial differential equations (PDE): In the simplest cases, the value of each agent is found by solving a backward Hamilton-Jacobi equation whereas the distribution of the agents' states evolves according to a forward Fokker-Planck equation. The "Master" equation (stated in the space of probability measures) subsumes the individual and collective behaviors. Finally, modeling, numerical analysis and scientific computing are crucial for the applications. French mathematicians play a world-leading role in the research on MFG: The terminology itself comes from a series of pioneering works by J.-M. Lasry and P.-L. Lions who introduced most of the key ideas for the mathematical analysis of MFG; the last conference on MFG was held last June in Paris and organized by Y. Achdou, P. Cardaliaguet and J.-M. Lasry. As testifies the proposal, the number of researchers working on MFG in France (and also abroad) is extremely fast-growing, not only because the theoretical aspects are exciting and challenging, but also because MFG models find more and more applications. The aim of the project is to better coordinate the French mathematical research on MFG and to achieve significant progress in the theory and its applications.

The partners of the project are the CEREMADE laboratory (Paris Dauphine), the IRMAR laboratory (Rennes I), the university of Nice and of Tours.

ANR MoHyCon: 2017-2021

The MoHyCon project is related to the analysis and simulation of numerical methods for multiscale models of semiconductors. As almost all current electronic technology involves the use of semiconductors, there is a strong interest for modeling and simulating the behavior of such devices, which was recently reinforced by the development of organic semiconductors used for example in solar panels or in mobile phones and television screens (among others).

There exists a hierarchy of semiconductors models, including mainly three classes, which correspond to different scales of observation: microscopic, mesoscopic and macroscopic. At the microscopic scale, the particles are described one by one, leading to a huge system almost impossible to study, both theoretically and numerically. Within MoHyCon, we are then rather interested in the two other scales. The considered models at the mesoscopic scale are kinetic, of Boltzmann type, describing a distribution of particles submitted to an electric field. These models describe accurately the behavior of the semiconductor, but can be intricate and highly time and resource consuming to solve numerically. Thus, when the mean free path becomes small, it is preferable to consider fluid models, describing macroscopic quantities. Depending on the considered number of moments, various models can be obtained. The more common ones are the energy transport model, describing the densities of electron and energy, and the more simple drift-diffusion model, where the temperature is assumed to be a given function of the electron density.

In this project and provided this context, our aim is to construct and study rigorously numerical methods for these multiscale models. To this end, we will consider two distinct approaches: "Asymptotic Preserving" (AP) methods and coupling methods. The idea of AP methods is to design only one scheme which will be able to treat accurately every scale, without imposing restrictive stability conditions on the discretization parameters. Regarding the coupling methods, they consist in decomposing the domain into different regions on which the more relevant model (kinetic or macroscopic) will be considered. After locating the kinetic and fluid domains, the main difficulty is to obtain correct coupling conditions at each interface between two regions.

Considering the AP approach, our aim is to construct schemes for the linear Boltzmann equation for semiconductors, asymptotic preserving at the limit given by the drift-diffusion model. We will start with a very simplified model, with a linearized BGK collision operator. The constructed scheme will tend to an implicit discretization of Scharfetter-Gummel type for the drift-diffusion equation. The main objective will be then to lead a complete and rigorous study of the AP property by adapting to the discrete framework some continuous techniques: establish a discrete dissipation property yielding uniform estimates on the approximate solution which will allow to pass to the diffusion limit in the scheme.

As regards the second approach, the aim is to build a hybrid model coupling the kinetic equation on weakly collisional regions with macroscopic models on the remaining domain. We will first study specifically discretizations for energy-transport model, since we are then going to couple them with kinetic and other macroscopic schemes in our hybrid method. A particular attention will be paid to the implementation in order to obtain a robust and efficient code.

This ANR project is headed by M. Bessemoulin (CNRS, Nantes).

9.3. International Initiatives

9.3.1. Inria International Labs

IPL FRATRES MINGuS is associated to IPL FRATRES which started in june 2015. The aim of this project is to organize Inria teams activities which develop mathematical and numerical tools in magnetically confined nuclear fusion. The ambition is to prepare the next generation of numerical modeling methodologies able to use in an optimal way the processing capabilities of modern massively parallel architectures. This objective requires close collaboration between a) applied mathematicians and physicists that develop and study mathematical models of PDE; b) numerical analysts developing approximation schemes; c) specialists of algorithmics proposing solvers and libraries using the many levels of parallelism offered by the modern architecture and d) computer scientists. The project road map ambitions to contribute in close connection with National and European initiatives devoted to nuclear Fusion to the improvement and design of numerical simulation technologies applied to plasma physics and in particular to the ITER project for magnetic confinement fusion.

The IPL FRATRES ends at the end of the year. The final meeting was organized in november during which X. Zhao (former IPL postdoc), G. Morel (IPL postdoc) and N. Crouseilles have presented the kinetic activities of the IPL project.

Postdoc

- Xiaofei Zhao has been hired as a postdoc, under the supervision of Nicolas Crouseilles and Sever Hirstoaga (Inria-Nancy). His contract started in october 2016 and ended in september 2017. Xiaofei Zhao is now postdoc in the MINGuS team.
- Guillaume Morel has been hired as a postdoc, under the supervision of Nicolas Crouseilles and Michel Mehrenberger (AMU). His contract started in october 2018 and ended in september 2019.

9.3.2. Inria Associate Teams Not Involved in an Inria International Labs

ANTIpODE

Title: Asymptotic Numerical meThods for Oscillatory partial Differential Equations with uncertainties

International Partner (Institution - Laboratory - Researcher):

University of Wisconsin-Madison, USA (United States) - DEPARTMENT MATHEMATICS - Shi Jin

Start year: 2018

See also: https://team.inria.fr/antipode/

The proposed associate team assembles the Inria team MINGuS and the research group led by Prof. Shi Jin from the Department of Mathematics at the University of Wisconsin, Madison. The main scientific objective of ANTIpODE consists in marrying uniformly accurate and uncertainty quantification techniques for multi-scale PDEs with uncertain data. Multi-scale models, as those originating e.g. from the simulation of plasma fusion or from quantum models, indeed often come with uncertainties. The main scope of this proposal is thus (i) the development of uniformly accurate schemes for PDEs where space and time high oscillations co-exist and (ii) their extension to models with uncertainties. Applications to plasmas (Vlasov equations) and graphene (quantum models) are of paramount importance to the project.

9.3.3. Inria International Partners

Informal International Partners The members of the MINGuS have several interactions with the following partners

- the group of W. Bao (university of Singapore).
- university of Maryland (S. Cerrai)
- university of Ferrare (G. Dimarco)
- university of Madison-Wisconsin (S. Jin and Q. Li)
- university of Geneva (G. Vilmart)

9.4. International Research Visitors

9.4.1. Visits of International Scientists

The following scientists came in Rennes to visit some MINGuS members

- Fernando Casas (Professor, University of Jaume I, Castellon, Spain): invited professor for five months, from september 2018 to january 2019, funded by the Labex (CHL) Center Henri Lebesgue, collaboration on dissipative equations, splitting methods and gradient descent methods with P. Chartier, N. Crouseilles, M. Lemou and F. Méhats.
- Molei Tao (Associate professor, Georgia Tech, Atlanta, USA): eigth-days visit in november, funded by the ANR Moorise, collaboration on highly-oscillatory differential equations with P. Chartier, M. Lemou and F. Méhats.
- Di Fang (PhD student, University of Wisconsin, USA): ten-days visit in december 2019, funded by the associated team ANTIpODE, collaboration on highly-oscillatory differential equations with P. Chartier, M. Lemou and F. Méhats.

9.4.2. Visits to International Teams

9.4.2.1. Research Stays Abroad

Here we list the members of the team who did some research stays abroad

- P. Chartier, N. Crouseilles, M. Lemou, F. Méhats and P. Navaro: invited working visit (Eric Sonnendrücker), Max Planck Institute, Garching, Germany, March 19th-23th, 2018.
- P. Chartier, M. Lemou and F. Méhats: working visit, university of Madison-Wisconsin, USA, october 10th-20th, 2018.
- P. Chartier: invited working visit (Christophe Besse), University of Toulouse 3, Toulouse, February 26th-28th, 2018.
- P. Chartier: invited working visit (Gilles Vilmart), University of Geneva, Geneva, Switzerland, January 19th-26th, 2018.
- P. Chartier: invited working visit (Ander Murua), University of the Basque Country, San-Sebastian, Spain, January 15th-19th, 2018.
- N. Crouseilles: invited working visit (Giacomo Dimarco), University of Ferrare, Italy, october 2018.
- A. Debussche: invited working visit (Sandra Cerrai), university of Maryland, USA, april 2018.
- M. Lemou and F. Méhats: invited working visit (Ana Maria Luz), University Federal of Fluminence, Niteroi, Brazil, April 3th-8th, 2018.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

10.1.1.1. Member of the Organizing Committees

- F. Castella organized the workshop "Multiscale numerical methods", Saint-Malo, december 13-15 2017. [15 participants]
- P. Chartier, M. Lemou and F. Méhats organized the MOONRISE Workshop on "Numerical methods for multi-scale PDEs", Institut d'études scientifiques de Cargèse, Cargèse, France, September 3th-7th, 2018.
- P. Chartier, N. Crouseilles, M. Lemou and F. Méhats organized the workshop Henri Lebesgue Center on "Geometric and multi-scale methods for kinetic equations", Rennes, June 12th-15th, 2018.
- A. Crestetto organized the "journée d'analyse Rennes-Nantes", january 2018, ENS Rennes.
- A. Crestetto organized the workshop Henri Lebesgue Center on "mathematical models in health sciences".
- N. Crouseilles co-organized the weekly seminar "Mathematic and applications", ENS Rennes.
- A. Debussche organized the workshop on Stochastic partial differential equations, may 2018, CIRM, Marseille, France.

10.1.2. Scientific Events Selection

10.1.2.1. Member of the Conference Program Committees

- N. Crouseilles was member of the scientific committee of the SMAI-2019 conference organized by the university of Nantes.
- E. Faou was head of organization of the semester *scientific computing* sponsored by the Labex Lebesgue (2 international summer schools, 7 workshops and international conferences), january-july 2018.

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

- P. Chartier is member of the editorial committee of Mathematical Modelling and Numerical Analysis (M2AN).
- A. Debussche is editor in chief of Stochastic Partial Differential Equations: analysis and computations (2013-).
- A. Debussche is associate editor of Differential and Integral Equations (2002-2018).
- A. Debussche is associate editor of Potential Analysis (2011-2018).
- A. Debussche is associate editor of ESAIM:PROC (2012-).
- A. Debussche is associate editor of Journal of Evolution Equation (2014-).
- A. Debussche is associate editor of Applied Mathematics & Optimization (2014-).
- A. Debussche is associate editor of SIAM JUQ (2017-).
- A. Debussche is member of the editorial board of the collection "Mathématiques & Applications de la SMAI", Springer.
- M. Lemou is member of the editorial committee of "Communications in Mathematical Sciences" (CMS).

10.1.3.2. Reviewer - Reviewing Activities

The members of the IPSO team are reviewers for almost all the journals in which they publish (SIAM, JCP, CPDE, CMP, ARMA, JSP, JSC, JMAA, ANM, JCAM, NMPDE, Numer. Math., $\cdots$)

10.1.4. Invited Talks

- J. Bernier: invited speaker at the workshop "dynamics of Hamiltonian PDEs", february 2018, La Thuile, Italy.
- J. Bernier: invited speaker at the workshop "NABUCO", february 2018, Toulouse, France.
- P. Chartier: invited speaker at the 12th AIMS Conference on Dynamical Systems, Differential Equations and Application, July 5 - July 9, 2018 Taipei, Taiwan.
- A. Crestetto: invited speaker at the conference ABPDEIII, august 2018, Lille, France.
- A. Crestetto: invited speaker at the workshop on "Numerical Methods for Multiscale PDEs", september 2018, Cargèse, France.
- N. Crouseilles: invited talk at the Workshop Numerical Aspects of Hyperbolic Balance Laws and Related Problems, University of Ferrara, Italy, April 2018.
- N. Crouseilles: invited talk at the "Journée Scientifique Inria", june 27-28, Bordeaux, France.
- N. Crouseilles: invited speaker at the 12th AIMS Conference on Dynamical Systems, Differential Equations and Application, July 5 - July 9, 2018 Taipei, Taiwan.
- N. Crouseilles: invited speaker at the worskhop "Integrating the Integrators for Nonlinear Evolution Equations: from Analysis to Numerical Methods, High-Performance-Computing and Applications", december 2018, BIRS, Banff, Canada.
- A. Desbussche: invited speaker "Aziz lecture", university of Maryland, april 2018, USA.
- A. Desbussche: invited speaker at the "school on PDEs", Riemann international school of mathematics, july 2018, Varese, Italy.
- E. Faou: invited plenary speaker at the conference on "Mathematics of Wave Phenomena", august 2018, KIT Karlsruhe, Germany.
- E. Faou: invited speaker at the worskhop "Integrating the Integrators for Nonlinear Evolution Equations: from Analysis to Numerical Methods, High-Performance-Computing and Applications", december 2018, BIRS, Banff, Canada.
- M. Lemou: invited speaker at the 12th AIMS Conference on Dynamical Systems, Differential Equations and Application, July 5 - July 9, 2018 Taipei, Taiwan.
- M. Lemou: invited speaker at the Ki-Net Conference on "Mathematical and Numerical Aspects of Quantum Dynamics", June 19-21, 2018. CSCAMM, University of Maryland, USA.
- M. Lemou: invited speaker at "Premier Congrès Franco-Marocain de Mathématiques Appliquées", april 16-20 2018, Marrakech, Maroc.
- F. Méhats: invited talk at the Workshop Numerical Aspects of Hyperbolic Balance Laws and Related Problems, University of Ferrara, Italy, April 2018.
- F. Méhats: 7 hours mini course on Averaging Techniques, Colloque Inter' Actions, Lyon, May 2018.
- F. Méhats: talk at the day Space-Time Multiscale Methods (Atelier MePhy), Paris, June 2018.
- F. Méhats: talk at the University of Wisconsin, October 2018.
- F. Méhats: talk at a workshop on dispersive equations, October 2018.
- G. Morel: invited speaker at the Numkin workshop, october 2018, Max Planck Institute, Garching, Germany.
- P. Navaro: invited speaker at the CANUM 2018, France.
- P. Navaro: invited speaker at the IPL meeting, France.
- P. Navaro: invited speaker at the Finist'R meeting, France.
- X. Zhao: invited speaker at the workshop on "Numerics for cosmology - the Schrödinger method", may 22nd-25th 2018, WPI Vienna, Austria.

- X. Zhao: invited speaker at the workshop on "weak turbulence, kinetic problems and related", july 14-15 2018, Wuhan university, China.
- X. Zhao: invited speaker at the "Forum on modern challenges in modelling and numerics", july 20th-21st 2018, Beijing CSRC, China.
- X. Zhao: invited speaker at the "Modern Numerics on Nonlinear Wave and Dispersive Equations and Related Problems", august 8th-9th 2018, Chengdu university, China.
- X. Zhao: invited speaker at the Numkin workshop, october 2018, Max Planck Institute, Garching, Germany.
- X. Zhao: invited speaker at the workshop on "Numerical Methods for Multiscale PDEs", september 2018, Cargèse, France.

10.1.5. Scientific Expertise

- P. Chartier was member of the hiring committee CR2-Inria (Bordeaux).
- P. Chartier was member of the hiring committee the Inria promotion committee (DR1-DR0).
- A. Crestetto was member of the hiring committee AGPR-2018, ENS Rennes, France.
- A. Crestetto was the vice-head of the hiring committee of the MCF position, university of Nantes, France.
- A. Debussche was reviewer for Austrian Science Fundation (FWF).
- E. Faou is the member of the CNU 26.
- E. Faou was member of the hiring committee of the professor position, university of Dijon, France.

10.1.6. Research Administration

- F. Castella was member of the evaluation HCERES committee of the MICS laboratory (Centrale-Supélec).
- P. Chartier is the vice-head of science (DSA) of the Rennes Inria-Center until the end of 2018,
- P. Chartier is member of the national evaluation committee (CE) of Inria until the end of 2018.
- P. Chartier is member of the direction committee (ED) of the Rennes Inria-Center until the end of 2018
- P. Chartier is member of the G6-committee (bureau of the direction committee) Inria-Rennes-Bretagne-Atlantique center until the end of 2018.
- A. Crestetto is member of the mathematics department council, university of Nantes.
- A. Crestetto is member of the scientific council of the sciences unity, university of Nantes.
- A. Crestetto is the SMAI correspondent for the mathematics laboratory of Nantes.
- N. Crouseilles is member of the scientific council of ENS Rennes.
- N. Crouseilles is member of the mathematic laboratory (IRMAR) council.
- N. Crouseilles is member of the Fédération de Fusion council (University of Rennes representative).
- A. Debussche is member of the scientific council of the Fédération Denis Poisson.
- A. Debussche is member of the administrative council of ENS Paris-Saclay.
- A. Debussche if scientific vice-deputy and international relations of ENS Rennes.
- A. Debussche is vice-head ot the Centre Henri Lebesgue.
- A. Debussche is vice-head of the Lebesgue agency for Mathematic and Innovation.
- E. Faou is the AMIES correspondent for Inria Bretagne Atlantique.
- E. Faou is member of the scientific council of the Pôle Universitaire Léonard de Vinci.
- M. Lemou is the head of the IRMAR team "Analyse numérique" composed of 48 members.
- M. Lemou is member of the scientific council of ENS Rennes.

- M. Lemou is member of the scientific council of the LABEX Lebesgue.
- P. Navaro is member of the national network "calcul" http://calcul.math.cnrs.fr. This network is well known for interdisciplinarity of CNRS dedicated to technological aspects of scientific computing (programming, optimization, architectures, $\cdots$).

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence:

- P. Chartier: lecture "Ordinary differential equations" of the first year (Licence) of the Magistère, Ecole Normale Supérieure de Rennes (24 hours).
- N. Crouseilles: lecture on "Numerical methods" of L3, ENS Rennes (30h).

Master:

- F. Castella: lecture on "Numerical methods" of M1, University of Rennes 1 (50 hours).
- P. Chartier: lecture on "Numerical methods for kinetic equations" of the third year (M2) of the Magistère, University of Rennes 1 (12 hours).
- A. Crestetto: lecture on "Numerical methods for incompressible fluids" of M2, university of Nantes (62h).
- N. Crouseilles: lecture on "Numerical methods for PDEs" of M1, ENS Rennes (30h).
- E. Faou: lecture on "numerical integration of Hamiltonian PDEs" of M2, university of Rennes 1 (30h).
- M. Lemou: lecture on "elliptic PDEs" of the second year (M1) of the Magistère, University of Rennes 1 (36 hours).
- M. Lemou: lecture on "fundamental tools for PDEs" of the third year (M2) of the Magistère, University of Rennes 1 (12 hours).
- M. Lemou: lecture on "Numerical tools for kinetic equations" of the third year (M2) of the Magistère, University of Rennes 1 (12 hours).
- P. Navaro: lecture on "numerical tools for Big Data", master, university of Rennes 2.
- P. Navaro: lecture on "Python programming", master, ENSAI.

Doctorat:

- F. Casas: lecture for PhD students " A (very) Short Introduction to Geometric Numerical Integration and Splitting Methods", (4h).

10.2.2. Supervision

PhD: M. Fontaine, Modèles mathématiques de type HMF : stabilité et méthodes numériques autour d'états staionnaires, ENS Rennes, july 13th, advisors: M. Lemou and F. Méhats.

PhD: M. jugal Nguepedja Nankep, Modélisation stochastique de systèmes biologiques multi-échelles et inhomogènes en espace, ENS Rennes, march 22th, advisor: A. Debussche.

PhD in progress: J. Bernier, Mathematical and numerical anaysis of nonlinear transport equations, started in september 2016, advisors: N. Crouseilles and E. Faou.

PhD in progress: A. Rosello, "Approximation-diffusion pour des équations cinétiques pour les modèles de type spray", started in september 2016, advisors: A. Debussche and J. Vovelle (CNRS, Lyon).

PhD in progress: J. Massot, "Modélisation et simulation de plasmas chauds", started in october 2018, advisors: A. Crestetto (Nantes) and N. Crouseilles.

PhD in progress: L. Trémant, "Méthodes d'analyse asymptotique et d'approximation numérique de modèles dissipatifs multi-échelles : EDO à variété centrale et modèles cinétiques", started in october 2018, advisors: P. Chartier and M. Lemou.

10.2.3. Juries

- F. Castella: reviewer of the thesis and member of the PhD committee of P. Terrier (university of Paris Est and CEA).
- A. Crestetto: member of the PhD committee of the thesis of M. Stauffert (university of Versailles).
- N. Crouseilles: reviewer of the thesis and member of the PhD comittee of N. Bouzat (university of Strasbourg, Inria and CEA).
- N. Crouseilles: member of the PhD comittee of M. Malo (ENS Rennes).
- A. Debussche: reviewer of the HDR and member of the HDR committee of G. Dujardin (Inria, university of Lille).
- A. Debussche: member of the PhD committee of M. Tomasevic (university of Nice).
- A. Debussche: member of the PhD committee of P. Tsatsoulis (university of Warwick).
- E. Faou: reviewer of the thesis and member of the PhD committee of M.-S. Dupuy (university of Paris Diderot).
- E. Faou: member of the HDR committee of G. Dujardin (Inria, university of Lille).

10.3. Popularization

10.3.1. Interventions

- A. Crestetto: talks on "traffic flows" in several educational institutions within the framework of "Essentielles", university of Nantes.
- N. Crouseilles: welcoming of two schoolchildren at IRMAR laboratory.
- N. Crouseilles: welcoming of a student within the framework of the modulus "visit a researcher" (3 meetings).

10.3.2. Internal action

- P. Navaro: Python training (3 days) at the CNRS delegation of Lille, april 2018.
- P. Navaro: Python training (3 days) for the Rennes DSI, june 2018.

11. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

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[2] S. BAUMSTARK, E. FAOU, K. SCHRATZ. *Uniformly accurate exponential-type integrators for Klein-Gordon equations with asymptotic convergence to classical splitting schemes in the nonlinear Schrödinger limit*, in "Mathematics of Computation", 2018, vol. 87, n° 311, pp. 1227-1254, https://arxiv.org/abs/1606.04652 [*DOI :* 10.1090/MCOM/3263], https://hal.archives-ouvertes.fr/hal-01331949

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[9] P. CHARTIER, M. LEMOU, F. MÉHATS, G. VILMART. *A new class of uniformly accurate numerical schemes for highly oscillatory evolution equations*, in "Foundations of Computational Mathematics", 2018, https://hal.archives-ouvertes.fr/hal-01666472

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