Activity Report 2018

Project-Team REALOPT

Reformulations based algorithms for Combinatorial Optimization

IN COLLABORATION WITH: Institut de Mathématiques de Bordeaux (IMB), Laboratoire Bordelais de Recherche en Informatique (LaBRI)

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2. Overall Objectives

2.1. Overall Objectives

Keywords: Reformulation techniques in Mixed Integer Programming (MIP), Polyhedral approaches (cut generation), Robust Optimization, Approximation Algorithms, Extended formulations, Lagrangian Relaxation (Column Generation) based algorithms, Dantzig and Benders Decomposition, Primal Heuristics, Graph Theory, Constraint Programming.

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.

Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications along variable fixing based on reduced cost, Lagrangian decomposition methods to produce powerful relaxations, and Bender's decomposition to project the formulation, reducing the problem to the important decision variables, and to implement multi-level programming that models a hierarchy of decision levels or recourse decision in the case of data adjustment, primal heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum, with specific strong strategies for the selection on the sequence of fixings. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random (or dynamically adjusted) data by way of modeling expectation (stochastic approaches) or worst-case behavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

(*i*) Methodologies: To design tight formulations for specific combinatorial optimization problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. To develop generic methods based on such strong formulations by handling their large scale dynamically. To generalize algorithmic features that have proven efficient in enhancing performance of exact optimization approaches. To develop approximation schemes with proven optimality gap and low computational complexity. More broadly, to contribute to theoretical and methodological developments of exact and approximate approaches in combinatorial optimization, while extending the scope of applications and their scale.

(*ii*) Problem solving: To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact, approximate, and heuristic methods.

(*iii*) Software platform & Transfer: To provide prototypes of modelers and solvers based on generic software tools that build on our research developments, writing code that serves as the proof-of-concept of the genericity and efficiency of our approaches, while transferring our research findings to internal and external users.

3. Research Program

3.1. Introduction

Integer Programming Graph Theory Decomposition Approaches Polyhedral Approaches Quadratic Programming Approaches constraint programming.

Combinatorial optimization is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm* : (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a factional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, decomposition and reformulation techniques in mixed integer programing, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems, High Performance and Cloud Computing.

3.2. Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [58]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if X is a mixed integer program: $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$ where $P = \{x \in \mathbb{R}^{n+p} : Ax \le b\}$ with matrix

$(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$, then $conv(X)$ is a polyhedron that can be described in terms of linear constraints, i.e. it writes as $conv(X) = \{x \in \mathbb{R}^{n+p} : C\,x \leq d\}$ for some matrix $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$ although the dimension m' is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem $\min\{cx : x \in X\}$ and solving the *separation problem* over the associated polyhedron $conv(X)$: if $\widetilde{x} \notin conv(X)$, find a linear inequality $\pi\,x \geq \pi_0$ satisfied by all points in $conv(X)$ but violated by $\widetilde{x}$. Hence, for NP-hard problems, one can not hope to get a compact description of $conv(X)$ nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of $conv(X)$ and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities $C\,x \leq d$ can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

3.3. Decomposition-and-reformulation-approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

3.4. Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use variable fixing techniques, primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...). Such methods are likely to produce good quality solutions only if the integer programming formulation is of top quality, i.e., if its LP relaxation provides a good approximation of the IP solution.

In the same line, variable fixing techniques, that are essential in reducing the size of large scale problems, rely on good quality approximations: either tight formulations or tight relaxation solvers (as a dynamic program combined with state space relaxation). Then if the dual bound derives when the variable is fixed to one exceeds the incubent solution value, the variable can be fixed to zero and hence removed from the problem. The process can be apply sequentially by refining the degree of relaxation.

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems. Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

3.5. Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization account for the expected realization of random data and optimize an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behavior of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

3.6. Approximation Algorithms

In some contexts, obtaining an exact solution to an optimization problem is not feasible: when instances are too large, or when decisions need to be taken rapidly. Since most of the combinatorial optimization problems are NP-hard, another direction to obtain good quality solutions in reasonable time is to focus on **approximation algorithms**. The definition of approximation algorithms is based on the notion of input set $\mathcal{I}$ and each $I \in \mathcal{I}$ defines a solution space $\mathcal{S}_I$. For a minimization problem $\min_{x \in \mathcal{S}_I} f(x)$, an algorithm $\mathcal{A}$ is an α-approximation algorithm if it provides a solution within α of the optimal solution for all instances in the input set:

$$\forall I \in \mathcal{I}, \quad f(\mathcal{A}(I)) \leq \alpha \min_{x \in \mathcal{S}_I} f(x) = f^*(I)$$

The objective is to search for polynomial algorithms, with approximation ratios as close to 1 as possible. Such algorithms are called *worst-case* approximation algorithms, because the performance guarantee is expressed over all possible inputs of the problem. The design of these algorithms have strong links with the enumeration techniques described above: since computing $f^*(I)$ is an NP-hard problem, it is often required to derive **strong a priori bounds** on the optimal solution value which can afterward be compared to estimations of the value of the solution produced. In many cases, it is also possible to build α-approximate solutions by a careful rounding of a solution obtained from the linear relaxation of an integer formulation of the problem. Members of the team have expertise in designing and evaluating approximation algorithms for resource allocation in computer systems, using a variety of techniques, such as dual approximation (where a guess of the optimal value f^* is provided, and $\mathcal{A}$ either provides a solution within αf^*, or guarantees that no solution of value f^* or less exists), or resource augmentation (where an approximation is obtained by relaxing some of the constraints of the problem).

3.7. Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

4. Application Domains

4.1. Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form "at least two paths link each pair of terminals". We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [46]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today's optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronics system installation cost. We developed and compared several decomposition approaches [73], [71], [70] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route [69]. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [51], [52].

The Dial-a-Ride Problem is a variant of the pickup and delivery problem with time windows, where the user inconvenience must be taken into account. In [62], ride time and customer waiting time are modeled through both constraints and an associated penalty in the objective function. We develop a column generation approach, dynamically generating feasible vehicle routes. Handling ride time constraints explicitly in the pricing problem solver requires specific developments. Our dynamic programming approach for pricing problem makes use of a heuristic dominance rule and a heuristic enumeration procedure, which in turns implies that our overall branch-and-price procedure is a heuristic. However, in practice our heuristic solutions are experimentally very close to exact solutions and our approach is numerically competitive in terms of computation times.

In [60], [59], we consider the problem of covering an urban area with sectors under additional constraints. We adapt the aggregation method to our column generation algorithm and focus on the problem of disaggregating the dual solution returned by the aggregated master problem.

We studied several time dependent formulations for the unit demand vehicle routing problem [36], [35]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We also worked on the p-median problem, applying the matching theory to develop an efficient algorithm in Y-free graphs and to provide a simple polyhedral characterization of the problem and therefore a simple linear formulation [68] simplifying results from Baiou and Barahona.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, "less than truckload" trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithmes based on the column generation approach. These algorithmes have been tested on a set of real-life instances coming from a real Russian freight transportation company. Our algorithms have been faster on these instances than the current solution approach being used by the company.

4.2. Packing and Covering Problems

Realopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [67]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developed a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [24] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem [54]: we faced a multiple-class integer knapsack problem with setups [53] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory [48], [47], [50], [49]. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2D knapsack problem that uses our 2D packing feasibility check. We are currently developing exact optimization tools for glass-cutting problems in a collaboration with Saint-Gobain [29]. This 2D-3stage-Guillotine cut problems are very hard to solve given the scale of the instance we have to deal with. Moreover one has to issue cutting patterns that avoid the defaults that

are present in the glass sheet that are used as raw material. There are extra sequencing constraints regarding the production that make the problem even more complex.

We have also organized a European challenge on packing with society Renault: see https://paginas.fe.up.pt/~esicup/extern/esicup-12thMeeting/pmwiki.php?n=Conference.Info. This challenge is about loading trucks under practical constraints.

4.3. Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a branch-and-price algorithm where periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem, and the global planning of customer visits is coordinated by the master program [55]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [57].

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [61] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximize the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [45] and exact algorithms [26]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [25]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [63]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [65]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [66], [64].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [23], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [22] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [30] provide demonstrably stronger formulations for some problem classes than any previously proposed. We are now working on planning phytosanitary treatments in vineries.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [41], [40], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [31]. We considered train timetabling problems and their re-optimization after a perturbation in the network [43], [42]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF [37], [38], [39]. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem [44]. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge addressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem [32].

4.4. Resource Allocation for High Performance and Cloud Computing

In the context of numerical simulations on high performance machines, optimizing data locality and resource usage is very important for faster execution times and lower energy consumption. This optimization can be seen as a special case of scheduling problem on parallel resource, with several challenges. First, instances are typically large: a large matrix factorization (with 50×50 blocks) involves about $30 \cdot 10^3$ tasks. Then, HPC platforms consist of heterogeneous and unrelated resources, what is known to make scheduling problems hard to approximate. Finally, due to co-scheduling effects and shared communication resources, it is not realistic to accurately model the exact duration of tasks. All these observations make it impossible to rely on static optimal solutions, and HPC applications have gone from simple generic static allocations to runtime dynamic scheduling strategies that make their decisions based on the current state of the platform (the location of input data), the expected transfer and running times for the tasks, and some affinity and priority information that have possibly been computed offline. In this context, we are strongly involved in the design of scheduling strategies for the StarPU runtime, with two goals: proving that it is possible to design approximation algorithms whose complexity is extremely small (typically sub-linear in the number of ready tasks), and show that they can be used in practice with good performance results. We are pursuing collaborations both with teams developing the StarPU system (Storm) by designing algorithms for the generic scheduling problems [28], and with teams developing linear algebra algorithms over the runtime (Hiepacs), by proposing specialized algorithms for specific cases. For example, in the case of linear algebra applications on heterogeneous platforms, we have considered the combinatorial optimization problem associated to matrix multiplication, that is amenable to partitioning the unit square into zones of prescribed areas while minimizing the overall size of the boundaries. We have improved the best known approximation ratio to 1.15 in [27] and we have shown that the resulting distribution schemes can indeed be used to design efficient implementations using StarPU in [34].

5. Highlights of the Year

5.1. Highlights of the Year

François Vanderbeck was chair of the organizing committee of ISMP'2018. ISMP is the triennial international congress of mathematical optimization, where scientists from all over the world as well as industrial practitioners of mathematical optimization meet in order to present their most recent developments and results and to discuss new challenges from theory and practice. It is the symposium of the Mathematical Optimization Society (MOS). More than 1900 scientists attended the conference this year in Bordeaux.

Olivier Beaumont was the program chair of the IEEE-ACM HiPC conference held in Bangalore in December 2018.

The team decided to develop an open-source platform, called coluna, to allow the scientific committee to use our state-of-the-art algorithms for extended formulations.

A first spinoff company is being created by RealOpt members.

6. New Software and Platforms

6.1. BaPCod

A generic Branch-And-Price-And-Cut Code

KEYWORDS: Column Generation - Branch-and-Price - Branch-and-Cut - Mixed Integer Programming - Mathematical Optimization - Benders Decomposition - Dantzig-Wolfe Decomposition - Extended Formulation

FUNCTIONAL DESCRIPTION: BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithms, Benders approaches, network flow and dynamic programming algorithms. These methods can be combined in several hybrid algorithms to produce exact or approximate solutions (primal solutions with a bound on the deviation to the optimum).

- Participants: Artur Alves Pessoa, Boris Detienne, Eduardo Uchoa Barboza, Franck Labat, François Clautiaux, Francois Vanderbeck, Halil Sen, Issam Tahiri, Michael Poss, Pierre Pesneau, Romain Leguay and Ruslan Sadykov
- Partners: Université de Bordeaux - CNRS - IPB - Universidade Federal Fluminense
- Contact: Francois Vanderbeck
- URL: https://wiki.bordeaux.inria.fr/realopt/pmwiki.php/Project/BaPCod

6.2. WineryPlanning

- Participants: Agnes Le Roux, Alexis Toullat, Francois Vanderbeck, Issam Tahiri and Ruslan Sadykov
- Contact: Francois Vanderbeck

6.3. ORTOJ

Operation Research Tools Under Julia

KEYWORDS: Modeling - Processing - Dashboard

FUNCTIONAL DESCRIPTION: This set of tools currently includes : 1) BlockJuMP.jl: extension of JuMP to model decomposable mathematical programs (using either Benders or Dantzig-Wolfe decomposition paradign) 2) Scanner.jl: a default data parser to ease the reading of the input data in the form that they are often encountered in operational research. 3) BenchmarkUtils.jl: Tools to ease the setup of numerical experiments to benchmark algorithmic feature performances. The test automation permits to quickly calibrate the parameters of an arbitrary algorithm control function.

- Participants: Francois Vanderbeck, Guillaume Marques, Issam Tahiri and Ruslan Sadykov
- Contact: Issam Tahiri

6.4. pmtool

KEYWORDS: Scheduling - Task scheduling - StarPU - Heterogeneity - GPGPU - Performance analysis

FUNCTIONAL DESCRIPTION: Analyse post-mortem the behavior of StarPU applications. Provide lower bounds on makespan. Study the performance of different schedulers in a simple context. Provide implementations of many scheduling algorithms from the literature

NEWS OF THE YEAR: Included many new algorithms, in particular online algorithms Better integration with StarPU by accepting .rec files as input

- Participant: Lionel Eyraud-Dubois
- Contact: Lionel Eyraud-Dubois
- Publications: Approximation Proofs of a Fast and Efficient List Scheduling Algorithm for Task-Based Runtime Systems on Multicores and GPUs - Fast Approximation Algorithms for Task-Based Runtime Systems
- URL: https://gitlab.inria.fr/eyrauddu/pmtool

6.5. Platforms

6.5.1. Bapcod

We have developed a stabilized Benders' decomposition in our generic library BapCod. This allowed to produce state-of-the-art results for an energy production planning.

7. New Results

7.1. Improving Branch-and-Price Methods

We have made progress on stabilization techniques and math-heuristics that are essential components for generic Branch-and-Price methods.

The convergence of a column generation algorithm can be improved in practice by using stabilization techniques. Smoothing and proximal methods based on penalizing the deviation from the incumbent dual solution have become standards of the domain. Interpreting column generation as cutting plane strategies in the dual problem, we have analyzed [6] the mechanisms on which stabilization relies. In particular, the link is established between smoothing and in-out separation strategies to derive generic convergence properties. For penalty function methods as well as for smoothing, we describe proposals for parameter self-adjusting schemes. Such schemes make initial parameter tuning less of an issue as corrections are made dynamically. Such adjustments also allow to adapt the parameters to the phase of the algorithm. Extensive test reports validate our self-adjusting parameter scheme and highlight their performances. Our results also show that using smoothing in combination with penalty function yields a cumulative effect on convergence speed-ups.

Math heuristics have become an essential component in mixed integer programming (MIP) solvers. Extending MIP based heuristics, we have studied [8] generic procedures to build primal solutions in the context of a branch-and-price approach. As the Dantzig-Wolfe reformulation of a problem is typically tighter than that of the original compact formulation, heuristics based on rounding its linear programing (LP) solution can be more competitive. We focus on the so-called diving methods that used re-optimization after each LP rounding. We explore combination with diversification- intensification paradigms such as Limited Discrepancy Search, sub-MIPing, relaxation induced neighbourhood search, local branching, and strong branching. The dynamic generation of variables inherent to a column generation approach requires specific adaptation of heuristic paradigms. We manage to use simple strategies to get around these technical issues. Our numerical results on generalized assignment, cutting stock, and vertex coloring problems sets new benchmarks, highlighting the performance of diving heuristics as generic procedures in a column generation context and producing better solutions than state-of-the-art specialized heuristics in some cases.

7.2. Routing Problems

In [7] we deal with the Minimum Latency Problem (MLP), another variant of the well-known Traveling Salesman Problem in which the objective is to minimize the sum of waiting times of customers. This problem arises in many applications where customer satisfaction is more important than the total time spent by the server. This paper presents a novel branch-and-price algorithm for MLP that strongly relies on new features for the *ng*-path relaxation, namely: (1) a new labeling algorithm with an enhanced dominance rule named multiple partial label dominance; (2) a generalized definition of *ng*-sets in terms of arcs, instead of nodes; and (3) a strategy for decreasing *ng*-set sizes when those sets are being dynamically chosen. Also, other elements of efficient exact algorithms for vehicle routing problems are incorporated into our method, such as reduced cost fixing, dual stabilization, route enumeration and strong branching. Computational experiments over TSPLIB instances are reported, showing that several instances not solved by the current state-of-the-art method can now be solved.

We also considered a family of Vehicle Routing Problem (VRP) variants that generalize the classical Capacitated VRP by taking into account the possibility that vehicles differ by capacity, costs, depot allocation, or even by the subset of customers that they can visit. In [5] we propose a branch-cut-and-price algorithm that adapts advanced features found in the best performing exact algorithms for homogeneous fleet VRPs. The original contributions include: (i) the use of Extended Capacity Cuts, defined over a pseudo-polynomially large extended formulation, together with Rank-1 Cuts, defined over the Set Partitioning Formulation; (ii) the concept of vehicle-type dependent memory for Rank-1 Cuts; and (iii) a new family of lifted Extended Capacity Cuts that takes advantage of the vehicle-type dependent route enumeration. The algorithm was extensively tested in instances of the literature and was shown to be significantly better than previous exact algorithms, finding optimal solutions for many instances with up to 200 customers and also for some larger instances. Several new best solutions were found too.

We examined the robust counterpart of the classical Capacitated Vehicle Routing Problem (CVRP) in [13], [20]. We considered two types of uncertainty sets for the customer demands: the classical budget polytope introduced by Bertsimas and Sim (2003), and a partitioned budget polytope proposed by Gounaris et al. (2013). We showed that using the set-partitioning formulation it is possible to reformulate our problem as a deterministic heterogeneous vehicle routing problem. Thus, many state-of-the-art techniques for exactly solving deterministic VRPs can be applied for the robust counterpart, and a modern branch-and-cut-and-price algorithm can be adapted to our setting by keeping the number of pricing subproblems strictly polynomial. More importantly, we introduced new techniques to significantly improve the efficiency of the algorithm. We present analytical conditions under which a pricing subproblem is infeasible. This result is general and can be applied to other combinatorial optimization problems with knapsack uncertainty. We also introduced robust capacity cuts which are provably stronger than the ones known in the literature. Finally, a fast iterated local search algorithm was proposed to obtain heuristic solutions for the problem. Using our branch-and-cut-and-price algorithm incorporating existing and new techniques, we were able to solve to optimality all but one open instances from the literature.

In [14], we have generalized our Branch-Cut-and-Price algorithm to solve other Vehicle Routing and related combinatorial optimization problems, as Generalized Assignment, Bin Packing, and Vector Packing. Our generic approach outperformed several problem specific algorithms.

7.3. Scheduling and Clustering Problems

In [19] we consider the unrelated parallel machine scheduling problem with setup times to minimize a general objective function. In this work we present a novel exact algorithm that is capable of solving this problem $R|r_j, s_{ij}^k| \sum f_j(C_j)$ and the large class of problems that can be derived as particular cases from it. The proposed algorithm consists of a branch-cut-and-price approach that combines several features such as non-robust cuts, strong branching, reduced cost fixing and dual stabilization. To our knowledge, this is the first exact algorithm for unrelated machines with earliness and/or tardiness criteria that can solve consistently instances with more than 20 jobs. We report improved bounds for instances of problems $R|r_j, s_{ij}^k| \sum w_j' E_j + w_j T_j$ and $R|| \sum w_j' E_j + w_j T_j$ with up to 80 and 120 jobs, respectively.

A cross-docking terminal is a transshipment facility in supply chains, where products transported by inbound trucks are unloaded at inbound doors, sorted, and reloaded on outbound trucks at outbound doors. In [16], we address the truck-to-door scheduling problem at a multi-door cross-docking terminal where temporary storage is considered. We propose two types of time-indexed formulation for the problem to assign trucks to dock doors and determine their arrival and departure times so that tadiness and earliness as well as unsatisfied demand are minimized. We examine the effectiveness of the proposed formulations by numerical experiment.

7.4. Scheduling and placement for HPC

In High Performance Computing, heterogeneity is now the norm with specialized accelerators like GPUs providing efficient computational power. Resulting complexity led to the development of task-based runtime systems, where complex computations are described as task graphs, and scheduling decisions are made at run-time to perform load balancing between all resources of the platforms. In [2], we consider the problem of developing good scheduling strategies, even at the scale of a single node, and analyzing them both theoretically and in practice is expected to have a very high impact on the performance of current HPC systems. The special case of two kinds of resources, typically CPUs and GPUs is already of great practical interest. The scheduling policy Hetero-Prio has been proposed in the context of fast multipole computations (FMM), and has been extended to general task graphs with very promising results. In this paper, we provide a theoretical study of the performance of HeteroPrio, by proving approximation bounds compared to the optimal schedule, both in the case of independent tasks and in the case of general task graphs. Interestingly, our results establish that spoliation (a technique that enables resources to restart uncompleted tasks on another resource) is enough to prove bounded approximation ratios for a list scheduling algorithm on two unrelated resources, which is known to be impossible otherwise. This result holds true both for independent and dependent tasks graphs. Additionally, we provide an experimental evaluation of HeteroPrio on real task graphs from dense linear algebra computation, that establishes its strong performance in practice.

In [1], we consider the problem of partitioning a matrix into a set of sub-matrices, that has received increased attention recently and is crucial when considering dense linear algebra and kernels with similar communication patterns on heterogeneous platforms. The problem of load balancing and minimizing communication is traditionally reducible to an optimization problem that involves partitioning a square into rectangles. This problem has been proven to be NP-Complete for an arbitrary number of partitions. In this paper, we present recent approaches that relax the restriction that all partitions be rectangles. The first approach uses an original mathematical technique to find the exact optimal partitioning. Due to the complexity of the technique, it has been developed for a small number of partitions only. However, even at a small scale, the optimal partitions found by this approach are often non-rectangular and sometimes non-intuitive. The second approach is the study of approximate partitioning methods by recursive partitioning algorithms. In particular we use the work on optimal partitioning to improve preexisting algorithms. In this paper we discuss the different perspectives it opens and present two algorithms, SNRPP which is a $\sqrt{3/2}$ approximation, and NRPP which is a $2/\sqrt{3}$ approximation. While sub-optimal, this approach works for an arbitrary number of partitions. We use the first exact approach to analyze how close to the known optimal solutions the NRRP algorithm is for small numbers of partitions.

In [12], we consider the problem of data allocation when performing matrix multiplication on a heterogeneous node, with multicores and GPUs. Classical (cyclic) allocations designed for homogeneous settings are not appropriate, but the advent of task-based runtime systems makes it possible to use more general allocations. Previous theoretical work has proposed square and cube partitioning algorithms aimed at minimizing data movement for matrix multiplication. We propose techniques to adapt these continuous square partitionings to allocating discrete tiles of a matrix, and strategies to adapt the static allocation at run-time. We use these techniques in an implementation of Matrix Multiplication based on the StarPU runtime system, and we show through extensive experiments that this implementation allows to consistently obtain a lower communication volume while improving slightly the execution time, compared to standard state-of-the-art dynamic strategies.

7.5. Convergence between HPC and Data Science

In [11] paper we concentrate on a crucial parameter for efficiency in Big Data and HPC applications: data locality. We focus on the scheduling of a set of independent tasks, each depending on an input file. We assume that each of these input files has been replicated several times and placed in local storage of different nodes of a cluster, similarly of what we can find on HDFS system for example. We consider two optimization problems, related to the two natural metrics: makespan optimization (under the constraint that only local tasks are allowed) and communication optimization (under the constraint of never letting a processor idle in order to optimize makespan). For both problems we investigate the performance of dynamic schedulers, in particular the basic greedy algorithm we can for example find in the default MapReduce scheduler. First we theoretically study its performance, with probabilistic models, and provide a lower bound for communication metric and asymptotic behaviour for both metrics. Second we propose simulations based on traces from a Hadoop cluster to compare the different dynamic schedulers and assess the expected behaviour obtained with the theoretical study.

In [10], we consider the use of Burst-Buffers, that are high throughput, small size intermediate storage systems typically based on SSDs or NVRAM that are designed to be used as a potential buffer between the computing nodes of a supercomputer and its main storage system consisting of hard drives. Their purpose is to absorb the bursts of I/O that many HPC applications experience (for example for saving checkpoints or data from intermediate results). In this paper, we propose a probabilistic model for evaluating the performance of Burst-Buffers. From a model of application and a data management strategy, we build a Markov chain based model of the system, that allows to quickly answer issues about dimensioning of the system: for a given set of applications, and for a given Burst-Buffer size and bandwidth, how often does the buffer overflow? We also provide extensive simulation results to validate our modeling approach.

7.6. Energy management

In [9], we consider energy management optimization problems in a future wherein an interaction with micro-grids has to be accounted for. We model this interaction through a set of contracts between the generation companies owning centralized assets and the micro-grids. We formulate a general stylized model that can, in principle, account for a variety of management questions such as unit-commitment. The resulting model, a bilevel stochastic mixed integer program will be numerically tackled through a novel preprocessing procedure. As a result the solution for the bilevel (or single leader multiple follower) problem will be neither " optimistic " nor " pessimistic ". We numerically evaluate the difference of the resulting solution with the " optimistic " solution. We also demonstrate the efficiency and potential of our methodology on a set of numerical instances.

7.7. Network Design Problems

The delivery of freight from manufacturing platforms to demand zones is often managed through one or more intermediate locations where storing, merging, transshipment and consolidation activities are performed. In [56], we design a Two-Echelon Distribution Network that helps synchronise different flows of product. Under demand uncertainty, our model integrates decisions on the locations and the size of second echelon facilities an decisions on the flows assignment between the echelons, and on delivery routes to serve the demand zones.

In [33], we study the k-edge-connected L-hop-constrained network design problem. Given a weighted graph $G = (V, E)$, a set D of pairs of nodes, two integers $L \geq 2$ and $k \geq 2$, the problem consists in finding a minimum weight subgraph of G containing at least k edge-disjoint paths of length at most L between every pair $\{s, t\} \in D$. We consider the problem in the case where $L = 2, 3$ and $|D| \geq 2$. We first discuss integer programming formulations introduced in the literature. Then, we introduce new integer programming formulations for the problem that are based on the transformation of the initial undirected graph into directed layered graphs. We present a theoretical comparison of these formulations in terms of LP-bound. Finally, these formulations are tested using CPLEX and compared in a computational study for $k = 3, 4, 5$.

In [72], we consider a multi-layer network design model arising from a real-life telecommunication application where traffic routing decisions imply the installation of expensive nodal equipment. Customer requests come in the form of bandwidth reservations for a given origin destination pair. Bandwidth demands are

expressed as multiples of nominal granularities. Each request must be single-path routed. Grooming several requests on the same wavelength and multiplexing wavelengths in the same optical stream allow a more efficient use of network capacity. However, each addition or withdrawal of a request from a wavelength requires optical to electrical conversion and the use of cross-connect equipment with expensive ports of high densities. The objective is to minimize the number of required ports of the cross-connect equipment. We deal with backbone optical networks, therefore with networks with a moderate number of nodes (14 to 20) but thousands of requests. Further difficulties arise from the symmetries in wavelength assignment and traffic loading. Traditional multi-commodity network flow approaches are not suited for this problem. Instead, four alternative models relying on Dantzig-Wolfe and/or Benders' decomposition are introduced and compared. The formulations are strengthened using symmetry breaking restrictions, variable domain reduction, zero-one discretization of integer variables, and cutting planes. The resulting dual bounds are compared to the values of primal solutions obtained through hierarchical optimization and rounding procedures. For realistic size instances, our best approaches provide solutions with optimality gap of approximately 5% on average in around two hours of computing time.

7.8. Packing and Cutting Problems

The two-dimensional knapsack problem consists in packing a set of small rectangular items into a given large rectangle while maximizing the total reward associated with selected items. In [3], we restrict our attention to packings that emanate from a k-stage guillotine-cut process. We introduce a generic model where a knapsack solution is represented by a flow in a directed acyclic hypergraph. This hypergraph model derives from a forward labeling dynamic programming recursion that enumerates all non-dominated feasible cutting patterns. To reduce the hypergraph size, we make use of further dominance rules and a filtering procedure based on Lagrangian reduced costs fixing of hyperarcs. Our hypergraph model is (incrementally) extended to account for explicit bounds on the number of copies of each item. Our exact forward labeling algorithm is numerically compared to solving the max-cost flow model in the base hyper-graph with side constraints to model production bounds. Benchmarks are reported on instances from the literature and on datasets derived from a real-world application.

Also we consider a variant of two-dimensional guillotine cutting-stock problem that arises when different bills of order (or batches) are considered consecutively. The raw material leftover of the last cutting pattern is not counted as waste as it can be reused for cutting the next batch. The objective is thus to maximize the length of the leftover. In [21] we propose a diving heuristic based on a Dantzig-Wolfe reformulation solved by column generation in which the pricing problem is solved using dynamic programming (DP). This DP generates so-called non-proper columns, *i.e.* cutting patterns that cannot participate in a feasible integer solution of the problem. We show how to adapt the standard diving heuristic to this "non-proper" case while keeping its effectiveness. We also introduce the partial enumeration technique, which is designed to reduce the number of non-proper patterns in the solution space of the dynamic program. This technique helps to strengthen the lower bounds obtained by column generation and improve the quality of solutions found by the diving heuristic. Computational results are reported and compared on classical benchmarks from the literature as well as on new instances inspired from industrial data. According to these results, proposed diving algorithms outperform constructive and evolutionary heuristics.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

We have an on-going contract with SNCF on scheduling of rolling-stock. The PhD thesis of Mohamed Benkirane is part of this contract.

Following the PhD thesis of Rodolphe Griset, our collaboration with EDF continues through a four months contract whose goal is to investigate the possibility of developing an operational prototype (called Fenix) for strategic planning of nuclear plant outages. Two scientific questions are raised. The first one concerns the new mechanisms of management of the power capacity market on the French power grid. The second one is about a new model of the stock variation during a refueling operation, which requires information of several previous production campaigns.

We also have a new contract with RTE to develop strategies inspired from stochastic gradient methods to speed-up Benders' decomposition.

9. Partnerships and Cooperations

9.1. Regional Initiatives

- **HPC scalable ecosystem**: This project is funded by Region Nouvelle Aquitaine for 4 years, starting in 2018. It is coordinated by Emmanuel Agullo (Hiepacs Inria project-team) and it gathers researchers from Inria, Inra, LaBRI but also from UPPA (University of Pau), Pprime Institute (Poitiers), CEA and Airbus. It aims at building a convergent approach between numerical simulation and HPC on the one hand, and Data Science and Big Data on the other hand. The goal is to study the feasibility of approaches based on runtime schedulers to achieve a high level of scalability.

- **SysNum Cluster** SysNum is a Cluster of Excellence of Bordeaux Idex that aims at bringing Bordeaux academic players in the digital sciences closer to each other around large-scale distributed digital systems. The cluster is organized around 4 methodological axes (Interconnected object systems; Reliability and safety; Modeling and numerical systems; Massive and heterogeneous data) and 3 application platforms around major societal issues (ecology, mobile systems, interconnected objects and data analysis).

  - François Clautiaux is leading the methodological WP on Interconnected object systems. Understanding and controlling the complexity of systems of interconnected objects is a major challenge for both industrial and everyday life applications. We think, in particular, to fields like robotics, car industry, energy distribution or smart buildings, where it is essential to tackle autonomous heterogeneous objects and to develop robust control tools to optimize their interconnections. Our research in this direction will be developed within three interconnected tasks.

  - Olivier Beaumont is leading the methodological WP on Modeling and numerical systems. Mathematical modeling and numerical simulation are by now highly important tools currently used in physics, chemistry, biology, medicine and humanities. The forthcoming development of exaflopic computers should allow to simulate with High Performance Computing new multi-scale and multi-physics phenomena, generating massive data (High Performance Data Analytics). One of the major challenges of the years to come is thus both to build new robust numerical modeling tools, working on several millions processors computers and to be able to accommodate on a single platform both HPC and BigData applications.

9.2. European Initiatives

9.2.1. FP7 & H2020 Projects

- **EXDCI-2**: EXDCI-2 is a H2020 project (type: CSA – Coordination & support action), coordinated by PRACE for a duration of 30 months, starting on March 2018.

  Through the joint action of PRACE and ETP4HPC, EXDCI-2 mobilises the European HPC stakeholders. The project participates in the support of the European HPC Ecosystem with two main goals:

- Development and advocacy of a competitive European HPC Exascale Strategy by supporting the implementation of a common European HPC strategy, open to synergistic areas including High Performance Data Analytics (HPDA) and Artificial Intelligence (AI). The expertise of the partners and stakeholders will permit to elaborate a transversal prospective vision.
- Coordination of the stakeholder community for European HPC at the Exascale through joint community structuring and synchronisation. This entails ensuring EXDCI-2 stakeholder representation in the main development of the European HPC eco-system towards Exascale, such as: The development of relationships with other ecosystems including upstream technologies as photonics and electronics, High Performance Embedded Computing (HiPEAC) and Big Data (BDVA) In the context of the upcoming European Data Infrastructure (EDI) a road mapping activity toward future converged HPC, HPDA and AI needs and new services from PRACE users communities and CoE The continuation of BDEC activities, for international participation of European stakeholders on the integration from edge computing to HPC, including Data Analytics and AI The mapping and analysis of related national and international R&I research agendas EXDCI-2 gives particular attention to creating synergies with the CoEs – for example with the CSA FocusCoE – and to building on the outcomes of FET HPC projects.

Olivier Beaumont is involved the task that will work on building a consensus based transversal vision and aims at proposing Ecosystem level concepts. Recommendations implementing the vision will be proposed. The work will benefit from the CoE connection established in the first EXDCI CSA. It is on purpose implemented on the first year of the CSA in order to allow it to be taken into account in the other WP.

9.3. International Initiatives

9.3.1. Inria International Partners

9.3.1.1. Informal International Partners

We continue close collaboration with the LOGIS laboratory (Universidade Federal Fluminense, Niteroi, Brazil) after the end of the Inria Associate Team SAMBA. In 2018, we had two two-weeks visits of Prof. Eduardo Uchoa. Prof. Artur Pessoa spent his sabbatical of 11 months with us.

Orlando Rivera Letelier is pursuing a co-tutelle thesis (with Universidad Adolfo Ibáñez, Peñalolén, Santiago, Chile)

9.4. International Research Visitors

Prof. Artur Pessoa (Universidade Federal Fluminense, Niteroi, Brazil) spent his sabbatical of 11 months with us (February–December 2018).

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organisation

10.1.1.1. General Chair, Scientific Chair

The Team has organized ISMP, the major Mathematical Programming conference (around 1900 people). ISMP is the triennial international congress of mathematical optimization, where scientists from all over the world as well as industrial practitioners of mathematical optimization meet in order to present their most recent developments and results and to discuss new challenges from theory and practice. It is the symposium of the Mathematical Optimization Society (MOS).

https://ismp2018.sciencesconf.org/

François Vanderbeck was Chair of the conference.

10.1.1.2. Member of the Organizing Committees

All members of the team have been member of ISMP Organizing Committee.

10.1.2. Scientific Events Selection

10.1.2.1. Chair of Conference Program Committees

Olivier Beaumont: Program Chair HIPC https://hipc.org

10.1.2.2. Member of the Conference Program Committees

- Lionel Eyraud-Dubois: Euro-Par'18, ICPP'18,
  - IPDPS'18 http://www.ipdps.org
  - Euro-Par'18 https://europar2018.org
  - ICPP'18 http://oaciss.uoregon.edu/icpp18/index.php
- Pierre Pesneau: ISCO 2018
- Ruslan Sadykov, Boris Detienne, Pierre Pesneau, Lionel Eyraud-Dubois, Olivier Beaumont: https://ismp2018.sciencesconf.org
- François Clautiaux: http://roadef2018.labsticc.fr/wp/
- Olivier Beaumont:
  - SuperComputing'18 https://sc18.supercomputing.org
  - IPDPS'18 (primary PC member) http://www.ipdps.org
  - IPDPSW'18 http://www.ipdps.org/ipdps2018/2018_call_for_workshops.html
  - HeteroPar'18 https://hcl.ucd.ie/heteropar2018/
  - HPML'18 https://hpml2018.github.io

10.1.2.3. Reviewer

- Lionel Eyraud-Dubois: SC

10.1.3. Journal

10.1.3.1. Reviewer - Reviewing Activities

- Lionel Eyraud-Dubois: JOSH, TPDS
- François Clautiaux: European Journal of Operational Research, International Transactions on Operations Research, Asia-Pacific Journal of Operational Research
- Pierre Pesneau : Discrete Applied Math.
- Ruslan Sadykov: Transportation Science, European Journal of Operational Research, Journal of Scheduling, Operations Research Letters, Mathematical Programming Computation
- Olivier Beaumont: TPDS, IJHPCA

10.1.4. Invited Talks

- François Clautiaux: Invited seminar at Séminaire parisien d'optimisation (December 10th 2018)
- François Clautiaux: Invited talk at MEXICO working group (November 13th, 2018)
- Olivier Beaumont: Invited talk at the 13th Scheduling for Large Scale Systems Workshop, Lawrence Berkeley National Laboratory, California http://scheduling-workshop.tk
- Ruslan Sadykov: Invited talk at the seminar of the Faculty of Economics and Business, KU Leuven, Belgium (June 6th, 2018)

10.1.5. Leadership within the Scientific Community

François Clautiaux is Secretary of ROADEF, the French OR association

10.1.6. Research Administration

- Olivier Beaumont and François Clautiaux are WorkPackage leaders of the Idex Cluster SysNum
  https://sysnum.labex.u-bordeaux.fr/en/
- Olivier Beaumont is the head of Commission Jeunes Chercheurs and Commission Délégations at Inria Bordeaux Sud-Ouest.

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence : François Clautiaux, Grands Domaines de l'Optimisation

Master : François Clautiaux, Gestion des Opérations et Planification de la Production, 58 heqTD, M2, Université de Bordeaux

Master : François Clautiaux, Combinatoire et logistique, 29 heqTD, M2, Université de Bordeaux

Master : François Clautiaux, Introduction à la programmation en variables entières, 29 heqTD, M2, Université de Bordeaux

Master : François Clautiaux, Outils logiciels pour l'optimisation, 29 heqTD, M2, Université de Bordeaux

Master: François Clautiaux, Combinatoire et routage, 15 heqTD, ENSEIRB INPB

Master : Lionel Eyraud-Dubois et Olivier Beaumont, Approximation et BigData, 29 heqTD, M2, Université de Bordeaux

Licence : Pierre Pesneau, Grands Domaines de l'Optimisation, L1, Université de Bordeaux

Licence : Pierre Pesneau, Programmation pour le calcul scientifique, 24 heqTD, L2, Université de Bordeaux

Licence : Pierre Pesneau, Optimisation, 59 heqTD, L2, Université de Bordeaux

Master : Pierre Pesneau, Algorithmique et Programmation 1, 28 heqTD, M1, Université de Bordeaux

Master : Pierre Pesneau, Algorithmique et Programmation 2, 29 heqTD, M1, Université de Bordeaux

Master : Pierre Pesneau, Optimisation dans les graphes, 15 heqTD, M1, Université de Bordeaux

Master : Pierre Pesneau, Programmation linéaire, 15 heqTD, M1, Université de Bordeaux

DUT Informatique : Pierre Pesneau, Recherche Opérationnelle, 24 heqTD, IUT de Bordeaux

Master : Ruslan Sadykov, Introduction to Constraint Programming, 29 heqTD, M2, Université de Bordeaux

10.2.2. Supervision

PhD : Thomas Bellitto, Walks, Transitions and Geometric Distances in Graphs. 27/08/2018, Arnaud Pêcher (dir) and Christine Bachoc (dir).

PhD : Rodolphe Griset, Robust planning in Electricity production, 15/11/2018, Boris Detienne (dir) and François Vanderbeck (dir).

PhD : Quentin Viaud, Mathematical Programming Methods for Complex Cutting Problems, Université de Bordeaux, 11/12/2018, François Clautiaux (dir), Ruslan Sadykov (dir), François Vanderbeck (co-dir)

PhD : Jérémy Guillot, Résolution exacte de problèmes de couverture par arborescences sous contraintes de capacité, 18/12/2018, François Clautiaux (dir) and Pierre Pesneau (dir).

PhD in progress : Alena Shilova, Scheduling for Deep Learning Frameworks from October 2018, Olivier Beaumont (dir) and Alexis Joly (dir)

PhD in progress: Tobias Castanet, Use of Replication in Distributed Games from September 2018, Olivier Beaumont (dir), Nicolas Hanusse (dir) and Corentin Travers (dir).

PhD in progress : Imen Ben Mohamed, Location routing problems, from October 2015, Walid Klibi (dir), Ruslan Sadykov (dir), François Vanderbeck (co-dir).

PhD in progress : Guillaume Marques, Planification de tournées de véhicules avec transbordement en logistique urbaine : approches basées sur les méthodes exactes de l'optimisation mathématique, from September 2017, Ruslan Sadykov (dir) and François Vanderbeck (co-dir)

PhD in progress : Gaël Guillot, Aggregation and disaggregation methods for hard combinatorial problems, from November 2017, François Clautiaux (dir) and Boris Detienne (dir).

PhD in progress : Orlando Rivera Letelier, Bin Packing Problem with Generalized Time Lags, from May 2018, François Clautiaux (dir) and Ruslan Sadykov (co-dir), a co-tutelle with Universidad Adolfo Ibáñez, Peñalolén, Santiago, Chile.

10.2.3. Juries

- François Clautiaux HDR : Mahdi Moeini (Toulouse, rapporteur) ; PhD thesis: E´milie Joannopoulous (Rennes, rapporteur), Dehia Ait-Ferhat (Grenoble, rapporteur), Stefania Pan (Paris 13, rapporteur), Mikae¨l Capelle (Toulouse, examinateur), Pierre-Antoine Morin (Toulouse, examinateur)
- Ruslan Sadykov : PhD thesis of Daniel Kowalczyk (KU Leuven, Belgium)

10.3. Popularization

Organizations

- Local events: "Journée emploi maths et interaction 2018". This day aims to bring together students, researchers and practitioners in mathematics in the Bordeaux area. https://uf-mi.u-bordeaux.fr/sites/jemi/

Interventions

- National events: Fête de la Science in Bordeaux (P. Pesneau and F. Clautiaux)
- RealOpt participated to the events related to the anniversary of Inria BSO

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