Activity Report 2018

Team TRIPOP

Modélisation, simulation et commande des systèmes dynamiques non lisses

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions).

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Creation of the Team: 2018 January 01

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       A6.4.3. - Observability and Controlability

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2. Overall Objectives

2.1. Introduction

The joint research team, TRIPOP, between Inria Grenoble Rhône–Alpes, Grenoble INP and CNRS, part of the Laboratoire Jean Kuntzmann (LJK UMR 5224) is a follow up of the BIPOP team (2003–2017). The team is mainly concerned by the modeling, the mathematical analysis, the simulation and the control of nonsmooth dynamical systems. Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, i.e., systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini unilateral contact, set-valued (Coulomb-like) friction and impacts. In Electronics, examples are found in switched electrical circuits with ideal components (diodes, switches, transistors). In Control, nonsmooth systems arise in the sliding mode control theory and in optimal control. A lot of examples can also be found in cyber-physical systems (hybrid systems), in transportation sciences, in mathematical biology or in finance.

2.2. General scope and motivations

Nonsmooth dynamics concerns the study of the time evolution of systems that are not smooth in the mathematical sense, *i.e.*, systems that are characterized by a lack of differentiability, either of the mappings in their formulations, or of their solutions with respect to time. The class of nonsmooth dynamical systems recovers a large variety of dynamical systems that arise in many applications. The term "nonsmooth", as the term "nonlinear", does not precisely define the scope of the systems we are interested in but, and most importantly, they are characterized by the mathematical and numerical properties that they share. To give more insight of what are nonsmooth dynamical systems, we give in the sequel a very brief introduction of their salient features. For more details, we refer to [1], [2] [61], [78], [94], [63], [39].

2.2.1. A *flavor of nonsmooth dynamical systems*

As a *first* illustration, let us consider a linear finite-dimensional system described by its state $x(t) \in \mathbb{R}^n$ over a time-interval $t \in [0, T]$:

$$\dot{x}(t) = Ax(t) + a, \quad A \in \mathbb{R}^{n \times n}, a \in \mathbb{R}^n, \tag{1}$$

subjected to a set of m inequality (unilateral) constraints:

$$y(t) = Cx(t) + c \geq 0, \qquad C \in I\!R^{m \times n}, c \in I\!R^m. \tag{2}$$

If the constraints are physical constraints, a standard modeling approach is to augment the dynamics in (1) by an input vector $\lambda(t) \in I\!R^m$ that plays the role of a Lagrange multiplier vector. The multiplier restricts the trajectory of the system in order to respect the constraints. Furthermore, as in the continuous optimization theory, the multiplier must be signed and must vanish if the constraint is not active. This is usually formulated as a complementarity condition:

$$0 \leq y(t) \perp \lambda(t) \geq 0, \tag{3}$$

which models the one-sided effect of the inequality constraints. The notation $y \geq 0$ holds component–wise and $y \perp \lambda$ means $y^T \lambda = 0$. All together we end up with a Linear Complementarity System (LCS) of the form,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ 0 \leq y(t) \perp \lambda(t) \geq 0 \end{cases} \tag{4}$$

where $B \in I\!R^{n \times m}$ is the matrix that models the input generated by the constraints. In a more general way, the constraints may also involve the Lagrange multiplier,

$$y(t) = Cx(t) + c + D\lambda(t) \geq 0, \qquad D \in I\!R^{m \times m}, \tag{5}$$

leading to a general definition of LCS as

$$\begin{cases} \dot{x}(t) = A\,x(t) + a + B\,\lambda(t) \\ y(t) = C\,x(t) + c + D\,\lambda(t) \\ 0 \leq y(t) \perp \lambda(t) \geq 0. \end{cases} \tag{6}$$
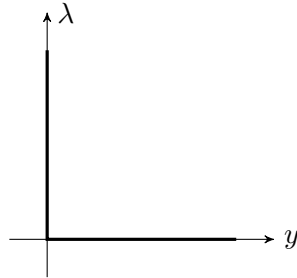

Figure 1. Complementarity condition $0 \leq y \perp \lambda \geq 0$.

The complementarity condition, illustrated in Figure 1 is the archetype of a nonsmooth graph that we extensively use in nonsmooth dynamics. The mapping $y \mapsto \lambda$ is a multi-valued (set-valued) mapping, that is nonsmooth at the origin. It has a lot of interesting mathematical properties and reformulations that come mainly from convex analysis and variational inequality theory. Let us introduce the indicator function of $I\!R_+$ as

$$\Psi_{\boldsymbol{R}_+}(x) = \begin{cases} 0 \text{ if } x \geq 0, \\ +\infty \text{ if } x < 0. \end{cases} \tag{7}$$

This function is convex, proper and can be sub-differentiated [67]. The definition of the subdifferential of a convex function $f : I\!R^m \to I\!R$ is defined as:

$$\partial f(x) = \{x^{\stackrel{\star}{\rightthreetimes}} \in I\!R^m \mid f(z) \geq f(x) + (z - x)^\top x^{\stackrel{\star}{\rightthreetimes}}, \forall z\}. \tag{8}$$

A basic result of convex analysis reads as

$$0 \leq y \perp \lambda \geq 0 \longleftrightarrow -\lambda \in \partial \Psi_{\boldsymbol{R}_+}(y) \tag{9}$$

that gives a first functional meaning to the set-valued mapping $y \mapsto \lambda$. Another interpretation of $\partial \Psi_{\boldsymbol{R}_+}$ *is based on* the normal cone to a closed and nonempty convex set C:

$$N_C(x) = \{v \in I\!R^m \mid v^\top(z - x) \leq 0 \text{ for all } z \in K\}. \tag{10}$$

It is easy to check that $\partial \Psi_{\boldsymbol{R}_+} = N_{\boldsymbol{R}_+}(x)$ and it follows that

$$0 \leq y \perp \lambda \geq 0 \longleftrightarrow -\lambda \in N_{\boldsymbol{R}_+}(y). \tag{11}$$

Finally, the definition of the normal cone yields a variational inequality:

$$0 \leq y \perp \lambda \geq 0 \longleftrightarrow \lambda^\top(y - z) \leq 0, \forall z \geq 0. \tag{12}$$

The relations (11) and (12) allow one to formulate the complementarity system with $D = 0$ as a differential inclusion based on a normal cone (see (15)) or as a differential variational inequality. By extending the definition to other types of convex functions, possibly nonsmooth, and using more general variational inequalities, the same framework applies to the nonsmooth laws depicted in Figure 2 that includes the case of piecewise smooth systems.

The mathematical concept of solutions depends strongly on the nature of the matrix quadruplet (A, B, C, D) in (6). If D is a positive definite matrix (or a P-matrix), the Linear Complementarity problem

$$0 \leq Cx + c + D\lambda \perp \lambda \geq 0, \tag{13}$$

admits a unique solution $\lambda(x)$ which is a Lipschitz continuous mapping. It follows that the Ordinary Differential Equation (ODE)

$$\dot{x}(t) = Ax(t) + a + B\lambda(x(t)), \tag{14}$$

is a standard ODE with a Lipschitz right-hand side with a C^1 solution for the initial value problem. If $D = 0$, the system can be written as a differential inclusion in a normal cone as

$$-\dot{x}(t) + Ax(t) + a \in BN_{\mathbf{R}_+}(Cx(t)), \tag{15}$$

that admits a solution that is absolutely continuous if CB is a definite positive matrix and the initial condition satisfies the constraints. The time derivative $\dot{x}(t)$ and the multiplier $\lambda(t)$ may have jumps and are generally considered as functions of bounded variations. If $CB = 0$, the order of nonsmoothness increases and the Lagrange multiplier may contain Dirac atoms and must be considered as a measure. Higher–order index, or higher relative degree systems yield solutions in terms of distributions and derivatives of distributions [33].

A lot of variants can be derived from the basic form of linear complementarity systems, by changing the form of the dynamics including nonlinear terms or by changing the complementarity relation by other multivalued maps. In particular the nonnegative orthant may be replaced by any convex closed cone $K \subset I\!\!R^m$ leading to complementarity over cones

$$K^{\star} \ni y \perp \lambda \in K, \tag{16}$$

where $K^{\star}$ its dual cone given by

$$K^{\star} = \{x \in I\!\!R^m \mid x^\top y \geq 0 \text{ for all } y \in K\}. \tag{17}$$

In Figure 2, we illustrate some other basic maps that can used for defining the relation between λ and y. The saturation map, depicted in Figure 2(a) is a single valued continuous function which is an archetype of piecewise smooth map. In Figure 2(b), the relay multi-function is illustrated. If the upper and the lower limits of λ are respectively equal to 1 and -1, we obtain the multivalued sign function defined as

$$\mathrm{Sgn}\,(y) = \begin{cases} 1, & y > 0 \\ [-1,1], & y = 0 \\ -1, & y < 0. \end{cases} \tag{18}$$

Using again convex analysis, the multivalued sign function may be formulated as an inclusion into a normal cone as

$$\lambda \in \mathrm{Sgn}\,(y) \longleftrightarrow y \in N_{[-1,1]}(\lambda). \tag{19}$$

More generally, any system of the type,

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + a \\ -\lambda(t) \in \mathrm{Sgn}(y(t)), \end{cases} \tag{20}$$

can reformulated in terms of the following set-valued system

$$\begin{cases} \dot{x}(t) = Ax(t) + a + B\lambda(t) \\ y(t) = Cx(t) + c \\ -y(t) \in N_{[-1,1]^m}(\lambda(t)). \end{cases} \tag{21}$$

The system (21) appears in a lot of applications; among them, we can cite the sliding mode control, electrical circuits with relay and Zener diodes [29], or mechanical systems with friction [31].
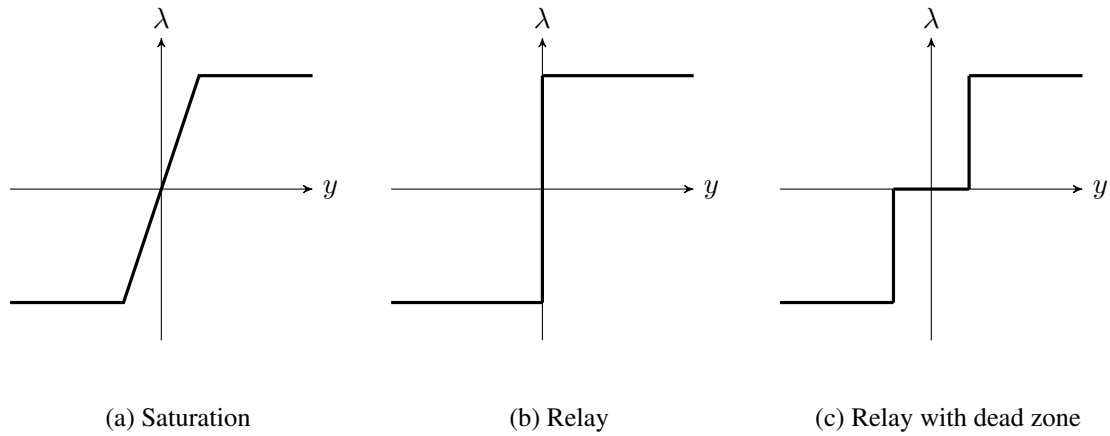
(a) Saturation   (b) Relay   (c) Relay with dead zone

Figure 2. Examples of multivalued piecewise linear models

Though this class of systems seems to be rather specific, it includes as well more general dynamical systems such as piecewise smooth systems and discontinuous ordinary differential equations. Indeed, the system (20) for scalars y and λ can be viewed as a discontinuous differential equation:

$$\dot{x}(t) = \begin{cases} Ax + a + B & \text{if} \quad Cx + c > 0 \\ Ax + a - B & \text{if} \quad Cx + c < 0. \end{cases} \tag{22}$$

One of the most well-known mathematical framework to deal with such systems is the Filippov theory [61] that embed the discontinuous differential equations into a differential inclusion. In the case of a single discontinuity surface given in our example by $S = \{x \mid Cx + c = 0\}$, the Filippov differential inclusion based on the convex hull of the vector fields in the neighborhood of S is equivalent to the use of the multivalued sign function in (20). Conversely, as it has been shown in [37], a piecewise smooth system can be formulated as a nonsmooth system *based on products of multivalued sign functions.*

2.2.2. Nonsmooth Dynamical systems in the large

Generally, the nonsmooth dynamical systems we propose to study mainly concern systems that possess the following features:

1. A nonsmooth formulation of the constitutive/behavioral laws that define the system. Examples of nonsmooth formulations are piecewise smooth functions, multi–valued functions, inequality constraints, yielding various definitions of dynamical systems such as piecewise smooth systems, discontinuous ordinary differential equations, complementarity systems, projected dynamical systems, evolution or differential variational inequalities and differential inclusions (into normal cones). Fundamental mathematical tools come from convex analysis [87], [68], [67], complementarity theory [57], and variational inequalities theory [60].

2. A concept of solutions that does not require continuously differentiable functions of time. For instance, absolutely continuous, Lipschitz continuous functions or functions of local bounded variation are the basis for solution concepts. Measures or distributions are also solutions of interest for differential inclusions or evolution variational inequalities.

2.2.3. Nonsmooth systems versus hybrid systems

The nonsmooth dynamical systems we are dealing with, have a nonempty intersection with hybrid systems and cyber-physical systems, as it is briefly discussed in Sect. 3.2.4. Like in hybrid systems, nonsmooth dynamical systems define continuous–time dynamics that can be identified to modes separated by guards, defined by the

constraints. However, the strong mathematical structure of nonsmooth dynamical systems allows us to state results on the following points:

1. *Mathematical concept of solutions: well-posedness (existence, and possibly, uniqueness properties, (dis)continuous dependence on initial conditions).*

2. *Dynamical systems theoretic properties: existence of invariants (equilibria, limit cycles, periodic solutions,...) and their stability, existence of oscillations, periodic and quasi-periodic solutions and propagation of waves.*

3. *Control theoretic properties: passivity, controllability, observability, stabilization, robustness.*

These latter properties, that are common for smooth nonlinear dynamical systems, distinguish the nonsmooth dynamical systems from the very general definition of hybrid or cyber-physical systems [42], [66]. Indeed, it is difficult to give a precise mathematical concept of solutions for hybrid systems since the general definition of hybrid automata is usually too loose.

2.2.4. *Numerical methods for nonsmooth dynamical systems*

To conclude this brief exposition of nonsmooth dynamical systems, let us recall an important fact related to numerical methods. Beyond their intrinsic mathematical interest, and the fact that they model real physical systems, using nonsmooth dynamical systems as a model is interesting, because it exists a large set of robust and efficient numerical techniques to simulate them. Without entering into deeper details, let us give two examples of these techniques:

- *Numerical time integration methods: convergence, efficiency (order of consistency, stability, symplectic properties).* For the nonsmooth dynamical systems described above, there exist event–capturing time–stepping schemes with strong mathematical results. These schemes have the ability to numerically integrate the initial value problem without performing an event location, but by capturing the event within a time step. We call an event, or a transition, every change into the index set of the active constraints in the complementarity formulation or in the normal cone inclusion. Hence these schemes are able to simulate systems with a huge number of transitions or even worth finite accumulation of events (Zeno behavior). Furthermore, the schemes are not suffering from the weaknesses of the standard schemes based on a regularization (smoothing) of the multi-valued mapping resulting in stiff ordinary differential equations. *For the time–integration of the initial value problem (IVP), or Cauchy problem, a lot of improvements of the standard time–stepping schemes for nonsmooth dynamics (Moreau–Jean time-stepping scheme) have been proposed in the last decade, in terms of accuracy and dissipation properties [26], [27], [88], [89], [28], [56], [52], [90], [54]. An important part of these schemes has been developed by members of the BIPOP team and has been implemented in the Siconos software (see Sect. 5.1).*

- *Numerical solution procedure for the time–discretized problem, mainly through well-identified problems studied in the optimization and mathematical programming community.* Another very interesting feature is the fact that the discretized problem that we have to solve at each time–step is generally a well-known problem in optimization. For instance, for LCSs, we have to solve a linear complementarity problem [57] for which there exist efficient solvers in the literature. Comparing to the brute force algorithm with exponential complexity that consists in enumerating all the possible modes, the algorithms for linear complementarity problem have polynomial complexity when the problem is monotone.

In the Axis 2 of the research program (see Sect. 3.3), we propose to perform new research on the geometric time-integration schemes of nonsmooth dynamical systems, to develop new integration schemes for Boundary Value Problem (BVP), and to work on specific methods for two time-discretized problems: the Mathematical Program with Equilibrium Constraints (MPEC) for optimal control and Second Order Cone Complementarity Problems (SOCCP) for discrete frictional contact systems.

3. Research Program

3.1. Introduction

In this section, we develop our scientific program. In the framework of nonsmooth dynamical systems, the activities of the project–team will be on focused on the following research axes:

- *Axis 1: Modeling and analysis (detailed in Sect. 3.2).*
- *Axis 2: Numerical methods and simulation (detailed in Sect. 3.3).*
- *Axis 3: Automatic Control (detailed in Sect. 3.4)*

These research axes will be developed with a strong emphasis on the software development and the industrial transfer that are detailed respectively in Sect. 5.1 and Sect. 7.1.

3.2. Axis 1: Modeling and analysis

This axis is dedicated to the modeling and the mathematical analysis of nonsmooth dynamical systems. It consists of four main directions. Two directions are in the continuation of BIPOP activities: 1) multibody vibro-impact systems (Sect. 3.2.1) and 2) excitable systems (Sect. 3.2.2). Two directions are completely new with respect to BIPOP: 3) Nonsmooth geomechanics and natural hazards assessment (Sect. 3.2.3) and 4) Cyber-physical systems (hybrid systems) (Sect. 3.2.4).

3.2.1. *Multibody vibro-impact systems*

Participants: B. Brogliato, F. Bourrier, G. James, V. Acary

- *Multiple impacts with or without friction :* there are many different approaches to model collisions, especially simultaneous impacts (so-called multiple impacts) [84]. One of our objectives is on one hand to determine the range of application of the models (for instance, when can one use "simplified" rigid contact models relying on kinematic, kinetic or energetic coefficients of restitution?) on typical benchmark examples (chains of aligned beads, rocking block systems). On the other hand, try to take advantage of the new results on nonlinear waves phenomena, to better understand multiple impacts in 2D and 3D granular systems. The study of multiple impacts with (unilateral) nonlinear visco-elastic models (Simon-Hunt-Crossley, Kuwabara-Kono), or visco-elasto-plastic models (assemblies of springs, dashpots and dry friction elements), is also a topic of interest, since these models are widely used.

- *Artificial or manufactured or ordered granular crystals, meta-materials :* Granular metamaterials (or more general nonlinear mechanical metamaterials) offer many perspectives for the passive control of waves originating from impacts or vibrations. The analysis of waves in such systems is delicate due to spatial discreteness, nonlinearity and non-smoothness of contact laws [86], [72], [73], [79]. We will use a variety of approaches, both theoretical (e.g. bifurcation theory, modulation equations) and numerical, in order to describe nonlinear waves in such systems, with special emphasis on energy localization phenomena (excitation of solitary waves, fronts, breathers).

- *Systems with clearances, modeling of friction :* joint clearances in kinematic chains deserve specific analysis, especially concerning friction modeling [40]. Indeed contacts in joints are often conformal, which involve large contact surfaces between bodies. Lubrication models should also be investigated.

- *Painlevé paradoxes :* the goal is to extend the results in [65], which deal with single-contact systems, to multi-contact systems. One central difficulty here is the understanding and the analysis of singularities that may occur in sliding regimes of motion.

As a continuation of the work in the BIPOP team, our software code, Siconos (see Sect. 5.1) will be our favorite software platform for the integration of these new modeling results.

3.2.2. *Excitable systems*

Participants: A. Tonnelier, G. James

An excitable system elicits a strong response when the applied perturbation is greater than a threshold [81], [82], [45], [91]. This property has been clearly identified in numerous natural and physical systems. In mechanical systems, non-monotonic friction law (of spinodal-type) leads to excitability. Similar behavior may be found in electrical systems such as active compounds of neuristor type. Models of excitable systems incorporate strong non-linearities that can be captured by non-smooth dynamical systems. Two properties are deeply associated with excitable systems: oscillations and propagation of nonlinear waves (autowaves in coupled excitable systems). We aim at understanding these two dynamical states in excitable systems through theoretical analysis and numerical simulations. Specifically we plan to study:

- Threshold-like models in biology: spiking neurons, gene networks.
- Frictional contact oscillators (slider block, Burridge-Knopoff model).
- Dynamics of active electrical devices : memristors, neuristors.

3.2.3. *Nonsmooth geomechanics and natural hazards assessment*

Participants: F. Bourrier, B. Brogliato, G. James, V. Acary

- *Rockfall impact modeling :* Trajectory analysis of falling rocks during rockfall events is limited by a rough modeling of the impact phase [47], [46], [77]. The goal of this work is to better understand the link between local impact laws at contact with refined geometries and the efficient impact laws written for a point mass with a full reset map. A continuum of models in terms of accuracy and complexity will be also developed for the trajectory studies. In particular, nonsmooth models of rolling friction, or rolling resistance will be developed and formulated using optimization problems.

- *Experimental validation :* The participation of IRSTEA with F. Bourrier makes possible the experimental validation of models and simulations through comparisons with real data. IRSTEA has a large experience of lab and in-situ experiments for rockfall trajectories modeling [47], [46]. It is a unique opportunity to vstrengthen our model and to prove that nonsmooth modeling of impacts is reliable for such experiments and forecast of natural hazards.

- *Rock fracturing :* When a rock falls from a steep cliff, it stores a large amount of kinetic energy that is partly dissipated though the impact with the ground. If the ground is composed of rocks and the kinetic energy is sufficiently high, the probability of the fracture of the rock is high and yields an extra amount of dissipated energy but also an increase of the number of blocks that fall. In this item, we want to use the capability of the nonsmooth dynamical framework for modeling cohesion and fracture [74], [38] to propose new impact models.

- *Rock/forest interaction :* To prevent damages and incidents to infrastructures, a smart use of the forest is one of the ways to control trajectories (decrease of the run-out distance, jump heights and the energy) of the rocks that fall under gravity [58], [59]. From the modeling point of view and to be able to improve the protective function of the forest, an accurate modeling of impacts between rocks and trees is required. Due to the aspect ratio of the trees, they must be considered as flexible bodies that may be damaged by the impact. This new aspect offers interesting modeling research perspectives.

More generally, our collaboration with IRSTEA opens new long term perspectives on granular flows applications such as debris and mud flows, granular avalanches and the design of structural protections. *The numerical methods that go with these new modeling approaches will be implemented in our software code, Siconos (see Sect. 5.1)*

3.2.4. *Cyber-physical systems (hybrid systems)*

Participants: V. Acary, B. Brogliato, C. Prieur, A. Tonnelier

Nonsmooth systems have a non-empty intersection with hybrid systems and cyber–physical systems. However, nonsmooth systems enjoy strong mathematical properties (concept of solutions, existence and uniqueness) and efficient numerical tools. This is often the result of the fact that nonsmooth dynamical systems are models of physical systems, and then, take advantage of their intrinsic property (conservation or dissipation of energy, passivity, stability). A standard example is a circuit with n ideal diodes. From the hybrid point of view, this circuit is a piecewise smooth dynamical system with 2^n modes, that can be quite cumbersome to enumerate in order to determinate the current mode. As a nonsmooth system, this circuit can be formulated as a complementarity system for which there exist efficient time–stepping schemes and polynomial time algorithms for the computation of the current mode. The key idea of this research action is to take benefit of this observation to improve the hybrid system modeling tools.

Research actions: There are two main actions in this research direction that will be implemented in the framework of the Inria Project Lab (IPL " Modeliscale", see https://team.inria.fr/modeliscale/ for partners and details of the research program):

●*Structural analysis of multimode DAE :* When a hybrid system is described by a Differential Algebraic Equation (DAE) with different differential indices in each continuous mode, the structural analysis has to be completely rethought. In particular, the re-initialization rule, when a switching occurs from a mode to another one, has to be consistently designed. We propose in this action to use our knowledge in complementarity and (distribution) differential inclusions [33] to design consistent re-initialization rule for systems with nonuniform relative degree vector $(r_1, r_2, ..., r_m)$ and $r_i \neq r_j, i \neq j$.

●*Cyber–physical in hybrid systems modeling languages :* Nowadays, some hybrid modeling languages and tools are widely used to describe and to simulate hybrid systems (MODELICA, SIMULINK, and see [55] for references therein). Nevertheless, the compilers and the simulation engines behind these languages and tools suffer from several serious weaknesses (failure, weird output or huge sensitivity to simulation parameters), especially when some components, that are standard in nonsmooth dynamics, are introduced (piecewise smooth characteristic, unilateral constraints and complementarity condition, relay characteristic, saturation, dead zone, ...). One of the main reasons is the fact that most of the compilers reduce the hybrid system to a set of smooth modes modeled by differential algebraic equations and some guards and reinitialization rules between these modes. Sliding mode and Zeno–behaviour are really harsh for hybrid systems and relatively simple for nonsmooth systems. With B. Caillaud (Inria HYCOMES) and M. Pouzet (Inria PARKAS), we propose to improve this situation by implementing a module able to identify/describe nonsmooth elements and to efficiently handle them with SICONOS as the simulation engine. They have already carried out a first implementation [53] in Zelus, a synchronous language for hybrid systems http://zelus.di.ens.fr. Removing the weaknesses related to the nonsmoothness of solutions should improve hybrid systems towards robustness and certification.

●*A general solver for piecewise smooth systems* This direction is the continuation of the promising result on modeling and the simulation of piecewise smooth systems [37]. As for general hybrid automata, the notion or concept of solutions is not rigorously defined from the mathematical point of view. For piecewise smooth systems, multiplicity of solutions can happen and sliding solutions are common. The objective is to recast general piecewise smooth systems in the framework of differential inclusions with Aizerman–Pyatnitskii extension [37], [61]. This operation provides a precise meaning to the concept of solutions. Starting from this point, the goal is to design and study an efficient numerical solver (time–integration scheme and optimization solver) based on an equivalent formulation as mixed complementarity systems of differential variational inequalities. We are currently discussing the issues in the mathematical analysis. The goal is to prove the convergence of the time–stepping scheme to get an existence theorem. With this work, we should also be able to discuss the general Lyapunov stability of stationary points of piecewise smooth systems.

3.3. Axis 2: Numerical methods and simulation

This axis is dedicated to the numerical methods and simulation for nonsmooth dynamical systems. As we mentioned in the introduction, the standard numerical methods have been largely improved in terms of accuracy and dissipation properties in the last decade. Nevertheless, the question of the geometric

time–integration techniques remains largely open. It constitutes the objective of the first research direction in Sect. 3.3.1. Beside the standard IVP, the question of normal mode analysis for nonsmooth systems is also a research topic that emerged in the recent years. More generally, the goal of the second research direction (Sect. 3.3.2) is to develop numerical methods to solve boundary value problems in the nonsmooth framework. This will serve as a basis for the computation of the stability and numerical continuation of invariants. Finally, once the time-integration method is chosen, it remains to solve the one-step nonsmooth problem, which is, most of time, a numerical optimization problem. In Sect. 3.3.3, we propose to study two specific problems with a lot of applications: the Mathematical Program with Equilibrium Constraints (MPEC) for optimal control, and Second Order Cone Complementarity Problems (SOCCP) for discrete frictional contact systems. After some possible prototypes in scripting languages (Python and Matlab), we will be attentive that all these developments of numerical methods will be integrated in Siconos.

3.3.1. *Geometric time–integration schemes for nonsmooth Initial Value Problem (IVP)*

Participants: V. Acary, B. Brogliato, G. James, F. Pérignon

The objective of this research item is to continue to improve classical time–stepping schemes for nonsmooth systems to ensure some qualitative properties in discrete-time. In particular, the following points will be developed

- Conservative and dissipative systems. The question of the energy conservation and the preservation of dissipativity properties in the Willems sense [64] will be pursued and extended to new kinds of systems (nonlinear mechanical systems with nonlinear potential energy, systems with limited differentiability (rigid impacts vs. compliant models)).

- Lie–group integration schemes for finite rotations for the multi-body systems extending recent progresses in that directions for smooth systems [43].

- Conservation and preservation of the dispersion properties of the (non)-dispersive system.

3.3.2. *Stability and numerical continuation of invariants*

Participants: G. James, V. Acary, A. Tonnelier, F. Pérignon,

By invariants, we mean equilibria, periodic solutions, limit cycles or waves. Our preliminary work on this subject raised the following research perspectives:

- Computation of periodic solutions of discrete mechanical systems . The modal analysis, *i.e.*, a spectral decomposition of the problem into linear normal modes is one of the basic tools for mechanical engineers to study dynamic response and resonance phenomena of an elastic structure. Since several years, the concept of nonlinear normal modes [75], that is closely related to the computation of quasi-periodic solutions that live in a nonlinear manifold, has emerged as the nonlinear extension of the modal analysis. One of the fundamental question is: what remains valid if we add unilateral contact conditions ? The computation of nonsmooth modes amounts to computing periodic solutions, performing the parametric continuation of solution branches and studying the stability of these branches. This calls for time integration schemes for IVP an BVP that satisfy some geometric criteria: conservation of energy, reduced numerical dispersion, symplecticity as we described before. Though the question of conservation of energy for unilateral contact has been discussed in [28], the other questions remain open. For the shooting technique and the study of stability, we need to compute the Jacobian matrix of the flow with respect to initial conditions, the so-called saltation matrix [76], [85] for nonsmooth flows. The eigenvalues of this matrix are the Floquet multipliers that give some information on the stability of the periodic solutions. The question of an efficient computation of this matrix is also an open question. For the continuation, the question is also largely open since the continuity of the solutions with respect to the parameters is not ensured.

- Extension to elastic continuum media . This is a difficult task. First of all, the question of the mathematical model for the dynamic continuum problem with unilateral contact raises some problems of well–posedness. For instance, the need for an impact law is not clear in some cases. If we perform

a semi–discretization in space with classical techniques (Finite Element Methods, Finite Difference Schemes), we obtain a discrete system for which the impact law is needed. Besides all the difficulties that we enumerate for discrete systems in the previous paragraph, the space discretization also induces numerical dispersion that may destroy the periodic solutions or renders their computation difficult. The main targeted applications for this research are cable–systems, string musical instruments, and seismic response of electrical circuit breakers with Schneider Electric.

- Computation of solutions of nonsmooth time Boundary Value Problems (BVP) (collocation, shooting) . The technique developed in the two previous items can serve as a basis for the development of more general solvers for nonsmooth BVP that can be for instance found when we solve optimal control problems by direct or indirect methods, or the computation of nonlinear waves. Two directions can be envisaged:

    – Shooting and multiple shooting techniques. In such methods, we reformulate the BVP into a sequence of IVPs that are iterated through a Newton based technique. This implies the computation of Jacobians for nonsmooth flows, the question of the continuity w.r.t to initial condition and the use of semi-smooth Newton methods.

    – Finite differences and collocations techniques. In such methods, the discretization will result into a large sparse optimization problems to solve. The open questions are as follows: a) the study of convergence, b) how to locally improve the order if the solution is locally smooth, and c) how to take benefit of spectral methods.

- Continuation techniques of solutions with respect to a parameter. Standard continuation technique requires smoothness. What types of methods can be extended in the nonsmooth case (arc-length technique, nonsmooth (semi-smooth) Newton, Asymptotical Numerical Methods (ANM))

3.3.3. Numerical optimization for discrete nonsmooth problems

Participants: V. Acary, M. Brémond, F. Pérignon, B. Brogliato, C. Prieur.

- Mathematical Program with Equilibrium Constraints (MPEC) for optimal control . The discrete problem that arises in nonsmooth optimal control is generally a MPEC [92]. This problem is intrinsically nonconvex and potentially nonsmooth. Its study from a theoretical point of view has started 10 years ago but there is no consensus for its numerical solving. The goal is to work with world experts of this problem (in particular M. Ferris from Wisconsin University) to develop dedicated algorithms for solving MPEC, and provide to the optimization community challenging problems.

- Second Order Cone Complementarity Problems (SOCCP) for discrete frictional systems : After some extensive comparisons of existing solvers on a large collection of examples [36], [30], the numerical treatment of constraints redundancy by the proximal point technique and the augmented Lagrangian formulation seems to be a promising path for designing new methods. From the comparison results, it appears that the redundancy of constraints prevents the use of second order methods such as semi–smooth Newton methods or interior point methods. With P. Armand (XLIM, U. de Limoges), we propose to adapt recent advances for regularizing constraints for the quadratic problem [62] for the second-order cone complementarity problem. The other question is the improvement of the efficiency of the algorithms by using accelerated schemes for the proximal gradient method that come from large–scale machine learning and image processing problems. Learning from the experience in large–scale machine learning and image processing problems, the accelerated version of the classical gradient algorithm [83] and the proximal point algorithm [44], and many of their further extensions, could be of interest for solving discrete frictional contact problems. Following the visit of Y. Kanno (University of Tokyo) and his preliminary experience on frictionless problems, we will extend its use to frictional contact problem. When we face large-scale problems, the main available solvers is based on a Gauss–Seidel strategy that is intrinsically sequential. Accelerated first-order methods could be a good alternative to take benefit of the distributed scientific computing architectures.

3.4. Axis 3: Automatic Control

Participants: B. Brogliato, C. Prieur, V. Acary

This last axis is dedicated to the automatic control of nonsmooth dynamical systems, or the nonsmooth control of smooth systems. The first item concerns the discrete-time sliding mode control for which significant results on the implicit implementation have been obtained in the BIPOP team. The idea is to pursue this research towards state observers and differentiators (Sect 3.4.1). The second direction concerns the optimal control which brings of nonsmoothness in their solution and their formulation. After the preliminary work in BIPOP on the quadratic optimal control of Linear Complementarity systems(LCS), we propose to go further to the minimal time problem, to impacting systems and optimal control with state constraints (Sect. 3.4.2). In Sect 3.4.3, the objective is to study the control of nonsmooth systems that contain unilateral constraint, impact and friction. The targeted systems are cable–driven systems, multi-body systems with clearances and granular materials. In Sect 3.4.4, we will continue our work on the higher order Moreau sweeping process. Up to now, the work of BIPOP was restricted to finite-dimensional systems. In Sect 3.4.5, we propose to extend our approach to the control of elastic structures subjected to contact unilateral constraints.

It is noteworthy that most of the problems listed below, will make strong use of the numerical tools analyzed in Axis 2, and of the Modeling analysis of Axis 1. For instance all optimal control problems yield BVPs. Control of granular materials will undoubtedly use models and numerical simulation developed in Axis 1 and 2. And so on. It has to be stressed that the type of nonsmooth models we are working with, deserve specific numerical algorithms which cannot be found in commercial software packages. One of the goals is to continue to extend our software package Siconos, and in particular the siconos/control toolbox with these developments.

3.4.1. Discrete-time Sliding-Mode Control (SMC) and State Observers (SMSO)

- *SMSO, exact differentiators:* we have introduced and obtained significant results on the implicit discretization of various classes of sliding-mode controllers [32], [34], [69], [80], [49], with successful experimental validations [70], [69], [71], [93]. Our objective is to prove that the implicit discretization can also bring advantages for sliding-mode state observers and Levant's exact differentiators, compared with the usual explicit digital implementation that generates chattering. In particular the implicit discretization guarantees Lyapunov stability and finite-time convergence properties which are absent in explicit methods.

- *High-Order SMC (HOSMC):* this family of controllers has become quite popular in the sliding-mode scientific community since its introduction by Levant in the nineties. We want here to continue the study of implicit discretization of HOSMC (twisting, super-twisting algorithms) and especially we would like to investigate the comparisons between classical (first order) SMC and HOSMC, when both are implicitly discretized, in terms of performance, accuracy, chattering suppression. Another topic of interest is stabilization in finite-time of systems with impacts and unilateral constraints, in a discrete-time setting.

3.4.2. Optimal Control

- *Linear Complementarity Systems (LCS) :* With the PhD thesis of A. Vieira, we have started to study the quadratic optimal control of LCS. Our objective is to go further with minimum-time problems. Applications of LCS are mainly in electrical circuits with set-valued components such as ideal diodes, transistors, *etc*. Such problems naturally yield MPEC when numerical solvers are sought. It is therefore intimately linked with Axis 2 objectives.

- *Impacting systems :* the optimal control of mechanical systems with unilateral constraints and impacts, largely remains an open issue. The problem can be tackled from various approaches: vibro-impact systems (no persistent contact modes) that may be transformed into discrete-time mappings *via* the impact Poincaré map; or the classical integral action minimization (Bolza problem) subjected to the complementarity Lagrangian dynamics including impacts.

- *State constraints, generalized control :* this problem differs from the previous two, since it yields Pontryagin's first order necessary conditions that take the form of an LCS with higher relative degree between the complementarity variables. This is related to the numerical techniques for the higher order sweeping process [33].

3.4.3. Control of nonsmooth discrete Lagrangian systems

- *Cable–driven systems:* these systems are typically different from the cable-car systems, and are closer in their mechanical structure to so-called tensegrity structures. The objective is to actuate a system *via* cables supposed in a first instance to be flexible (slack mode) but non-extensible in their longitudinal direction. This gives rise to complementarity conditions, one big difference with usual complementarity Lagrangian systems being that the control actions operate directly in one of the complementary variables (and not in the smooth dynamics as in cable-car systems). Therefore both the cable models and the control properties are expected to differ a lot from what we may use for cableway systems (for which guaranteeing a positive cable tension is usually not an issue, hence avoiding slack modes, but the deformation of the cables due to the nacelles and cables weights, is an important factor). Tethered systems are a close topic.

- *Multi-body systems with clearances:* our approach is to use models of clearances with dynamical impact effects, *i.e.* within Lagrangian complementarity systems. Such systems are strongly under-actuated due to mechanical play at the joints. However their structure, as underactuated systems, is quite different from what has been usually considered in the Robotics and Control literature. In the recent past we have proposed a thorough numerical robustness analysis of various feedback collocated and non-collocated controllers (PD, linearization, passivity-based). We propose here to investigate specific control strategies tailored to such underactuated systems [48].

- *Granular systems:* the context is the feedback control of granular materials. To fix the ideas, one may think of a "juggling" system whose "object" (uncontrolled) part consists of a chain of aligned beads. Once the modeling step has been fixed (choice of a suitable multiple impact law), one has to determine the output to be controlled: all the beads, some of the beads, the chain's center of mass (position, velocity, vibrational magnitude and frequency), *etc*. Then we aim at investigating which type of controller may be used (output or state feedback, "classical" or sinusoidal input with feedback through the magnitude and frequency) and especially which variables may be measured/observed (positions and/or velocities of all or some of the beads, position and/or velocity of the chain's center of gravity). This topic follows previous results we obtained on the control of juggling systems [50], with increasing complexity of the "object"'s dynamics. The next step would be to extend to 2D and then 3D granular materials. Applications concern vibrators, screening, transport in mining and manufacturing processes.

- *Stability of structures:* our objective here is to study the stability of stacked blocks in 2D or 3D, and the influence on the observed behavior (numerically and/or analytically) of the contact/impact model.

3.4.4. Switching LCS and DAEs, higher-order sweeping process (HOSwP)

- We have gained a strong experience in the field of complementarity systems and distribution differential inclusions [33], [51], that may be seen as some kind of switching DAEs. We plan to go further with non-autonomous HOSwP with switching feedback inputs and non-uniform vector relative degrees. Switching linear complementarity systems can also be studied, though the exact relationships between both point of views remain unclear at the present time. This axis of research is closely related to cyber-physical systems in section 3.2.

3.4.5. Control of Elastic (Visco-plastic) systems with contact, impact and friction

- *Stabilization, trajectory tracking:* until now we have focused on the stability and the feedback control of systems of rigid bodies. The proposal here is to study the stabilization of flexible systems (for instance, a "simple" beam) subjected to unilateral contacts with or without set-valued friction

(contacts with obstacles, or impacts with external objects line particle/beam impacts). This gives rise to varying (in time and space) boundary conditions. The best choice of a good contact law is a hard topic discussed in the literature.

- *Cableway systems (STRMTG, POMA):* cable-car systems present challenging control problems because they usually are underactuated systems, with large flexibilities and deformations. Simplified models of cables should be used (Ritz-Galerkin approach), and two main classes of systems may be considered: those with moving cable and only actuator at the station, and those with fixed cable but actuated nacelles. It is expected that they possess quite different control properties and thus deserve separate studies. The nonsmoothness arises mainly from the passage of the nacelles on the pylons, which induces frictional effects and impacts. It may certainly be considered as a nonsmooth set-valued disturbance within the overall control problem.

4. Application Domains

4.1. Applications Domains

Nonsmooth dynamical systems arise in a lot of application fields. *We briefly expose here some applications that have been treated in the BIPOP team and that we will continue in the TRIPOP team, as a validation for the research axes and also in terms of transfer (Sect. 7.1).* In mechanics, the main instances of nonsmooth dynamical systems are multibody systems with Signorini's unilateral contact, set-valued (Coulomb-like) friction and impacts, or in continuum mechanics, ideal plasticity, fracture or damage. Some illustrations are given in Figure 4(a-f). Other instances of nonsmooth dynamical systems can also be found in electrical circuits with ideal components (see Figure 4(g)) and in control theory, mainly with sliding mode control and variable structure systems (see Figure 4(h)). More generally, every time a piecewise, possibly set–valued, model of systems is invoked, we end up with a nonsmooth system. This is the case, for instance, for hybrid systems in nonlinear control or for piecewise linear modeling of gene regulatory networks in mathematical biology (see Figure 4(i)). Another common example of nonsmooth dynamics is also found when the vector field of a dynamical system is defined as a solution of an optimization problem under constraints, or a variational inequality. Examples of this kind are found in the optimal control theory, in dynamic Nash equilibrium or in the theory of dynamic flows over networks.

5. New Software and Platforms

5.1. Platforms: SICONOS

5.1.1. Platform A : SICONOS

Participants: Vincent Acary, Maurice Brémond, Olivier Huber, Franck Pérignon.

In the framework of the FP5 European project Siconos (2002-2006), Bipop was the leader of the Work Package 2 (WP2), dedicated to the numerical methods and the software design for nonsmooth dynamical systems. This has given rise to the platform SICONOS which is the main software development task in the team. The aim of this work is to provide a common platform for the simulation, modeling, analysis and control of abstract nonsmooth dynamical systems. Besides usual quality attributes for scientific computing software, we want to provide a common framework for various scientific fields, to be able to rely on the existing developments (numerical algorithms, description and modeling software), to support exchanges and comparisons of methods, to disseminate the know-how to other fields of research and industry, and to take into account the diversity of users (end-users, algorithm developers, framework builders) in building expert interfaces in Python and end-user front-end through Scilab.
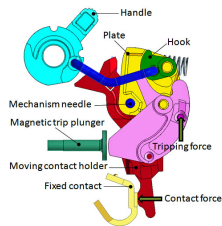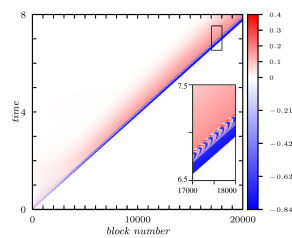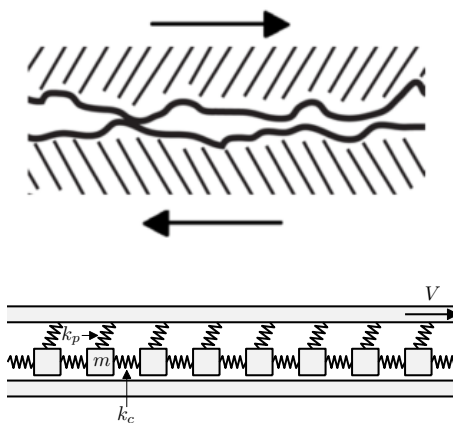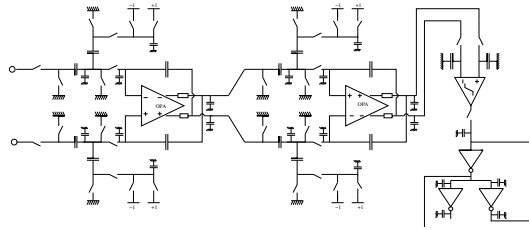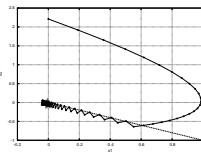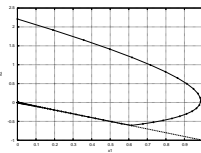
(a) Circuit breakers mechanisms [40]

(b) Granular flows
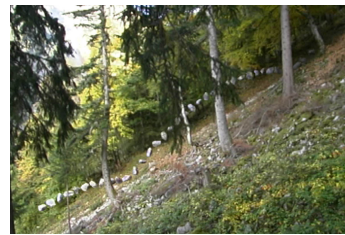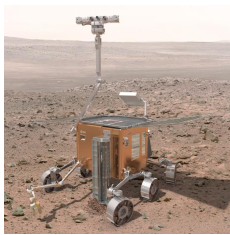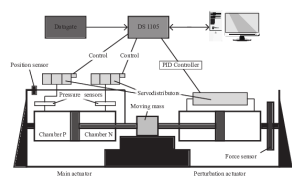

(c) Robots (ESA ExoMars Rover [35])
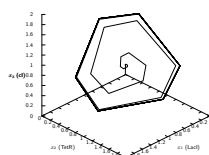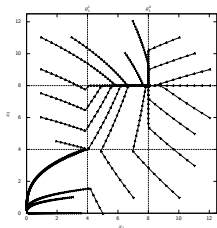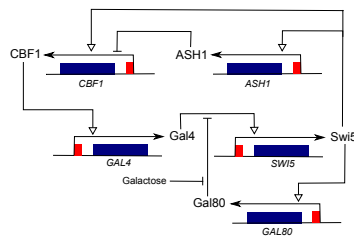

(d) Rockfall [47], [46], [59]

(e) Frictional interface and solitary waves in the Burridge-Knopoff model [81]

(a) Switched electrical circuits (delta-sigma converter) [29]

(b) Sliding mode control [32], [34], [70], [71], [80]

After the requirement elicitation phase, the Siconos Software project has been divided into 5 work packages which are identified to software products:

1. SICONOS/NUMERICS This library contains a set of numerical algorithms, already well identified, to solve non smooth dynamical systems. This library is written in low-level languages (C,F77) in order to ensure numerical efficiency and the use of standard libraries (Blas, Lapack, ...)

2. SICONOS/KERNEL This module is an object-oriented structure (C++) for the modeling and the simulation of abstract dynamical systems. It provides the users with a set of classes to describe their nonsmooth dynamical system (dynamical systems, intercations, nonsmooth laws, ...) and to perform a numerical time integration and solving.

3. SICONOS/FRONT-END. This module is mainly an auto-generated wrapper in Python which provides a user-friendly interface to the Siconos libraries. A scilab interface is also provided in the Front-End module.

4. SICONOS/CONTROL This part is devoted to the implementation of control strategies of non smooth dynamical systems.

5. SICONOS/MECHANICS. This part is dedicated to the modeling and the simulation of multi-body systems with 3D contacts, impacts and Coulomb's friction. It uses the Siconos/Kernel as simulation engine but relies on a industrial CAD library (OpenCascade and pythonOCC) to deal with complex body geometries and to compute the contact locations and distances between B-Rep description and on Bullet for contact detection between meshes.

Further informations may be found at http://siconos.gforge.inria.fr/

6. New Results

6.1. Nonlinear waves in granular chains

Participants: Guillaume James, Bernard Brogliato, Kirill Vorotnikov.

Granular chains made of aligned beads interacting by contact (e.g. Newton's cradle) are widely studied in the context of impact dynamics and acoustic metamaterials. In order to describe the response of such systems to impacts or vibrations, it is important to analyze different wave effects such as the propagation of localized compression pulses (solitary waves) or oscillations (traveling breathers), or the scattering of vibrations through the chain. Such phenomena are strongly influenced by contact nonlinearities (Hertz force), spatial inhomogeneities and dissipation.

In the work [22], we analyze the Kuwabara-Kono (KK) model for contact damping, and we develop new approximations of this model which are efficient for the simulation of multiple impacts. The KK model is a simplified viscoelastic contact model derived from continuum mechanics, which allows for simpler calibration (using material parameters instead of phenomenological ones), but its numerical simulation requires a careful treatment due to its non-Lipschitzian character. Using different dissipative time-discretizations of the conservative Hertz model, we show that numerical dissipation can be tuned properly in order to reproduce the physical dissipation of the KK model and associated wave effects. This result is obtained analytically in the limit of small time steps (using methods from backward analysis) and is numerically validated for larger time steps. The resulting schemes turn out to provide good approximations of impact propagation even for relatively large time steps.

In reference [8], we analyze the discrete p-Schrödinger equation, an envelope equation that describes small oscillations in a Newton's cradle. In the limit when the exponent of the contact force lies slightly above unity, we derive three different continuum limits of the model which allow us to approximate the profiles of traveling breather solutions. One model consists of a logarithmic nonlinear Schrödinger equation which leads to a Gaussian approximation, and the two other are fully nonlinear degenerate Schrödinger equations which provide compacton approximations. These approximations are numerically validated by Newton-type computations. In the opposite (vibroimpact) limit when the exponent of the contact force is large, we obtain an analytical approximation of solitary waves in the form of a compacton.

6.2. Periodic motions of coupled impact oscillators

Participants: Guillaume James, Vincent Acary, Franck Pérignon.

In the work [17], we study the existence and stability of time-periodic oscillations in an infinite chain of linearly coupled impact oscillators, for rigid impacts without energy dissipation. We reformulate the search of periodic solutions as a boundary value problem incorporating unilateral constraints. This formulation, together with an appropriate notion of nondegenerate modes, allows us to construct nonsmooth modes of oscillations (spatially localized or extended) when the oscillators are weakly coupled (this approach is an adaptation of the idea of 'anticontinuum' limit to the nonsmooth setting). In this framework, we show the existence of exact solutions (in particular, we check the condition of non-penetration of the obstacle) for an arbitrary number of impacting particles. Different solution branches corresponding to stable or unstable breathers, multibreathers and nonsmooth normal modes are found. We provide a formula for the monodromy matrix that determines spectral stability of nonsmooth modes in the presence of simple impacts. These results are completed by a numerical computation of the time-periodic solutions at larger coupling, and the Siconos software is used to simulate the system and explore dynamical instabilities. The above approach is much more effective than numerical continuation of periodic solutions based on stiff compliant models, which leads to stiff ODEs and costly numerical continuation.

6.3. Solitary waves in the excitable Burridge-Knopoff model

Participants: Guillaume James, Jose Eduardo Morales Morales, Arnaud Tonnelier.

The Burridge-Knopoff model is a lattice differential equation describing a chain of blocks connected by springs and pulled over a surface. This model was originally introduced to investigate nonlinear effects arising in the dynamics of earthquake faults. One of the main ingredients of the model is a nonlinear velocity-dependent friction force between the blocks and the fixed surface. We introduce a simplified piecewise linear friction law (reminiscent of the McKean nonlinearity for excitable cells) which allows us to obtain analytical expression of solitary waves and study some of their qualitative properties, such as wavespeed and propagation failure. These results have been published in [11].

We have obtained an existence theorem for solitary waves in the Burridge-Knopoff model. Our approach uses a piecewise-linear friction force combined with a weak coupling strength. Using asymptotic arguments, we show that trial solutions, obtained semi-analytically, satisfy, for some parameter set, the inequality constraints associated with the threshold conditions. An approximation of the wave profile is obtained and a minimal wave speed is derived.

6.4. Signal propagation along excitable chains

Participant: Arnaud Tonnelier.

Nonlinear self-sustained waves, or *autowaves*, have been identified in a large class of discrete excitable media. We have proposed a simple continuous-time threshold model for wave propagation in excitable media. The ability of the resulting transmission line to convey a one-bit signal is investigated. Existence and multistability of signals where two successive units share the same waveform is established. We show that, depending on the connectivity of the transmission line, an arbitrary number of distinct signals can be transmitted. More precisely, we prove that, for a one-dimensional information channel with nth-neighbor interactions, a n-fold degeneracy of the speed curve induces the coexistence of $2n$ propagating signals, n of which are stable and allow n distinct symbols transmission. The influence of model parameters (time constants, coupling strength and connectivity) on the traveling signal properties is analyzed. This work is almost finished and is going to be submitted.

6.5. Numerical analysis of multibody mechanical systems with constraints

This scientific theme concerns the numerical analysis of mechanical systems with bilateral and unilateral constraints, with or without friction [1]. They form a particular class of dynamical systems whose simulation requires the development of specific methods for analysis and dedicated simulators [6].

6.5.1. *Multibody systems with clearances (dynamic backlash)*
Participants: Vincent Acary, Bernard Brogliato.

The PhD thesis of N. Akadkhar under contract with Schneider Electric has concerned the numerical simulation of mechanical systems with unilateral constraints and friction, where the presence of clearances in imperfect joints plays a crucial role. A first work deals with four-bar planar mechanisms with clearances at the joints, which induce unilateral constraints and impacts, rendering the dynamics nonsmooth. The objective is to determine sets of parameters (clearance value, restitution coefficients, friction coefficients) such that the system's trajectories stay in a neighborhood of the ideal mechanism (*i.e.* without clearance) trajectories. The analysis is based on numerical simulations obtained with the projected Moreau-Jean time-stepping scheme. Circuits breakers with 3D joint clearances have been studied in [3] [41] where it is demonstrated that the nonsmooth dynamics approach as coded in our software SICONOS, allows a very good prediction of the system's dynamics, with experimental validation. An overview of various approaches for the feedback control of multibody systems with joint clearances is proposed in [4].

6.5.2. *Generalized-α scheme for nonsmooth multibody systems.*
Participant: Vincent Acary.

This work [16] concerns a formalism for the transient simulation of nonsmooth dynamic mechanical systems composed of rigid and flexible bodies, kinematic joints and frictionless contact conditions. The proposed algorithm guarantees the exact satisfaction of the bilateral and unilateral constraints both at position and velocity levels. Thus, it significantly differs from penalty techniques since no penetration is allowed. The numerical scheme is obtained in two main steps. Firstly, a splitting method is used to isolate the contributions of impacts, which shall be integrated with only first-order accuracy, from smooth contributions which can be integrated using a higher order scheme. Secondly, following the idea of Gear, Gupta and Leimkuhler, the equation of motion is reformulated so that the bilateral and unilateral constraints appear both at position and velocity levels. After time discretization, the equation of motion involves two complementarity conditions and it can be solved at each time step using a monolithic semi-smooth Newton method. The numerical behaviour of the proposed method is studied and compared to other approaches for a number of numerical examples. Itis shown that the formulation offers a unified and valid approach for the description of contactconditions between rigid bodies as well as between flexible bodies.

6.5.3. *Mechanics of musical instruments with contact and impacts.*
Participants: Vincent Acary, Franck Pérignon.

Collisions in musical string instruments play a fundamental role in explaining the sound production in various instruments such as sitars, tanpuras and electric basses. Contacts occuring during the vibration provide a nonlinear effect which shapes a specific tone due to energy transfers and enriches the hearing experience. As such, they must be carefully simulated for the purpose of physically-based sound synthesis. Most of the numerical methods presented in the literature rely on a compliant modeling of the contact force between the string and the obstacle. In this contribution, numerical methods from nonsmooth contact dynamics are used to integrate the problem in time. A Moreau-Jean time-stepping scheme is combined with an exact scheme for phases with no contact, thus controlling the numerical dispersion. Results for a two-point bridge mimicking a tanpura and an electric bass are presented, showing the ability of the method to deal efficiently with such problems while invoking, as compared to a compliant approach, less modelling parameters and a reduced computational burden [7].

6.5.4. *Numerical solvers for frictional contact problems.*
Participants: Vincent Acary, Maurice Brémond.

In [15] report, we review several formulations of the discrete frictional contact problem that arises in space and time discretized mechanical systems with unilateral contact and three-dimensional Coulomb's friction. Most of these formulations are well–known concepts in the optimization community, or more generally, in the mathematical programming community. To cite a few, the discrete frictional contact problem can be formulated as variational inequalities, generalized or semi–smooth equations, second–order cone complementarity problems, or as optimization problems such as quadratic programming problems over second-order cones. Thanks to these multiple formulations, various numerical methods emerge naturally for solving the problem. We review the main numerical techniques that are well-known in the literature and we also propose new applications of methods such as the fixed point and extra-gradient methods with self-adaptive step rules for variational inequalities or the proximal point algorithm for generalized equations. All these numerical techniques are compared over a large set of test examples using performance profiles. One of the main conclusion is that there is no universal solver. Nevertheless, we are able to give some hints to choose a solver with respect to the main characteristics of the set of tests

6.5.4.1. Impact laws in chains of aligned balls

In [18] several "classical" multiple-impact laws are compared on chains of aligned balls: Moreau's law, the binary collision law, and the LZB approach [2]. Short analyses of these laws are made, and thorough comparisons are led numerically. It is concluded that both Moreau and the binary collision laws, furnish good results (in terms of predictability) only in very particular cases of elasticity coefficien, contact stiffenesses ratios, and mass ratios.

6.6. Analysis and Control of Set-Valued Systems

Participants: Bernard Brogliato, Christophe Prieur, Alexandre Vieira.

6.6.1. *Higher-order sweeping process*

This work [5] continues our previous results in [33], to the case when exogeneous terms are present in both the unilateral constraint, and in the dynamics. A suitable change of state variables allows one to recast the dynamics in a format that is close to the autonomous case, so that the well-posedness issues (existence and uniqueness of solutions) is shown (see the preprint [20] for a complete analysis, which in fact differs only slightly from the original one in [33]). The link with switching DAEs is made.

6.6.2. *Robust sliding-mode control: continuous and discrete-time*

This work [10] concerns the robust control of linear time-invariant systems, subjected to nonlinear varying state dependent disturbances as well as parameter uncertainties. A specific set-valued class of sliding-mode controllers is designed, and its discretization (with the implicit method introduced in [32], [34]) is analysed. One difficulty is that the parameter uncertainties, as well as the discretization, create unmatched disturbances. Stability and convergence results are proved. Let us mention also [9] that corrects a slight mistake in [80]. In the same way it is worth citing [14], [14] which continues the analysis of the implicit discretization of set-valued systems, this time oriented towards the consistency of time-discretizations for homogeneous systems, with one discontinuity at zero (sometimes called quasi-continuous, strangely enough).

6.6.3. *Evolution variational inequalities*

In [13] we continue our previous works on well-posedness and stabilization/control of a class of set-valued systems, that take the form of evolution variational inequalities. Dissipativity is then a key property. Regulation with state and output feedback, viability issues, are solved, with absolutely continuous and bounded variations solutions. Applications are in power converters.

6.6.4. Optimal control of LCS

The quadratic and minimum time optimal control of LCS as in (6) is tackled in [24], [25]. This work relies on the seminal results by Guo and ye (SIAM 2016), and aims at particularizing their results for LCS, so that they become numerically tractable and one can compute optimal controllers and optimal trajectories. The basic idea is to take advantage of the complementarity, to construct linear complementarity problems in the Pontryagin's necessary conditions which can then be integrated numerically, without having to guess a priori the switching instants (the optimal controller can eb discontinuous and the optimal trajectories can visit several modes of the complementarity conditions).

7. Bilateral Contracts and Grants with Industry

7.1. Schneider Electric

This action started in 2001 with my post-doc co–supported by Schneider Electric and CNRS. With some brief interruptions, this action is still active and should further continue. It concerns mainly the simulation and modeling of multi–body systems with contact, friction and impacts with the application for the virtual prototyping of electrical circuit breakers. During these years, various forms of collaborations have been held. Two PhD thesis have been granted by Schneider Electric (D.E. Taha and N. Akhakdar) accompanied with research contracts between Inria and Schneider Electric. Schneider Electric participated also the ANR project Saladyn as a main partner. Without going into deep details of the various actions over the years, the major success of this collaboration is the statistical tolerance analysis of the functional requirements of the circuit breakers with respect to clearance in joints and geometrical tolerances on the parts. Starting from the geometrical descriptions (CAD files) of a mechanism with prescribed tolerances on the manufacturing process, we perform worst-case analysis and Monte–Carlo simulations of the circuit breaker with Siconos and we record the variations in the functional requirements. The difficulty in such simulations are the modeling of contact with friction that models the joints with clearances. The results of these analysis enable Schneider Electric to define the manufacturing precision that has a huge impact of the production cost (Schneider Electric produces several millions of C60-type circuit breaker per year). Note that it is not possible to perform such simulations with the existing software codes of the market. At the beginning, our interlocutor at Schneider Electric was the innovation (R&D) department. Now, we are working and discussing with the business unit, Division Power and Dinnov (M. Abadie, E. Boumediene, X. Herreros) in charge of designing and producing the circuit–breakers. The targeted users are the R&D engineers of Schneider Electric that use simulation tools for designing new models or improving existing circuit breakers. This collaboration continues with new modeling and simulation challenges (flexible parts, multiple impact laws) with the CIFRE PhD of Rami Sayoud.

7.2. STRMTG

We have recently started with STRMTG a research contract about modelling, simulation and control of cable-transport systems. In such systems, the question of the coupling between the nonlinear dynamics of cables and their supports with unilateral contact and friction appears now to be determinant in order to increase the performances of the cableway systems, especially for urban transportation systems.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

B. Brogliato coordinates the ANR project Digitslid (PRC, ANR-18-CE40-0008-01), Differentiateurs et commandes homogenes par modes glissants en temps discret: l'approche implicite. Partners: LS2N (Ecole Centrale de Nantes), Inria Lille Nord Europe (team Non-A-Post), and Tripop. October 2018-September 2021. 12 participants overall (3 post-doc students recruited by the project, 3 Ph.D. students supported by other means). Total financial support by the ANR: 338 362 euros (100 762 for Tripop, 18 months of post-doc to be recruited in 2019).

8.1.2. *FUI Modeliscale.*

https://team.inria.fr/modeliscale/

The ModeliScale FUI focuses on the modeling, simulation and analysis of large cyber-physical systems. It federates the research activities of several teams, covering a broad spectrum of topics, namely hybrid systems modeling & verification, numerical analysis, programming language design and automatic control. Our research agenda includes the following tracks:

- New compilation techniques for Modelica modelers: structural analysis of multimode DAE (Differential Algebraic Equations) systems, modular compilation, combining state-machines and non-smooth dynamical systems (complementarity dynamical systems and Filippov differential inclusions), contract-based specification of cyber-physical systems requirements, requirements capture using under-/over-determined DAE systems.
- Simulation of large cyber-physical systems: distributed simulation, discretization methods for non-smooth dynamical systems, space-/time-adaptive discretization methods for multimode DAE systems, quantized state solvers (QSS).
- Guaranteed numerics: guaranteed simulation of non-smooth and hybrid dynamical systems, numerical methods preserving invariant properties of hybrid systems, contract-based reasoning methods.

8.1.3. *Inria Project Lab (IPL): ModeliScale, Languages and Compilation for Cyber-Physical System Design*

https://team.inria.fr/modeliscale/

The project gathers researchers from three Inria teams, and from three other research labs in Grenoble and Paris area.

Table 1. Member of IPL Modeliscale

Name	Team	Inria Center or Laboratory
Vincent Acary	Bipop	Inria Grenoble Rhône Alpes
Bernard Brogliato		
Albert Benveniste	Hycomes Inria Rennes	
Benoît Caillaud		Bretagne Atlantique
Khalil Ghorbal		
Marc Pouzet	Parkas	ENS
Tim Bourke		Inria Paris
Goran Frehse	Tempo	Verimag-univ. Grenoble Alpes
Antoine Girard		L2S-CNRS, Saclay
Eric Goubault	Cosynus	LIX, École Polytechnique,
Sylvie Putot		Saclay

The main objective of ModeliScale is to advance modeling technologies (languages, compile-time analyses, simulation techniques) for CPS combining physical interactions, communication layers and software components. We believe that mastering CPS comprising thousands to millions of components requires radical changes of paradigms. For instance, modeling techniques must be revised, especially when physics is involved. Modeling languages must be enhanced to cope with larger models. This can only be done by combining new compilation techniques (to master the structural complexity of models) with new mathematical tools (new numerical methods, in particular).

ModeliScale gathers a broad scope of experts in programming language design and compilation (reactive synchronous programming), numerical solvers (nonsmooth dynamical systems) and hybrid systems modeling and analysis (guaranteed simulation, verification). The research program is carried out in close cooperation with the Modelica community as well as industrial partners, namely, Dassault Systèmes as a Modelica/FMI tool vendor, and EDF and Engie as end users.

8.2. International Research Visitors

8.2.1. Visits of International Scientists

- Mathias Legrand (McGill University, Mechanical Engineering).

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Journal

9.1.1.1. Member of the Editorial Boards

- B. Brogliato is Associate Editor at Nonlinear Analysis: Hybrid Systems.
- B. Brogliato is Associate Editor at ASME Journal of Computational and Nonlinear Dynamics.

9.1.1.2. Reviewer - Reviewing Activities

- A. Tonnelier has been reviewer for PRE (Physical Review E) and DCDS (Discrete and Continuous Dynamical Systems).
- G. James has been reviewer for IMA J. Appl. Math.
- V. Acary has been for Optimization Methods and Software, Nonlinear Dynamics, Applied Mechanics Reviews, IEEE Transactions on Robotics, Multibody Systems Dynamics, International Journal for Numerical Methods in Engineering, IFAC Conference on Modelling, Identification and Control of Nonlinear Systems, CDC 2018, SIMPAR 2018.
- B. Brogliato has been reviewer for IEEE Transactions on Automatic Control, IEEE Transactions on Robotics, IEEE Control Systems Letters, Automatica, SIAM Journal on Control and Optimization, Multibody System Dynamics.

9.1.2. Invited Talks

- G. James : invited talk at the International Symposium on Intrinsic Localized Modes (Kyoto, 01/18), speaker at the Conference on Advances in Nonsmooth Mechanics (Bristol, 06/18), lecture at the thematic school Méthodes de Dynamique Non-Linéaire pour l'Ingénierie des Structures, GDR Dynolin (Fréjus, 05/18).
- V. Acary : lecture at the thematic school Méthodes de Dynamique Non-Linéaire pour l'Ingénierie des Structures, GDR Dynolin (Fréjus, 05/18).

9.1.3. Leadership within the Scientific Community

- V. Acary is coordinator with R. Leine of the Europe Network for Nonsmooth Dynamics http://ennsd.gforge.inria.fr/.

9.1.4. Research Administration

- A. Tonnelier is member of the CED and the CLHSCT.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Licence : G. James, Introduction to Dynamical Systems in UE MAP201, 18 hETD, L1, Université Grenoble Alpes.
- Licence : G. James, Normed Vector Spaces, 25 hETD, L2, Prépa INP, Grenoble.
- Master : G. James, Numerical Methods, 94 hETD, M1, Grenoble INP - Ensimag (1st year).
- Master : G. James, Dynamical Systems, 45 hETD, M1, Grenoble INP - Ensimag (2nd year).
- Master : Vincent Acary, 17H éq TD Systèmes dynamiques, ENSIMAG 2A.
- Master : Vincent Acary, 12H éq TD Systèmes dynamiques, Master ACSYON. Université de Limoges.

9.2.2. Supervision

Rami Sayoub, Influence of vibrations on multibody systems, September 2018, V. Acary and B. Brogliato.

Charlélie Bertrand, Mechanical model fro cable vibrations, October 2018, V. Acary and C.H. Lamarque.

Christelle Kozaily. Structural analysis for multi-mode DAE systems, Octobre 2018, V. Acary and B. Caillaud.

Alexandre Vieira. Commande optimale de systèmes linéaires de complémentarité, université Grenoble Alpes, 24 septembre 2018, B. Brogliato and C. Prieur.

9.2.3. Juries

- Vincent Acary, president of Ph.D. Thesis committee of Thomas Catterou (22 October 2018), Universite Aix–Marseille.
- Bernard Brogliato, examinateur of Ph.D. thesis of C. Beneux (12 July 2018), Université Lorraine.

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Major publications by the team in recent years

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