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Project-Team CAGE

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

RESEARCH CENTER
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THEME
Optimization and control of dynamic systems

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Project-Team CAGE

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2. Overall Objectives

2.1. Overall Objectives

CAGE's activities take place in the field of mathematical control theory, with applications in three main directions: geometric models for vision, control of quantum mechanical systems, and control of systems with uncertain dynamics.

The relations between control theory and geometry of vision rely on the notion of sub-Riemannian structure, a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful control theoretical interpretation. We recall that nonholonomicity refers to the property of a velocity constraint that cannot be recast as a state constraint. In the language of differential geometry, a sub-Riemannian structure is a (possibly rank-varying) Lie bracket generating distribution endowed with a smoothly varying norm.

Sub-Riemannian geometry, and in particular the theory of associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the recent literature (including by members of our team). Our contributions to this field are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

Control theory is one of the components of the forthcoming quantum revolution [1], since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance). The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. Time scales analysis is important for evaluation approaches based on adiabatic approximation theory, which is well-known to improve the robustness of the control strategy. CAGE works for the improvement of evaluation and design tools for efficient quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces.

Simultaneous control of a continuum of systems with slightly different dynamics is a typical problem in quantum mechanics and also a special case of the third applicative axis to which CAGE is contributing: control of systems with uncertain dynamics. The slightly different dynamics can indeed be seen as uncertainties in the system to be controlled, and simultaneous control rephrased in terms of a robustness task. Robustification, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of

[1] As anticipated by the recent launch of the FET Flagship on Quantum Technologies

controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. Our contributions to this research field concern both stabilization (either asymptotic or in finite time) and optimal control, where redundancies and probabilistic tools can be introduced to offset uncertainties.

3. Research Program

3.1. Research domain

The activities of CAGE are part of the research in the wide area of control theory. This nowadays mature discipline is still the subject of intensive research because of its crucial role in a vast array of applications.

More specifically, our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [156]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [120]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [159], [119], [147]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [143], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [137] and models for neural activity [106]. Therapy analysis from the point of view of optimal control has also attracted a great attention [140].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [153].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [107].

3.2. Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, observability, optimal control... [70], [111]. The emphasis of such a geometric approach to control theory is put on intrinsic properties of the systems and it is particularly well adapted to study nonlinear and nonholonomic phenomena.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [93]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting from 2009 [96] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [152], [139]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [132], [122], those based on the differentiation of nonlinear flows such as the **return method** [101], [100], and those exploiting the **differential flatness** of the system [105].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;

- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [68] or shape optimization [76]. Examples of the second type are inactivation principles in human motricity [79] or neurogeometrical models for image representation of the primary visual cortex in mammals [90].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [130], geometric measure theory [72] and hypoelliptic operator theory [82].

The geometric control approach has historically been related to the development of finite-dimensional control theory. However, its impact in the analysis of distributed parameter control systems and in particular systems of controlled partial differential equations has been growing in the last decades, complementing analytical and numerical approaches, providing dynamical, qualitative and intrinsic insight [99]. CAGE's ambition is to be at the core of this development in the years to come.

4. Application Domains

4.1. First axis: Geometry of vision

A suggestive application of sub-Riemannian geometry and in particular of hypoelliptic diffusion comes from a model of geometry of vision describing the functional architecture of the primary visual cortex V1. In 1958, Hubel and Wiesel (Nobel in 1981) observed that the visual cortex V1 is endowed with the so-called **pinwheel structure**, characterized by neurons grouped into orientation columns, that are sensible both to positions and directions [110]. The mathematical rephrasing of this discovery is that the visual cortex lifts an image from $\mathbf{R}^2$ into the bundle of directions of the plane [97], [136], [138], [109].

A simplified version of the model can be described as follows: neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli at a given point of the retina and for a given direction on it. The retina is modeled by the real plane, i.e., each point is represented by a pair $(x, y) \in \mathbb{R}^2$, while the directions at a given point are modeled by the projective line, i.e. an element θ of the projective line P^1. Hence, the primary visual cortex V1 is modeled by the so called projective tangent bundle $\mathrm{PTR}^2 = \mathbf{R}^2 \times \mathbf{P}^1$. From a neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them being sensitive to stimuli at a given point (x, y) with any direction.

Orientation columns are connected between them in two different ways. The first kind of connections are the vertical (inhibitory) ones, which connect orientation columns belonging to the same hypercolumn and sensible to similar directions. The second kind of connections are the horizontal (excitatory) connections, which connect neurons belonging to different (but not too far) hypercolumns and sensible to the same directions. The resulting metric structure is sub-Riemannian and the model obtained in this way provides a convincing explanation in terms of sub-Riemannian geodesics of gestalt phenomena such as Kanizsa illusory contours.

The sub-Riemannian model for image representation of V1 has a great potential of yielding powerful bio-inspired image processing algorithms [104], [90]. Image inpainting, for instance, can be implemented by reconstructing an incomplete image by activating orientation columns in the missing regions in accordance with sub-Riemannian non-isotropic constraints. The process intrinsically defines an hypoelliptic heat equation on PTR^2 which can be integrated numerically using non-commutative Fourier analysis on a suitable semidiscretization of the group of roto-translations of the plane [88].

We have been working on the model and its software implementation since 2012. This work has been supported by several project, as the ERC starting grant GeCoMethods and the ERC Proof of Concept ARTIV1 of U. Boscain, and the ANR GCM.

A parallel approach that we will pursue and combine with this first one is based on **pattern matching in the group of diffeomorphisms**. We want to extend this approach, already explored in the Riemannian setting [151], [127], to the general sub-Riemannian framework. The paradigm of the approach is the following: consider a distortable object, more or less rigid, discretized into a certain number of points. One may track its distortion by considering the paths drawn by these points. One would however like to know how the object itself (and not its discretized version) has been distorted. The study in [151], [127] shed light on the importance of Riemannian geometry in this kind of problem. In particular, they study the Riemannian submersion obtained by making the group of diffeomorphisms act transitively on the manifold formed by the points of the discretization, minimizing a certain energy so as to take into account the whole object. Settled as such, the problem is Riemannian, but if one considers objects involving connections, or submitted to nonholonomic constraints, like in medical imaging where one tracks the motions of organs, then one comes up with a sub-Riemannian problem. The transitive group is then far bigger, and the aim is to lift curves submitted to these nonholonomic constraints into curves in the set of diffeomorphisms satisfying the corresponding constraints, in a unique way and minimizing an energy (giving rise to a sub-Riemannian structure).

4.2. Second axis: Quantum control

The goal of quantum control is to design efficient protocols for tuning the occupation probabilities of the energy levels of a system. This task is crucial in atomic and molecular physics, with applications ranging from photochemistry to nuclear magnetic resonance and quantum computing. A quantum system may be controlled by exciting it with one or several external fields, such as magnetic or electric fields. The goal of quantum control theory is to adapt the tools originally developed by control theory and to develop new specific strategies that tackle and exploit the features of quantum dynamics (probabilistic nature of wavefunctions and density operators, measure and wavefunction collapse, decoherence, ...). A rich variety of relevant models for controlled quantum dynamics exist, encompassing low-dimensional models (e.g., single-spin systems) and PDEs alike, with deterministic and stochastic components, making it a rich and exciting area of research in control theory.

The controllability of quantum system is a well-established topic when the state space is finite-dimensional [102], thanks to general controllability methods for left-invariant control systems on compact Lie groups [92], [112]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [154]. Nevertheless, weaker controllability properties, such as approximate controllability or controllability between eigenstates of the internal Hamiltonian (which are the most relevant physical states), may hold. In certain cases, when the state space is a function space on a 1D manifold, some rather precise description of the set of reachable states has been provided [77]. A similar description for higher-dimensional manifolds seems intractable and at the moment only approximate controllability results are available [128], [134], [113]. The most widely applicable tests for controllability of quantum systems in infinite-dimensional Hilbert spaces are based on the **Lie–Galerkin technique** [96], [85], [86]. They allow, in particular, to show that the controllability property is generic among this class of systems [125].

A family of algorithms which are specific to quantum systems are those based on adiabatic evolution [158], [157], [116]. The basic principle of adiabatic control is that the flow of a slowly varying Hamiltonian can be approximated (up to a phase factor) by a quasi-static evolution, with a precision proportional to the velocity of variation of the Hamiltonian. The advantage of the **adiabatic approach** is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty. The paradigm is based on the adiabatic perturbation theory developed in mathematical physics [83], [133], [150], where it plays an important role for understanding molecular dynamics. Approximation theory by adiabatic perturbation can be used to describe the evolution of the occupation probabilities of the energy levels of a slowly varying Hamiltonian. Results from the last 15 years, including those by members of our team [64], [89], have highlighted the effectiveness of control techniques based on adiabatic path following.

4.3. Third axis: Stability and uncertain dynamics

Switched and hybrid systems constitute a broad framework for the description of the heterogeneous aspects of systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [142], energy management [135] and congestion control [126].

Even if both controllability [146] and observability [114] of switched and hybrid systems have attracted much research efforts, the central role in their study is played by the problem of stability and stabilizability. The goal is to determine whether a dynamical or a control system whose evolution is influenced by a time-dependent signal is uniformly stable or can be uniformly stabilized [119], [147]. Uniformity is considered with respect to all signals in a given class. Stability of switched systems lead to several interesting phenomena. For example, even when all the subsystems corresponding to a constant switching law are exponentially stable, the switched systems may have divergent trajectories for certain switching signals [118]. This fact illustrates the fact that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the class of switching signals which is considered.

The most common class of switching signals which has been considered in the literature is made of all piecewise constant signals. In this case uniform stability of the system is equivalent to the existence of a common quadratic Lyapunov function [129]. Moreover, provided that the system has finitely many modes, the Lyapunov function can be taken polyhedral or polynomial [80], [81], [103]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see the surveys [121], [141] and the references therein). It is known, however, that the existence of a common quadratic Lyapunov function is not necessary for the global uniform exponential stability of a linear switched system with finitely many modes. Moreover, there exists no uniform upper bound on the minimal degree of a common polynomial Lyapunov function [124]. More refined tools rely on multiple and non-monotone Lyapunov functions [91]. Let us also mention linear switched systems technics based on the analysis of the Lie algebra generated by the matrices corresponding to the modes of the system [67].

For systems evolving in the plane, more geometrical tests apply, and yield a complete characterization of the stability [84], [73]. Such a geometric approach also yields sufficient conditions for uniform stability in the linear planar case [87].

In many situations, it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Switching rules can also be imposed, for instance, by a timed automata. When constraints apply, the common Lyapunov function approach becomes conservative and new tools have to be developed to give more detailed characterizations of stable and unstable systems.

Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. One then looks at the corresponding Lyapunov exponents, whose existence is established by the multiplicative ergodic theorem. The interest of this approach is that probabilistic stability analysis filters out highly ‘exceptional’ worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [78], [98], [155].

4.4. Joint theoretical core

The theoretical questions raised by the different applicative area will be pooled in a research axis on the transversal aspects of geometric control theory and sub-Riemannian structures.

We recall that sub-Riemannian geometry is a generalization of Riemannian geometry, whose birth dates back to Carathéodory's seminal paper on the foundations of Carnot thermodynamics [94], followed by E. Cartan's address at the International Congress of Mathematicians in Bologna [95]. In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with a variety of motivations and ramifications in several parts of pure and applied mathematics. Let us mention geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics and optimal control (motion planning, robotics, nonholonomic mechanics, quantum control) [62], [63].

One of the main open problems in sub-Riemannian geometry concerns the regularity of length-minimizers [65], [131]. Length-minimizers are solutions to a variational problem with constraints and satisfy a first-order necessary condition resulting from the Pontryagin Maximum Principle (PMP). Solutions of the PMP are either *normal* or *abnormal*. Normal length-minimizer are well-known to be smooth, i.e., C^∞, as it follows by the Hamiltonian nature of the PMP. The question of regularity is then reduced to abnormal length-minimizers. If the sub-Riemannian structure has step 2, then abnormal length-minimizers can be excluded and thus every length-minimizer is smooth. For step 3 structures, the situation is already more complicated and smoothness of length-minimizers is known only for Carnot groups [115], [149]. The question of regularity of length-minimizers is not restricted to the smoothness in the C^∞ sense. A recent result prove that length-minimizers, for sub-Riemannian structures of any step, cannot have corner-like singularities [108]. When the sub-Riemannian structure is analytic, more is known on the size of the set of points where a length-minimizer can lose analyticity [148], regardless of the rank and of the step of the distribution.

An interesting set of recent results in sub-Riemannian geometry concerns the extension to such a setting of the Riemannian notion of sectional curvature. The curvature operator can be introduced in terms of the symplectic invariants of the Jacobi curve [69], [117], [66], a curve in the Lagrange Grassmannian related to the linearization of the Hamiltonian flow. Alternative approaches to curvatures in metric spaces are based either on the associated heat equation and the generalization of the curvature-dimension inequality [74], [75] or on optimal transport and the generalization of Ricci curvature [145], [144], [123], [71].

5. New Software and Platforms

5.1. BOCOP

Boite à Outils pour le Contrôle OPtimal

KEYWORDS: Dynamic Optimization - Identification - Biology - Numerical optimization - Energy management - Transportation

FUNCTIONAL DESCRIPTION: Bocop is an open-source toolbox for solving optimal control problems, with collaborations with industrial and academic partners. Optimal control (optimization of dynamical systems governed by differential equations) has numerous applications in transportation, energy, process optimization, energy and biology. Bocop includes a module for parameter identification and a graphical interface, and runs under Linux / Windows / Mac.

RELEASE FUNCTIONAL DESCRIPTION: Handling of delay systems Alternate automatic differentiation tool: CppAD Update for CMake and MinGW (windows version)

- Participants: Benjamin Heymann, Virgile Andréani, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Pierre Martinon
- URL: http://bocop.org

5.2. Bocop HJB

KEYWORDS: Optimal control - Stochastic optimization - Global optimization

FUNCTIONAL DESCRIPTION: Toolbox for stochastic or deterministic optimal control, dynamic programming / HJB approach.

RELEASE FUNCTIONAL DESCRIPTION: User interface State jumps for switched systems Explicit handling of final conditions Computation of state probability density (fiste step to mean field games)

- Participants: Benjamin Heymann, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Joseph Frédéric Bonnans
- URL: http://bocop.org

6. New Results

6.1. Geometry of vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In [12] we propose a variational model for joint image reconstruction and motion estimation applicable to spatiotemporal imaging. This model consists of two parts, one that conducts image reconstruction in a static setting and another that estimates the motion by solving a sequence of coupled indirect image registration problems, each formulated within the large deformation diffeomorphic metric mapping framework. The proposed model is compared against alternative approaches (optical flow based model and diffeomorphic motion models). Next, we derive efficient algorithms for a time-discretized setting and show that the optimal solution of the time-discretized formulation is consistent with that of the time-continuous one. The complexity of the algorithm is characterized and we conclude by giving some numerical examples in 2D space + time tomography with very sparse and/or highly noisy data.
- The article [16] presents a method to incorporate a deformation prior in image reconstruction via the formalism of deformation modules. The framework of deformation modules allows to build diffeomorphic deformations that satisfy a given structure. The idea is to register a template image against the indirectly observed data via a modular deformation, incorporating this way the deformation prior in the reconstruction method. We show that this is a well-defined regularization method (proving existence, stability and convergence) and present numerical examples of reconstruction from 2-D tomographic simulations and partially-observed images.
- The article [28] adapts the framework of metamorphosis to the resolution of inverse problems with shape prior. The metamorphosis framework allows to transform an image via a balance between geometrical deformations and changes in intensities (that can for instance correspond to the appearance of a new structure). The idea developed here is to reconstruct an image from noisy and indirect observations by registering, via metamorphosis, a template to the observed data. Unlike a registration with only geometrical changes, this framework gives good results when intensities of the template are poorly chosen. We show that this method is a well-defined regularization method (proving existence, stability and convergence) and present several numerical examples.
- In [8] we prove the C^1 regularity for a class of abnormal length-minimizers in rank 2 sub-Riemannian structures. As a consequence of our result, all length-minimizers for rank 2 sub-Riemannian structures of step up to 4 are of class C^1
- In [33] we show that, for a sub-Laplacian Δ on a 3-dimensional manifold M, no point interaction centered at a point $q_0 \in M$ exists.

- In [39] we consider a one-parameter family of Grushin-type singularities on surfaces, and discuss the possible diffusions that extend Brownian motion to the singularity. This gives a quick proof and clear intuition for the fact that heat can only cross the singularity for an intermediate range of the parameter. When crossing is possible and the singularity consists of one point, we give a complete description of these diffusions, and we describe a “best" extension, which respects the isometry group of the surface and also realizes the unique symmetric one-point extension of the Brownian motion, in the sense of Chen-Fukushima. This extension, however, does not correspond to the bridging extension, which was introduced by Boscain-Prandi, when they previously considered self-adjoint extensions of the Laplace-Beltrami operator on the Riemannian part for these surfaces. We clarify that several of the extensions they considered induce diffusions that are carried by the Marin compactification at the singularity, which is much larger than the (one-point) metric completion. In the case when the singularity is more than one-point, a complete classification of diffusions extending Brownian motion would be unwieldy. Nonetheless, we again describe a “best" extension which respects the isometry group, and in this case, this diffusion corresponds to the bridging extension. A prominent role is played by Bessel processes (of every real dimension) and the classical theory of one-dimensional diffusions and their boundary conditions.
- In [50] we study the notion of geodesic curvature of smooth horizontal curves parametrized by arc length in the Heisenberg group, that is the simplest sub-Riemannian structure. Our goal is to give a metric interpretation of this notion of geodesic curvature as the first corrective term in the Taylor expansion of the distance between two close points of the curve.

We would also like to mention the monograph [30] and the PhD thesis of Mathieu Kohli [3].

6.2. Quantum control: new results

Let us list here our new results in quantum control theory.

- In [29], we discuss the compatibility between the rotating-wave and the adiabatic approximations for controlled quantum systems. Although the paper focuses on applications to two-level quantum systems, the main results apply in higher dimension. Under some suitable hypotheses on the time scales, the two approximations can be combined. As a natural consequence of this, it is possible to design control laws achieving transitions of states between two energy levels of the Hamiltonian that are robust with respect to inhomogeneities of the amplitude of the control input.
- In [34] we study one-parametric perturbations of finite dimensional real Hamiltonians depending on two controls, and we show that generically in the space of Hamiltonians, conical intersections of eigenvalues can degenerate into semi-conical intersections of eigenvalues. Then, through the use of normal forms, we study the problem of ensemble controllability between the eigenstates of a generic Hamiltonian.
- In [35] we discuss which controllability properties of classical Hamiltonian systems are preserved after quantization. We discuss some necessary and some sufficient conditions for small-time controllability of classical systems and quantum systems using the WKB method. In particular, we investigate the conjecture that if the classical system is not small-time controllable, then the corresponding quantum system is not small-time controllable either.
- In [40] we study the controllability problem for a symmetric-top molecule, both for its classical and quantum rotational dynamics. As controlled fields we consider three orthogonally polarized electric fields which interact with the electric dipole of the molecule. We characterize the controllability in terms of the dipole position: when it lies along the symmetry axis of the molecule nor the classical neither the quantum dynamics are controllable, due to the presence of a conserved quantity, the third component of the total angular momentum; when it lies in the orthogonal plane to the symmetry axis, a quantum symmetry arises, due to the superposition of symmetric states, which as no classical counterpart. If the dipole is neither along the symmetry axis nor orthogonal to it, controllability for the classical dynamics and approximate controllability for the quantum dynamics is proved to hold.

We would also like to mention the defense of the PhD thesis of Nicolas Augier (not yet on TEL) on the subject.

6.3. Stability and uncertain dynamics: new results

Let us list here our new results about stability and stabilization of control systems, on the properties of systems with uncertain dynamics.

- In an open channel, a hydraulic jump is an abrupt transition between a torrential (super-critical) flow and a fluvial (subcritical) flow. In [9] hydraulic jumps are represented by discontinuous shock solutions of hyperbolic Saint-Venant equations. Using a Lyapunov approach, we prove that we can stabilize the state of the system in H^2-norm as well as the hydraulic jump location, with simple feedback boundary controls and an arbitrary decay rate, by appropriately choosing the gains of the feedback boundary controls.
- In [10], we study the exponential stabilization of a shock steady state for the inviscid Burgers equation on a bounded interval. Our analysis relies on the construction of an explicit strict control Lyapunov function. We prove that by appropriately choosing the feedback boundary conditions, we can stabilize the state as well as the shock location to the desired steady state in H^2-norm, with an arbitrary decay rate.
- We develop in [19] a method ensuring robustness properties to bang-bang strategies , for general nonlinear control systems. Our main idea is to add bang arcs in the form of needle-like variations of the control. With such bang-bang controls having additional degrees of freedom, steering the control system to some given target amounts to solving an overdetermined nonlinear shooting problem, what we do by developing a least-square approach. In turn, we design a criterion to measure the quality of robustness of the bang-bang strategy, based on the singular values of the end-point mapping, and which we optimize. Our approach thus shows that redundancy implies robustness, and we show how to achieve some compromises in practice, by applying it to the attitude control of a 3d rigid body.
- Partial stability characterizes dynamical systems for which only a part of the state variables exhibits a stable behavior. In his book on partial stability, Vorotnikov proposed a sufficient condition to establish this property through a Lyapunov-like function whose total derivative is upper-bounded by a negative definite function involving only the sub-state of interest. In [20], we show with a simple two-dimensional system that this statement is wrong in general. More precisely, we show that the convergence rate of the relevant state variables may not be uniform in the initial state. We also discuss the impact of this lack of uniformity on the connected issue of robustness with respect to exogenous disturbances.
- The paper [21] elaborates control strategies to prevent clustering effects in opinion formation models. This is the exact opposite of numerous situations encountered in the literature where, on the contrary, one seeks controls promoting consensus. In order to promote declustering, instead of using the classical variance that does not capture well the phenomenon of dispersion, we introduce an entropy-type functional that is adapted to measuring pairwise distances between agents. We then focus on a Hegselmann-Krause-type system and design declustering sparse controls both in finite-dimensional and kinetic models. We provide general conditions characterizing whether clustering can be avoided as function of the initial data. Such results include the description of black holes (where complete collapse to consensus is not avoidable), safety zones (where the control can keep the system far from clustering), basins of attraction (attractive zones around the clustering set) and collapse prevention (when convergence to the clustering set can be avoided).
- The goal of [23] is to compute a boundary control of reaction-diffusion partial differential equation. The boundary control is subject to a constant delay, whereas the equation may be unstable without any control. For this system equivalent to a parabolic equation coupled with a transport equation, a prediction-based control is explicitly computed. To do that we decompose the infinite-dimensional system into two parts: one finite-dimensional unstable part, and one stable infinite-dimensional part. An finite-dimensional delay controller is computed for the unstable part, and it is shown that this controller succeeds in stabilizing the whole partial differential equation. The proof is based on a

an explicit form of the classical Artstein transformation, and an appropriate Lyapunov function. A numerical simulation illustrate the constructive design method.

- Given a linear control system in a Hilbert space with a bounded control operator, we establish in [26] a characterization of exponential stabilizability in terms of an observability inequality. Such dual characterizations are well known for exact (null) controllability. Our approach exploits classical Fenchel duality arguments and, in turn, leads to characterizations in terms of observability inequalities of approximately null controllability and of α-null controllability. We comment on the relationships between those various concepts, at the light of the observability inequalities that characterize them.
- In [37] we propose an extension of the theory of control sets to the case of inputs satisfying a dwell-time constraint. Although the class of such inputs is not closed under concatenation, we propose a suitably modified definition of control sets that allows to recover some important properties known in the concatenable case. In particular we apply the control set construction to dwell-time linear switched systems, characterizing their maximal Lyapunov exponent looking only at trajectories whose angular component is periodic. We also use such a construction to characterize supports of invariant measures for random switched systems with dwell-time constraints.
- In [41] we study asymptotic stability of continuous-time systems with mode-dependent guaranteed dwell time. These systems are reformulated as special cases of a general class of mixed (discrete-continuous) linear switching systems on graphs, in which some modes correspond to discrete actions and some others correspond to continuous-time evolutions. Each discrete action has its own positive weight which accounts for its time-duration. We develop a theory of stability for the mixed systems; in particular, we prove the existence of an invariant Lyapunov norm for mixed systems on graphs and study its structure in various cases, including discrete-time systems for which discrete actions have inhomogeneous time durations. This allows us to adapt recent methods for the joint spectral radius computation (Gripenberg's algorithm and the Invariant Polytope Algorithm) to compute the Lyapunov exponent of mixed systems on graphs.
- Given a discrete-time linear switched system associated with a finite set of matrices, we consider the measures of its asymptotic behavior given by, on the one hand, its deterministic joint spectral radius and, on the other hand, its probabilistic joint spectral radius for Markov random switching signals with given transition matrix and corresponding invariant probability. In [42], we investigate the cases of equality between the two measures.
- In [45] we address the question of the exponential stability for the C^1 norm of general 1-D quasilinear systems with source terms under boundary conditions. To reach this aim, we introduce the notion of basic C^1 Lyapunov functions, a generic kind of exponentially decreasing function whose existence ensures the exponential stability of the system for the C^1 norm. We show that the existence of a basic C^1 Lyapunov function is subject to two conditions: an interior condition, intrinsic to the system, and a condition on the boundary controls. We give explicit sufficient interior and boundary conditions such that the system is exponentially stable for the C^1 norm and we show that the interior condition is also necessary to the existence of a basic C^1 Lyapunov function. Finally, we show that the results conducted in this article are also true under the same conditions for the exponential stability in the C^p norm, for any $p \geq 1$.
- In [46] we study the exponential stability for the C^1 norm of general 2×2 1-D quasilinear hyperbolic systems with source terms and boundary controls. When the eigenvalues of the system have the same sign, any nonuniform steady-state can be stabilized using boundary feedbacks that only depend on measurements at the boundaries and we give explicit conditions on the gain of the feedback. In other cases, we exhibit a simple numerical criterion for the existence of basic C^1 Lyapunov function, a natural candidate for a Lyapunov function to ensure exponential stability for the C^1 norm.
- In [47] we study the exponential stability in the H^2 norm of the nonlinear Saint-Venant (or shallow water) equations with arbitrary friction and slope using a single Proportional-Integral (PI) control

at one end of the channel. Using a local dissipative entropy we find a simple and explicit condition on the gain the PI control to ensure the exponential stability of any steady-states. This condition is independent of the slope, the friction, the length of the river, the inflow disturbance and, more surprisingly, the steady-state considered. When the inflow disturbance is time-dependent and no steady-state exist, we still have the Input-to-State stability of the system, and we show that changing slightly the PI control enables to recover the exponential stability of slowly varying trajectories.

- In [48], we address the problem of the exponential stability of density-velocity systems with boundary conditions. Density-velocity systems are omnipresent in physics as they encompass all systems that consist in a flux conservation and a momentum equation. In this paper we show that any such system can be stabilized exponentially quickly in the H^2 norm using simple local feedbacks, provided a condition on the source term which holds for most physical systems, even when it is not dissipative. Besides, the feedback laws obtained only depends on the target values at the boundaries, which implies that they do not depend on the expression of the source term or the force applied on the system and makes them very easy to implement in practice and robust to model errors. For instance, for a river modeled by Saint-Venant equations this means that the feedback laws do not require any information on the friction model, the slope or the shape of the channel considered. This feat is obtained by showing the existence of a basic H^2 Lyapunov functions and we apply it to numerous systems: the general Saint-Venant equations, the isentropic Euler equations, the motion of water in rigid-pipe, the osmosis phenomenon, etc.
- The general context of [56] is the feedback control of an infinite-dimensional system so that the closed-loop system satisfies a fading-memory property and achieves the setpoint tracking of a given reference signal. More specifically , this paper is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional reaction-diffusion equation with a delayed right Dirichlet boundary control. In this setting, the studied reaction-diffusion equation might be either open-loop stable or unstable. The proposed control strategy goes as follows. First, a finite-dimensional truncated model that captures the unstable dynamics of the original infinite-dimensional system is obtained via spectral decomposition. The truncated model is then augmented by an integral component on the tracking error of the left Neumann trace. After resorting to the Artstein transformation to handle the control input delay, the PI controller is designed by pole shifting. Stability of the resulting closed-loop infinite-dimensional system, consisting of the original reaction-diffusion equation with the PI controller, is then established thanks to an adequate Lyapunov function. In the case of a time-varying reference input and a time-varying distributed disturbance, our stability result takes the form of an exponential Input-to-State Stability (ISS) estimate with fading memory. Finally, another exponential ISS estimate with fading memory is established for the tracking performance of the reference signal by the system output. In particular, these results assess the setpoint regulation of the left Neumann trace in the presence of distributed perturbations that converge to a steady-state value and with a time-derivative that converges to zero. Numerical simulations are carried out to illustrate the efficiency of our control strategy.
- There exist many ways to stabilize an infinite-dimensional linear autonomous control systems when it is possible. Anyway, finding an exponentially stabilizing feedback control that is as simple as possible may be a challenge. The Riccati theory provides a nice feedback control but may be computationally demanding when considering a discretization scheme. Proper Orthogonal Decomposition (POD) offers a popular way to reduce large-dimensional systems. In [59], we establish that, under appropriate spectral assumptions, an exponentially stabilizing feedback Riccati control designed from a POD finite-dimensional approximation of the system stabilizes as well the infinite-dimensional control system.
- In [60] we consider a 1-D linear transport equation on the interval $(0, L)$, with an internal scalar control. We prove that if the system is controllable in a periodic Sobolev space of order greater than 1, then the system can be stabilized in finite time, and we give an explicit feedback law.
- In [61] we use the backstepping method to study the stabilization of a 1-D linear transport equation on the interval $(0, L)$, by controlling the scalar amplitude of a piecewise regular function of the space

variable in the source term. We prove that if the system is controllable in a periodic Sobolev space of order greater than 1, then the system can be stabilized exponentially in that space and, for any given decay rate, we give an explicit feedback law that achieves that decay rate.

6.4. Controllability: new results

Let us list here our new results on controllability beyond the quantum control framework.

- In [13], we study approximate and exact controllability of linear difference equations using as a basic tool a representation formula for its solution in terms of the initial condition, the control, and some suitable matrix coefficients. When the delays are commensurable, approximate and exact controllability are equivalent and can be characterized by a Kalman criterion. The paper focuses on providing characterizations of approximate and exact controllability without the commensurability assumption. In the case of two-dimensional systems with two delays, we obtain an explicit characterization of approximate and exact controllability in terms of the parameters of the problem. In the general setting, we prove that approximate controllability from zero to constant states is equivalent to approximate controllability in L^2. The corresponding result for exact controllability is true at least for two-dimensional systems with two delays.
- In [14] we consider the 2D incompressible Navier-Stokes equation in a rectangle with the usual no-slip boundary condition prescribed on the upper and lower boundaries. We prove that for any positive time, for any finite energy initial data, there exist controls on the left and right boundaries and a distributed force, which can be chosen arbitrarily small in any Sobolev norm in space, such that the corresponding solution is at rest at the given final time. Our work improves earlier results where the distributed force is small only in a negative Sobolev space. It is a further step towards an answer to Jacques-Louis Lions' question about the small-time global exact boundary controllability of the Navier-Stokes equation with the no-slip boundary condition, for which no distributed force is allowed. Our analysis relies on the well-prepared dissipation method already used for Burgers and for Navier-Stokes in the case of the Navier slip-with-friction boundary condition. In order to handle the larger boundary layers associated with the no-slip boundary condition, we perform a preliminary regularization into analytic functions with arbitrarily large analytic radius and prove a long-time nonlinear Cauchy-Kovalevskaya estimate relying only on horizontal analyticity.
- We consider the wave equation on a closed Riemannian manifold. We observe the restriction of the solutions to a measurable subset ω along a time interval $[0,T]$ with $T>0$. It is well known that, if ω is open and if the pair (ω,T) satisfies the Geometric Control Condition then an observability inequality is satisfied, comparing the total energy of solutions to their energy localized in $\omega\times(0,T)$. The observability constant $C_T(\omega)$ is then defined as the infimum over the set of all nontrivial solutions of the wave equation of the ratio of localized energy of solutions over their total energy. In [17], we provide estimates of the observability constant based on a low/high frequency splitting procedure allowing us to derive general geometric conditions guaranteeing that the wave equation is observable on a measurable subset ω. We also establish that, as $T\to+\infty$, the ratio $C_T(\omega)/T$ converges to the minimum of two quantities: the first one is of a spectral nature and involves the Laplacian eigenfunctions, the second one is of a geometric nature and involves the average time spent in ω by Riemannian geodesics.
- In [22] we consider the problem of controlling parabolic semilinear equations arising in population dynamics, either in finite time or infinite time. These are the monostable and bistable equations on $(0,L)$ for a density of individuals $0\leq y(t,x)\leq 1$, with Dirichlet controls taking their values in $[0,1]$. We prove that the system can never be steered to extinction (steady state 0) or invasion (steady state 1) in finite time, but is asymptotically controllable to 1 independently of the size L, and to 0 if the length L of the interval domain is less than some threshold value L^*, which can be computed from transcendental integrals. In the bistable case, controlling to the other homogeneous steady state $0<\theta<1$ is much more intricate. We rely on a staircase control strategy to prove that θ can be reached in finite time if and only if $L<L^*$. The phase plane analysis of those equations

is instrumental in the whole process. It allows us to read obstacles to controllability, compute the threshold value for domain size as well as design the path of steady states for the control strategy.

- The paper [27] deals with the controllability problem of a linearized Korteweg-de Vries equation on bounded interval. The system has a homogeneous Dirichlet boundary condition and a homogeneous Neumann boundary condition at the right end-points of the interval, a non homogeneous Dirichlet boundary condition at the left end-point which is the control. We prove the null controllability by using a backstepping approach, a method usually used to handle stabilization problems.
- The paper [44] is devoted to the controllability of a general linear hyperbolic system in one space dimension using boundary controls on one side. Under precise and generic assumptions on the boundary conditions on the other side, we previously established the optimal time for the null and the exact controllability for this system for a generic source term. In this work, we prove the null-controllability for any time greater than the optimal time and for any source term. Similar results for the exact controllability are also discussed.
- Given any measurable subset ω of a closed Riemannian manifold and given any $T > 0$, we study in [49] the smallest average time over $[0, T]$ spent by all geodesic rays in ω. This quantity appears naturally when studying observability properties for the wave equation on M, with ω as an observation subset.
- Our goal is to study controllability and observability properties of the 1D heat equation with internal control (or observation) set an interval of size $\epsilon \to 0$. For any ϵ fixed, the heat equation is controllable in any time $T > 0$. It is known that depending on arithmetic properties of the center of the interval, there may exist a minimal time of pointwise control of the heat equation. We relate these two phenomena in [54].
- Our goal in [55] is to relate the observation (or control) of the wave equation on observation domains which evolve in time with some dynamical properties of the geodesic flow. In comparison to the case of static domains of observation, we show that the observability of the wave equation in any dimension of space can be improved by allowing the domain of observation to move.
- In [57] we consider the controllability problem for finite-dimensional linear autonomous control systems with nonnegative controls. Despite the Kalman condition, the unilateral nonnegativity control constraint may cause a positive minimal controllability time. When this happens, we prove that, if the matrix of the system has a real eigenvalue, then there is a minimal time control in the space of Radon measures, which consists of a finite sum of Dirac impulses. When all eigenvalues are real, this control is unique and the number of impulses is less than half the dimension of the space. We also focus on the control system corresponding to a finite-difference spatial discretization of the one-dimensional heat equation with Dirichlet boundary controls, and we provide numerical simulations.

Let us also mention t the book chapter [31], which has been published this year.

6.5. Optimal control: new results

Let us list here our new results in optimal control theory beyond the sub-Riemannian framework.

- In order to determine the optimal strategy to run a race on a curved track according to the lane number, we introduce in [7] a model based on differential equations for the velocity, the propulsive force and the anaerobic energy which takes into account the centrifugal force. This allows us to analyze numerically the different strategies according to the types of track since different designs of tracks lead to straights of different lengths. In particular, we find that the tracks with shorter straights lead to better performances, while the double bend track with the longest straight leads to the worst performances and the biggest difference between lanes. Then for a race with two runners, we introduce a psychological interaction: there is an attraction to follow someone just ahead, but after being overtaken, there is a delay before any benefit from this interaction occurs. We provide numerical simulations in different cases. Overall, the results agree with the IAAF rules for lane draws in competition, where the highest ranked athletes get the center lanes, the next ones the outside lanes, while the lowest ranked athletes get the inside lanes.

- Consider a general nonlinear optimal control problem in finite dimension, with constant state and/or control delays. By the Pontryagin Maximum Principle, any optimal trajectory is the projection of a Pontryagin extremal. In [11] we establish that, under appropriate assumptions, Pontryagin extremals depend continuously on the parameter delays, for adequate topologies. The proof of the continuity of the trajectory and of the control is quite easy, however, for the adjoint vector, the proof requires a much finer analysis. The continuity property of the adjoint with respect to the parameter delay opens a new perspective for the numerical implementation of indirect methods, such as the shooting method. We also discuss the sharpness of our assumptions.
- In [15] we are concerned about the controllability of a general linear hyperbolic system in one space dimension using boundary controls on one side. More precisely, we establish the optimal time for the null and exact controllability of the hyperbolic system under some is a generic setting. We also present examples which yield that the generic requirement is necessary. Our approach is based on the backstepping method paying a special attention on the construction of the kernel and the selection of controls.
- A new approach to estimate traffic energy consumption via traffic data aggregation in (speed,acceleration) probability distributions is proposed in [18]. The aggregation is done on each segment composing the road network. In order to reduce data occupancy, clustering techniques are used to obtain meaningful classes of traffic conditions. Different times of the day with similar speed patterns and traffic behavior are thus grouped together in a single cluster. Different energy consumption models based on the aggregated data are proposed to estimate the energy consumption of the vehicles in the road network. For validation purposes, a microscopic traffic simulator is used to generate the data and compare the estimated energy consumption to the reference one. A thorough sensitivity analysis with respect to the parameters of the proposed method (i.e. number of clusters, size of the distributions support, etc.) is also conducted in simulation. Finally, a real-life scenario using floating car data is analyzed to evaluate the applicability and the robustness of the proposed method.
- In [24] we consider a spectral optimal design problem involving the Neumann traces of the Dirichlet-Laplacian eigenfunctions on a smooth bounded open subset Ω of $\mathbf{R}^n$. The cost functional measures the amount of energy that Dirichlet eigenfunctions concentrate on the boundary and that can be recovered with a bounded density function. We first prove that, assuming a L^1 constraint on densities, the so-called *Rellich functions* maximize this functional.Motivated by several issues in shape optimization or observation theory where it is relevant to deal with bounded densities, and noticing that the L^∞-norm of *Rellich functions* may be large, depending on the shape of Ω, we analyze the effect of adding pointwise constraints when maximizing the same functional. We investigate the optimality of *bang-bang* functions and *Rellich densities* for this problem. We also deal with similar issues for a close problem, where the cost functional is replaced by a spectral approximation. Finally, this study is completed by the investigation of particular geometries and is illustrated by several numerical simulations.
- In [25] we consider the task of solving an aircraft trajectory optimization problem where the system dynamics have been estimated from recorded data. Additionally, we want to avoid optimized trajectories that go too far away from the domain occupied by the data, since the model validity is not guaranteed outside this region. This motivates the need for a proximity indicator between a given trajectory and a set of reference trajectories. In this presentation, we propose such an indicator based on a parametric estimator of the training set density. We then introduce it as a penalty term in the optimal control problem. Our approach is illustrated with an aircraft minimal consumption problem and recorded data from real flights. We observe in our numerical results the expected trade-off between the consumption and the penalty term.
- In [36] we study how bad can be the singularities of a time-optimal trajectory of a generic control

affine system. In the case where the control is scalar and belongs to a closed interval it was recently shown that singularities cannot be, generically, worse than finite order accumulations of Fuller points, with order of accumulation lower than a bound depending only on the dimension of the manifold where the system is set. We extend here such a result to the case where the control has an even number of scalar components and belongs to a closed ball.

- In [38] we develop a geometric analysis and a numerical algorithm, based on indirect methods, to solve optimal guidance of endo-atmospheric launch vehicle systems under mixed control-state constraints. Two main difficulties are addressed. First, we tackle the presence of Euler singularities by introducing a representation of the configuration manifold in appropriate local charts. In these local coordinates, not only the problem is free from Euler singularities but also it can be recast as an optimal control problem with only pure control constraints. The second issue concerns the initialization of the shooting method. We introduce a strategy which combines indirect methods with homotopies, thus providing high accuracy. We illustrate the efficiency of our approach by numerical simulations on missile interception problems under challenging scenarios.
- We introduce and study in [51] the turnpike property for time-varying shapes, within the viewpoint of optimal control. We focus here on second-order linear parabolic equations where the shape acts as a source term and we seek the optimal time-varying shape that minimizes a quadratic criterion. We first establish existence of optimal solutions under some appropriate sufficient conditions. We then provide necessary conditions for optimality in terms of adjoint equations and, using the concept of strict dissipativity, we prove that state and adjoint satisfy the measure-turnpike property, meaning that the extremal time-varying solution remains essentially close to the optimal solution of an associated static problem. We show that the optimal shape enjoys the exponential turnpike property in term of Hausdorff distance for a Mayer quadratic cost. We illustrate the turnpike phenomenon in optimal shape design with several numerical simulations.
- The work [52] proposes a new approach to optimize the consumption of a hybrid electric vehicle taking into account the traffic conditions. The method is based on a bi-level decomposition in order to make the implementation suitable for online use. The offline lower level computes cost maps thanks to a stochastic optimization that considers the influence of traffic, in terms of speed/acceleration probability distributions. At the online upper level, a deterministic optimization computes the ideal state of charge at the end of each road segment, using the computed cost maps. Since the high computational cost due to the uncertainty of traffic conditions has been managed at the lower level, the upper level is fast enough to be used online in the vehicle. Errors due to discretization and computation in the proposed algorithm have been studied. Finally, we present numerical simulations using actual traffic data, and compare the proposed bi-level method to a deterministic optimization with perfect information about traffic conditions. The solutions show a reasonable over-consumption compared with deterministic optimization, and manageable computational times for both the offline and online parts.
- An extension of the bi-level optimization for the energy management of hybrid electric vehicles (HEVs) proposed in [52] to the eco-routing problem is presented in [53]. Using the knowledge of traffic conditions over the entire road network, we search both the optimal path and state of charge trajectory. This problem results in finding the shortest path on a weighted graph whose nodes are (position, state of charge) pairs for the vehicle, the edge cost being evaluated thanks to the cost maps from optimization at the 'micro' level of a bi-level decomposition. The error due to the discretization of the state of charge is proven to be linear if the cost maps are Lipschitz. The classical A^* algorithm is used to solve the problem, with a heuristic based on a lower bound of the energy needed to complete the travel. The eco-routing method is validated by numerical simulations and compared to the fastest path on a synthetic road network.
- In [58] we study a driftless system on a three-dimensional manifold driven by two scalar controls. We assume that each scalar control has an independent bound on its modulus and we prove that, locally around every point where the controlled vector fields satisfy some suitable nondegeneracy Lie bracket condition, every time-optimal trajectory has at most five bang or singular arcs. The result

is obtained using first-and second-order necessary conditions for optimality.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

Contract CIFRE with ArianeGroup (les Mureaux), 2019–2021, funding the thesis of A. Nayet. Participants : M. Cerf (ArianeGroup), E. Trélat (coordinator).

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. ANR

- ANR SRGI, for *Sub-Riemannian Geometry and Interactions*, coordinated by **Emmanuel Trélat**, started in 2015 and runs until 2020. Other partners: Toulon University and Grenoble University. SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.
- ANR Finite4SoS, for *Commande et estimation en temps fini pour les Systèmes de Systèmes*, coordinated by Wilfrid Perruquetti, started in 2015 and run up to this year. Other partners: Inria Lille, CAOR - ARMINES. Finite4SoS aims at developing a new promising framework to address control and estimation issues of Systems of Systems subject to model diversity, while achieving robustness as well as severe time response constraints.
- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and runs until 2021. Other partners: Lorraine University. QUACO aims at contributing to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improve the efficiency of control algorithms for MRI.

8.2. European Initiatives

8.2.1. FP7 & H2020 Projects

Program: H2020-EU.1.3.1. - Fostering new skills by means of excellent initial training of researchers

Call for proposal: MSCA-ITN-2017 - Innovative Training Networks

Project acronym: QUSCO

Project title: Quantum-enhanced Sensing via Quantum Control

Duration: From November 2017 to October 2021.

Coordinator: Christiane Koch

Coordinator for the participant Inria: Ugo Boscain

Abstract: Quantum technologies aim to exploit quantum coherence and entanglement, the two essential elements of quantum physics. Successful implementation of quantum technologies faces the challenge to preserve the relevant nonclassical features at the level of device operation. It is thus deeply linked to the ability to control open quantum systems. The currently closest to market quantum technologies are quantum communication and quantum sensing. The latter holds the promise of reaching unprecedented sensitivity, with the potential to revolutionize medical imaging or structure determination in biology or the controlled construction of novel quantum materials. Quantum control manipulates dynamical processes at the atomic or molecular scale by means of specially tailored external electromagnetic fields. The purpose of QuSCo is to demonstrate the enabling capability of quantum control for quantum sensing and quantum measurement, advancing this field by systematic use of quantum control methods. QuSCo will establish quantum control as a vital part for progress in quantum technologies. QuSCo will expose its students, at the same time, to fundamental questions of quantum mechanics and practical issues of specific applications. Albeit challenging, this reflects our view of the best possible training that the field of quantum technologies can offer. Training in scientific skills is based on the demonstrated tradition of excellence in research of the consortium. It will be complemented by training in communication and commercialization. The latter builds on strong industry participation whereas the former existing expertise on visualization and gamification and combines it with more traditional means of outreach to realize target audience specific public engagement strategies.

8.3. International Research Visitors

8.3.1. Internships

Rosa Kowalewski made an internship under the supervision of Barbara Gris from January to May 2019.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events: Organisation

9.1.1.1. Member of the Organizing Committees

- Ugo Boscain, Jean-Michel Coron, and Mario Sigalotti organized the workshop “Quantum day: analysis and control”, December 16, Paris
- Ugo Boscain and Emmanuel Trélat organized the minisymposium “Degenerate diffusion processes and their control” at Equadiff 2019, July 8-12, Leiden, The Netherlands
- Ugo Boscain organized (with Aleksey Kostenko and Konstantin Pankrashkin) the workshop “Self-adjoint Extensions in New Settings”, October 6–12, Mathematisches Forschungsinstitut Oberwolfach, Germany
- Barbara Gris organized the minisymposium “Analyse des formes anatomiques” at Smai, May 13–17, Guidel Plages (Morbihan)
- Barbara Gris organized the workshop “Shape analysis in biology”, November 21–22, Paris
- Emmanuel Trélat organized (with Roland Herzog) the workshop “New Trends in PDE-constrained Optimization”, October 14–18, RICAM, Linz, Austria

9.1.2. Scientific Events: Selection

9.1.2.1. Member of the Conference Program Committees

Emmanuel Trélat was member of the Conference Program Committee of the SIAM Conference on Control and its Applications (CT 2019), Chengdu, China.

9.1.3. Journal

9.1.3.1. Member of the Editorial Boards

- Ugo Boscain is Associate editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Member of the editorial board of Journal of Evolution Equations
- Jean-Michel Coron is Member of the editorial board of Asymptotic Analysis
- Jean-Michel Coron is Member of the editorial board of ESAIM : Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Member of the editorial board of Applied Mathematics Research Express
- Jean-Michel Coron is Member of the editorial board of Advances in Differential Equations
- Jean-Michel Coron is Member of the editorial board of Math. Control Signals Systems
- Jean-Michel Coron is Member of the editorial board of Annales de l'IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of ESAIM : Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM : Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Syst. Cont. Letters
- Emmanuel Trélat is Associate editor of J. Dynam. Cont. Syst.
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM Math. Modelling Num. Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of J. Optim. Theory Appl.
- Emmanuel Trélat is Associate editor of Math. Control Related fields

9.1.4. Invited Talks

- Ugo Boscain was invited speaker at the School and Workshop "Random Matrix Theory and Point Processes", Trieste, Italy, September.
- Barbara Gris was invited speaker at "Young researchers Imaging Seminars", IHP, Paris, February.
- Barbara Gris was invited speaker at GDR MAMOVI, Tours, February.
- Barbara Gris was invited speaker at "Information Geometry", Toulouse, October.
- Emmanuel Trélat was plenary speaker at "Equadiff 2019", Leiden, The Netherlands, July.
- Emmanuel Trélat was invited speaker at "Colloquium du Laboratoire de Mathématiques d'Avignon", Mars.
- Emmanuel Trélat was invited speaker at "DEA 2019", Krakow, Poland, September.
- Emmanuel Trélat was invited speaker at "Quantization in Symplectic Geometry", Cologne, Germany, July.
- Emmanuel Trélat was invited speaker at "Mathematical Models and Methods in Earth and Space Science", Rome, Italy, March.
- Emmanuel Trélat was invited speaker at "Sub-Riemannian Geometry and beyond", Jyväskylä, Finland, February.

9.1.5. Leadership within the Scientific Community

Emmanuel Trélat was director of the Fondation Sciences Mathématiques de Paris (FSMP) until June.

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

- Ugo Boscain thought “Controllability of Closed Quantum Systems” to PhD students at the first QUSCO School, Saarbrucken, Germany.
- Ugo Boscain thought “Automatic Control” (with Mazyar Mirrahimi) at Ecole Polytechnique
- Ugo Boscain thought “MODAL of applied mathematics. Contrôle de modèles dynamiques” at Ecole Polytechnique
- Ugo Boscain thought “Control theory and applications in Quantum Mechanics” to PhD students at SISSA, Trieste, Italy
- Mario Sigalotti thought “Geometric Control Theory” to PhD students at “29th Jyväskylä Summer School”, Jyväskylä, Finland
- Mario Sigalotti thought “Équations d’évolution, stabilité et contrôle” to M1 students at Sorbonne Université
- Emmanuel Trélat thought “Control in finite and infinite dimension” to M2 students at Sorbonne Université

9.2.2. Supervision

PhD: Nicolas Augier, “Contrôle adiabatique des systèmes quantiques", September 2019, supervisors: Ugo Boscain, Mario Sigalotti.

PhD: Amaury Hayat, “ Stabilisation de systèmes hyperboliques non-linéaires en dimension un d’espace", May 2019, supervisors: Jean-Michel Coron and Sébastien Boyaval

PhD: Mathieu Kohli, “On the notion of geodesic curvature in sub-Riemannian geometry", September 2019, supervisors: Davide Barilari, Ugo Boscain.

PhD: Jakub Orłowski, “Adaptive control of time-delay systems to counteract pathological brain oscillations", December 2019, supervisors: Antoine Chaillet, and Mario Sigalotti.

PhD: Shengquan Xiang, “Stabilisation rapide d’équations de Burgers et de Korteweg-de Vrie”, June 2019, supervisor: Jean-Michel Coron.

PhD: Christophe Zhang, “Contrôle et stabilisation internes de systèmes hyperboliques 1-D”, October 2019, supervisor: Jean-Michel Coron.

PhD in progress: Gontran Lance, started in September 2018, supervisors: Emmanuel Trélat and Enrique Zuazua.

PhD in progress: Cyril Letrouit, “Équation des ondes sous-riemanniennes", started in September 2019, supervisor Emmanuel Trélat.

PhD in progress: Emilio Molina, “Application of optimal control techniques to natural resources management”, started in September 2018, supervisors: Pierre Martinon, Héctor Ramírez, and Mario Sigalotti.

PhD in progress: Eugenio Pozzoli, “Adiabatic Control of Open Quantum Systems", started in September 2018, supervisors: Ugo Boscain and Mario Sigalotti.

PhD in progress: Rémi Robin, “Orbit spaces of Lie groups and applications to quantum control", started in September 2019, supervisors: Ugo Boscain and Mario Sigalotti.

9.2.3. Juries

- Ugo Boscain and Mario Sigalotti were supervisors and members of the jury of the PhD thesis of Nicolas Augier, Université Paris Saclay
- Ugo Boscain was co-supervisor and member of the jury of the PhD thesis of Mathieu Kohli, Université Paris Saclay

- Jean-Michel Coron was supervisor and member of the jury of the PhD thesis of Christophe Zhang, Sorbonne Université.
- Jean-Michel Coron was supervisor and member of the jury of the PhD thesis of Shengquan Xiang, Sorbonne Université.
- Mario Sigalotti was opponent of the PhD thesis of Eero Hakavuori, Jyväskylä University, Finland
- Mario Sigalotti was president and member of the jury of the PhD thesis of Gerardo Cardona, École des Mines, Paris.
- Emmanuel Trélat was referee and member of the jury of the HDR of Patrick Martinez, Université de Toulouse.
- Emmanuel Trélat was member of the jury of the HDR of Max Cerf, Sorbonne Université.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of Mathieu Granzotto, Université de Lorraine.
- Emmanuel Trélat was member of the jury of the PhD thesis of Christophe Zhang, Sorbonne Université.
- Emmanuel Trélat was president and member of the jury of the PhD thesis of Armand Koenig, Université de Nice.
- Emmanuel Trélat was member of the jury of the PhD thesis of Shengquan Xiang, Sorbonne Université.
- Emmanuel Trélat was member of the jury of the PhD thesis of Nicolas Hegoburu, Université de Bordeaux.

9.3. Popularization

9.3.1. Internal or external Inria responsibilities

Emmanuel Trélat is member of the Comité d'Honneur du Comité International des Jeux Mathématiques

9.3.2. Articles and contents

- The work [7] by Pierre Martinon has been popularized in the article "Athlétisme, affaire Dreyfus, itinéraire des éboueurs... Comment les maths irriguent notre monde" by Soline Roy, appeared in *Le Figaro*, 1/1/2019.
- Emmanuel Trélat has been interviewed, as director of the Fondation Sciences Mathématiques de Paris, in *Le Figaro*, 2/1/2019.

9.3.3. Interventions

- Emmanuel Trélat gave a cycle of lectures at "Sciences et Société", Nancy
- Emmanuel Trélat gave a lecture at Bibliothèque Nationale de France (BNF) (https://vimeo.com/313123529)
- Emmanuel Trélat gave a general public lecture at "Mois de l'optimisation", Limoges
- Emmanuel Trélat gave a general public lecture in Padua, Italy

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[2] A. HAYAT. *Stabilization of 1D nonlinear hyperbolic systems by boundary controls*, Sorbonne Université , UPMC, May 2019, https://tel.archives-ouvertes.fr/tel-02274457

[3] M. KOHLI. *On the notion of geodesic curvature in sub-Riemannian geometry*, Université Paris-Saclay, September 2019, https://tel.archives-ouvertes.fr/tel-02325081

[4] J. ORŁOWSKI. *Adaptive control of time-delay systems to counteract pathological brain oscillations*, Université Paris-Saclay (ComUE), December 2019

[5] S. XIANG. *Stabilisation rapide d'e´quations de Burgers et de Korteweg-de Vrie*, Sorbonne Université, June 2019

[6] C. ZHANG. *Internal control and stabilization of some 1-D hyperbolic systems*, Sorbonne Université, October 2019, https://tel.archives-ouvertes.fr/tel-02464011

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[7] A. AFTALION, P. MARTINON. *Optimizing running a race on a curved track*, in "PLoS ONE", September 2019, vol. 14, n° 9, 23 p. , https://arxiv.org/abs/1811.12321 [*DOI :* 10.1371/JOURNAL.PONE.0221572], https://hal.inria.fr/hal-01936993

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