Activity Report 2019

Project-Team COMMANDS

Control, Optimization, Models, Methods and Applications for Nonlinear Dynamical Systems

IN COLLABORATION WITH: Centre de Mathématiques Appliquées (CMAP)

RESEARCH CENTER
Saclay - Île-de-France

THEME
Optimization and control of dynamic systems

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Project-Team COMMANDS

Creation of the Project-Team: 2009 January 01

Keywords:

Computer Science and Digital Science:

A6.2.1. - Numerical analysis of PDE and ODE
A6.2.6. - Optimization
A6.2.7. - High performance computing
A6.3.2. - Data assimilation
A6.4.1. - Deterministic control
A6.4.2. - Stochastic control

Other Research Topics and Application Domains:

B4.4. - Energy delivery
B4.4.1. - Smart grids
B7.1.2. - Road traffic
B7.1.3. - Air traffic
B7.2.1. - Smart vehicles

1. Team, Visitors, External Collaborators

Research Scientists

Joseph Frederic Bonnans [Team leader, Inria, Senior Researcher, HDR]
Laurent Pfeiffer [Inria, Researcher, from Oct 2019]

Post-Doctoral Fellows

Justina Gianatti [Inria, Post-Doctoral Fellow, until Apr 2019]
Saeed Hadikhanloo [Inria, Post-Doctoral Fellow, until Jan 2019]
Davin Glen Lunz [Inria, Post-Doctoral Fellow, from Nov 2019]

PhD Students

Guillaume Bonnet [Univ Paris-Sud, PhD Student]
Pierre Lavigne [École polytechnique, PhD Student]
Arthur Le Rhun [IFPEN, PhD Student]

Technical staff

Hélèna Kutniak [Inria, Engineer, until Jan 2019]

Administrative Assistants

Ines Dumontier [Inria, Administrative Assistant, until May 2019]
Hanadi Dib [Inria, Administrative Assistant,since May 2019]

External Collaborators

Axel Kröner [U. Humboldt, until Mar 2019]
Laurent Pfeiffer [U. Graz, from Jun 2019 until Sep 2019]

2. Overall Objectives

2.1. Scientific directions

Commands is a team devoted to dynamic optimization, both for deterministic and stochastic systems. This includes the following approaches: trajectory optimization, deterministic and stochastic optimal control, stochastic programming, dynamic programming and Hamilton-Jacobi-Bellman equation.

Our aim is to derive new and powerful algorithms for solving numerically these problems, with applications in several industrial fields. While the numerical aspects are the core of our approach it happens that the study of convergence of these algorithms and the verification of their well-posedness and accuracy raises interesting and difficult theoretical questions, such as, for trajectory optimization: qualification conditions and second-order optimality condition, well-posedness of the shooting algorithm, estimates for discretization errors; for the Hamilton-Jacobi-Bellman approach: accuracy estimates, strong uniqueness principles when state constraints are present, for stochastic programming problems: sensitivity analysis.

2.2. Industrial impact

For many years the team members have been deeply involved in various industrial applications, often in the framework of PhD theses. The Commands team itself has dealt since its foundation in 2009 with several types of applications:

- Space vehicle trajectories, in collaboration with CNES, the French space agency.
- Aeronautics, in collaboration with the startup Safety Line.
- Production, management, storage and trading of energy resources, in collaboration with Edf, ex-Gdf and Total.
- Energy management for hybrid vehicles, in collaboration with Renault and Ifpen.

We give more details in the Bilateral contracts section.

3. Research Program

3.1. Historical aspects

The roots of deterministic optimal control are the “classical” theory of the calculus of variations, illustrated by the work of Newton, Bernoulli, Euler, and Lagrange (whose famous multipliers were introduced in [24]), with improvements due to the “Chicago school”, Bliss [16] during the first part of the 20th century, and by the notion of relaxed problem and generalized solution (Young [29]).

Trajectory optimization really started with the spectacular achievement done by Pontryagin’s group [28] during the fifties, by stating, for general optimal control problems, nonlocal optimality conditions generalizing those of Weierstrass. This motivated the application to many industrial problems (see the classical books by Bryson and Ho [20], Leitmann [26], Lee and Markus [25], Ioffe and Tihomirov [23]).

Dynamic programming was introduced and systematically studied by R. Bellman during the fifties. The HJB equation, whose solution is the value function of the (parameterized) optimal control problem, is a variant of the classical Hamilton-Jacobi equation of mechanics for the case of dynamics parameterized by a control variable. It may be viewed as a differential form of the dynamic programming principle. This nonlinear first-order PDE appears to be well-posed in the framework of *viscosity solutions* introduced by Crandall and Lions [21]. The theoretical contributions in this direction did not cease growing, see the books by Barles [14] and Bardi and Capuzzo-Dolcetta [13].

3.2. Trajectory optimization

The so-called *direct methods* consist in an optimization of the trajectory, after having discretized time, by a nonlinear programming solver that possibly takes into account the dynamic structure. So the two main problems are the choice of the discretization and the nonlinear programming algorithm. A third problem is the possibility of refinement of the discretization once after solving on a coarser grid.

In the *full discretization approach*, general Runge-Kutta schemes with different values of control for each inner step are used. This allows to obtain and control high orders of precision, see Hager [22], Bonnans [17]. In the *indirect* approach, the control is eliminated thanks to Pontryagin's maximum principle. One has then to solve the two-points boundary value problem (with differential variables state and costate) by a single or multiple shooting method. The questions are here the choice of a discretization scheme for the integration of the boundary value problem, of a (possibly globalized) Newton type algorithm for solving the resulting finite dimensional problem in IR^n (n is the number of state variables), and a methodology for finding an initial point.

3.3. Hamilton-Jacobi-Bellman approach

This approach consists in calculating the value function associated with the optimal control problem, and then synthesizing the feedback control and the optimal trajectory using Pontryagin's principle. The method has the great particular advantage of reaching directly the global optimum, which can be very interesting when the problem is not convex.

Optimal stochastic control problems occur when the dynamical system is uncertain. A decision typically has to be taken at each time, while realizations of future events are unknown (but some information is given on their distribution of probabilities). In particular, problems of economic nature deal with large uncertainties (on prices, production and demand). Specific examples are the portfolio selection problems in a market with risky and non-risky assets, super-replication with uncertain volatility, management of power resources (dams, gas). Air traffic control is another example of such problems.

For solving stochastic control problems, we studied the so-called Generalized Finite Differences (GFD), that allow to choose at any node, the stencil approximating the diffusion matrix up to a certain threshold [19]. Determining the stencil and the associated coefficients boils down to a quadratic program to be solved at each point of the grid, and for each control. This is definitely expensive, with the exception of special structures where the coefficients can be computed at low cost. For two dimensional systems, we designed a (very) fast algorithm for computing the coefficients of the GFD scheme, based on the Stern-Brocot tree [18].

4. Application Domains

4.1. Energy management for hybrid vehicles

In collaboraton with Ifpen and in the framework of A. Le Rhun's thesis, we have developped a methodology for the optimal energy management for hybrid vehicles, based on a statistical analysis of the traffic. See [12], [12], [7].

4.2. Biological cells culture

In collaboration with the Inbio team (Inst. Pasteur and Inria) we started to study the optimization of protein production based on cell culture.

5. Highlights of the Year

5.1. Highlights of the Year

We have now a strong involvement in the study of mean-field games (MFG) and their application to distributed energy production problems. In the paper [3] we study MFG equilibria with coupling of the agents through a price function (see more in the 'New Results' section). In the framework of the PhD of Pierre Lavigne we currently study discrete-time models with risk-averse agents. Both directions take advantange of the recent recruitment of Laurent Pfeiffer as “chargé de recherche”, and of a starting collaboration with Jameson Graber (Baylor University, Texas).

6. New Software and Platforms

6.1. BOCOP

Boite à Outils pour le Contrôle OPtimal

KEYWORDS: Dynamic Optimization - Identification - Biology - Numerical optimization - Energy management - Transportation

FUNCTIONAL DESCRIPTION: Bocop is an open-source toolbox for solving optimal control problems, with collaborations with industrial and academic partners. Optimal control (optimization of dynamical systems governed by differential equations) has numerous applications in transportation, energy, process optimization, energy and biology. Bocop includes a module for parameter identification and a graphical interface, and runs under Linux / Windows / Mac.

RELEASE FUNCTIONAL DESCRIPTION: Handling of delay systems Alternate automatic differentiation tool: CppAD Update for CMake and MinGW (windows version)

- Participants: Benjamin Heymann, Virgile Andréani, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Pierre Martinon
- URL: http://bocop.org

6.2. Bocop HJB

KEYWORDS: Optimal control - Stochastic optimization - Global optimization

FUNCTIONAL DESCRIPTION: Toolbox for stochastic or deterministic optimal control, dynamic programming / HJB approach.

RELEASE FUNCTIONAL DESCRIPTION: User interface State jumps for switched systems Explicit handling of final conditions Computation of state probability density (fiste step to mean field games)

- Participants: Benjamin Heymann, Jinyan Liu, Joseph Frédéric Bonnans and Pierre Martinon
- Contact: Joseph Frédéric Bonnans
- URL: http://bocop.org

6.3. Bocop Avion

KEYWORDS: Optimization - Aeronautics

FUNCTIONAL DESCRIPTION: Optimize the climb speeds and associated fuel consumption for the flight planning of civil airplanes.

NEWS OF THE YEAR: Improved atmosphere model 2D interpolations for temperature and wind data

- Participants: Gregorutti Baptiste, Cindie Andrieu, Anamaria Lupu, Joseph Frédéric Bonnans, Karim Tekkal, Pierre Jouniaux and Pierre Martinon
- Partner: Safety Line
- Contact: Pierre Martinon
- URL: http://www.safety-line.fr

6.4. Bocop HJB Avion

KEYWORDS: Optimization - Aeronautics

FUNCTIONAL DESCRIPTION: Optimize the climb and cruising trajectory of flight by a HJB approach.

NEWS OF THE YEAR: First demonstrator for cruise flight deployed at Safety Line

- Participants: Pierre Martinon, Joseph Frédéric Bonnans, Jinyan Liu, Gregorutti Baptiste and Anamaria Lupu
- Partner: Safety Line
- Contact: Pierre Martinon
- URL: http://www.safety-line.fr

7. New Results

7.1. Stochastic control and HJB equations

7.1.1. Monotone and second order consistent schemes for the Pucci and Monge-Ampere equations

In [9] we introduce a new strategy for the design of second-order accurate discretiza-tions of non-linear second order operators of Bellman type, which preserves degenerate ellipticity. The approach relies on Selling's formula, a tool from lattice geometry, and is applied to the Pucci and Monge-Ampere equations, discretized on a two dimensional cartesian grid. In the case of the Monge-Ampere equation, our work is related to both the stable formulation and the second order accurate scheme. Numerical experiments illustrate the robustness and the accuracy of the method.

7.1.2. Mean-field games of control

In [3], an existence result for a class of mean field games of controls is provided. In the considered model, the cost functional to be minimized by each agent involves a price depending at a given time on the controls of all agents and a congestion term. The existence of a classical solution is demonstrated with the Leray–Schauder theorem; the proof relies in particular on a priori bounds for the solution, which are obtained with the help of a potential formulation of the problem.

7.2. Optimal control of PDEs

7.2.1. Optimal Control of an Age-Structured System with State Constraints

In [10] we study an optimal control problem with state constraints where the state is given by an age-structured, abstract parabolic differential equation. We prove the existence and uniqueness of solution for the state equation and provide first and second parabolic estimates. We analyze the differentiability of the cost function and, based on the general theory of Lagrange multipliers, we give a first order optimality condition. We also define and analyze the regularity of the costate. Finally, we present a pregnancy model, where two coupled age-structured equations are involved, and we apply the obtained results to this case.

7.2.2. Feedback laws

The articles [4], [5], [6], co-written by L. Pfeiffer in the framework of his former position at the University of Graz, deal with the computation of feedback laws for stabilization problems of PDE systems. These problems are formulated as infinite-horizon optimal control problems.

In [5], we prove that the value function associated with bilinear stabilization problems (including some control problems of the Fokker-Planck equation) can be expanded as a Taylor expansion, where the second-order term is the solution to an algebraic Riccati equation and where the terms of order three and more are solutions to well-posed linear equations. These equations are obtained by successive differentiation of the HJB equation. A polynomial feedback law can be deduced from the Taylor approximation and its efficiency is analyzed. This approach generalizes the classical LQR-stabilization method.

In [4], we apply the methodology previously described to a stabilization problem of the 2D Navier-Stokes equation. Numerical results are provided.

In [6], we analyze an implementation of the Receding-Horizon Control method utilizing the Taylor expansion of the value function as a terminal cost. More precisely, we show that the method converges at an exponential rate with respect to the prediction horizon and the degree of the Taylor approximation.

7.3. Energy management for hybrid vehicles

7.3.1. A stochastic data-based traffic model applied to vehicles energy consumption estimation

In [7], a new approach to estimate traffic energy consumption via traffic data aggregation in (speed,acceleration) probability distributions is proposed. The aggregation is done on each segment composing the road network. In order to reduce data occupancy, clustering techniques are used to obtain meaningful classes of traffic conditions. Different times of the day with similar speed patterns and traffic behavior are thus grouped together in a single cluster. Different energy consumption models based on the aggregated data are proposed to estimate the energy consumption of the vehicles in the road network. For validation purposes, a microscopic traffic simulator is used to generate the data and compare the estimated energy consumption to the reference one. A thorough sensitivity analysis with respect to the parameters of the proposed method (i.e. number of clusters, size of the distributions support, etc.) is also conducted in simulation. Finally, a real-life scenario using floating car data is analyzed to evaluate the applicability and the robustness of the proposed method.

7.3.2. A bi-level energy management strategy for HEVs under probabilistic traffic conditions

In [11], we propose a new approach to optimize the consumption of a hybrid electric vehicle taking into account the traffic conditions. The method is based on a bi-level decomposition in order to make the implementation suitable for online use. The offline lower level computes cost maps thanks to a stochastic optimization that considers the influence of traffic, in terms of speed/acceleration probability distributions. At the online upper level, a deterministic optimization computes the ideal state of charge at the end of each road segment, using the computed cost maps. Since the high computational cost due to the uncertainty of traffic conditions has been managed at the lower level, the upper level is fast enough to be used online in the vehicle. Errors due to discretization and computation in the proposed algorithm have been studied. Finally, we present numerical simulations using actual traffic data, and compare the proposed bi-level method to a deterministic optimization with perfect information about traffic conditions. The solutions show a reasonable over-consumption compared with deterministic optimization, and manageable computational times for both the offline and online parts.

7.3.3. An Eco-routing algorithm for HEVs under traffic conditions

In [12], an extension of the bi-level optimization for the energy management of hybrid electric vehicles (HEVs) proposed above to the eco-routing problem is presented. Using the knowledge of traffic conditions over the entire road network, we search both the optimal path and state of charge trajectory. This problem results in finding the shortest path on a weighted graph whose nodes are (position, state of charge) pairs for the vehicle, the edge cost being evaluated thanks to the cost maps from optimization at the 'micro' level of a bi-level decomposition. The error due to the discretization of the state of charge is proven to be linear if the cost maps are Lipschitz. The classical $A*$ algorithm is used to solve the problem, with a heuristic based on a lower bound of the energy needed to complete the travel. The eco-routing method is validated by numerical simulations and compared to the fastest path on a synthetic road network.

8. Partnerships and Cooperations

8.1. National Initiatives

8.1.1. IPL

8.1.1.1. Algae in Silico

Inria Project Lab ALGAE IN SILICO (2014-2018) was dedicated to provide an integrated platform for numerical simulation of microalgae “from genes to industrial process“. Commands joined the project in 2017 to tackle the optimization aspects. Our previous collaborations with teams Modemic and Biocore on bioreactors [27], [15] have been renewed in this framework.

8.1.1.2. Cosy

Inria Project Lab COSY (started in 2017) aims at exploiting the potential of state-of-art biological modelling, control techniques, synthetic biology and experimental equipment to achieve a paradigm shift in control of microbial communities. More precisely, we plan to determine and implement control strategies to make heterogeneous communities diversify and interact in the most profitable manner. Study of yeast cells has started in collaboration with team Lifeware (G. Batt) in the framework of the PhD of V. Andreani, and is pursued in the Postdoc of D. Lunz (started Nov. 2019).

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events: Selection

9.1.1.1. Member of the Conference Program Committees

- F. Bonnans: PGMO Days, EDF’Lab Palaiseau,Dec. 3-4, 2019.

9.1.2. Journal

9.1.2.1. Member of the Editorial Boards

- F. Bonnans: Associate Editor: Math. & Appl. / Annals of the Academy of Romanian Scientists (AOSR)

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master :
F. Bonnans: *Numerical analysis of partial differential equations arising in finance and stochastic control*, 18h, M2, Ecole Polytechnique and U. Paris 6, France.
F. Bonnans: *Optimal control of ordinary differential equations*, 15h, M2, Optimization master (U. Paris-Saclay) and Ensta, France.
A. Kröner : Optimal control of partial differential equations, 20h, M2, Optimization master (U. Paris-Saclay), France.
L. Pfeiffer: *Optimal control of ordinary differential equations*, 18h, M2, Optimization master (U. Paris-Saclay) and Ensta, France.
L. Pfeiffer: *Optimisation continue et combinatoire*, 17h, Ensta, France.

9.2.2. Supervision

Finished PhD : A. Le Rhun, Optimal and robust control of hybrid vehicles. Started September 2016 (IFPEN fellowship), finished December 2019, F. Bonnans and P. Martinon.

PhD in progress : G. Bonnet, Efficient schemes for the Hamilton-Jacobi-Bellman equation. Started Oct. 2018. F. Bonnans and J.-M. Mirebeau, LMO, U. Orsay.

PhD in progress : P. Lavigne, Mathematical study of economic equilibria for renewable energy sources. Started Oct. 2018. F. Bonnans and L. Pfeiffer.

9.3. Popularization

- F. Bonnans: codirection of a joint Allistene-Ancre commission (contribution to the national strategy for research), Numerics and Energy committee (2017-2019).
- F. Bonnans: Dimitrie Pompeiu Prize Committee (Academy of Romanian Scientists).

10. Bibliography

Publications of the year

Articles in International Peer-Reviewed Journals

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Other Publications

[9] J. F. BONNANS, G. BONNET, J.-M. MIREBEAU. *Monotone and second order consistent schemes for the Pucci and Monge-Ampere equations*, November 2019, working paper or preprint, https://hal.archives-ouvertes.fr/hal-02383521

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