

Inria

IN PARTNERSHIP WITH:
Université de Lorraine

Activity Report 2019

Project-Team GAMBLE

Geometric Algorithms & Models Beyond the
Linear & Euclidean realm

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

RESEARCH CENTER
Nancy - Grand Est

THEME
**Algorithmics, Computer Algebra and
Cryptology**

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Project-Team GAMBLE

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A8.4. - Computer Algebra

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B1.2.3. - Computational neurosciences
B2.6. - Biological and medical imaging
B3.3. - Geosciences
B5.5. - Materials
B5.6. - Robotic systems
B5.7. - 3D printing
B6.2.2. - Radio technology

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2. Overall Objectives

2.1. Overall Objectives

Starting in the eighties, the emerging computational geometry community has put a lot of effort into designing and analyzing algorithms for geometric problems. The most commonly used framework was to study the worst-case theoretical complexity of geometric problems involving linear objects (points, lines, polyhedra...) in Euclidean spaces. This so-called *classical computational geometry* has some known limitations:

- Objects: dealing with objects only defined by linear equations.
- Ambient space: considering only Euclidean spaces.
- Complexity: worst-case complexities often do not capture realistic behaviour.
- Dimension: complexities are often exponential in the dimension.
- Robustness: ignoring degeneracies and rounding errors.

Even if these limitations have already got some attention from the community [44], a quick look at the flagship conference SoCG¹ proceedings shows that these topics still need a big effort.

It should be stressed that, in this document, the notion of certified algorithms is to be understood with respect to robustness issues. In other words, certification does not refer to programs that are proven correct with the help of mechanical proof assistants such as Coq, but to algorithms that are proven correct on paper even in the presence of degeneracies and computer-induced numerical rounding errors.

We address several of the above limitations:

• **Non-linear computational geometry.** Curved objects are ubiquitous in the world we live in. However, despite this ubiquity and decades of research in several communities, curved objects are far from being robustly and efficiently manipulated by geometric algorithms. Our work on, for instance, quadric intersections and certified drawing of plane curves has proven that dramatic improvements can be accomplished when the right mathematics and computer science concepts are put into motion. In this direction, many problems are fundamental and solutions have potential industrial impact in Computer Aided Design and Robotics for instance. Intersecting NURBS (Non-uniform rational basis splines) and meshing singular surfaces in a certified manner are important examples of such problems.

• **Non-Euclidean computational geometry.** Triangulations are central geometric data structures in many areas of science and engineering. Traditionally, their study has been limited to the Euclidean setting. Needs for triangulations in non-Euclidean settings have emerged in many areas dealing with objects whose sizes range from the nuclear to the astrophysical scale, and both in academia and in industry. It has become timely to extend the traditional focus on \mathbb{R}^d of computational geometry and encompass non-Euclidean spaces.

¹Symposium on Computational Geometry. <http://www.computational-geometry.org/>.

- **Probability in computational geometry.** The design of efficient algorithms is driven by the analysis of their complexity. Traditionally, worst-case input and sometimes uniform distributions are considered and many results in these settings have had a great influence on the domain. Nowadays, it is necessary to be more subtle and to prove new results in between these two extreme settings. For instance, smoothed analysis, which was introduced for the simplex algorithm and which we applied successfully to convex hulls, proves that such promising alternatives exist.

- **Discrete geometric structures.** Many geometric algorithms work, explicitly or implicitly, over discrete structures such as graphs, hypergraphs, lattices that are induced by the geometric input data. For example, convex hulls or straight-line graph drawing are essentially based on orientation predicates, and therefore operate on the so-called *order type* of the input point set. Order types are a subclass of oriented matroids that remains poorly understood: for instance, we do not even know how to sample this space with reasonable bias. One of our goals is to contribute to the development of these foundations by better understanding these discrete geometric structures.

3. Research Program

3.1. Non-linear computational geometry



Figure 1. Two views of the Whitney umbrella (on the left, the “stick” of the umbrella, i.e., the negative z -axis, is missing). Right picture from [\[Wikipedia\]](#), left picture from [\[Lachaud et al.\]](#).

As mentioned above, curved objects are ubiquitous in real world problems and in computer science and, despite this fact, there are very few problems on curved objects that admit robust and efficient algorithmic solutions without first discretizing the curved objects into meshes. Meshing curved objects induces a loss of accuracy which is sometimes not an issue but which can also be most problematic depending on the application. In addition, discretization induces a combinatorial explosion which could cause a loss in efficiency compared to a direct solution on the curved objects (as our work on quadrics has demonstrated with flying colors [\[50\]](#), [\[51\]](#), [\[52\]](#), [\[54\]](#), [\[58\]](#)). But it is also crucial to know that even the process of computing meshes that approximate curved objects is far from being resolved. As a matter of fact there is no algorithm capable of computing in practice meshes with certified topology of even rather simple singular 3D surfaces, due to the high constants in the theoretical complexity and the difficulty of handling degenerate cases. Part of the difficulty comes from the unintuitive fact that the structure of an algebraic object can be quite complicated, as depicted in the Whitney umbrella (see Figure 1), surface of equation $x^2 = y^2z$ on which the origin (the “special” point of the surface) is a vertex of the arrangement induced by the surface while the singular locus is simply the whole z -axis. Even in 2D, meshing an algebraic curve with the correct topology, that is in other words producing a correct drawing of the curve (without knowing where the domain of interest is), is a very difficult problem on which we have recently made important contributions [\[37\]](#), [\[38\]](#), [\[59\]](#).

It is thus to be understood that producing practical robust and efficient algorithmic solutions to geometric problems on curved objects is a challenge on all and even the most basic problems. The basicness and fundamentality of two problems we mentioned above on the intersection of 3D quadrics and on the drawing in a topologically certified way of plane algebraic curves show rather well that the domain is still in its infancy. And it should be stressed that these two sets of results were not anecdotal but flagship results produced during the lifetime of the VEGAS team (the team preceding GAMBLE).

There are many problems in this theme that are expected to have high long-term impacts. Intersecting NURBS (Non-uniform rational basis splines) in a certified way is an important problem in computer-aided design and manufacturing. As hinted above, meshing objects in a certified way is important when topology matters. The 2D case, that is essentially drawing plane curves with the correct topology, is a fundamental problem with far-reaching applications in research or R&D. Notice that on such elementary problems it is often difficult to predict the reach of the applications; as an example, we were astonished by the scope of the applications of our software on 3D quadric intersection ² which was used by researchers in, for instance, photochemistry, computer vision, statistics and mathematics.

3.2. Non-Euclidean computational geometry



Figure 2. Left: 3D mesh of a gyroid (triplly periodic surface) [61]. Right: Simulation of a periodic Delaunay triangulation of the hyperbolic plane [33].

Triangulations, in particular Delaunay triangulations, in the *Euclidean space* \mathbb{R}^d have been extensively studied throughout the 20th century and they are still a very active research topic. Their mathematical properties are now well understood, many algorithms to construct them have been proposed and analyzed (see the book of Aurenhammer *et al.* [32]). Some members of GAMBLE have been contributing to these algorithmic advances (see, e.g. [36], [68], [47], [35]); they have also contributed robust and efficient triangulation packages through the state-of-the-art Computational Geometry Algorithms Library **CGAL** whose impact extends far beyond computational geometry. Application fields include particle physics, fluid dynamics, shape matching, image processing, geometry processing, computer graphics, computer vision, shape reconstruction, mesh generation, virtual worlds, geophysics, and medical imaging. ³

It is fair to say that little has been done on non-Euclidean spaces, in spite of the large number of questions raised by application domains. Needs for simulations or modeling in a variety of domains ⁴ ranging from the infinitely small (nuclear matter, nano-structures, biological data) to the infinitely large (astrophysics) have led

²QI: [web](#).

³See [Projects using CGAL](#) for details.

⁴See [CGAL Prospective Workshop on Geometric Computing in Periodic Spaces](#), [Subdivide and Tile: Triangulating spaces for understanding the world](#), [Computational geometry in non-Euclidean spaces](#), [Shape Up 2015 : Exercises in Materials Geometry and Topology](#)

us to consider 3D periodic Delaunay triangulations, which can be seen as Delaunay triangulations in the 3D *flat torus*, quotient of \mathbb{R}^3 under the action of some group of translations [42]. This work has already yielded a fruitful collaboration with astrophysicists [55], [69] and new collaborations with physicists are emerging. To the best of our knowledge, our CGAL package [41] is the only publicly available software that computes Delaunay triangulations of a 3D flat torus, in the special case where the domain is cubic. This case, although restrictive, is already useful.⁵ We have also generalized this algorithm to the case of general d -dimensional compact flat manifolds [43]. As far as non-compact manifolds are concerned, past approaches, limited to the two-dimensional case, have stayed theoretical [60].

Interestingly, even for the simple case of triangulations on the *sphere*, the software packages that are currently available are far from offering satisfactory solutions in terms of robustness and efficiency [40].

Moreover, while our solution for computing triangulations in hyperbolic spaces can be considered as ultimate [33], the case of *hyperbolic manifolds* has hardly been explored. Hyperbolic manifolds are quotients of a hyperbolic space by some group of hyperbolic isometries. Their triangulations can be seen as hyperbolic periodic triangulations. Periodic hyperbolic triangulations and meshes appear for instance in geometric modeling [62], neuromathematics [45], or physics [65]. Even the case of the Bolza surface (a surface of genus 2, whose fundamental domain is the regular octagon in the hyperbolic plane) shows mathematical difficulties [34], [57].

3.3. Probability in computational geometry

In most computational geometry papers, algorithms are analyzed in the worst-case setting. This often yields too pessimistic complexities that arise only in pathological situations that are unlikely to occur in practice. On the other hand, probabilistic geometry provides analyses with great precision [63], [64], [39], but using hypotheses with much more randomness than in most realistic situations. We are developing new algorithmic designs improving state-of-the-art performance in random settings that are not overly simplified and that can thus reflect many realistic situations.

Twelve years ago, smooth analysis was introduced by Spielman and Teng analyzing the simplex algorithm by averaging on some noise on the data [67] (and they won the Gödel prize). In essence, this analysis smoothes the complexity around worst-case situations, thus avoiding pathological scenarios but without considering unrealistic randomness. In that sense, this method makes a bridge between full randomness and worst case situations by tuning the noise intensity. The analysis of computational geometry algorithms within this framework is still embryonic. To illustrate the difficulty of the problem, we started working in 2009 on the smooth analysis of the size of the convex hull of a point set, arguably the simplest computational geometry data structure; then, only one very rough result from 2004 existed [46] and we only obtained in 2015 breakthrough results, but still not definitive [49], [48], [53].

Another example of a problem of different flavor concerns Delaunay triangulations, which are rather ubiquitous in computational geometry. When Delaunay triangulations are computed for reconstructing meshes from point clouds coming from 3D scanners, the worst-case scenario is, again, too pessimistic and the full randomness hypothesis is clearly not adapted. Some results exist for “good samplings of generic surfaces” [31] but the big result that everybody wishes for is an analysis for random samples (without the extra assumptions hidden in the “good” sampling) of possibly non-generic surfaces.

Trade-offs between full randomness and worst case may also appear in other forms such as dependent distributions, or random distributions conditioned to be in some special configurations. Simulating these kinds of geometric distributions is currently out of reach for more than a few hundred points [56] although it has practical applications in physics or networks.

3.4. Discrete geometric structures

⁵See examples at [Projects using CGAL](#)

Our work on discrete geometric structures develops in several directions, each one probing a different type of structure. Although these objects appear unrelated at first sight, they can be tackled by the same set of probabilistic and topological tools.

A first research topic is the study of *Order types*. Order types are combinatorial encodings of finite (planar) point sets, recording for each triple of points the orientation (clockwise or counterclockwise) of the triangle they form. This already determines properties such as convex hulls or half-space depths, and the behaviour of algorithms based on orientation predicates. These properties for all (infinitely many) n -point sets can be studied through the finitely many order types of size n . Yet, this finite space is poorly understood: its estimated size leaves an exponential margin of error, no method is known to sample it without concentrating on a vanishingly small corner, the effect of pattern exclusion or VC dimension-type restrictions are unknown. These are all directions we actively investigate.

A second research topic is the study of *Embedded graphs and simplicial complexes*. Many topological structures can be effectively discretized, for instance combinatorial maps record homotopy classes of embedded graphs and simplicial complexes represent a large class of topological spaces. This raises many structural and algorithmic questions on these discrete structures; for example, given a closed walk in an embedded graph, can we find a cycle of the graph homotopic to that walk? (The complexity status of that problem is unknown.) Going in the other direction, some purely discrete structures can be given an associated topological space that reveals some of their properties (*e.g.* the Nerve theorem for intersection patterns). An open problem is for instance to obtain fractional Helly theorems for set system of bounded topological complexity.

Another research topic is that of *Sparse inclusion-exclusion formulas*. For any family of sets A_1, A_2, \dots, A_n , by the principle of inclusion-exclusion we have

$$\mathbb{1}_{\bigcup_{i=1}^n A_i} = \sum_{I \subseteq \{1,2,\dots,n\}} (-1)^{|I|+1} \mathbb{1}_{\bigcap_{i \in I} A_i} \quad (1)$$

where $\mathbb{1}_X$ is the indicator function of X . This formula is universal (it applies to any family of sets) but its number of summands grows exponentially with the number n of sets. When the sets are balls, the formula remains true if the summation is restricted to the regular triangulation; we proved that similar simplifications are possible whenever the Venn diagram of the A_i is sparse. There is much room for improvements, both for general set systems and for specific geometric settings. Another interesting problem (the subject of the PhD thesis of Galatée Hemery) is to combine these simplifications with the inclusion-exclusion algorithms developed, for instance, for graph coloring.

4. Application Domains

4.1. Applications of computational geometry

Many domains of science can benefit from the results developed by GAMBLE. Curves and surfaces are ubiquitous in all sciences to understand and interpret raw data as well as experimental results. Still, the non-linear problems we address are rather basic and fundamental, and it is often difficult to predict the impact of solutions in that area. The short-term industrial impact is likely to be small because, on basic problems, industries have used ad hoc solutions for decades and have thus got used to it. The example of our work on quadric intersection is typical: even though we were fully convinced that intersecting 3D quadrics is such an elementary/fundamental problem that it ought to be useful, we were the first to be astonished by the scope of the applications of our software ⁶ (which was the first and still is the only one —to our knowledge— to compute robustly and efficiently the intersection of 3D quadrics) which has been used by researchers in, for instance, photochemistry, computer vision, statistics, and mathematics. Our work on certified drawing of plane (algebraic) curves falls in the same category. It seems obvious that it is widely useful to be able to draw

⁶QI: [web](#).

curves correctly (recall also that part of the problem is to determine where to look in the plane) but it is quite hard to come up with specific examples of fields where this is relevant. A contrario, we know that certified meshing is critical in mechanical-design applications in robotics, which is a non-obvious application field. There, the singularities of a manipulator often have degrees higher than 10 and meshing the singular locus in a certified way is currently out of reach. As a result, researchers in robotics can only build physical prototypes for validating, or not, the approximate solutions given by non-certified numerical algorithms.

The fact that several of our pieces of software for computing non-Euclidean triangulations had already been requested by users long before they become public in CGAL is a good sign for their wide future impact. This will not come as a surprise, since most of the questions that we have been studying followed from discussions with researchers outside computer science and pure mathematics. Such researchers are either users of our algorithms and software, or we meet them in workshops. Let us only mention a few names here. Rien van de Weijgaert [55], [69] (astrophysicist, Groningen, NL) and Michael Schindler [66] (theoretical physicist, ENSPCI, CNRS, France) used our software for 3D periodic weighted triangulations. Stephen Hyde and Vanessa Robins (applied mathematics and physics at Australian National University) used our package for 3D periodic meshing. Olivier Faugeras (neuromathematics, Inria Sophia Antipolis) had come to us and mentioned his needs for good meshes of the Bolza surface [45] before we started to study them. Such contacts are very important both to get feedback about our research and to help us choose problems that are relevant for applications. These problems are at the same time challenging from the mathematical and algorithmic points of view. Note that our research and our software are generic, i.e., we are studying fundamental geometric questions, which do not depend on any specific application. This recipe has made the success of the CGAL library.

Probabilistic models for geometric data are widely used to model various situations ranging from cell phone distribution to quantum mechanics. The impact of our work on probabilistic distributions is twofold. On the one hand, our studies of properties of geometric objects built on such distributions will yield a better understanding of the above phenomena and has potential impact in many scientific domains. On the other hand, our work on simulations of probabilistic distributions will be used by other teams, more maths oriented, to study these distributions.

5. Highlights of the Year

5.1. Highlights of the Year

We are happy to report that some of our past work appeared this year in highly visible journals. Our proof that deciding *shellability* of simplicial complexes, a problem that was open for 40 years, was published in the Journal of the ACM [15], and our survey on *combinatorial geometry and topology and their applications* was published in the Bulletin of the AMS [13].

6. New Software and Platforms

6.1. CGAL Package : 2D periodic hyperbolic triangulations

KEYWORDS: Geometry - Delaunay triangulation - Hyperbolic space

FUNCTIONAL DESCRIPTION: This module implements the computation of Delaunay triangulations of the Bolza surface.

NEWS OF THE YEAR: Integration into CGAL 4.14

- Authors: Iordan Iordanov and Monique Teillaud
- Contact: Monique Teillaud
- Publication: [Implementing Delaunay Triangulations of the Bolza Surface](#)
- URL: <https://doc.cgal.org/latest/Manual/packages.html#PkgPeriodic4HyperbolicTriangulation2>

6.2. CGAL Package : 2D hyperbolic triangulations

KEYWORDS: Geometry - Delaunay triangulation - Hyperbolic space

FUNCTIONAL DESCRIPTION: This package implements the construction of Delaunay triangulations in the Poincaré disk model.

NEWS OF THE YEAR: Integration into CGAL 4.14

- Participants: Mikhail Bogdanov, Olivier Devillers, Jordan Iordanov and Monique Teillaud
- Contact: Monique Teillaud
- Publication: [Hyperbolic Delaunay Complexes and Voronoi Diagrams Made Practical](#)
- URL: <https://doc.cgal.org/latest/Manual/packages.html#PkgHyperbolicTriangulation2>

6.3. clenshaw

KEYWORDS: Numerical solver - Visualization - Polynomial equations

FUNCTIONAL DESCRIPTION: Clenshaw is a mixed C and python library that provides computation and plotting functions for the solutions of polynomial equations in the Taylor or the Chebyshev basis. The library is optimized for machine double precision and for numerically well-conditioned polynomials. In particular, it can find the roots of polynomials with random coefficients of degree one million.

- Contact: Guillaume Moroz
- URL: <https://gitlab.inria.fr/gmoro/clenshaw>

6.4. voxelize

KEYWORDS: Visualization - Curve plotting - Implicit surface - Polynomial equations

FUNCTIONAL DESCRIPTION: Voxelize is a C++ software to visualize the solutions of polynomial equations and inequalities. The software is optimized for high degree curves and surfaces. Internally, polynomials and sets of boxes are stored in the Compressed Sparse Fiber format. The output is either a mesh or a union of boxes written in the standard 3D file format ply.

RELEASE FUNCTIONAL DESCRIPTION: This is the first published version.

- Contact: Guillaume Moroz
- URL: <https://gitlab.inria.fr/gmoro/voxelize>

7. New Results

7.1. Non-Linear Computational Geometry

Participants: Laurent Dupont, Nuwan Herath Mudiyansele, George Krait, Sylvain Lazard, Viviane Ledoux, Guillaume Moroz, Marc Pouget.

7.1.1. Clustering Complex Zeros of Triangular Systems of Polynomials

This work, presented at the CASC'19 Conference [23], gives the first algorithm for finding a set of natural ϵ -clusters of complex zeros of a regular triangular system of polynomials within a given polybox in \mathbb{C}^n , for any given $\epsilon > 0$. Our algorithm is based on a recent near-optimal algorithm of Becker et al (2016) for clustering the complex roots of a univariate polynomial where the coefficients are represented by number oracles. Our algorithm is based on recursive subdivision. It is local, numeric, certified and handles solutions with multiplicity. Our implementation is compared to well-known homotopy solvers on various triangular systems. Our solver always gives correct answers, is often faster than the homotopy solvers that often give correct answers, and sometimes faster than the ones that give sometimes correct results.

In collaboration with R. Imbach and C. Yap (Courant Institute of Mathematical Sciences, New York University, USA).

7.1.2. Numerical Algorithm for the Topology of Singular Plane Curves

We are interested in computing the topology of plane singular curves. For this, the singular points must be isolated. Numerical methods for isolating singular points are efficient but not certified in general. We are interested in developing certified numerical algorithms for isolating the singularities. In order to do so, we restrict our attention to the special case of plane curves that are projections of smooth curves in higher dimensions. In this setting, we show that the singularities can be encoded by a regular square system whose isolation can be certified by numerical methods. This type of curves appears naturally in robotics applications and scientific visualization. This work was presented at the EuroCG'19 Conference [24].

7.1.3. Reliable Computation of the Singularities of the Projection in \mathbb{R}^3 of a Generic Surface of \mathbb{R}^4

Computing efficiently the singularities of surfaces embedded in \mathbb{R}^3 is a difficult problem, and most state-of-the-art approaches only handle the case of surfaces defined by polynomial equations. Let F and G be C^∞ functions from \mathbb{R}^4 to \mathbb{R} and $\mathcal{M} = \{(x, y, z, t) \in \mathbb{R}^4 \mid F(x, y, z, t) = G(x, y, z, t) = 0\}$ be the surface they define. Generically, the surface \mathcal{M} is smooth and its projection Ω in \mathbb{R}^3 is singular. After describing the types of singularities that appear generically in Ω , we design a numerically well-posed system that encodes them. This can be used to return a set of boxes that enclose the singularities of Ω as tightly as required. As opposed to state-of-the-art approaches, our approach is not restricted to polynomial mappings, and can handle trigonometric or exponential functions for example. This work was presented at the MACIS'19 Conference [19].

In collaboration with Sény Diatta (University Assane Seck of Ziguinchor, Senegal)

7.1.4. Evaluation of Chebyshev polynomials on intervals and application to root finding

In approximation theory, it is standard to approximate functions by polynomials expressed in the Chebyshev basis. Evaluating a polynomial f of degree n given in the Chebyshev basis can be done in $O(n)$ arithmetic operations using the Clenshaw algorithm. Unfortunately, the evaluation of f on an interval I using the Clenshaw algorithm with interval arithmetic returns an interval of width exponential in n . We describe a variant of the Clenshaw algorithm based on ball arithmetic that returns an interval of width quadratic in n for an interval of small enough width. As an application, our variant of the Clenshaw algorithm can be used to design an efficient root finding algorithm. This work was presented at the MACIS'19 Conference [21].

7.1.5. Using Maple to analyse parallel robots

We present the SIROPA Maple Library which has been designed to study serial and parallel manipulators at the conception level. We show how modern algorithms in Computer Algebra can be used to study the workspace, the joint space but also the existence of some physical capabilities w.r.t. to some design parameters left as degree of freedom for the designer of the robot. This work was presented at the Maple Conference 2019 [18].

In collaboration with Philippe Wenger, Damien Chablat (Laboratoire des Sciences du Numérique de Nantes, UMR CNRS 6004) and Fabrice Rouillier (project team OURAGAN)

7.2. Non-Euclidean Computational Geometry

Participants: Vincent Despré, Yan Garito, Elies Harington, Benedikt Kolbe, Georg Osang, Monique Teillaud, Gert Vegter.

7.2.1. Flipping Geometric Triangulations on Hyperbolic Surfaces

We consider geometric triangulations of surfaces, i.e., triangulations whose edges can be realized by disjoint locally geodesic segments. We prove that the flip graph of geometric triangulations with fixed vertices of a flat torus or a closed hyperbolic surface is connected. We give upper bounds on the number of edge flips that are necessary to transform any geometric triangulation on such a surface into a Delaunay triangulation [28].

In collaboration with Jean-Marc Schlenker (University of Luxembourg).

7.2.2. Computing the Geometric Intersection Number of Curves

The geometric intersection number of a curve on a surface is the minimal number of self-intersections of any homotopic curve, i.e. of any curve obtained by continuous deformation. Given a curve c represented by a closed walk of length at most ℓ on a combinatorial surface of complexity n we describe simple algorithms to compute the geometric intersection number of c in $O(n + \ell^2)$ time, construct a curve homotopic to c that realizes this geometric intersection number in $O(n + \ell^4)$ time, decide if the geometric intersection number of c is zero, i.e. if c is homotopic to a simple curve, in $O(n + \ell \log(\ell))$ time [14].

In collaboration with Francis Lazarus (University of Grenoble).

7.3. Probabilistic Analysis of Geometric Data Structures and Algorithms

Participants: Olivier Devillers, Charles Duménil, Xavier Goaoc, Fernand Kuiebove Pefireko, Ji Won Park.

7.3.1. Expected Complexity of Routing in Θ_6 and Half- Θ_6 Graphs

We study online routing algorithms on the Θ_6 -graph and the half- Θ_6 -graph (which is equivalent to a variant of the Delaunay triangulation). Given a source vertex s and a target vertex t in the Θ_6 -graph (resp. half- Θ_6 -graph), there exists a deterministic online routing algorithm that finds a path from s to t whose length is at most $2st$ (resp. $2.89st$) which is optimal in the worst case [Bose et al., SIAM J. on Computing, 44(6)]. We propose alternative, slightly simpler routing algorithms that are optimal in the worst case and for which we provide an analysis of the average routing ratio for the Θ_6 -graph and half- Θ_6 -graph defined on a Poisson point process. For the Θ_6 -graph, our online routing algorithm has an expected routing ratio of 1.161 (when s and t are random) and a maximum expected routing ratio of 1.22 (maximum for fixed s and t where all other points are random), much better than the worst-case routing ratio of 2. For the half- Θ_6 -graph, our memoryless online routing algorithm has an expected routing ratio of 1.43 and a maximum expected routing ratio of 1.58. Our online routing algorithm that uses a constant amount of additional memory has an expected routing ratio of 1.34 and a maximum expected routing ratio of 1.40. The additional memory is only used to remember the coordinates of the starting point of the route. Both of these algorithms have an expected routing ratio that is much better than their worst-case routing ratio of 2.89 [27].

In collaboration with Prosenjit Bose (University Carleton) and JeanLou De Carufel (University of Ottawa)

7.3.2. A Poisson sample of a smooth surface is a good sample

The complexity of the 3D-Delaunay triangulation (tetrahedralization) of n points distributed on a surface ranges from linear to quadratic. When the points are a deterministic good sample of a smooth compact generic surface, the size of the Delaunay triangulation is $O(n \log n)$. Using this result, we prove that when points are Poisson distributed on a surface under the same hypothesis, whose expected number of vertices is λ , the expected size is $O(\lambda \log_2 \lambda)$ [22].

7.3.3. On Order Types of Random Point Sets

Let P be a set of n random points chosen uniformly in the unit square. We examine the typical resolution of the order type of P . First, we show that with high probability, P can be rounded to the grid of step $\frac{1}{n^{3+\epsilon}}$ without changing its order type. Second, we study algorithms for determining the order type of a point set in terms of the number of coordinate bits they require to know. We give an algorithm that requires on average $4n \log_2 n + O(n)$ bits to determine the order type of P , and show that any algorithm requires at least $4n \log_2 n - O(n \log \log n)$ bits. Both results extend to more general models of random point sets [29].

In collaboration with Philippe Duchon (Université de Bordeaux) and Marc Glisse (project team DATASHAPE).

7.3.4. Randomized incremental construction of Delaunay triangulations of nice point sets

Randomized incremental construction (RIC) is one of the most important paradigms for building geometric data structures. Clarkson and Shor developed a general theory that led to numerous algorithms that are both simple and efficient in theory and in practice. Randomized incremental constructions are most of the time space and time optimal in the worst-case, as exemplified by the construction of convex hulls, Delaunay triangulations and arrangements of line segments. However, the worst-case scenario occurs rarely in practice and we would like to understand how RIC behaves when the input is nice in the sense that the associated output is significantly smaller than in the worst-case. For example, it is known that the Delaunay triangulations of nicely distributed points on polyhedral surfaces in \mathbb{E}^3 has linear complexity, as opposed to a worst-case quadratic complexity. The standard analysis does not provide accurate bounds on the complexity of such cases and we aim at establishing such bounds. More precisely, we will show that, in the case of nicely distributed points on polyhedral surfaces, the complexity of the usual RIC is $O(n \log n)$ which is optimal. In other words, without any modification, RIC nicely adapts to good cases of practical value. Our proofs also work for some other notions of nicely distributed point sets, such as (ϵ, κ) -samples. Along the way, we prove a probabilistic lemma for sampling without replacement, which may be of independent interest [16], [26].

In collaboration with Jean-Daniel Boissonnat, Kunal Dutta and Marc Glisse (project team DATASHAPE).

7.3.5. Random polytopes and the wet part for arbitrary probability distributions

We examine how the measure and the number of vertices of the convex hull of a random sample of n points from an arbitrary probability measure in \mathbb{R}^d relates to the wet part of that measure. This extends classical results for the uniform distribution from a convex set [Bárány and Larman 1988]. The lower bound of Bárány and Larman continues to hold in the general setting, but the upper bound must be relaxed by a factor of $\log n$. We show by an example that this is tight [25].

In collaboration with Imre Barany (Rényi Institute of Mathematics) Matthieu Fradelizi (Laboratoire d'Analyse et de Mathématiques Appliquées) Alfredo Hubard (Laboratoire d'Informatique Gaspard-Monge) Günter Rote (Institut für Informatik, Berlin)

7.4. Discrete Geometric structures

Participants: Xavier Goaoc, Galatée Hemery Vaglica.

7.4.1. Shatter functions with polynomial growth rates

We study how a single value of the shatter function of a set system restricts its asymptotic growth. Along the way, we refute a conjecture of Bondy and Hajnal which generalizes Sauer's Lemma. [12]

7.4.2. The discrete yet ubiquitous theorems of Caratheodory, Helly, Sperner, Tucker, and Tverberg

We discuss five discrete results: the lemmas of Sperner and Tucker from combinatorial topology and the theorems of Carathéodory, Helly, and Tverberg from combinatorial geometry. We explore their connections and emphasize their broad impact in application areas such as game theory, graph theory, mathematical optimization, computational geometry, etc. [13]

7.4.3. Shellability is NP-complete

We prove that for every $d \geq 2$, deciding if a pure, d -dimensional, simplicial complex is shellable is NP-hard, hence NP-complete. This resolves a question raised, e.g., by Danaraj and Klee in 1978. Our reduction also yields that for every $d \geq 2$ and $k \geq 0$, deciding if a pure, d -dimensional, simplicial complex is k -decomposable is NP-hard. For $d \geq 3$, both problems remain NP-hard when restricted to contractible pure d -dimensional complexes. Another simple corollary of our result is that it is NP-hard to decide whether a given poset is CL-shellable. [15]

7.4.4. An Experimental Study of Forbidden Patterns in Geometric Permutations by Combinatorial Lifting

We study the problem of deciding if a given triple of permutations can be realized as geometric permutations of disjoint convex sets in \mathbb{R}^3 . We show that this question, which is equivalent to deciding the emptiness of certain semi-algebraic sets bounded by cubic polynomials, can be "lifted" to a purely combinatorial problem. We propose an effective algorithm for that problem, and use it to gain new insights into the structure of geometric permutations. [20]

7.5. Classical Computational Geometry

Participants: Olivier Devillers, Sylvain Lazard, Leo Valque.

7.5.1. Rounding Meshes

Let \mathcal{P} be a set of n polygons in \mathbb{R}^3 , each of constant complexity and with pairwise disjoint interiors. We previously proposed [5] a rounding algorithm that maps \mathcal{P} to a simplicial complex \mathcal{Q} whose vertices have integer coordinates such that every face of \mathcal{P} is mapped to a set of faces (or edges or vertices) of \mathcal{Q} and the mapping from \mathcal{P} to \mathcal{Q} can be built through a continuous motion of the faces such that (i) the L_∞ Hausdorff distance between a face and its image during the motion is at most $3/2$ and (ii) if two points become equal during the motion they remain equal through the rest of the motion. We developed [30] the first implementation of this algorithm, which is also the first implementation for rounding a mesh on a grid (whose fineness is independent of the input size) while preserving reasonable geometric and topological properties. We also provided some insight that this algorithm and implementation have practical average complexity in $O(n\sqrt{n})$ on "real data", which has to be compared to its $O(n^{15})$ worst-case time complexity. Our implementation is still too slow to be used in practice but it provides a good proof of concept.

7.5.2. Hardness results on Voronoi, Laguerre and Apollonius diagrams

We show that converting Apollonius and Laguerre diagrams from an already built Voronoi diagram of a set of n points in 2D requires at least $\Omega(n \log n)$ computation time. We also show that converting an Apollonius diagram of a set of n weighted points in 2D from a Laguerre diagram and vice-versa requires at least $\Omega(n \log n)$ computation time as well. Furthermore, we present a very simple randomized incremental construction algorithm that takes expected $O(n \log n)$ computation time to build an Apollonius diagram of non-overlapping circles in 2D [17].

In collaboration with Kevin Buchin (TU Eindhoven), Pedro de Castro (University Pernambuco), and Menelaos Karavelas (University Heraklion).

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

- Company: WATERLOO MAPLE INC
Duration: 2 years
Participants: GAMBLE and OURAGAN Inria teams
Abstract: A two-years licence and cooperation agreement was signed on April 1st, 2018 between WATERLOO MAPLE INC., Ontario, Canada (represented by Laurent Bernardin, its Executive Vice President Products and Solutions) and Inria. On the Inria side, this contract involves the teams GAMBLE and OURAGAN (Paris), and it is coordinated by Fabrice Rouillier (OURAGAN).
F. Rouillier and GAMBLE are the developers of the ISOTOP software for the computation of topology of curves. One objective of the contract is to transfer a version of ISOTOP to WATERLOO MAPLE INC.

- Company: GEOMETRYFACTORY
Duration: permanent
Participants: Inria and GEOMETRYFACTORY
Abstract: CGAL packages developed in GAMBLE are commercialized by GEOMETRYFACTORY.

9. Partnerships and Cooperations

9.1. National Initiatives

9.1.1. ANR SoS

Project title: Structures on Surfaces

Duration: 4 years

Starting Date: April 1st, 2018

Coordinator: Monique Teillaud

Participants:

- Gamble project-team, Inria.
- LIGM (Laboratoire d'Informatique Gaspard Monge), Université Paris-Est Marne-la-Vallée. Local Coordinator: Éric Colin de Verdière.
- RMATH (Mathematics Research Unit), University of Luxembourg. National Coordinator: Hugo Parlier

SoS is co-funded by ANR (ANR-17-CE40-0033) and FNR (INTER/ANR/16/11554412/SoS) as a PRCI (Projet de Recherche Collaborative Internationale).

The central theme of this project is the study of geometric and combinatorial structures related to surfaces and their moduli. Even though they work on common themes, there is a real gap between communities working in geometric topology and computational geometry and SoS aims to create a long-lasting bridge between them. Beyond a common interest, techniques from both ends are relevant and the potential gain in perspective from long-term collaborations is truly thrilling.

In particular, SoS aims to extend the scope of computational geometry, a field at the interface between mathematics and computer science that develops algorithms for geometric problems, to a variety of unexplored contexts. During the last two decades, research in computational geometry has gained wide impact through CGAL, the Computational Geometry Algorithms Library. In parallel, the needs for non-Euclidean geometries are arising, e.g., in geometric modeling, neuromathematics, or physics. Our goal is to develop computational geometry for some of these non-Euclidean spaces and make these developments readily available for users in academy and industry.

To reach this aim, SoS will follow an interdisciplinary approach, gathering researchers whose expertise cover a large range of mathematics, algorithms and software. A mathematical study of the objects considered will be performed, together with the design of algorithms when applicable. Algorithms will be analyzed both in theory and in practice after prototype implementations, which will be improved whenever it makes sense to target longer-term integration into CGAL.

Our main objects of study will be Delaunay triangulations and circle patterns on surfaces, polyhedral geometry, and systems of disjoint curves and graphs on surfaces.

Project website: <https://members.loria.fr/Monique.Teillaud/collab/SoS/>.

9.1.2. ANR Aspag

Project title: Analyse et Simulation Probabilistes d'Algorithmes Géométriques

Duration: 4 years

Starting date: January 1st, 2018

Coordinator: Olivier Devillers

Participants:

- Gamble project-team, Inria.
- Labri (Laboratoire Bordelais de Recherche en Informatique), Université de Bordeaux. Local Coordinator: Philippe Duchon.
- Laboratoire de Mathématiques Raphaël Salem, Université de Rouen. Local Coordinator: Pierre Calka.
- LAMA (Laboratoire d'Analyse et de Mathématiques Appliquées), Université Paris-Est Marne-la-Vallée. Local Coordinator: Matthieu Fradelizi

Abstract: The ASPAG projet is funded by ANR under number ANR-17-CE40-0017 .

The analysis and processing of geometric data has become routine in a variety of human activities ranging from computer-aided design in manufacturing to the tracking of animal trajectories in ecology or geographic information systems in GPS navigation devices. Geometric algorithms and probabilistic geometric models are crucial to the treatment of all this geometric data, yet the current available knowledge is in various ways much too limited: many models are far from matching real data, and the analyses are not always relevant in practical contexts. One of the reasons for this state of affairs is that the breadth of expertise required is spread among different scientific communities (computational geometry, analysis of algorithms and stochastic geometry) that historically had very little interaction. The Aspaga project brings together experts of these communities to address the problem of geometric data. We will more specifically work on the following three interdependent directions.

(1) Dependent point sets: One of the main issues of most models is the core assumption that the data points are independent and follow the same underlying distribution. Although this may be relevant in some contexts, the independence assumption is too strong for many applications.

(2) Simulation of geometric structures: The phenomena studied in (1) involve intricate random geometric structures subject to new models or constraints. A natural first step would be to build up our understanding and identify plausible conjectures through simulation. Perhaps surprisingly, the tools for an effective simulation of such complex geometric systems still need to be developed.

(3) Understanding geometric algorithms: the analysis of algorithms is an essential step in assessing the strengths and weaknesses of algorithmic principles, and is crucial to guide the choices made when designing a complex data processing pipeline. Any analysis must strike a balance between realism and tractability; the current analyses of many geometric algorithms are notoriously unrealistic. Aside from the purely scientific objectives, one of the main goals of Aspaga is to bring the communities closer in the long term. As a consequence, the funding of the project is crucial to ensure that the members of the consortium will be able to interact on a very regular basis, a necessary condition for significant progress on the above challenges.

Project website: <https://members.loria.fr/Olivier.Devillers/aspaga/>.

9.1.3. ANR MinMax

Project title: MIN-MAX

Duration: 4 years

Starting date: 2019

Coordinator: Stéphane Sabourau (Université Paris-Est Créteil)

Participants:

- Université Paris Est Créteil, Laboratoire d'Analyse et de Mathématiques Appliquées (LAMA). Local coordinator: Stéphane Sabourau
- Université de Tours, Institut Denis Poisson. Local coordinator: Laurent Mazet. This node includes two participants from Nancy, Benoît Daniel (IECL) and Xavier Goaoc (Loria, GAMBLE).

Abstract: The MinMax projet is funded by ANR under number ANR-19-CE40-0014

This collaborative research project aims to bring together researchers from various areas – namely, geometry and topology, minimal surface theory and geometric analysis, and computational geometry and algorithms – to work on a precise theme around min-max constructions and waist estimates.

9.1.4. Institut Universitaire de France

Xavier Goaoc was appointed *junior member* of the Institut Universitaire de France, a grant supporting a reduction in teaching duties and funding.

Starting Date: October 1st, 2014.

Duration: 5 years.

9.2. International Initiatives

9.2.1. Inria Associate Teams Not Involved in an Inria International Labs

9.2.1.1. TRIP

Title: Triangulation and Random Incremental Paths

International Partner (Institution - Laboratory - Researcher):

Carleton University (Canada) - CGLab - Prosenjit Bose

Start year: 2018

See also: <https://members.loria.fr/Olivier.Devillers/trip/>

The two teams are specialists of Delaunay triangulation with a focus on computation algorithms on the French side and routing on the Canadian side. We plan to attack several problems where the two teams are complementary:

- Stretch factor of the Delaunay triangulation in 3D.
- Probabilistic analysis of Theta-graphs and Yao-graphs.
- Smoothed analysis of a walk in Delaunay triangulation.
- Walking in/on surfaces.
- Routing un non-Euclidean spaces.

9.2.1.2. Astonishing

Title: ASSociate Team On Non-ISH euclIdeaN Geometry

International Partner (Institution - Laboratory - Researcher):

University of Groningen (Netherlands) - Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence - Gert Vegter

Start year: 2017

See also: <https://members.loria.fr/Monique.Teillaud/collab/Astonishing/>

Some research directions in computational geometry have hardly been explored. The spaces in which most algorithms have been designed are the Euclidean spaces \mathbb{R}^d . To extend further the scope of applicability of computational geometry, other spaces must be considered, as shown by the concrete needs expressed by our contacts in various fields as well as in the literature. Delaunay triangulations in non-Euclidean spaces are required, e.g., in geometric modeling, neuromathematics, or physics. Topological problems for curves and graphs on surfaces arise in various applications in computer graphics and road map design. Providing robust implementations of these results is a key towards their reusability in more applied fields. We aim at studying various structures and algorithms in other spaces than \mathbb{R}^d , from a computational geometry viewpoint. Proposing algorithms operating in such spaces requires a prior deep study of the mathematical properties of the objects considered, which raises new fundamental and difficult questions that we want to tackle.

9.3. International Research Visitors

9.3.1. Visits of International Scientists

Gert Vegter (University of Groningen, NL) spent two weeks in GAMBLE in the context of the Astonishing associate team.

Matthijs Ebbens (University of Groningen, NL) spent one week in GAMBLE in the context of the Astonishing associate team.

Hugo Parlier (University of Luxembourg) spent two days in GAMBLE in the context of the ANR project SoS.

Erin Wolf Chambers (Saint Louis University, USA) spent two days in GAMBLE

Vanessa Robins (Australian National University) spent two days in GAMBLE

Andreas Holmsen (KAIST, South Korea) and Zuzanna Patáková (IST Austria, Vienna) spent a week in GAMBLE

9.3.2. Visits to International Teams

Olivier Devillers and Monique Teillaud spent one week in June at the Computational Geometry Lab of Carleton University <http://cglab.ca/> in the context of the TRIP associate team.

Vincent Despré spent a total of three week during 2019 at the Mathematical Research Unit of the University of Luxembourg in the context of the ANR SoS project.

Sylvain Lazard spent two weeks in September at the Computational Geometry Lab of Carleton University <http://cglab.ca/> in the context of the TRIP associate team.

Monique Teillaud spent two weeks at Bernoulli Institute for Mathematics, Computer Science and Artificial Intelligence of the University of Groningen in the context of the Astonishing associate team.

Monique Teillaud spent two days at University of Luxembourg in the context of the ANR SoS project

Xavier Goaoc spent one week at UNAM Queretaro, in Mexico.

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events Organization

10.1.1.1. Member of the Organizing Committees

Sylvain Lazard organized with S. Whitesides (Victoria University) the **18th Workshop on Computational Geometry** at the Bellairs Research Institute of McGill University in Feb. (1 week workshop on invitation).

Olivier Devillers organized the Trip-Aspag **Mini-workshop on routing in triangulations**, October 21-25 in Nancy.

10.1.2. Scientific Events Selection

10.1.2.1. Member of the Conference Program Committees

Guillaume Moroz was in the program committee of the **Maple Conference 2019**

Xavier Goaoc was on the organizing committee of the **Rouen probability meeting**

Xavier Goaoc was on the program committee of the **Iranian conference on Computational geometry**

Xavier Goaoc was on the scientific committee of the **Séminaire Francilien de Géométrie Algorithmique et Combinatoire**

10.1.2.2. Reviewer

All members of the team are regular reviewers for the conferences of our field, namely the *Symposium on Computational Geometry* (SoCG) and the *International Symposium on Symbolic and Algebraic Computation* (ISSAC) and also SODA, CCCG, EuroCG.

10.1.3. Journal

10.1.3.1. Member of the Editorial Boards

Monique Teillaud is a managing editor of JoCG, *Journal of Computational Geometry* and a member of the editorial board of IJCGA, *International Journal of Computational Geometry and Applications*.

Marc Pouget and Monique Teillaud are members of the CGAL editorial board.

10.1.3.2. Reviewer - Reviewing Activities

All members of the team are regular reviewers for the journals of our field, namely *Discrete and Computational Geometry* (DCG), *Journal of Computational Geometry* (JoCG), *International Journal on Computational Geometry and Applications* (IJCGA), *Journal on Symbolic Computations* (JSC), *SIAM Journal on Computing* (SICOMP), *Mathematics in Computer Science* (MCS), etc.

10.1.4. Invited Talks

Olivier Devillers and Monique Teillaud gave talks at the workshop [New Horizons in Computational Geometry and Topology](#)

Monique Teillaud gave a talk at the [Celebration for the CNRS Silver medal of Claire Mathieu](#)

10.1.5. Leadership within the Scientific Community

10.1.5.1. Steering Committees

Monique Teillaud is chairing the [Steering Committee](#) of the Symposium on Computational Geometry (SoCG).

10.1.6. Research Administration

10.1.6.1. Hiring committees

Sylvain Lazard was vice chair of the hiring committee for researchers (CRCN) of Inria Nancy - Grand Est.

Monique Teillaud was a member of the hiring committee for a Professor position at Université Paris Est Marne-la-Vallée.

10.1.6.2. National committees

M. Teillaud is a member of the working group for the BIL, *Base d'Information des Logiciels* of Inria.

10.1.6.3. Local Committees and Responsibilities

O. Devillers: Elected member to *Pole AM2I* the council that gathers labs in mathematics, computer science, and control theory at *Université de Lorraine*.

L. Dupont is responsible of Fablab of IUT Charlemagne, Université de Lorraine (since 2018, November). Member of *Comité Information Edition Scientifique* of LORIA.

X. Goaoc is a member of the council of the *Fédération Charles Hermite* since sep. 2018.

S. Lazard: Head of the PhD and Post-doc hiring committee for Inria Nancy-Grand Est (since 2009). Member of the *Bureau de la mention informatique* of the *École Doctorale IAEM* (since 2009). Head of the *Mission Jeunes Chercheurs* for Inria national (since 2018). Head of the Department Algo at LORIA (since 2014). Member of the *Conseil Scientifique* of LORIA (since 2014).

G. Moroz is head of the *Comité des utilisateurs des moyens informatiques* (since nov. 2019). He is member of the CDT, *Commission de développement technologique*, of Inria Nancy - Grand Est (since 2018). He is member of the CLHSC *Comité local d'hygiène, de sécurité et des conditions de travail* of Inria Nancy - Grand Est (since jan. 2019).

M. Pouget is an elected member of the *Comité de centre*, and is secretary of the board of AGOS-Nancy.

M. Teillaud is “Chargée de Mission” as Scientific Advisor for Technologic Development for Inria Nancy-Grand Est. She is a member of the *Conseil de Laboratoire* of LORIA.

10.1.6.4. Websites

M. Teillaud is maintaining the Computational Geometry Web Pages <http://www.computational-geometry.org/>, hosted by Inria Nancy - Grand Est. This site offers general interest information for the computational geometry community, in particular the Web proceedings of the Video Review of Computational Geometry, part of the Annual/international Symposium on Computational Geometry.

10.2. Teaching - Supervision - Juries

10.2.1. Committees

L. Dupont is the secretary of *Commission Pédagogique Nationale Carrières Sociales / Information-Communication / Métiers du Multimédia et de l'Internet* (2017-2022).

L. Dupont: Head of the Bachelor diploma *Licence Professionnelle Animation des Communautés et Réseaux Socionumériques*, Université de Lorraine.

10.2.2. Teaching

Master: Olivier Devillers, *Modèles d'environnements, planification de trajectoires*, 18h, M2 AVR, Université de Lorraine. <https://members.loria.fr/Olivier.Devillers/master/>

Master: Vincent Després, *Algorithmique*, 48h, M1, Polytech Nancy, France.

Master: Vincent Després, *Programmation réseau*, 60h, M1, Polytech Nancy, France.

Master: Vincent Després, *Architecture avancée*, 20h, M1, Polytech Nancy, France.

Master: Vincent Després, *Architecture Java EE*, 72h, M1, Polytech Nancy, France.

Licence: Charles Duménil, *Algorithmique et programmation avancée*, 10h, M2, FST, Université de Lorraine, France.

Licence: Charles Duménil, *Découverte de l'informatique*, 88h, L1, Polytech Nancy, Université de Lorraine, France.

Licence: Charles Duménil, *Logiciels scientifiques*, 8h, L3, Polytech Nancy, Université de Lorraine, France.

Licence: Laurent Dupont, *Web development*, 35h, L2, Université de Lorraine, France.

Licence: Laurent Dupont, *Web development*, 150h, L2, Université de Lorraine, France.

Licence: Laurent Dupont *Web development and Social networks* 100h L3, Université de Lorraine, France.

Licence: Xavier Goac, *Programmation*, 20 HETD, L3, École des Mines de Nancy, France.

Master: Xavier Goac, *Algorithms*, 32 HETD, M1, École des Mines de Nancy, France.

Master: Xavier Goac, *Computer architecture*, 32+24 HETD, M1, École des Mines de Nancy + Polytech Nancy, France.

Licence: Galatée Hemery, *Programmation*, 52 HETD, L3, École des Mines de Nancy, France.

Licence: Sylvain Lazard, *Algorithms and Complexity*, 20h, L3, Université de Lorraine, France.

Master: Marc Pouget, *Introduction to computational geometry*, 10.5h, M2, École Nationale Supérieure de Géologie, France.

10.2.3. Supervision

PhD: Iordan Iordanov, Delaunay triangulations of a family of symmetric hyperbolic surfaces in practice, defended March 12th, supervised by Monique Teillaud [11].

PhD in progress: Sény Diatta, Complexité du calcul de la topologie d'une courbe dans l'espace et d'une surface, started in Nov. 2014, supervised by Daouda Niang Diatta, Marie-Françoise Roy and Guillaume Moroz.

PhD in progress: Charles Duménil, Probabilistic analysis of geometric structures, started in Oct. 2016, supervised by Olivier Devillers.

PhD in progress: George Krait, Topology of singular curves and surfaces, applications to visualization and robotics, started in Nov. 2017, supervised by Sylvain Lazard, Guillaume Moroz and Marc Pouget.

PhD in progress: Galatée Hemery, Algorithmic and geometric aspects of inclusion-exclusion, started in Sep. 2018, supervised by Xavier Goaoc and Éric Colin de Verdière (UPEM).

PhD in progress: Nuwan Herath, Fast algorithm for the visualization of surfaces, started in Nov. 2019, supervised by Sylvain Lazard, Guillaume Moroz and Marc Pouget.

10.2.4. Juries

M. Teillaud was a member of the PhD committee of Iordan Iordanov (Université de Lorraine)

X. Goaoc was on the reading and defense committees of the habilitation defense of Arnau Padrol (IMJ, Université Paris Sorbonne)

10.3. Popularization

10.3.1. Education

G. Moroz is member of the Mathematics Olympiades committee of the Nancy-Metz academy.

10.3.2. Interventions

L. Dupont participated in several events of popularization of computer science:

Day #4 FAN Project, April 23th, Inria project, adult audience.

ISN day, March 7th, adult continuing education of computer science for high-school teachers.

Atelier Google, April 13th, popularization of computer science, general audience.

Atelier Google, December 7th, popularization of computer science, general audience.

11. Bibliography

Major publications by the team in recent years

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