IN PARTNERSHIP WITH:
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Ecole des Ponts ParisTech

Université Paris-Est Marne-la-Vallée

Activity Report 2019

Project-Team MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

RESEARCH CENTER
Paris

THEME
Stochastic approaches

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Project-Team MATHRISK

Creation of the Team: 2012 January 01, updated into Project-Team: 2013 January 01

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1. Team, Visitors, External Collaborators

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2. Overall Objectives

2.1. Overall Objectives

The Inria project team **MathRisk** team was created in 2013. It is the follow-up of the MathFi project team founded in 2000. MathFi was focused on financial mathematics, in particular on computational methods for pricing and hedging increasingly complex financial products. The 2007 global financial crisis and its "aftermath crisis" has abruptly highlighted the critical importance of a better understanding and management of risk. The project **MathRisk** has been reoriented towards mathematical handling of risk, and addresses broad research topics embracing risk measurement and risk management, modeling and optimization in quantitative finance, but also in other related domains where risk control is paramount. The project team **MathRisk** aims both at producing mathematical tools and models in these domains, and developing collaborations with various institutions involved in risk control. Quantitative finance remains for the project an important source of mathematical problems and applications. Indeed, the pressure of new legislation leads to a massive reorientation of research priorities, and the interest of analysts shifted to risk control preoccupation.

The scientific issues related to quantitative finance we consider include systemic risk and contagion modeling, robust finance, market frictions, counterparty and liquidity risk, assets dependence modeling, market microstructure modeling and price impact. In this context, models must take into account the multidimensional feature and various market imperfections. They are much more demanding mathematically and numerically, and require the development of risk measures taking into account incompleteness issues, model uncertainties, interplay between information and performance and various defaults.

Besides, financial institutions, submitted to more stringent regulatory legislations such as FRTB or XVA computation, are facing practical implementation challenges which still need to be solved. Research focused on numerical efficiency remains strongly needed in this context, renewing the interest for the numerical platform Premia (http://www.premia.fr) that Mathrisk is developing in collaboration with a consortium of financial institutions.

While these themes arise naturally in the world of quantitative finance, a number of these issues and mathematical tools are also relevant to the treatment of risk in other areas as economy, social insurance and sustainable development, of fundamental importance in today's society. In these contexts, the management of risk appears at different time scales, from high frequency data to long term life insurance management, raising challenging renewed modeling and numerical issues.

The **MathRisk** project is strongly involved in the development of new mathematical methods and numerical algorithms. Mathematical tools include stochastic modeling, stochastic analysis, in particular stochastic (partial) differential equations and various aspects of stochastic control and optimal stopping of these equations, nonlinear expectations, Malliavin calculus, stochastic optimization, dynamic game theory, random graphs, martingale optimal transport (especially in relation to numerical considerations), long time behavior of Markov processes (with applications to Monte-Carlo methods) and generally advanced numerical methods for effective solutions.

3. Research Program

3.1. Risk management: modeling and optimization

3.1.1. Contagion modeling and systemic risk

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. Interconnected systems are subject to contagion in time of distress. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [45] and A. Minca [80]) or mean field interaction models (Garnier-Papanicolaou-Yang [73]).

We have contributed in the last years to the research on the control of contagion in financial systems in the framework of random graph models : In [46], [81], [5], A. Sulem with A. Minca and H. Amini consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph. They study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects. The paper [5] provides in particular important insight on the relation between the value of a financial system, connectivity and optimal intervention.

The results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity. This is studied in [58], where network formation for a large set of financial institutions represented as nodes is investigated. Linkages are source of income, and at the same time they bear the risk of contagion, which is endogeneous and depends on the strategies of all nodes in the system. The optimal connectivity of the nodes results from a game. Existence of an equilibrium in the system and stability properties is studied. The results suggest that financial stability is best described in terms of the mechanism of network formation than in terms of simple statistics of the network topology like the average connectivity.

3.1.2. Liquidity risk and Market Microstructure

Liquidity risk is the risk arising from the difficulty of selling (or buying) an asset. Usually, assets are quoted on a market with a Limit Order Book (LOB) that registers all the waiting limit buy and sell orders for this asset. The bid (resp. ask) price is the most expensive (resp. cheapest) waiting buy or sell order. If a trader wants to sell a single asset, he will sell it at the bid price, but if he wants to sell a large quantity of assets, he will have to sell them at a lower price in order to match further waiting buy orders. This creates an extra cost, and raises important issues. From a short-term perspective (from few minutes to some days), it may be interesting to split the selling order and to focus on finding optimal selling strategies. This requires to model the market microstructure, i.e. how the market reacts in a short time-scale to execution orders. From a long-term perspective (typically, one month or more), one has to understand how this cost modifies portfolio managing strategies (especially delta-hedging or optimal investment strategies). At this time-scale, there is no need to model precisely the market microstructure, but one has to specify how the liquidity costs aggregate.

For rather liquid assets, liquidity risk is usually taken into account via price impact models which describe how a (large) trader influences the asset prices. Then, one is typically interested in the optimal execution problem: how to buy/sell a given amount of assets optimally within a given deadline. This issue is directly related to the existence of statistical arbitrage or Price Manipulation Strategies (PMS). Most of price impact models deal with single assets. A. Alfonsi, F. Klöck and A. Schied [44] have proposed a multi-assets price impact model that extends previous works. Price impact models are usually relevant when trading at an intermediary frequency (say every hour). At a lower frequency, price impact is usually ignored while at a high frequency (every minute or second), one has to take into account the other traders and the price jumps, tick by tick. Midpoint price models are thus usually preferred at this time scale. With P. Blanc, Alfonsi [3] has proposed a model that makes a bridge between these two types of model: they have considered an (Obizhaeva and Wang) price impact model, in which the flow of market orders generated by the other traders is given by an exogeneous process. They have shown that Price Manipulation Strategies exist when the flow of order is a compound Poisson process. However, modeling this flow by a mutually exciting Hawkes process with a particular parametrization allows them to exclude these PMS. Besides, the optimal execution strategy is explicit in this model. A practical implementation is given in [40].

3.1.3. Dependence modeling

- Calibration of stochastic and local volatility models. The volatility is a key concept in modern mathematical finance, and an indicator of market stability. Risk management and associated instruments depend strongly on the volatility, and volatility modeling is a crucial issue in the finance industry. Of particular importance is the assets *dependence* modeling.

By Gyongy's theorem, a local and stochastic volatility model is calibrated to the market prices of all call options with positive maturities and strikes if its local volatility function is equal to the ratio of the Dupire local volatility function over the root conditional mean square of the stochastic volatility factor given the spot value. This leads to a SDE nonlinear in the sense of McKean. Particle methods based on a kernel approximation of the conditional expectation, as presented by Guyon and Henry-Labordère [74], provide an efficient calibration procedure even if some calibration errors may appear when the range of the stochastic volatility factor is very large. But so far, no existence result is available for the SDE nonlinear in the sense of McKean. In the particular case when the local volatility function is equal to the inverse of the root conditional mean square of the stochastic volatility factor multiplied by the spot value given this value and the interest rate is zero, the solution to the SDE is a fake Brownian motion. When the stochastic volatility factor is a constant (over time) random variable taking finitely many values and the range of its square is not too large, B. Jourdain and A. Zhou proved existence to the associated Fokker-Planck equation [21]. Thanks to results obtained by Figalli in [69], they deduced existence of a new class of fake Brownian motions. They extended these results to the special case of the LSV model called Regime Switching Local Volatility, when the stochastic volatility factor is a jump process taking finitely many values and with jump intensities depending on the spot level.

- Interest rates modeling. Affine term structure models have been popularized by Dai and Singleton [59], Duffie, Filipovic and Schachermayer [60]. They consider vector affine diffusions (the coordinates are usually called factors) and assume that the short interest rate is a linear combination of these factors. A model of this kind is the Linear Gaussian Model (LGM) that considers a vector Ornstein-Uhlenbeck diffusions for the factors, see El Karoui and Lacoste [68]. A. Alfonsi et al. [37] have proposed an extension of this model, when the instantaneous covariation between the factors is given by a Wishart process. Doing so, the model keeps its affine structure and tractability while generating smiles for option prices. A price expansion around the LGM is obtained for Caplet and Swaption prices.

3.1.4. Robust finance

- Numerical Methods for Martingale Optimal Transport problems.

The Martingale Optimal Transport (MOT) problem introduced in [57] has received a recent attention in finance since it gives model-free hedges and bounds on the prices of exotic options. The market prices of liquid call and put options give the marginal distributions of the underlying asset at each traded maturity. Under the simplifying assumption that the risk-free rate is zero, these probability measures are in increasing convex order, since by Strassen's theorem this property is equivalent to the existence of a martingale measure with the right marginal distributions. For an exotic payoff function of the values of the underlying on the time-grid given by these maturities, the model-free upper-bound (resp. lower-bound) for the price consistent with these marginal distributions is given by the following martingale optimal transport problem : maximize (resp.

minimize) the integral of the payoff with respect to the martingale measure over all martingale measures with the right marginal distributions. Super-hedging (resp. sub-hedging) strategies are obtained by solving the dual problem. With J. Corbetta, A. Alfonsi and B. Jourdain [12] have studied sampling methods preserving the convex order for two probability measures μ and ν on $\mathbf{R}^d$, with ν dominating μ.

Their method is the first generic approach to tackle the martingale optimal transport problem numerically and can also be applied to several marginals.
- Robust option pricing in financial markets with imperfections.

A. Sulem, M.C. Quenez and R. Dumitrescu have studied robust pricing in an imperfect financial market with default. The market imperfections are taken into account via the nonlinearity of the wealth dynamics. In this setting, the pricing system is expressed as a nonlinear g-expectation $\mathcal{E}^g$ induced by a nonlinear BSDE with nonlinear driver g and default jump (see [61]). A large class of imperfect market models can fit in this framework, including imperfections coming from different borrowing and lending interest rates, taxes on profits from risky investments, or from the trading impact of a large investor seller on the market prices and the default probability. Pricing and superhedging issues for American and game options in this context and their links with optimal stopping problems and Dynkin games with nonlinear expectation have been studied. These issues have also been addressed in the case of model uncertainty, in particular uncertainty on the default probability. The seller's robust price of a game option has been characterized as the value function of a Dynkin game under $\mathcal{E}^g$ expectation as well as the solution of a nonlinear doubly reflected BSDE in [9]. Existence of robust superhedging strategies has been studied. The buyer's point of view and arbitrage issues have also been studied in this context.

In a Markovian framework, the results of the paper [8] on combined optimal stopping/stochastic control with $\mathcal{E}^g$ expectation allows us to address American nonlinear option pricing when the payoff function is only Borelian and when there is ambiguity both on the drift and the volatility of the underlying asset price process. Robust optimal stopping of dynamic risk measures induced by BSDEs with jumps with model ambiguity is studied in [83].

3.2. Perspectives in Stochastic Analysis

3.2.1. Optimal transport and longtime behavior of Markov processes

The dissipation of general convex entropies for continuous time Markov processes can be described in terms of backward martingales with respect to the tail filtration. The relative entropy is the expected value of a backward submartingale. In the case of (non necessarily reversible) Markov diffusion processes, J. Fontbona and B. Jourdain [71] used Girsanov theory to explicit the Doob-Meyer decomposition of this submartingale. They deduced a stochastic analogue of the well known entropy dissipation formula, which is valid for general convex entropies, including the total variation distance. Under additional regularity assumptions, and using Itô's calculus and ideas of Arnold, Carlen and Ju [47], they obtained a new Bakry-Emery criterion which ensures exponential convergence of the entropy to 0. This criterion is non-intrinsic since it depends on the square root of the diffusion matrix, and cannot be written only in terms of the diffusion matrix itself. They provided examples where the classic Bakry Emery criterion fails, but their non-intrinsic criterion applies without modifying the law of the diffusion process.

With J. Corbetta, A. Alfonsi and B. Jourdain have studied the time derivative of the Wasserstein distance between the marginals of two Markov processes [11]. The Kantorovich duality leads to a natural candidate for this derivative. Up to the sign, it is the sum of the integrals with respect to each of the two marginals of the corresponding generator applied to the corresponding Kantorovich potential. For pure jump processes with bounded intensity of jumps, J. Corbetta, A. Alfonsi and B. Jourdain [41] proved that the evolution of the Wasserstein distance is actually given by this candidate. In dimension one, they showed that this remains true for Piecewise Deterministic Markov Processes. They applied the formula to estimate the exponential decrease rate of the Wasserstein distance between the marginals of two birth and death processes with the same generator in terms of the Wasserstein curvature.

3.2.2. Mean-field systems: modeling and control

- Mean-field limits of systems of interacting particles. In [77], B. Jourdain and his former PhD student J. Reygner have studied a mean-field version of rank-based models of equity markets such as the Atlas model introduced by Fernholz in the framework of Stochastic Portfolio Theory. They obtained an asymptotic description of the market when the number of companies grows to infinity. Then, they discussed the long-term capital distribution, recovering the Pareto-like shape of capital distribution curves usually derived from empirical studies, and providing a new description of the phase transition phenomenon observed by Chatterjee and Pal. They have also studied multitype sticky particle systems which can be obtained as vanishing noise limits of multitype rank-based diffusions (see [76]). Under a uniform strict hyperbolicity assumption on the characteristic fields, they constructed a multitype version of the sticky particle dynamics. In [78], they obtain the optimal rate of convergence as the number of particles grows to infinity of the approximate solutions to the diagonal hyperbolic system based on multitype sticky particles and on easy to compute time discretizations of these dynamics.

In [72], N. Fournier and B. Jourdain are interested in the two-dimensional Keller-Segel partial differential equation. This equation is a model for chemotaxis (and for Newtonian gravitational interaction).

- Mean field control and Stochastic Differential Games (SDGs). To handle situations where controls are chosen by several agents who interact in various ways, one may use the theory of Stochastic Differential Games (SDGs). Forward–Backward SDG and stochastic control under Model Uncertainty are studied in [84] by A. Sulem and B. Øksendal. Also of interest are large population games, where each player interacts with the average effect of the others and individually has negligible effect on the overall population. Such an interaction pattern may be modeled by mean field coupling and this leads to the study of mean-field stochastic control and related SDGs. A. Sulem, Y. Hu and B. Øksendal have studied singular mean field control problems and singular mean field two-players stochastic differential games [75]. Both sufficient and necessary conditions for the optimal controls and for the Nash equilibrium are obtained. Under some assumptions, the optimality conditions for singular mean-field control are reduced to a reflected Skorohod problem. Applications to optimal irreversible investments under uncertainty have been investigated. Predictive mean-field equations as a model for prices influenced by beliefs about the future are studied in [86].

3.2.3. Stochastic control and optimal stopping (games) under nonlinear expectation

M.C. Quenez and A. Sulem have studied optimal stopping with nonlinear expectation $\mathcal{E}^g$ induced by a BSDE with jumps with nonlinear driver g and irregular obstacle/payoff (see [83]). In particular, they characterize the value function as the solution of a reflected BSDE. This property is used in [67] to address American option pricing in markets with imperfections. The Markovian case is treated in [64] when the payoff function is continuous.

In [8], M.C. Quenez, A. Sulem and R. Dumitrescu study a combined optimal control/stopping problem under nonlinear expectation $\mathcal{E}^g$ in a Markovian framework when the terminal reward function is only Borelian. In this case, the value function u associated with this problem is irregular in general. They establish a *weak* dynamic programming principle (DPP), from which they derive that the upper and lower semi-continuous envelopes of u are the sub- and super- *viscosity solution* of an associated nonlinear Hamilton-Jacobi-Bellman variational inequality.

The problem of a generalized Dynkin game problem with nonlinear expectation $\mathcal{E}^g$ is addressed in [65]. Under Mokobodzki's condition, we establish the existence of a value function for this game, and characterize this value as the solution of a doubly reflected BSDE. The results of this work are used in [9] to solve the problem of game option pricing in markets with imperfections.

A generalized mixed game problem when the players have two actions: continuous control and stopping is studied in a Markovian framework in [66]. In this work, dynamic programming principles (DPP) are established: a strong DPP is proved in the case of a regular obstacle and a weak one in the irregular case. Using these DPPs, links with parabolic partial integro-differential Hamilton-Jacobi- Bellman variational inequalities with two obstacles are obtained.

With B. Øksendal and C. Fontana, A. Sulem has contributed on the issues of robust utility maximization [85], [86], and relations between information and performance [70].

3.2.4. Generalized Malliavin calculus

Vlad Bally has extended the stochastic differential calculus built by P. Malliavin which allows one to obtain integration by parts and associated regularity probability laws. In collaboration with L. Caramellino (Tor Vegata University, Roma), V. Bally has developed an abstract version of Malliavin calculus based on a splitting method (see [49]). It concerns random variables with law locally lower bounded by the Lebesgue measure (the so-called Doeblin's condition). Such random variables may be represented as a sum of a "smooth" random variable plus a rest. Based on this smooth part, he achieves a stochastic calculus which is inspired from Malliavin calculus [6]. An interesting application of such a calculus is to prove convergence for irregular test functions (total variation distance and more generally, distribution distance) in some more or less classical frameworks as the Central Limit Theorem, local versions of the CLT and moreover, general stochastic polynomials [53]. An exciting application concerns the number of roots of trigonometric polynomials with random coefficients [15]. Using Kac Rice lemma in this framework one comes back to a multidimensional CLT and employs Edgeworth expansions of order three for irregular test functions in order to study the mean and the variance of the number of roots. Another application concerns U statistics associated to polynomial functions. The techniques of generalized Malliavin calculus developed in [49] are applied in for the approximation of Markov processes (see [56] and [55]). On the other hand, using the classical Malliavin calculus, V. Bally in collaboration with L. Caramellino and P. Pigato studied some subtle phenomena related to diffusion processes, as short time behavior and estimates of tubes probabilities (see [51], [52], [50]).

3.3. Numerical Probability

Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. This activity in the MathRisk team is strongly related to the development of the Premia software.

3.3.1. Simulation of stochastic differential equations

3.3.1.1. - Weak convergence of the Euler scheme in optimal transport distances.

With A. Kohatsu-Higa, A. Alfonsi and B. Jourdain [4] have proved using optimal transport tools that the Wasserstein distance between the time marginals of an elliptic SDE and its Euler discretization with N steps is not larger than $C\sqrt{\log(N)}/N$. The logarithmic factor may is removed when the uniform time-grid is replaced by a grid still counting N points but refined near the origin of times.

3.3.1.2. - Strong convergence properties of the Ninomiya Victoir scheme and multilevel Monte-Carlo estimators.

With their former PhD student, A. Al Gerbi, E. Clément and B. Jourdain [1] have proved strong convergence with order $1/2$ of the Ninomiya-Victoir scheme which is known to exhibit order 2 of weak convergence [82]. This study was aimed at analysing the use of this scheme either at each level or only at the finest level of a multilevel Monte Carlo estimator : indeed, the variance of a multilevel Monte Carlo estimator is related to the strong error between the two schemes used in the coarse and fine grids at each level. In [38], they proved that the order of strong convergence of the crude Ninomiya Victoir scheme is improved to 1 when the vector fields corresponding to each Brownian coordinate in the SDE commute, and in [39], they studied the error introduced by discretizing the ordinary differential equations involved in the Ninomiya-Victoir scheme.

3.3.1.3. - Non-asymptotic error bounds for the multilevel Monte Carlo Euler method.

A. Kebaier and B. Jourdain are interested in deriving non-asymptotic error bounds for the multilevel Monte Carlo method. As a first step, they dealt in [20] with the explicit Euler discretization of stochastic differential equations with a constant diffusion coefficient. They obtained Gaussian-type concentration. To do so, they used the Clark-Ocone representation formula and derived bounds for the moment generating functions of the squared difference between a crude Euler scheme and a finer one and of the squared difference of their Malliavin derivatives. The estimation of such differences is much more complicated than the one of a single Euler scheme contribution and explains why they suppose the diffusion coefficient to be constant. This assumption ensures boundedness of the Malliavin derivatives of both the SDE and its Euler scheme.

3.3.1.4. - Computation of sensibilities of integrals with respect to the invariant measure.

In [48], R. Assaraf, B. Jourdain, T. Lelièvre and R. Roux considered the solution to a stochastic differential equation with constant diffusion coefficient and with a drift function which depends smoothly on some real parameter λ, and admitting a unique invariant measure for any value of λ around $\lambda = 0$. Their aim was to compute the derivative with respect to λ of averages with respect to the invariant measure, at $\lambda = 0$. They analyzed a numerical method which consists in simulating the process at $\lambda = 0$ together with its derivative with respect to λ on a long time horizon. They gave sufficient conditions implying uniform-in-time square integrability of this derivative. This allows in particular to compute efficiently the derivative with respect to λ of the mean of an observable through Monte Carlo simulations.

3.3.1.5. - Approximation of doubly reflected Backward stochastic differential equations.

R. Dumitrescu and C. Labart have studied the discrete time approximation scheme for the solution of a doubly reflected Backward Stochastic Differential Equation with jumps, driven by a Brownian motion and an independent compensated Poisson process [63], [62].

3.3.1.6. - Parametrix methods.

V. Bally and A. Kohatsu-Higa have recently proposed an unbiased estimator based on the parametrix method to compute expectations of functions of a given SDE ([54]). This method is very general, and A. Alfonsi, A. Kohastu-Higa and M. Hayashi [42] have applied it to the case of one-dimensional reflected diffusions. In this case, the estimator can be obtained explicitly by using the scheme of Lépingle [79] and is quite simple to implement. It is compared to other simulation methods for reflected SDEs.

3.3.2. Estimation of the parameters of a Wishart process

A. Alfonsi, A. Kebaier and C. Rey [43] have computed the Maximum Likelihood Estimator for the Wishart process and studied its convergence in the ergodic and in some non ergodic cases. In the ergodic case, which is the most relevant for applications, they obtain the standard square-root convergence. In the non ergodic case, the analysis rely on refined results for the Laplace transform of Wishart processes, which are of independent interest.

3.3.3. Optimal stopping and American options

In joint work with A. Bouselmi, D. Lamberton studied the asymptotic behavior of the exercise boundary near maturity for American put options in exponential Lévy models. In [7], they deal with jump-diffusion models, and establish that, in some cases, the behavior differs from the classical Black and Scholes setting. D. Lamberton has also worked on the binomial approximation of the American put. The conjectured rate of convergence is $O(1/n)$ where n is the number of time periods. He was able to derive a $O((\ln n)^{\alpha}/n)$ bound, where the exponent α is related to the asymptotic behavior of the exercise boundary near maturity.

4. Application Domains

4.1. Financial Mathematics, Insurance

The domains of application are quantitative finance and insurance with emphasis on risk modeling and control. In particular, Mathrisk focuses on dependence modeling, systemic risk, market microstructure modeling and risk measures.

5. Highlights of the Year

5.1. Highlights of the Year

5.1.1. Awards

Aurélien Alfonsi : Award for the Best Young Researcher in Finance and Insurance, Europlace Institute of Finance and SCOR Corporate Foundation.

5.1.2. Publications

B. Øksendal and A. Sulem. *Applied Stochastic Control of Jump Diffusions*. 3rd edition (436 pages) 2019, Universitext, Springer Verlag, Berlin, Heidelberg, New York [23].

6. New Software and Platforms

6.1. PREMIA

KEYWORDS: Financial products - Computational finance - Option pricing

SCIENTIFIC DESCRIPTION: The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing.

Premia is thus a powerful tool to assist Research and Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods without starting from scratch.

Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: 1 - to be a powerful testing platform for comparing different numerical methods between each other, 2 - to build a link between professional financial teams and academic researchers, 3 - to provide a useful teaching support for Master and PhD students in mathematical finance.

FUNCTIONAL DESCRIPTION: Premia is a software designed for option pricing, hedging and financial model calibration.

- Participants: Agnes Sulem, Antonino Zanette, Aurélien Alfonsi, Benjamin Jourdain, Jérôme Lelong and Bernard Lapeyre
- Partners: Inria - Ecole des Ponts ParisTech - Université Paris-Est
- Contact: Agnes Sulem
- URL: http://www.premia.fr

6.2. Premia

6.2.1. Development of the quantitative platform Premia in 2019

Premia 21 has been delivered to the Premia Consortium on March 21th 2019.
In this version, the following algorithms have been implemented:

6.2.1.1. Machine Learning, Risk Management, Risk model, Insurance

- Machine Learning for Quantitative Finance: Fast Derivative Pricing, Hedging and Fitting. J. De Spiegeleer, D. B. Madan, S. Reyners, W. Schoutens
  Quantitative Finance 2018
- Gaussian Process Regression for Pricing Variable Annuities with Stochastic Volatility and Interest Rate. L.Goudenege A.Molent A.Zanette 2019
- Machine Learning for Pricing American Options in High Dimension. L.Goudenege A.Molent A.Zanette 2019
- Neural networks for American options. L.Goudenege
- Sampling of probability measures in the convex order and approximation of Martingale Optimal Transport problems. A. Alfonsi, J. Corbetta, B. Jourdain
- Computing Credit Valuation Adjustment solving coupled PIDEs in the Bates model. L. Goudenege, A. Molent, A. Zanette
  Computational Management Science, to appear 2019
- Valuation of variable annuities with Guaranteed Minimum Withdrawal Benefit under stochastic interest rate. P.V.Shevchenko, X.Luo
  Insurance: Mathematics and Economics, Volume 76, 2017

6.2.1.2. Equity Derivatives

- An adjoint method for the exact calibration of Stochastic Local Volatility models. M.Wins, K.J.in 't Hout.
  Journal of Computational Science, Volume 24, 2018
- Discretization of class of diffusions nonlinear in the sense of McKean including the calibrated LVSV model. B. Jourdain, A. Zhou
- On an efficient multiple time-step Monte Carlo simulation of the SABR model. A. Leitao, L.A. Grzelak, C.W. Oosterlee
  Applied Mathematics and Computation, Vol. 293, 2017
- Robust Barrier Option Pricing by Frame Projection under Exponential Levy Dynamics. J.L. Kirkby
  Applied Mathematical Finance, Volume 24, Issue 4, 2017
- Ultra-Fast Pricing Barrier Options and CDSs. S.Levendorskii
  International Journal of Theoretical and Applied Finance, 20(5), 2017
- Efficient Binomial tree for the discretization of the CEV model. L.Caramellino, E.Lombardo.
- Pricing path-dependent Bermudan options using Wiener chaos expansion: an embarrassingly parallel approach. J.Lelong

We benefited from the help of the engineer Pierre-Guillaume Raverdy.

7. New Results

7.1. Control of systemic risk in a dynamic framework

Agnès Sulem, Andreea Minca (Cornell University), Hamed Amini (J. Mack Robinson College of Business, Georgia State University) and Rui Chen have studied a Dynamic Contagion Risk Model With Recovery Features [27]. In this paper, they introduce threshold growth in the classical threshold contagion model, in which nodes have downward jumps when there is a failure of a neighboring node. Choosing the configuration model as underlying graph, they prove fluid limits for the baseline model, as well as extensions to the directed case, state-dependent inter-arrival times and the case of growth driven by upward jumps. They obtain explicit ruin probabilities for the nodes according to their characteristics: initial threshold and in- (and out-) degree. They then allow nodes to choose their connectivity by trading off link benefits and contagion risk. They define a rational equilibrium concept in which nodes choose their connectivity according to an expected failure probability of any given link, and then impose condition that the expected failure probability coincides with the actual failure probability under the optimal connectivity. Existence of an asymptotic equilibrium is shown as well as convergence of the sequence of equilibria on the finite networks. In particular, these results show that systems with higher overall growth may have higher failure probability in equilibrium.

The results have been presented in Lisbon at the COMPLEX NETWORKS 2019 conference. Rui Chen has defended his thesis in July 2019 on this topic [10].

7.2. Mean-field BSDEs and systemic risk measures

Agnès Sulem with her PhD student Rui Chen, Andreea Minca and Roxana Dumitrescu have studied mean-field BSDEs with a generalized mean-field operator that can capture the average intensity in an inhomogeneous random graph. Comparison and strict comparison results have been obtained. Based on these, they interpret the BSDE solution as a global dynamic risk measure that can account for the intensity of system interactions and therefore incorporate systemic risk. Using Fenchel-Legendre transforms, they establish a dual representation for the expectation of the risk measure, and exhibit its dependence on the mean-field operator [31].

7.3. Risk management in finance and insurance

7.3.1. Option pricing in a non-linear incomplete market model with default

Agnès Sulem has studied with Miryana Grigorova (University of Leeds) and Marie-Claire Quenez (Université Paris Denis Diderot) superhedging prices and the associated superhedging strategies for both European and American options (see [33] and [32] in a non-linear incomplete market model with default. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow non-linear dynamics with a non-linear driver f.

7.3.2. Neural network regression for Bermudan option pricing

The pricing of Bermudan options amounts to solving a dynamic programming principle, in which the main difficulty, especially in high dimension, comes from the conditional expectation involved in the computation of the continuation value. These conditional expectations are classically computed by regression techniques on a finite dimensional vector space. In [36], Bernard Lapeyre and Jérôme Lelong study neural networks approximations of conditional expectations. They prove the convergence of the well-known Longstaff and Schwartz algorithm when the standard least-square regression is replaced by a neural network approximation. They illustrate the numerical efficiency of neural networks as an alternative to standard regression methods for approximating conditional expectations on several numerical examples.

7.3.3. Hybrid numerical method for option pricing

With Giulia Terenzi, Lucia Caramellino (Tor Vegata University), and Maya Briani (CNR Roma), Antonino Zanette develop and study stability properties of a hybrid approximation of functionals of the Bates jump model with stochastic interest rate that uses a tree method in the direction of the volatility and the interest rate and a finite-difference approach in order to handle the underlying asset price process. They also propose hybrid simulations for the model, following a binomial tree in the direction of both the volatility and the interest rate, and a space-continuous approximation for the underlying asset price process coming from a Euler–Maruyama type scheme. They test their numerical schemes by computing European and American option prices [17].

7.3.4. American options

With his PhD student Giulia Terenzi, Damien Lamberton has been working on American options in Heston's model [22]. He is currently preparing his contribution to a winter school on "Theory and practice of optimal stopping and free boundary problems" (cf. https://conferences.leeds.ac.uk/osfbp/).

7.3.5. Solvency Capital Requirement in Insurance

A. Alfonsi has obtained a grant from AXA Foundation on a Joint Research Initiative with a team of AXA France working on the strategic asset allocation. This team has to make recommendations on the investment over some assets classes as, for example, equity, real estate or bonds. In order to do that, each side of the balance sheet (assets and liabilities) is modeled in order to take into account their own dynamics but also their interactions. Given that the insurance products are long time contracts, the projections of the company's margins have to be done considering long maturities. When doing simulations to assess investment policies, it is necessary to take into account the SCR which is the amount of cash that has to be settled to manage the portfolio. Typically, the computation of the future values of the SCR involve expectations under conditional laws, which is greedy in computation time.

A. Alfonsi and his PhD student A. Cherchali have developed a model of the ALM management of insurance companies that takes into account the regulatory constraints on life-insurance [25]. We now focus on developing Multilevel Monte-Carlo methods to approximate the SCR (Solvency Capital Requirement).

7.3.6. Pricing and hedging variable annuities of GMWB type in advanced stochastic models

Antonino Zanette with Ludovic Goudenège (Ecole Centrale de Paris) and Andrea Molent (University of Udine) study the valuation of a particular type of variable annuity called GMWB when advanced stochastic models are considered. As remarked by Yang and Dai (Insur Math Econ 52(2):231–242, 2013), and Dai et al. (Insur Math Econ 64:364–379, 2015), the Black–Scholes framework seems to be inappropriate for such a long maturity products. Also Chen et al. (Insur Math Econ 43(1):165–173, 2008) show that the price of GMWB variable annuities is very sensitive to the interest rate and the volatility parameters. They propose here to use a stochastic volatility model (the Heston model) and a Black–Scholes model with stochastic interest rate (the Black–Scholes Hull–White model). For this purpose, they consider four numerical methods: a hybrid tree-finite difference method, a hybrid tree-Monte Carlo method, an ADI finite difference scheme and a Standard Monte Carlo method. These approaches are employed to determine the no-arbitrage fee for a popular version of the GMWB contract and to calculate the Greeks used in hedging. Both constant withdrawal and dynamic withdrawal strategies are considered. Numerical results are presented, which demonstrate the sensitivity of the no-arbitrage fee to economic and contractual assumptions as well as the different features of the proposed numerical methods [18].

7.4. Stochastic Analysis and probabilistic numerical methods

7.4.1. Particles approximation of mean-field SDEs

O. Bencheikh and Benjamin Jourdain analysed the rate of convergence of a system of N interacting particles with mean-field rank based interaction in the drift coefficient and constant diffusion coefficient [16], [30]. They first adapted arguments by Kolli and Shkolnikhov to check trajectorial propagation of chaos with optimal rate $N^{-1/2}$ to the associated stochastic differential equations nonlinear in the sense of McKean. They next relaxed the assumption needed by Bossy to check convergence in $L^1(\mathbf{R})$ of the empirical cumulative distribution function of the Euler discretization with step h of the particle system to the solution of a one dimensional viscous scalar conservation law with rate $\mathcal{O}\left(\frac{1}{\sqrt{N}}+h\right)$. Last, they proved that the bias of this stochastic particle method behaves in $\mathcal{O}\left(\frac{1}{N}+h\right)$, which is confirmed by numerical experiments.

7.4.2. Abstract Malliavin calculus and convergence in total variation

In collaboration with L. Caramellino (University Tor Vergata) and with G. Poly (University of Rennes), V. Bally has settled a Malliavin type calculus for a general class of random variables, which are not supposed to be Gaussian (as it is the case in the standard Malliavin calculus). This is an alternative to the Γ calculus settled by Bakry, Gentile and Ledoux. The main application is the estimate in total variation distance of the error in general convergence theorems. This is done in [29].

7.4.3. Invariance principles

As an application of the methodology mentioned above, V. Bally and coauthors have studied several limit theorems of Central Limit type - (see [14] and [15]. In particular they have estimate the total variation distance between random polynomials on one hand, and proved an universality principle for the variance of the number of roots of trigonometric polynomials with random coefficients, on the other hand.

7.4.4. Regularity of the low of the solution of jump type equations

V. Bally, L. Caramellino and G. Poly obtained some new regularity results for the solution of the 2 dimensional Bolzmann equation (see [13]). Moreover, in collaboration with L. Caramellino and A. Kohatsu Higa, V. Bally has started a research program on the regularity of the solutions of jump type equations. A first result in this sense is contained in [28].

7.4.5. Approximation of ARCH models

Benjamin Jourdain and Gilles Pagès (LPSM) are interested in proposing approximations of a sequence of probability measures in the convex order by finitely supported probability measures still in the convex order [35]. They propose to alternate transitions according to a martingale Markov kernel mapping a probability measure in the sequence to the next and dual quantization steps. In the case of ARCH models and in particular of the Euler scheme of a driftless Brownian diffusion, the noise has to be truncated to enable the dual quantization step. They analyze the error between the original ARCH model and its approximation with truncated noise and exhibit conditions under which the latter is dominated by the former in the convex order at the level of sample-paths. Last, they analyse the error of the scheme combining the dual quantization steps with truncation of the noise according to primal quantization.

7.4.6. Convergence of metadynamics

By drawing a parallel between metadynamics and self interacting models for polymers, B. Jourdain, T. Lelièvre (Cermics / ENPC) and P.-A. Zitt (LAMA) study the longtime convergence of the original metadynamics algorithm in the adiabatic setting, namely when the dynamics along the collective variables decouples from the dynamics along the other degrees of freedom. They also discuss the bias which is introduced when the adiabatic assumption does not holds [34].

7.4.7. Optimal transport

With V. Ehrlacher, D. Lombardi and R. Coyaud, Aurelien Alfonsi isworking on numerical a approximations of the optimal transport between two (or more) probability measures [26].

7.4.8. Generic approximation schemes for Markov semigroups.

A. Alfonsi and V. Bally have produced a general approximation scheme for Markov semigroups, based on random grids. This is a new approach to approximation schemes which is an alternative to the multi level method and the Romberg method [24].

7.4.9. Approximation with rough paths

A. Alfonsi and A. Kebaier are working on the approximation of some processes with rough paths.

8. Bilateral Contracts and Grants with Industry

8.1. Bilateral Contracts with Industry

- Consortium PREMIA, Natixis - Inria
- Consortium PREMIA, Crédit Agricole Corporate Investment Bank (CA - CIB) - Inria
- AXA Joint Research Initiative on Numerical methods for the ALM, from September 2017 to August 2020. PhD grant of Adel Cherchali, Supervisor: A. Alfonsi.
- CIFRE agreement Milliman company/Ecole des Ponts (http://fr.milliman.com),

  PhD thesis of Sophian Mehalla (started November 2017) on "Interest rate risk modeling for insurance companies", Supervisor: Bernard Lapeyre.
- Collaboration with IRT Systemx

  PhD grant of Adrien Touboul (started November 2017) on "Uncertainty computation in a graph of physical simulations", Supervisors: Bernard Lapeyre and Julien Reygner.

8.2. Grants with Industry

Chair X-ENPC-SU-Société Générale "Financial Risks" of the Risk fondation : A. Alfonsi, B. Jourdain, B. Lapeyre

9. Partnerships and Cooperations

9.1. National Initiatives

- ANR Cosmos 2015-2018, Participant: B. Jourdain ; Partners : Ecole des Ponts, Telecom, INIRIA Rennes and IBPC
- Labex Bezout
  http://bezout.univ-paris-est.fr

9.1.1. Competitivity Clusters

Pôle Finance Innovation

9.2. International Initiatives

9.2.1. Inria International Partners

9.2.1.1. Informal International Partners

- Center of Excellence program in Mathematics and Life Sciences at the Department of Mathematics, University of Oslo, Norway, (B. Øksendal).
- Cornell University, ORIE department (Andreea Minca)
- Roma Tor Vergata University (Lucia Caramellino)
- Ritsumeikan University (A. Kohatsu-Higa).

9.3. International Research Visitors

9.3.1. Visits of International Scientists

- Oleg Kudryavtsev (Rostov University, Russia)
- B. Stemper (Weierstrass Institute, Berlin)
- A. Kohatsu Higa (Ritsumeikan University)
- Justin Kirkby (Georgia Institute of Technology, Atlanta)
- Xiao Wei (Beijing University)
- Anton Arnold (TU Vienna)

9.3.1.1. Internships

- Baba Abdel Hamid, Inria
- Asma Sassi, Inria

9.3.2. Visits to International Teams

9.3.2.1. Research Stays Abroad

In the period 15.05 - 15. 06. 2019 Vlad Bally was an invited professor at the University Tor Vergata, Roma. Here he gave a course of 20h entitled "Integration by Parts and Convergence in Total Variation".

10. Dissemination

10.1. Promoting Scientific Activities

10.1.1. Scientific Events: Organisation

10.1.1.1. Member of the Organizing Committees

- A. Alfonsi

  Co-organizer of the working group seminar of MathRisk “Méthodes stochastiques et finance”. http://cermics.enpc.fr/~alfonsi/GTMSF.html
- V. Bally

  Organizer of the seminar of the LAMA laboratory of Université Paris-Est.
- A. Sulem

  Co-organizer of the seminar Inria-MathRisk /Université Paris Diderot LPSM “Numerical probability and mathematical finance”. https://www.lpsm.paris/mathfipronum/gt

10.1.2. Journal

10.1.2.1. Member of the Editorial Boards

- B. Jourdain

  Associate editor of:
  - ESAIM : Proceedings and Surveys
  - Stochastic Processes and their Applications (SPA)
- D. Lamberton

  Associate editor of:
  - Mathematical Finance,
  - Associate editor of ESAIM Probability & Statistics
- A. Sulem

  Associate editor of:
  - Journal of Mathematical Analysis and Applications (JMAA)
  - International Journal of Stochastic Analysis (IJSA)
  - SIAM Journal on Financial Mathematics (SIFIN)

10.1.2.2. Reviewer - Reviewing Activities

- B. Jourdain: Reviewer for *Mathematical Reviews*
- A. Sulem: Reviewer for *Mathematical Reviews*

10.1.3. Invited Talks

- Aurélien Alfonsi
  - 4th of April, 2019: "Lifted and Geometric Differentiability of the Squared Quadratic Wasserstein Distance", Séminaire MAD-Stat, Toulouse School of Economics.
  - 16th of May, 2019: "Approximation of OT problems with marginal moments contraints", Séminaire de probabilités et mathématiques financières, Evry.
  - 20th of June, 2019: "A generic construction for high order approximation schemes of semigroups using random grids", Mathematical and Computational Finance Seminar, Oxford.

  - 27th of September, 2019: "A generic construction for high order approximation schemes of semigroups using random grids", Séminaire Bachelier.
  - 24th of October, 2019: "A generic construction for high order approximation schemes of semigroups using random grids", GT Finance mathématique, probabilités numériques et statistique des processus, LPSM.
- Vlad Bally
  - The 22nd Conference of the Romanian Society of Probability and Statistics, Bucharest, May 10-11, Talk "Convergence in the CLT in distribution norms"
  - Analyse stochastique et thèmes connexes. 6-9 mai 2019, Bucarest, Roumanie. Talk "Regularity and estimates for the function solution of the 2D Bolzmann equation"
  - Perturbation Techniques in Stochastic Analysis and its Applications. Luminy 11-15 March. Talk "Regularity and estimates for the function solution of the 2D Bolzmann equation"
  - Conference Unirandom on random nodal sets, Rennes 9-13 September.
  - Conference on asymptotic expansions and Malliavin calculus. Talk:"Regularization lemmas and convergence in total variation".
- Benjamin Jourdain
  - Vienna Seminar in Mathematical Finance and Probability, 10 October 2019 : Differentiability of the squared quadratic Wasserstein distance
  - Inria-CWI workshop, Amsterdam, 17-18 September 2019 : Sampling of probability measures in the convex order and computation of robust option price bounds
  - MCMC 2019, Sydney, 8-12 July 2019 : Weak error analysis for some mean-field SDEs
  - Nine talks on contemporary optimal transport problems, Strasbourg, 4-5 July 2019: The inverse transform martingale coupling
  - 9th general AMAMEF conference, Paris, 11-14 June 2019 : The inverse transform martingale coupling
  - Nancy probability and statistics seminar, 9 May 2019 : A new family of martingale couplings in dimension one
  - Conference Perturbation Techniques in Stochastic Analaysis and its Applications, Marseille 11-15 March 2019 : 3h lectures entitled Stochastic Differential Equations with mean-field rank-based coefficients
- Agnès Sulem
  - Conference on Stochastic Analysis and Applications Risør, Norway, August 26-30 2019. *Nonlinear pricing in incomplete markets with default*.
  - Complex Networks 2919, December 10-12, 2019, Lisbon. (Conference with referee process), *A Dynamic Contagion Risk Model With Recovery Features*.

10.1.4. Research Administration

- A. Alfonsi
  - Deputy director of CERMICS laboratory until August,
  - Director of the CERMICS since September.
  - In charge of the Master “Finance and Data” at the Ecole des Ponts.
- V. Bally
  - Member of the LAMA committee
  - Responsible of the Master 2, option finance.
- Benjamin Jourdain
  - Head of the doctoral school MSTIC, University Paris-Est until February 2019

- D. Lamberton
  – Vice-president for research at Université Paris-Est Marne-la-Vallée
- A. Sulem
  – Member of the Scientific Committee of AMIES
  – Corresponding member of the Operational Committee for the assesment of Legal and Ethical risks (COERLE) at Inria Paris research center
  – Member of the Committee for Inria international Chairs

10.2. Teaching - Supervision - Juries

10.2.1. Teaching

Licence :

- A. Alfonsi: "Probabilités”, first year course at Ecole des Ponts.
- V. Bally : "Analyse Hilbertienne", Course L3, UPEMLV

Master :

- Aurélien Alfonsi

- “Données Haute Fréquence en finance”, lecture for the Master at UPEMLV.
- “Mesures de risque”, Master course of UPEMLV and Sorbonne Université.
- Professeur chargé de cours at Ecole Polytechnique.

- Vlad Bally

- "Taux d’Intêret", M2 Finance.
- "Calcul de Malliavin et applications en finance", M2 Finance
- "Analyse du risque" M2 Actuariat,
- "Processus Stochastiques" M2 Recherche

- Benjamin Jourdain

- course "Mathematical finance", 2nd year ENPC
- course "Monte-Carlo Markov chain methods and particle algorithms", Research Master Probabilités et Modèles Aléatoires, Sorbonne Université

- B. Jourdain, B. Lapeyre

- course "Monte-Carlo methods", 3rd year ENPC and Research Master Mathématiques et Application, university of Marne-la-Vallée

- J.-F. Delmas, B.Jourdain

- course "Jump processes with applications to energy markets", 3rd year ENPC and Research Master Mathématiques et Application, University of Marne-la-Vallée

- D. Lamberton

- "Calcul stochastique pour la finance", master 1 course, Université Paris-Est Marne-la-Vallée

- B. Lapeyre

- Monte-Carlo methods in quantitative finance, Master of Mathematics, University of Luxembourg,

- A. Sulem

- "PDE methods in Finance", Master of Mathematics, University of Luxembourg, 22 h lectures and responsible of the module "Numerical Methods in Finance".

10.2.2. Supervision

- PhD: Rui Chen, "Dynamic Optimal Control for Distress in Large Financial Networks and Mean field Systems with Jumps", Université Paris-Dauphine, defended on July 19th 2019, Supervisor: Agnès Sulem [10].
- PhD in progress :
  - Anas Bentaleb (started February 2018) : Mathematical techniques for expected exposure evaluation, Supervisor: B. Lapeyre.
  - Adel Cherchali, “Numerical methods for the ALM”, funded by Fondation AXA, started in September 2017, Supervisor: Aurélien Alfonsi
  - Rafaël Coyaud, “Deterministic ans stochastic numerical methods for multimarginal and martingale constraint optimal transport problems”, started in October 2017, Supervisor: Aurélien Alfonsi
  - Oumaima Bencheikh (started November 2017) "Acceleration of probabilistic particle methods", supervised by B.Jourdain
  - Ezechiel Kahn (started September 2018) "Functional inequalities for random matrices models", supervised by B. Jourdain and D. Chafai
  - Sophian Mehalla (started November 2017), CIFRE agreement Milliman company/Ecole des Ponts (http://fr.milliman.com, Supervisor: B. Lapeyre
  - William Margheriti (started January 2018) "Numerical methods for martingale optimal transport problems", supervised by J.-F. Delmas and B. Jourdain

10.2.3. Juries

- Aurélien Alfonsi:
  - Report and jury of the PhD thesis of Rui Chen, July 19, Paris Dauphine.
  - Report and jury of the PhD thesis of Babacar Diallo, December 9, Evry University.
- Benjamin Jourdain:
  - Report and jury of the PhD of Yating Liu, defended on December 3, Sorbonne University
  - PhD of Victor Marx, defended on October 25, University Nice Côte d’Azur
  - PhD of Nicolas Thomas, defended on June 20, Sorbonne University
- Agnès Sulem:
  - Report and jury of the HdR thesis *"Gestion optimale de Portefeuille: Du contrôle Sensible au Risque, à la Finance Comportementale et à la Science des Données* Sebastien Lleo, CNAM, Paris, 4 February 2019
  - PhD of Cyril Benezet, defended on 5 November 2019, LPSM, Université Paris-Diderot, (Chair of the Committee)
  - Member of the Committee for the recruitment of a Professor in applied mathematics, finance and numerical probability, Laboratoire de probabilités (LPSM), Université Paris-Diderot, Spring 2019.

11. Bibliography

Major publications by the team in recent years

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Publications of the year

Doctoral Dissertations and Habilitation Theses

[10] R. CHEN. *Dynamic optimal control for distress large financial networks and Mean field systems with jumps*, Université Paris-Dauphine, July 2019, https://hal.inria.fr/tel-02434108

Articles in International Peer-Reviewed Journals

[11] A. ALFONSI, J. CORBETTA, B. JOURDAIN. *Sampling of one-dimensional probability measures in the convex order and computation of robust option price bounds*, in "International Journal of Theoretical and Applied Finance", 2019, vol. 22, n^{o} 3, This paper is an updated version of a part of the paper https://hal.archives-ouvertes.fr/hal-01589581 (or https://arxiv.org/pdf/1709.05287.pdf) [*DOI :* 10.1142/S021902491950002X], https://hal-enpc.archives-ouvertes.fr/hal-01963507

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