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Activity Report 2019

Project-Team MINGUS

MultI-scale Numerical Geometric Schemes

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

RESEARCH CENTER
Rennes - Bretagne-Atlantique

THEME
Numerical schemes and simulations

Table of contents

Project-Team MINGUS

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Keywords:

Computer Science and Digital Science:

A6.1.1. - Continuous Modeling (PDE, ODE)
A6.1.2. - Stochastic Modeling
A6.1.4. - Multiscale modeling
A6.2.1. - Numerical analysis of PDE and ODE
A6.2.7. - High performance computing

Other Research Topics and Application Domains:

B4.2.2. - Fusion
B5.11. - Quantum systems
B9.5.2. - Mathematics

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2. Overall Objectives

2.1. Presentation

In applications involving complex physics, such as plasmas and nanotechnologies, numerical simulations serve as a prediction tool supplementing real experiments and are largely endorsed by engineers or researchers. Their performances rely not only on computational power, but also on the efficiency of the underlying numerical method and the complexity of the underlying models. The contribution of applied mathematics is then required, on the one hand for a better understanding of qualitative properties and a better identification of the different regimes present in the model, and on the other hand, for a more sounded construction of new models based on asymptotic analysis. This mathematical analysis is expected to greatly impact the design of *multiscale* numerical schemes.

The proposed research group MINGUS will be dedicated to the mathematical and numerical analysis of (possibly stochastic) partial differential equations (PDEs), originating from plasma physics and nanotechnologies, with emphasis on *multiscale* phenomena either of **highly-oscillatory**, of **dissipative** or **stochastic** types. These equations can be also encountered in applications to rarefied gas dynamics, radiative transfer, population dynamics or laser propagation, for which the multiscale character is modelled by a scale physical parameter ε.

Producing accurate solutions of multiscale equations is extremely challenging owing to severe restrictions to the numerical methods imposed by fast (or stiff) dynamics. *Ad-hoc* numerical methods should aim at capturing the slow dynamics solely, instead of resolving finely the stiff dynamics at a formidable computational cost. At the other end of the spectrum, the separation of scales -as required for numerical efficiency- is envisaged in asymptotic techniques, whose purpose is to describe the model in the limit where the small parameter ε tends to zero. MINGUS aspires to accommodate sophisticated tools of mathematical analysis and heuristic numerical methods in order to produce simultaneously rich asymptotic models and efficient numerical methods.
To be more specific, MINGUS aims at finding, implementing and analysing new multiscale numerical schemes for the following physically relevant multiscale problems:

- **Highly-oscillatory Schrödinger equation for nanoscale physics:** In quantum mechanics, the Schrödinger equation describes how the quantum state of some physical system changes with time. Its mathematical and numerical study is of paramount importance to fundamental and applied physics in general. We wish to specifically contribute to the mathematical modeling and the numerical simulation of confined quantum mechanical systems (in one or more space dimensions) possibly involving stochastic terms. Such systems are involved in quantum semi-conductors or atom-chips, as well as in cold atom physics (Bose-Einstein condensates) or laser propagation in optical fibers.

  The prototypical equation is written

$$\boxed{i\varepsilon\partial_t\psi^\varepsilon = \tfrac{\varepsilon^2}{\beta}\Delta\psi^\varepsilon + |\psi^\varepsilon|^2\psi^\varepsilon + \psi^\varepsilon\xi} \tag{1}$$

where the function $\psi^\varepsilon = \psi^\varepsilon(t,x) \in \mathbb{C}$ depends on time $t \geq 0$ and position $x \in \mathbb{R}^3$, $\xi = \xi(x,t)$ is a white noise and where the small parameter ε is the Planck's constant describing the microscopic/macroscopic ratio. The limit $\varepsilon \to 0$ is referred to as the semi-classical limit. The regime $\varepsilon = 1$ and $\beta \to 0$ (this can be for instance the relative length of the optical fiber) is highly-oscillatory. The noise ξ acts as a potential, it may represent several external perturbations. For instance temperature effects in Bose-Einstein condensation or amplification in optical fibers. The highly oscillatory regime combined with noise introduces new challenges in the design of efficient schemes.

- **Highly-oscillatory or highly-dissipative kinetic equations:** Plasma is sometimes considered as the fourth state of matter, obtained for example by bringing a gas to a very high temperature. A globally neutral gas of neutral and charged particles, called plasma, is then obtained and is described by a kinetic equation as soon as collective effects dominate as compared to binary collisions. A situation of major importance is magnetic fusion in which collisions are not predominant. In order to confine such a plasma in devices like tokamaks (ITER project) or stellarators, a large magnetic field is used to endow the charged particles with a cyclotronic motion around field lines. Note that kinetic models are also widely used for modeling plasmas in earth magnetosphere or in rarefied gas dynamics.

  Denoting $f^\varepsilon = f^\varepsilon(t,x,v) \in \mathbb{R}^+$ the distribution function of charged particles at time $t \geq 0$, position $x \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$, a typical kinetic equation for f^ε reads

$$\boxed{\partial_t f^\varepsilon + v \cdot \nabla_x f^\varepsilon + \left(E + \tfrac{1}{\varepsilon}(v \times B)\right) \cdot \nabla_v f^\varepsilon = \tfrac{1}{\beta} Q(f^\varepsilon) + f^\varepsilon m^\varepsilon} \tag{2}$$

  where (E,B) is the electro-magnetic field (which may itself depend on f through Maxwell's equations), m^ε is a random process (which may describe absorption or creation of particles) and Q is a collision operator. The dimensionless parameters ε, β are related to the cyclotronic frequency and the mean free path. Limits $\varepsilon \to 0$ and $\beta \to 0$ do not share the same character (the former is oscillatory and the latter is dissipative) and lead respectively to gyrokinetic and hydrodynamic models. The noise term m^ε is correlated in space and time. At the limit $\varepsilon \to 0$, it converges formally to a white noise and stochastic PDEs are obtained.

MINGUS project is the follow-up of IPSO, ending in december in 2017. IPSO original aim was to extend the analysis of geometric schemes from ODEs to PDEs. During the last evaluation period, IPSO also considered the numerical analysis of geometric schemes for (S)PDEs, possibly including multiscale phenomena. Breakthrough results [36], [38], [39], [42] have been recently obtained which deserve to be deepened and extended. It thus appears quite natural to build the MINGUS team upon these foundations.

The objective of MINGUS is twofold: the construction and the analysis of numerical schemes (such as "Uniformly Accurate numerical schemes", introduced by members of the IPSO project) for multiscale (S)PDEs originating from physics. In turn, this requires (i) a deep mathematical understanding of the (S)PDEs under consideration and (ii) a strong involvement into increasingly realistic problems, possibly resorting to parallel computing. For this aspect, we intend to benefit from the Inria Selalib software library which turns out to be the ideal complement of our activities.

3. Research Program

3.1. Research Program

The MINGUS project is devoted to the mathematical and numerical analysis of models arising in plasma physics and nanotechnology. The main goal is to construct and analyze numerical methods for the approximation of PDEs containing multiscale phenomena. Specific multiscale numerical schemes will be proposed and analyzed in different regimes (namely highly-oscillatory and dissipative). The ultimate goal is to dissociate the physical parameters (generically denoted by ε) from the numerical parameters (generically denoted by h) with a uniform accuracy. Such a task requires mathematical prerequisite of the PDEs.

Then, for a given stiff (highly-oscillatory or dissipative) PDE, the methodology of the MINGUS team will be the following

- Mathematical study of the asymptotic behavior of multiscale models.
  This part involves averaging and asymptotic analysis theory to derive asymptotic models, but also long-time behavior of the considered models.
- Construction and analysis of multiscale numerical schemes.
  This part is the core of the project and will be deeply inspired for the mathematical prerequisite. In particular, our ultimate goal is the design of *Uniformly Accurate* (UA) schemes, whose accuracy is independent of ε.
- Validation on physically relevant problems.
  The last goal of the MINGUS project is to validate the new numerical methods, not only on toy problems, but also on realistic models arising in physics of plasmas and nanotechnologies. We will benefit from the Selalib software library which will help us to scale-up our new numerical methods to complex physics.

3.1.1. Dissipative problems

In the dissipative context, the asymptotic analysis is quite well understood in the deterministic case and multiscale numerical methods have been developed in the last decades. Indeed, the so-called Asymptotic-Preserving schemes has retained a lot of attention all over the world, in particular in the context of collisional kinetic equations. But, there is still a lot of work to do if one is interesting in the derivation high order asymptotic models, which enable to capture the original solution for all time. Moreover, this analysis is still misunderstood when more complex systems are considered, involving non homogeneous relaxation rates or stochastic terms for instance. Following the methodology we aim at using, we first address the mathematical analysis before deriving multiscale efficient numerical methods.

A simple model of dissipative systems is governed by the following differential equations

$$\left\{ \begin{array}{rcll} \dfrac{dx^\varepsilon(t)}{dt} & = & \mathcal{G}(x^\varepsilon(t), y^\varepsilon(t)), & x^\varepsilon(0) = x_0, \\ \dfrac{dy^\varepsilon(t)}{dt} & = & -\dfrac{y^\varepsilon(t)}{\varepsilon} + \mathcal{H}(x^\varepsilon(t), y^\varepsilon(t)), & y^\varepsilon(0) = y_0, \end{array} \right. \tag{3}$$

for given initial condition $(x_0, y_0) \in \mathbb{R}^2$ and given smooth functions $\mathcal{G}, \mathcal{H}$ which possibly involve stochastic terms.

3.1.1.1. Asymptotic analysis of dissipative PDEs (F. Castella, P. Chartier, A. Debussche, E. Faou, M. Lemou)

Derivation of asymptotic problems

Our main goal is to analyze the asymptotic behavior of dissipative systems of the form (3) when ε goes to zero. The *center manifold theorem* [35] is of great interest but is largely unsatisfactory from the following points of view

- a constructive approach of h and x_0^ε is clearly important to identify the high-order asymptotic models: this would require expansions of the solution by means of B-series or word-series [37] allowing the derivation of error estimates between the original solution and the asymptotic one.
- a better approximation of the transient phase is strongly required to capture the solution for small time: extending the tools developed in averaging theory, the main goal is to construct a suitable change of variable which enables to approximate the original solution for all time.

Obviously, even at the ODE level, a deep mathematical analysis has to be performed to understand the asymptotic behavior of the solution of (3). But, the same questions arise at the PDE level. Indeed, one certainly expects that dissipative terms occurring in collisional kinetic equations (2) may be treated theoretically along this perspective. The key new point indeed is to see the center manifold theorem as a change of variable in the space on unknowns, while the standard point of view leads to considering the center manifold as an asymptotic object.

Stochastic PDEs
We aim at analyzing the asymptotic behavior of stochastic collisional kinetic problems, that is equation of the type (2). The noise can describe creation or absorption (as in (2)), but it may also be a forcing term or a random magnetic field. In the parabolic scaling, one expect to obtain parabolic SPDEs at the limit. More precisely, we want to understand the fluid limits of kinetic equations in the presence of noise. The noise is smooth and non delta correlated. It contains also a small parameter and after rescaling converges formally to white noise. Thus, this adds another scale in the multiscale analysis. Following the pioneering work [38], some substantial progresses have been done in this topic.

More realistic problems may be addressed such as high field limit describing sprays, or even hydrodynamic limit. The full Boltzmann equation is a very long term project and we wish to address simpler problems such as convergences of BGK models to a stochastic Stokes equation.

The main difficulty is that when the noise acts as a forcing term, which is a physically relevant situation, the equilibria are affected by the noise and we face difficulties similar to that of high field limit problems. Also, a good theory of averaging lemma in the presence of noise is lacking. The methods we use are generalization of the perturbed test function method to the infinite dimensional setting. We work at the level of the generator of the infinite dimensional process and prove convergence in the sense of the martingale problems. A further step is to analyse the speed of convergence. This is a prerequisite if one wants to design efficient schemes. This requires more refined tools and a good understanding of the Kolmogorov equation.

3.1.1.2. Numerical schemes for dissipative problems (All members)

The design of numerical schemes able to reproduce the transition from the microscopic to macroscopic scales largely matured with the emergence of the Asymptotic Preserving schemes which have been developed initially for collisional kinetic equations (actually, for solving (2) when $\beta \to 0$). Several techniques have flourished in the last decades. As said before, AP schemes entail limitations which we aim at overcoming by deriving

- AP numerical schemes whose numerical cost diminishes as $\beta \to 0$,
- Uniformly accurate numerical schemes, whose accuracy is independent of β.

Time diminishing methods
The main goal consists in merging Monte-Carlo techniques [33] with AP methods for handling *automatically* multiscale phenomena. As a result, we expect that the cost of the so-obtained method decreases when the asymptotic regime is approached; indeed, in the collisional (i.e. dissipative) regime, the deviational part becomes negligible so that a very few number of particles will be generated to sample it. A work in this direction has been done by members of the team.

We propose to build up a method which permits to realize the transition from the microscopic to the macroscopic description without domain decomposition strategies which normally oblige to fix and tune an interface in the physical space and some threshold parameters. Since it will permit to go over domain decomposition and AP techniques, this approach is a very promising research direction in the numerical approximation of multiscale kinetic problems arising in physics and engineering.

Uniformly accurate methods

To overcome the accuracy reduction observed in AP schemes for intermediate regimes, we intend to construct and analyse multiscale numerical schemes for (3) whose error is uniform with respect to ε. The construction of such a scheme requires a deep mathematical analysis as described above. Ideally one would like to develop schemes that preserve the center manifold (without computing the latter!) as well as schemes that resolve numerically the stiffness induced by the fast convergence to equilibrium (the so-called transient phase). First, our goal is to extend the strategy inspired by the central manifold theorem in the ODE case to the PDE context, in particular for collisional kinetic equations (2) when $\beta \to 0$. The design of Uniformly Accurate numerical schemes in this context would require to generalize two-scale techniques introduced in the framework of highly-oscillatory problems [36].

Multiscale numerical methods for stochastic PDEs
AP schemes have been developed recently for kinetic equations with noise in the context of Uncertainty Quantification UQ [41]. These two aspects (multiscale and UQ) are two domains which usually come within the competency of separate communities. UQ has drawn a lot of attention recently to control the propagation of data pollution; undoubtedly UQ has a lot of applications and one of our goals will be to study how sources of uncertainty are amplified or not by the multiscale character of the model. We also wish to go much further and by developing AP schemes when the noise is also rescaled and the limit is a white noise driven SPDE, as described in section (3.1.1.1). For simple nonlinear problem, this should not present much difficulties but new ideas will definitely be necessary for more complicated problems when noise deeply changes the asymptotic equation.

3.1.2. Highly-oscillatory problems

As a generic model for highly-oscillatory systems, we will consider the equation

$$\frac{du^\varepsilon(t)}{dt} = \mathcal{F}(t/\varepsilon, u^\varepsilon(t)), \quad u^\varepsilon(0) = u_0, \tag{4}$$

for a given u_0 and a given periodic function $\mathcal{F}$ (of period P w.r.t. its first variable) which possibly involves stochastic terms. Solution u^ε exhibits high-oscillations in time superimposed to a slow dynamics. Asymptotic techniques -resorting in the present context to *averaging* theory [45]- allow to decompose

$$u^\varepsilon(t) = \Phi_{t/\varepsilon} \circ \Psi_t \circ \Phi_0^{-1}(u_0), \tag{5}$$

into a fast solution component, the εP-periodic change of variable $\Phi_{t/\varepsilon}$, and a slow component, the flow Ψ_t of a non-stiff *averaged* differential equation. Although equation (5) can be satisfied only up to a small remainder, various methods have been recently introduced in situations where (4) is posed in $\mathbb{R}^n$ or for the Schrödinger equation (1).

In the asymptotic behavior $\varepsilon \to 0$, it can be advantageous to replace the original singularly perturbed model (for instance (1) or (2)) by an approximate model which does not contain stiffness any longer. Such reduced models can be derived using asymptotic analysis, namely averaging methods in the case of highly-oscillatory problems. In this project, we also plan to go beyond the mere derivation of limit models, by searching for better approximations of the original problem. This step is of mathematical interest *per se* but it also paves the way of the construction of multiscale numerical methods.

3.1.2.1. Asymptotic analysis of highly-oscillatory PDEs (All members)

Derivation of asymptotic problems
We intend to study the asymptotic behavior of highly-oscillatory evolution equations of the form (4) posed in an infinite dimensional Banach space.

Recently, the stroboscopic averaging has been extended to the PDE context, considering nonlinear Schrödinger equation (1) in the highly-oscillatory regime. A very exciting way would be to use this averaging strategy for highly-oscillatory kinetic problem (2) as those encountered in strongly magnetized plasmas. This turns out to be a very promising way to re-derive gyrokinetic models which are the basis of tokamak simulations in the physicists community. In contract with models derived in the literature (see [34]) which only capture the average with respect to the oscillations, this strategy allows for the complete recovery of the exact solution from the asymptotic (non stiff) model. This can be done by solving companion transport equation that stems naturally from the decomposition (5).

Long-time behavior of Hamiltonian systems

The study of long-time behavior of nonlinear Hamiltonian systems have received a lot of interest during the last decades. It enables to put in light some characteristic phenomena in complex situations, which are beyond the reach of numerical simulations. This kind of analysis is of great interest since it can provide very precise properties of the solution. In particular, we will focus on the dynamics of nonlinear PDEs when the initial condition is close to a stationary solution. Then, the long-time behavior of the solution is studied through mainly three axis

- *linear stability*: considering the linearized PDE, do we have stability of a stationary solution ? Do we have linear Landau damping around stable non homogeneous stationary states?
- *nonlinear stability*: under a criteria, do we have stability of a stationary solution in energy norm like in [42], and does this stability persist under numerical discretization? For example one of our goals is to address the question of the existence and stability of discrete travelling wave in space and time.
- do we have existence of damped solutions for the full nonlinear problem ? Around homogeneous stationary states, solutions scatter towards a modified stationary state (see [43], [39]). The question of existence of Landau damping effects around non homogeneous states is still open and is one of our main goal in the next future.

Asymptotic behavior of stochastic PDEs

The study of SPDEs has known a growing interest recently, in particular with the fields medal of M. Hairer in 2014. In many applications such as radiative transfer, molecular dynamics or simulation of optical fibers, part of the physical interactions are naturally modeled by adding supplementary random terms (the noise) to the initial deterministic equations. From the mathematical point of view, such terms change drastically the behavior of the system.

- In the presence of noise, highly-oscillatory dispersive equations presents new problems. In particular, to study stochastic averaging of the solution, the analysis of the long time behavior of stochastic dispersive equations is required, which is known to be a difficult problem in the general case. In some cases (for instance highly-oscillatory Schrödinger equation (1) with a time white noise in the regime $\varepsilon \ll 1$), it is however possible to perform the analysis and to obtain averaged stochastic equations. We plan to go further by considering more difficult problems, such as the convergence of a stochastic Klein-Gordon-Zakharov system to as stochastic nonlinear Schrödinger equation.
- The long-time behavior of stochastic Schrödinger equations is of great interest to analyze mathematically the validity of the Zakharov theory for wave turbulence (see [44]). The problem of wave turbulence can be viewed either as a deterministic Hamiltonian PDE with random initial data or a randomly forced PDEs where the stochastic forcing is concentrated in some part of the spectrum (in this sense it is expected to be a hypoelliptic problem). One of our goals is to test the validity the Zakharov equation, or at least to make rigorous the spectrum distribution spreading observed in the numerical experiments.

3.1.2.2. Numerical schemes for highly-oscillatory problems (All members)

This section proposes to explore numerical issues raised by highly-oscillatory nonlinear PDEs for which (4) is a prototype. Simulating a highly-oscillatory phenomenon usually requires to adapt the numerical parameters in order to solve the period of size ε so as to accurately simulate the solution over each period, resulting in a unacceptable execution cost. Then, it is highly desirable to derive numerical schemes able to advance the solution by a time step independent of ε. To do so, our goal is to construct *Uniformly Accurate* (UA) numerical schemes, for which the numerical error can be estimated by Ch^p (h being any numerical parameters) with C independent of ε and p the order of the numerical scheme.

Recently, such numerical methods have been proposed by members of the team in the highly-oscillatory context [36]. They are mainly based on a separation of the fast and slow variables, as suggested by the decomposition (5). An additional ingredient to prove the uniformly accuracy of the method for (4) relies on the search for an appropriate initial data which enables to make the problem smooth with respect to ε.

Such an approach is assuredly powerful since it provides a numerical method which enables to capture the high oscillations in time of the solution (and not only its average) even with a large time step. Moreover, in the asymptotic regime, the potential gain is of order $1/\varepsilon$ in comparison with standard methods, and finally averaged models are not able to capture the intermediate regime since they miss important information of the original problem. We are strongly convinced that this strategy should be further studied and extended to cope with some other problems. The ultimate goal being to construct a scheme for the original equation which degenerates automatically into a consistent approximation of the averaged model, without resolving it, the latter can be very difficult to solve.

- Space oscillations:
  When rapidly oscillating coefficients in **space** (*i.e.* terms of the form $a(x, x/\varepsilon)$) occur in elliptic or parabolic equations, homogenization theory and numerical homogenization are usually employed to handle the stiffness. However, these strategies are in general not accurate for all $\varepsilon \in]0, 1]$. Then, the construction of numerical schemes which are able to handle both regimes in an uniform way is of great interest. Separating fast and slow *spatial* scales merits to be explored in this context. The delicate issue is then to extend the choice suitable initial condition to an *appropriate choice of boundary conditions* of the augmented problem.
- Space-time oscillations:
  For more complex problems however, the recent proposed approaches fail since the main oscillations cannot be identified explicitly. This is the case for instance when the magnetic field B depends on t or x in (2) but also for many other physical problems. We then have to deal with the delicate issue of space-time oscillations, which is known to be a very difficult problem from a mathematical and a numerical point of view. To take into account the space-time mixing, a periodic motion has to be detected together with a phase S which possibly depends on the time and space variables. These techniques originate from **geometric optics** which is a very popular technique to handle highly-frequency waves.
- Geometrical properties:
  The questions related to the geometric aspects of multiscale numerical schemes are of crucial importance, in particular when long-time simulations are addressed (see [40]). Indeed, one of the main questions of geometric integration is whether intrinsic properties of the solution may be passed onto its numerical approximation. For instance, if the model under study is Hamiltonian, then the exact flow is symplectic, which motivates the design of symplectic numerical approximation. For practical simulations of Hamiltonian systems, symplectic methods are known to possess very nice properties (see [40]). It is important to combine multiscale techniques to geometric numerical integration. All the problems and equations we intend to deal with will be addressed with a view to preserve intrinsic geometric properties of the exact solutions and/or to approach the asymptotic limit of the system in presence of a small parameter. An example of a numerical method developed by members of the team is the multi-revolution method.
- Quasi-periodic case:

So far, numerical methods have been proposed for the periodic case with single frequency. However, the quasi-periodic case [1] is still misunderstood although many complex problems involve multi-frequencies. Even if the quasi-periodic averaging is doable from a theoretical point of view in the ODE case (see [45]), it is unclear how it can be extended to PDEs. One of the main obstacle being the requirement, usual for ODEs like (4), for $\mathcal{F}$ to be analytic in the periodic variables, an assumption which is clearly impossible to meet in the PDE setting. An even more challenging problem is then the design of numerical methods for this problem.

- extension to stochastic PDEs:
  All these questions will be revisited within the stochastic context. The mathematical study opens the way to the derivation of efficient multiscale numerical schemes for this kind of problems. We believe that the theory is now sufficiently well understood to address the derivation and numerical analysis of multiscale numerical schemes. Multi-revolution composition methods have been recently extended to highly-oscillatory stochastic differential equations The generalization of such multiscale numerical methods to SPDEs is of great interest. The analysis and simulation of numerical schemes for highly-oscillatory nonlinear stochastic Schrödinger equation under diffusion-approximation for instance will be one important objective for us. Finally, an important aspect concerns the quantification of uncertainties in highly-oscillatory kinetic or quantum models (due to an incomplete knowledge of coefficients or imprecise measurement of datas). The construction of efficient multiscale numerical methods which can handle multiple scales as well as random inputs have important engineering applications.

4. Application Domains

4.1. Application domains

The MINGUS project aims at applying the new numerical methods on realistic problems arising for instance in physics of nanotechnology and physics of plasmas. Therefore, in addition to efforts devoted to the design and the analysis of numerical methods, the inherent large size of the problems at hand requires advanced mathematical and computational methods which are hard to implement. Another application is concerned with population dynamics for which the main goal is to understand how the spatial propagation phenomena affect the demography of a population (plankton, parasite fungi, ...). Our activity is mostly at an early stage in the process of transfer to industry. However, all the models we use are physically relevant and all have applications in many areas (ITER, Bose-Einstein condensate, wave turbulence, optical tomography, transport phenomena, population dynamics, $\cdots$). As a consequence, our research aims at reaching theoretical physicists or computational scientists in various fields who have strong links with industrial applications. In order to tackle as realistic physical problems as possible, a fundamental aspect will consist in working on the realization of numerical methods and algorithms which are able to make an efficient use of a large number of processors. Then, it is essential for the numerical methods developed in the MINGuS project to be thought through this prism. We will benefit from the strong expertise of P. Navaro in scientific computing and more precisely on the Selalib software library (see description below). Below, we detail our main applications: first, the modeling and numerical approximation of magnetized plasmas is our major application and will require important efforts in terms of software developments to scale-up our multiscale methods; second, the transport of charged particles in nanostructures has very interesting applications (like graphene material), for which our contributions will mainly focus on dedicated problems; lastly, applications on population dynamics will be dedicated to mathematical modeling and some numerical validations.

[1] replacing t/ε by $t\omega/\varepsilon$ in (4), with $\omega \in \mathbb{R}^d$ a vector of non-resonant frequencies

4.1.1. Plasmas problems

The Selalib (SEmi-LAgrangian LIBrary) software library [2] is a modular library for kinetic and gyrokinetic simulations of plasmas in fusion energy devices. Selalib is a collection of fortran modules aimed at facilitating the development of kinetic simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities and modules to help parallelization (both MPI and OpenMP), as well as pre-packaged simulations. Its main objective is to develop a documented library implementing several numerical methods for the numerical approximation of kinetic models. Another objective of the library is to provide physicists with easy-to-use gyrokinetic solvers. It has been originally developed by E. Sonnendrücker and his collaborators in the past CALVI Inria project, and has played an important role in the activities of the IPL FRATRES. P. Navaro is one of the main software engineer of this library and as such he played an important daily role in its development and its portability on supercomputers. Though Selalib has reached a certain maturity some additional works are needed to make available by the community. There are currently discussions for a possible evolution of Selalib, namely the writing of a new release which will be available for free download. Obviously, the team will be involved in this process. At the scientific level, Selalib is of great interest for us since it provides a powerful tool with which we can test, validate and compare our new methods and algorithms (users level). Besides numerical algorithms the library provides low-level utilities, input-output modules as well as parallelization strategies dedicated to kinetic problems. Moreover, a collection of simulations for typical test cases (of increasing difficulties) with various discretization schemes supplements the library. This library turns out to be the ideal complement of our activities and it will help us to scale-up our numerical methods to high-dimensional kinetic problems. During the last years, several experiments have been successfully performed in this direction (especially with PhD students) and it is important for us that this approach remains throughout. Then, we intend to integrate several of the numerical methods developed by the team within the Selalib library, with the strong help of P. Navaro (contributors level). This work has important advantages: (i) it will improve our research codes (in terms of efficiency but also of software maintenance point of view); (ii) it will help us to promote our research by making our methods available to the research community.

4.1.2. Quantum problems

Nowadays, a great challenge consists in the downscaling at the nanometer scale of electronic components in order to improve speed and efficiency of semiconductor materials. In this task, modeling and numerical simulations play an important role in the determination of the limit size of the nanotransistors. At the nanoscale, quantum effects have to be considered and the Schrödinger equation is prominent equation in this context. In the so-called semiclassical regime or when the transport is strongly confined, the solution endows space-time highly oscillations which are very difficult to capture numerically. An important application is the modeling of charged particles transport in graphene. Graphene is a sheet of carbone made of a single layer of molecule, organized in a bidimensional honeycomb crystal. The transport of charged particles in this structure is usually performed by Dirac equation (which is the relativistic counterpart of the Schrödinger equation). Due to the unusual properties of graphene -at room temperature, electrons moving in graphene behave as massless relativistic particles- physicists and compagnies are nowadays actively studying this material. Here, predicting how the material properties are affected by the uncertainties in the hexagonal lattice structure or in external potentials, is a major issue.

4.1.3. Population dynamics

The main goal is to characterize how spatial propagation phenomena (diffusion, transport, advection, $\cdots$) affect the time evolution of the demography of a population. In collaboration with Y. Lagadeuc (ECOBIO, Rennes), this question has been studied for plankton. In this context, mathematical models have been proposed and it has been shown that the spatial dynamic (in this context, due to the marine current) which is fast compared to demographic scales, can strongly modify the demographic evolution of the plankton. In collaboration with Ecole d'Agronomie de Rennes, a mathematical study on the demography of a parasite fungi of plants has been performed. In this context, the demography is specific: the fungi can proliferate through sexual reproduction or through parthenogenesis. This two ways of reproduction give rise mathematically to quadratic and linear

[2] SELALIB, http://selalib.gforge.inria.fr

growth rates with respect to the population variable. The demography is then coupled with transport (transport of fungi spore by wind). Here, the goal is characterize the propagation of the fungi population by finding travelling waves solutions which are well adapted to describe the evolution of invasive fronts. Moreover, this approach enables to recover with a good agreement realistic examples (infection of ash or banana tree) for which experimental data are available. In these contexts, mathematical models are a powerful tool for biologists since measurements are very complicated to obtain and laboratory experiments hardly reproduce reality. The models derived are multiscale due to the nature of the underlying phenomena and the next step is to provide efficient numerical schemes.

5. New Software and Platforms

5.1. Selalib

SEmi-LAgrangian LIBrary

KEYWORDS: Plasma physics - Semilagrangian method - Parallel computing - Plasma turbulence

SCIENTIFIC DESCRIPTION: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.

FUNCTIONAL DESCRIPTION: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

- Partners: Max Planck Insitute - Garching - Université de Strasbourg
- Contact: Philippe Helluy
- URL: http://selalib.gforge.inria.fr/

6. New Results

6.1. New Results

Analysis of PDEs and SPDEs

In [17], we prove the nonlinear instability of inhomogeneous steady states solutions to the Hamiltonian Mean Field (HMF) model. We first study the linear instability of this model under a simple criterion by adapting the techniques developed by the authors recently. In a second part, we extend to the inhomogeneous case some techniques developed by the authors recently and prove a nonlinear instability result under the same criterion.

In [24], we consider the non linear wave equation (NLW) on the d-dimensional torus with a smooth nonlinearity of order at least two at the origin. We prove that, for almost any mass, small and smooth solutions of high Sobolev indices are stable up to arbitrary long times with respect to the size of the initial data. To prove this result we use a normal form transformation decomposing the dynamics into low and high frequencies with weak interactions. While the low part of the dynamics can be put under classical Birkhoff normal form, the high modes evolve according to a time dependent linear Hamiltonian system. We then control the global dynamics by using polynomial growth estimates for high modes and the preservation of Sobolev norms for the low modes. Our general strategy applies to any semi-linear Hamiltonian PDEs whose linear frequencies satisfy a very general non resonance condition. The (NLW) equation on a torus is a good example since the standard Birkhoff normal form applies only when $d = 1$ while our strategy applies in any dimension.

In [20], we study semigroups generated by accretive non-selfadjoint quadratic differential operators. We give a description of the polar decomposition of the associated evolution operators as products of a selfadjoint operator and a unitary operator. The selfadjoint parts turn out to be also evolution operators generated by time-dependent real-valued quadratic forms that are studied in details. As a byproduct of this decomposition, we give a geometric description of the regularizing properties of semigroups generated by accretive non-selfadjoint quadratic operators. Finally, by using the interpolation theory, we take advantage of this smoothing effect to establish subelliptic estimates enjoyed by quadratic operators.

In [16], we prove the nonlinear orbital stability of a large class of steady states solutions to the Hamiltonian Mean Field (HMF) system with a Poisson interaction potential. These steady states are obtained as minimizers of an energy functional under one, two or infinitely many constraints. The singularity of the Poisson potential prevents from a direct run of the general strategy which was based on generalized rearrangement techniques, and which has been recently extended to the case of the usual (smooth) cosine potential. Our strategy is rather based on variational techniques. However, due to the boundedness of the space domain, our variational problems do not enjoy the usual scaling invariances which are, in general, very important in the analysis of variational problems. To replace these scaling arguments, we introduce new transformations which, although specific to our context, remain somehow in the same spirit of rearrangements tools introduced in the references above. In particular, these transformations allow for the incorporation of an arbitrary number of constraints, and yield a stability result for a large class of steady states.

In [25], we study the Boltzmann equation with external forces, not necessarily deriving from a potential, in the incompressible Navier-Stokes perturbative regime. On the torus, we establish local-in-time, for any time, Cauchy theories that are independent of the Knudsen number in Sobolev spaces. The existence is proved around a time-dependent Maxwellian that behaves like the global equilibrium both as time grows and as the Knudsen number decreases. We combine hypocoercive properties of linearized Boltzmann operators with linearization around a time-dependent Maxwellian that catches the fluctuations of the characteristics trajectories due to the presence of the force. This uniform theory is sufficiently robust to derive the incompressible Navier-Stokes-Fourier system with an external force from the Boltzmann equation. Neither smallness, nor time-decaying assumption is required for the external force, nor a gradient form, and we deal with general hard potential and cutoff Boltzmann kernels. As a by-product the latest general theories for unit Knudsen number when the force is sufficiently small and decays in time are recovered.

In [15], we show how the methods recently applied by Debussche and Weber to solve the stochastic nonlinear Schrödinger equation on $\mathbb{T}^2$ can be enhanced to yield solutions on $\mathbb{R}^2$ if the non-linearity is weak enough. We prove that the solutions remains localized on compact time intervals which allows us to apply energy methods on the full space.

In [2], we provide in this work a local in time well-posedness result for a quasilinear generalized parabolic Anderson model in dimension two $\partial_t u + \Delta\Pi(u) = g(u)\xi$. The key idea of our approach is a simple transformation of the equation which allows to treat the problem as a semilinear problem. The analysis is done within the setting of paracontrolled calculus.

In [30], we consider the Burgers equation on $H = L^2(0,1)$ perturbed by white noise and the corresponding transition semigroup $P_t D\varphi$. We prove a new formula for $PtD\varphi$ (where $\varphi : H \to \mathbb{R}$ is bounded and Borel) which depends on φ but not on its derivative. Then we deduce some consequences for the invariant measure ν of P_t as its Fomin differentiability and an integration by parts formula which generalises the classical one for gaussian measures.

In [9], we deal with the validity of a large deviation principle for the two-dimensional Navier-Stokes equation, with periodic boundary conditions, perturbed by a Gaussian random forcing. We are here interested in the regime where both the strength of the noise and its correlation are vanishing, on a length scale ε and $\delta(\varepsilon)$, respectively, with $0 < \varepsilon, \delta(\varepsilon) << 1$. Depending on the relationship between ε and $\delta(\varepsilon)$ we will prove the validity of the large deviation principle in different functional spaces.

In [30], the authors consider the transition semigroup P_t of the Φ^4_2 stochastic quantisation on the torus $\mathbb{T}^2$ and prove the following new estimate

$$|DP_t\varphi(x)\cdot h| \leq ct^{-\beta}|h|_{C^{-s}}\|\varphi\|_0(1+|x|_{C^{-s}})^{\gamma},$$

for some α, β, γ, s positive. Thanks to this estimate, we show that cylindrical functions are a core for the corresponding Kolmogorov equation. Some consequences of this fact are discussed in a final remark.

In [32], we consider a particle system with a mean-field-type interaction perturbed by some common and individual noises. When the interacting kernels are sublinear and only locally Lipschitz-continuous, relying on arguments regarding the tightness of random measures in Wasserstein spaces, we are able to construct a weak solution of the corresponding limiting SPDE. In a setup where the diffusion coefficient on the environmental noise is bounded, this weak convergence can be turned into a strong $L^p(\Omega)$ convergence and the propagation of chaos for the particle system can be established. The systems considered include perturbations of the Cucker-Smale model for collective motion.

Numerical schemes

In [7], the asymptotic behavior of the solutions of the second order linearized Vlasov-Poisson system around homogeneous equilibria is derived. It provides a fine description of some nonlinear and multidimensional phenomena such as the existence of Best frequencies. Numerical results for the $1D \times 1D$ and $2D \times 2D$ Vlasov-Poisson system illustrate the effectiveness of this approach.

In [6], we consider the problem of existence and stability of solitary traveling waves for the one dimensional discrete non linear Schrödinger equation (DNLS) with cubic nonlinearity, near the continuous limit. We construct a family of solutions close to the continuous traveling waves and prove their stability over long times. Applying a modulation method, we also show that we can describe the dynamics near these discrete traveling waves over long times.

In [4], we consider the discrete nonlinear Schrödinger equations on a one dimensional lattice of mesh h, with a cubic focusing or defocusing nonlinearity. We prove a polynomial bound on the growth of the discrete Sobolev norms, uniformly with respect to the stepsize of the grid. This bound is based on a construction of higher modified energies.

The efficient numerical solution of many kinetic models in plasma physics is impeded by the stiffness of these systems. Exponential integrators are attractive in this context as they remove the CFL condition induced by the linear part of the system, which in practice is often the most stringent stability constraint. In the literature, these schemes have been found to perform well, e.g., for drift-kinetic problems. Despite their overall efficiency and their many favorable properties, most of the commonly used exponential integrators behave rather erratically in terms of the allowed time step size in some situations. This severely limits their utility and robustness. Our goal in [29] is to explain the observed behavior and suggest exponential methods that do not suffer from the stated deficiencies. To accomplish this we study the stability of exponential integrators for a linearized problem. This analysis shows that classic exponential integrators exhibit severe deficiencies in that regard. Based on the analysis conducted we propose to use Lawson methods, which can be shown not to suffer from the same stability issues. We confirm these results and demonstrate the efficiency of Lawson methods by performing numerical simulations for both the Vlasov-Poisson system and a drift-kinetic model of a ion temperature gradient instability.

In [18], a bracket structure is proposed for the laser-plasma interaction model introduced in the physical literature, and it is proved by direct calculations that the bracket is Poisson which satisfies the Jacobi identity. Then splitting methods in time are proposed based on the Poisson structure. For the quasi- relativistic case, the Hamiltonian splitting leads to three subsystems which can be solved exactly. The conservative splitting is proposed for the fully relativistic case, and three one-dimensional conservative subsystems are obtained. Combined with the splittings in time, in phase space discretization we use the Fourier spectral and finite volume methods. It is proved that the discrete charge and discrete Poisson equation are conserved by our numerical schemes. Numerically, some numerical experiments are conducted to verify good conservations for the charge, energy and Poisson equation.

In [26], the recent advances about the construction of a Trefftz Discontinuous Galerkin (TDG) method to a class of Friedrichs systems coming from linear transport with relaxation are presented in a comprehensive setting. Application to the $2D$ P_N model are discussed, together with the derivation of new high order convergence estimates and new numerical results for the P_1 and P_3 models. More numerical results in dimension 2 illustrate the theoretical properties.

In [8], we are concerned with a formulation of Magnus and Floquet-Magnus expansions for general nonlinear differential equations. To this aim, we introduce suitable continuous variable transformations generated by operators. As an application of the simple formulas so-obtained, we explicitly compute the first terms of the Floquet-Magnus expansion for the Van der Pol oscillator and the nonlinear Schrödinger equation on the torus.

The article [11] is devoted to the construction of numerical methods which remain insensitive to the smallness of the semiclassical parameter for the linear Schrödinger equation in the semiclassical limit. We specifically analyse the convergence behavior of the first-order splitting. Our main result is a proof of uniform accuracy. We illustrate the properties of our methods with simulations.

In [10], we consider the numerical solution of highly-oscillatory Vlasov and Vlasov-Poisson equations with non-homogeneous magnetic field. Designed in the spirit of recent uniformly accurate methods, our schemes remain insensitive to the stiffness of the problem, in terms of both accuracy and computational cost. The specific difficulty (and the resulting novelty of our approach) stems from the presence of a non-periodic oscillation, which necessitates a careful ad-hoc reformulation of the equations. Our results are illustrated numerically on several examples.

In the analysis of highly-oscillatory evolution problems, it is commonly assumed that a single frequency is present and that it is either constant or, at least, bounded from below by a strictly positive constant uniformly in time. Allowing for the possibility that the frequency actually depends on time and vanishes at some instants introduces additional difficulties from both the asymptotic analysis and numerical simulation points of view. This work [13] is a first step towards the resolution of these difficulties. In particular, we show that it is still possible in this situation to infer the asymptotic behaviour of the solution at the price of more intricate computations and we derive a second order uniformly accurate numerical method.

In [12], we introduce a new methodology to design uniformly accurate methods for oscillatory evolution equations. The targeted models are envisaged in a wide spectrum of regimes, from non-stiff to highly-oscillatory. Thanks to an averaging transformation, the stiffness of the problem is softened, allowing for standard schemes to retain their usual orders of convergence. Overall, high-order numerical approximations are obtained with errors and at a cost independent of the regime.

In [1], we present an asymptotic preserving scheme based on a micro-macro decomposition for stochastic linear transport equations in kinetic and diffusive regimes. We perform a mathematical analysis and prove that the scheme is uniformly stable with respect to the mean free path of the particles in the simple telegraph model and in the general case. We present several numerical tests which validate our scheme.

In [22], a splitting strategy is introduced to approximate two-dimensional rotation motions. Unlike standard approaches based on directional splitting which usually lead to a wrong angular velocity and then to large error, the splitting studied here turns out to be exact in time. Combined with spectral methods, the so-obtained numerical method is able to capture the solution to the associated partial differential equation with a very high accuracy. A complete numerical analysis of this method is given in this work. Then, the method is used to design highly accurate time integrators for Vlasov type equations: the Vlasov-Maxwell system and the Vlasov-HMF model. Finally , several numerical illustrations and comparisons with methods from the literature are discussed.

In [23], some exact splittings are proposed for inhomogeneous quadratic differential equations including, for example, transport equations, kinetic equations, and Schrödinger type equations with a rotation term. In this work, these exact splittings are combined with pseudo-spectral methods in space to illustrate their high accuracy and efficiency.

In [14], we develop a new class of numerical schemes for collisional kinetic equations in the diffusive regime. The first step consists in reformulating the problem by decomposing the solution in the time evolution of an

equilibrium state plus a perturbation. Then, the scheme combines a Monte Carlo solver for the perturbation with an Eulerian method for the equilibrium part, and is designed in such a way to be uniformly stable with respect to the diffusive scaling and to be consistent with the asymptotic diffusion equation. Moreover, since particles are only used to describe the perturbation part of the solution, the scheme becomes computationally less expensive - and is thus an asymptotically complexity diminishing scheme (ACDS) - as the solution approaches the equilibrium state due to the fact that the number of particles diminishes accordingly. This contrasts with standard methods for kinetic equations where the computational cost increases (or at least does not decrease) with the number of interactions. At the same time, the statistical error due to the Monte Carlo part of the solution decreases as the system approaches the equilibrium state: the method automatically degenerates to a solution of the macroscopic diffusion equation in the limit of infinite number of interactions. After a detailed description of the method, we perform several numerical tests and compare this new approach with classical numerical methods on various problems up to the full three dimensional case.

In [5], we revisit the old problem of compact finite difference approximations of the homogeneous Dirichlet problem in dimension 1. We design a large and natural set of schemes of arbitrary high order, and we equip this set with an algebraic structure. We give some general criteria of convergence and we apply them to obtain two new results. On the one hand, we use Padé approximant theory to construct, for each given order of consistency, the most efficient schemes and we prove their convergence. On the other hand, we use diophantine approximation theory to prove that almost all of these schemes are convergent at the same rate as the consistency order, up to some logarithmic correction.

In [28], we introduce a new Monte Carlo method for solving the Boltzmann model of rarefied gas dynamics. The method works by reformulating the original problem through a micro-macro decomposition and successively in solving a suitable equation for the perturbation from the local thermodynamic equilibrium. This equation is then discretized by using unconditionally stable exponential schemes in time which project the solution over the corresponding equilibrium state when the time step is sent to infinity. The Monte Carlo method is designed on this time integration method and it only describes the perturbation from the final state. In this way, the number of samples diminishes during the time evolution of the solution and when the final equilibrium state is reached, the number of statistical samples becomes automatically zero. The resulting method is computationally less expensive as the solution approaches the equilibrium state as opposite to standard methods for kinetic equations which computational cost increases with the number of interactions. At the same time, the statistical error decreases as the system approaches the equilibrium state. In a last part, we show the behaviors of this new approach in comparison with standard Monte Carlo techniques and in comparison with spectral methods on different prototype problems.

In [27], we consider the three dimensional Vlasov equation with an inhomogeneous, varying direction, strong magnetic field. Whenever the magnetic field has constant intensity, the oscillations generated by the stiff term are periodic. The homogenized model is then derived and several state-of-the-art multiscale methods, in combination with the Particle-In-Cell discretisation, are proposed for solving the Vlasov-Poisson equation. Their accuracy as much as their computational cost remain essentially independent of the strength of the magnetic field. The proposed schemes thus allow large computational steps, while the full gyro-motion can be restored by a linear interpolation in time. In the linear case, extensions are introduced for general magnetic field (varying intensity and direction). Eventually, numerical experiments are exposed to illustrate the efficiency of the methods and some long-term simulations are presented.

7. Bilateral Contracts and Grants with Industry

7.1. Bilateral Contracts with Industry

- Contrat with RAVEL (onne year, budget 15000 euros): this is a collaboration with the startup RAVEL on a one-year basis (with possible renewal at the end of the year). The objective is to study the mathematical fondations of artificial intelligence and in particular machine learning algorithms for data anonymized though homomorphic encryption.
  Participants: P. Chartier, M. Lemou and F. Méhats.

- Contract with Cailabs (6 months, budget 3000 euros): This collaboration aims at exploring the possibility of deriving new fiber optics devices based on neural networks architecture.
  Participants: P. Chartier, E. Faou, M. Lemou and F. Méhats.

8. Partnerships and Cooperations

8.1. Regional Initiatives

- M. Lemou and N. Crouseilles are head of the project "MUNIQ" of ENS Rennes. This two-years project (2018-2019) intends to gather multiscale numerical methods and uncertainty quantification techniques. The MINGuS members are P. Chartier, N. Crouseilles, M. Lemou and F. Méhats and colleagues from university of Madison-Wisconsin also belong to this project.

8.2. National Initiatives

8.2.1. ANR

8.2.1.1. MOONRISE: 2015-2019

Participants: François Castella, Philippe Chartier, Nicolas Crouseilles, Mohammed Lemou, Florian Méhats.

The project *Moonrise* submitted by Florian Méhats has been funded by the ANR for 4 years, for the period 2015-2019. This project aims at exploring modeling, mathematical and numerical issues originating from the presence of high-oscillations in nonlinear PDEs from the physics of nanotechnologies (quantum transport) and from the physics of plasmas (magnetized transport in tokamaks). The partners of the project are the IRMAR (Rennes), the IMT (Toulouse) and the CEA Cadarache. In the MINGuS team, François Castella, Philippe Chartier, Nicolas Crouseilles and Mohammed Lemou are members of the project Moonrise.

Postdocs

- Loïc Le Treust has been hired as a Postdoc, under the supervision of Philippe Chartier and Florian Méhats. His contract started in september 2015 and ended in august 2016. Loïc Le Treust is now assistant professor at the university of Marseille.
- Yong Zhang has been hired as a Postdoc, under the supervision of Philippe Chartier and Florian Méhats. His contract started in september 2015 and ended in august 2016. Yong Zhang is now professor at the Tianjin university (China).
- Xiaofei Zhao has been hired as a Postdoc from september 2015 to september 2016 under the supervision of Florian Méhats. Xiaofei Zhao is now postdoc assistant professor in the Wuhan University (China).

8.2.1.2. MFG: 2016-2020

Participant: Arnaud Debussche.

Mean Field Games (MFG) theory is a new and challenging mathematical topic which analyzes the dynamics of a very large number of interacting rational agents. Introduced ten years ago, the MFG models have been used in many areas such as, e.g., economics (heterogeneous agent models, growth modeling,...), finance (formation of volatility, models of bank runs,...), social sciences (crowd models, models of segregation) and engineering (data networks, energy systems...). Their importance comes from the fact that they are the simplest (“stochastic control"-type) models taking into account interactions between rational agents (thus getting beyond optimization), yet without entering into the issues of strategic interactions. MFG theory lies at the intersection of mean field theories (it studies systems with a very large number of agents), game theory, optimal control and stochastic analysis (the agents optimize a payoff in a possibly noisy setting), calculus of variations (MFG equilibria may arise as minima of suitable functionals) and partial differential equations (PDE): In the simplest cases, the value of each agent is found by solving a backward Hamilton-Jacobi equation whereas the distribution of the agents’ states evolves according to a forward Fokker-Planck equation.

The “Master" equation (stated in the space of probability measures) subsumes the individual and collective behaviors. Finally, modeling, numerical analysis and scientific computing are crucial for the applications. French mathematicians play a world-leading role in the research on MFG: The terminology itself comes from a series of pioneering works by J.-M. Lasry and P.-L. Lions who introduced most of the key ideas for the mathematical analysis of MFG; the last conference on MFG was held last June in Paris and organized by Y. Achdou, P. Cardaliaguet and J.-M. Lasry. As testifies the proposal, the number of researchers working on MFG in France (and also abroad) is extremely fast-growing, not only because the theoretical aspects are exciting and challenging, but also because MFG models find more and more applications. The aim of the project is to better coordinate the French mathematical research on MFG and to achieve significant progress in the theory and its applications.

The partners of the project are the CEREMADE laboratory (Paris Dauphine), the IRMAR laboratory (Rennes I), the university of Nice and of Tours.

8.2.1.3. ADA: 2019-2023

Participant: Arnaud Debussche.

The aim of this project is to treat multiscale models which are both infinite-dimensional and stochastic with a theoretic and computational approach. Multiscale analysis and multiscale numerical approximation for infinite-dimensional problems (partial differential equations) is an extensive part of contemporary mathematics, with such wide topics as hydrodynamic limits, homogenization, design of asymptotic-preserving scheme. Multiscale models in a random or stochastic context have been analysed and computed essentially in finite dimension (ordinary/stochastic differential equations), or in very specific areas, mainly the propagation of waves, of partial differential equations. The technical difficulties of our project are due to the stochastic aspect of the problems (this brings singular terms in the equations, which are difficult to understand with a pure PDE's analysis approach) and to their infinite-dimensional character, which typically raises compactness and computational issues. Our main fields of investigation are: stochastic hydrodynamic limit (for example for fluids), diffusion-approximation for dispersive equations, numerical approximation of stochastic multiscale equations in infinite dimension. Our aim is to create the new tools - analytical, probabilistic and numerical - which are required to understand a large class of stochastic multiscale partial differential equations. Various modelling issues require this indeed, and are pointing at a new class of mathematical problems that we wish to solve. We also intend to promote the kind of problems we are interested in, particularly among young researchers, but also to recognized experts, via schools, conference, and books.

The partners are ENS Lyon (coordinator J. Vovelle) and ENS Rennes (Coordinator A. Debussche).

8.2.2. Fédération de Recherche : Fusion par Confinement Magnétique

We are involved in the national research multidisciplinary group around magnetic fusion activities. As such, we answer to annual calls.

8.2.3. IPL SURF

A. Debussche and E. Faou are members of the IPL (Inria Project Lab) SURF: Sea Uncertainty Representation and Forecast. Head: Patrick Vidard.

8.2.4. AdT J-Plaff

This AdT started in october 2019 and will be finished in september 2021. An engineer has been hired (Y. Mocquard) to develop several packages in the Julia langage. The J-Plaff is shared with the Fluminance team.

8.3. European Initiatives

8.3.1. Collaborations in European Programs, Except FP7 & H2020

Program: Eurofusion

Project acronym: MAGYK

Project title:

Duration: january 2019-december 2020

Coordinator: E. Sonnendrücker

Other partners: Switzerland, Germany, France, Austria, Finland.

Abstract: This proposal is aimed at developing new models and algorithms that will be instrumental in enabling the efficient and reliable simulation of the full tokamak including the edge and scrape-off layer up to the wall with gyrokinetic or full kinetic models. It is based on a collaboration between applied mathematicians and fusion physicists that has already been very successful in a previous enabling research project and brings new ideas and techniques into the magnetic fusion community. New modelling and theoretical studies to extend the modern gyrokinetic theory up to the wall including boundary conditions will be addressed, and the limits of gyrokinetics will be assessed. New multiscale methods will enable to efficiently and robustly separate time scales, which will on the one hand make gyrokinetic codes more efficient and on the other hand enable full implicit kinetic simulations. Difficult algorithmic issues for handling the core to edge transition, the singularities at the O- and X-points will be addressed. And finally, pioneering work based on recent (deep) machine learning techniques will be performed, on the one hand to automatically identify a Partial Differential Equation (PDE) from the data, which can be used for verification and sensitivity analysis purposes, and on the other hand to develop reduced order models that will define a low- cost low-fidelity model based on the original high-fidelity gyrokinetic or kinetic model that can be used for parameter scans and uncertainty quantification.

8.4. International Initiatives

8.4.1. Inria Associate Teams Not Involved in an Inria International Labs

8.4.1.1. ANTIpODE

Title: Asymptotic Numerical meThods for Oscillatory partial Differential Equations with uncertainties

International Partner (Institution - Laboratory - Researcher):

University of Wisconsin-Madison, USA (United States)

Start year: 2018

See also: https://team.inria.fr/antipode/

The proposed associate team assembles the Inria team IPSO and the research group led by Prof. Shi Jin from the Department of Mathematics at the University of Wisconsin, Madison. The main scientific objective of ANTIpODE consists in marrying uniformly accurate and uncertainty quantification techniques for multi-scale PDEs with uncertain data. Multi-scale models, as those originating e.g. from the simulation of plasma fusion or from quantum models, indeed often come with uncertainties. The main scope of this proposal is thus (i) the development of uniformly accurate schemes for PDEs where space and time high oscillations co-exist and (ii) their extension to models with uncertainties. Applications to plasmas (Vlasov equations) and graphene (quantum models) are of paramount importance to the project.

8.4.2. Inria International Partners

8.4.2.1. Informal International Partners

The members of MINGuS have several interactions with the following partners

- Europe: University of Geneva (Switzerland), University of Jaume I (Spain), University of Basque Country (Spain), University of Innsbruck (Austria), University of Ferrare (Italy), Max Planck Institute (Germany), SNS Pisa (Italy)
- USA: Georgia Tech, University of Maryland, University of Wisconsin, NYU
- Asia: Chinese Academy of Science (China), University of Wuhan (China), shanghai jiao tong university (China), National University of Singapore (Singapore)

8.4.3. Participation in Other International Programs

- SIMONS project. Erwan Faou is one of the Principal investigators of the Simons Collaboration program *Wave Turbulence*. Head: Jalal Shatah (NYU).

8.5. International Research Visitors

8.5.1. Visits of International Scientists

- Fernando Casas (University of Jaume I, Spain) was invited in the MINGuS team during 6 months (september 2018 to february 2019), funded by the Labex (CHL) Center Henri Lebesgue.
- Yingzhe Li (University of Chinese Academy of Sciences, China) is visiting the IRMAR laboratory during one year (March 2019-February 2020) thanks to a chinese grant. He is currently a PhD student advised by Yajuan Sun, professor at CAS.
- Xiaofei Zhao (University of Wuhan, China) was invited in the MINGuS team during 2 weeks (july 2019).
- Yoshio Tsutsumi (Kyoto University, Japan) was invited in the IRMAR laboratory during 2 months (october-november 2019).

8.5.1.1. Internships

- G. Barrué: Master 2 internship, A. Debussche.
- Q. Chauleur: Master 2 internship, R. Carles (CNRS, Rennes) and E. Faou.
- U. Léauté: Master 1 internship, B. Boutin (University Rennes I and N. Crouseilles).
- A. V. Tuan: Master 2 internship, M. Lemou and F. Méhats.

8.5.2. Visits to International Teams

8.5.2.1. Sabbatical programme

P. Chartier was on a sabbatical visit from the 1st of February to the 30th of September 2019 at the University of the Basque Country, Spain.

8.5.2.2. Research Stays Abroad

- P. Chartier was invited by G. Vilmart, University of Geneva, Geneva, Switzerland, January 2019.
- P. Chartier was invited by F. Casas at the university of Jaume I, Castellon, Spain, July 2019.
- P. Chartier was Invited by Q. Li at the university of Wisconsin, Madison, USA, September 2019.
- P. Chartier was Invited by M. Tao at Georgia Tech, Atlanta, USA, August 2019.
- A. Debussche was invited by G. Da Prato at Scuola Normale Superiore, Pise, Italy, April 2019.
- E. Faou was a participant of the Semester *Geometry, compatibility and structure preservation in computational differential equations*, Isaac Newton Institute, Cambridge, UK (3 months stay, September-December 2019).
- M. Lemou was invited by J. Joudioux and L. Anderson, at the Albert Einstein Institute, Golm, Germany, February 2019.
- M. Lemou was invited by A. M. M. Luz at the Universidade Federal Fluminense, Rio de Janeiro, Brazil, April 2019.
- M. Lemou was invited by S. Jin at Shanghai Jiao Tong University, Shanghai, China, April 2019.
- M. Lemou was invited by J. Ben-Artzi at the university of Cardiff, Cardiff, UK, May 2019.
- M. Lemou was invited by G. Vilmart, University of Geneva, Geneva, Switzerland, January 2019.
- M. Lemou was Invited by Q. Li at the university of Wisconsin, Madison, USA, September 2019.
- M. Lemou was Invited by M. Tao at Georgia Tech, Atlanta, USA, August 2019.

- F. Méhats was invited by A. de la Luz at the Universidade Federal Fluminense, Rio de Janeiro, Brazil, April 2019.
- F. Méhats was invited by G. Vilmart, University of Geneva, Geneva, Switzerland, January 2019.

9. Dissemination

9.1. Promoting Scientific Activities

9.1.1. Scientific Events: Organisation

9.1.1.1. Member of the Organizing Committees

- F. Castella organized the MINGuS team meeting, Dinard, december 2019. [15 participants]
- P. Chartier, N. Crouseilles, M. Lemou and F. Méhats organized a workshop on "Asymptotic methods and numerical approximations of multi-scale evolution problems, and uncertainty quantification", ENS Rennes, May 2019. [30 participants]
- N. Crouseilles is co-organizer of the weekly seminar "Mathematic and applications" at ENS Rennes.
- P. Chartier and M. Lemou co-organized (with M. Thalhammer, university of Innsbruck) the mini-symposium "Advanced numerical methods for differential equations" in the ICIAM 2019 conference, Valencia, Spain, July 2019.
- F. Méhats co-organized (with W. Bao, National University of Singapore) the mini-symposium "Multiscale methods and analysis for oscillatory PDEs" in the Scicade conference, Innsbruck, Austria, July 2019.

9.1.1.2. Member of the Conference Program Committees

- N. Crouseilles was member of the scientific committee of the SMAI-19 Conference, June 2019.
- E. Faou was member of the scientific committee of the Scicade Conference, July 2019.

9.1.2. Journal

9.1.2.1. Member of the Editorial Boards

- P. Chartier is member of the editorial board of "Mathematical Modelling and Numerical Analysis" (M2AN).
- A. Debussche is editor in chief of Stochastic Partial Differential Equations: analysis and computations (2013-).
- A. Debussche is member of the editorial board of the collection "Mathématiques & Applications".
- A. Debussche is associate editor of Differential and Integral Equations (2002-19).
- A. Debussche is associate editor of Potential Analysis (2011-2019).
- A. Debussche is associate editor of Journal of Evolution Equation (2014-).
- A. Debussche is associate editor of Applied Mathematics & Optimization, SIAM JUQ (2016-2019).
- M. Lemou is member of the editorial committee of "Communications in Mathematical Sciences" (CMS).

9.1.2.2. Reviewer - Reviewing Activities

The members of the MINGuS team are reviewers for almost all the journals in which they publish (SIAM, JCP, CPDE, CMP, ARMA, JSP, JSC, JMAA, ANM, JCAM, NMPDE, Numer. Math., $\cdots$).

9.1.3. Invited Talks

- J. Bernier gave a talk in the seminar of Columbia university, New York, USA, June 2019.
- P. Chartier gave a talk in the workshop *Nonlinear Evolution Equations: Analysis and Numerics*, organized by M. Hochbruck, H. Koch, S.-J. Oh, and A. Ostermann, Oberwolfach, Germany, February 2019.
- P. Chartier gave a talk in the workshop HaLu, Gran Sasso Science Institute (GSSI) School of Advanced Studies, L'Aquila, Italy, June 2019.
- P. Chartier gave a talk in the ICIAM conference, Valencia, Spain, July 2019.
- P. Chartier gave a talk in the Scicade conference, University of Innsbruck, Austria, July 2019.
- N. Crouseilles gave a talk at the workshop *Quantum and Kinetic Transport*, Shanghai, China, April 2019.
- N. Crouseilles gave a talk in the workshop *Numerical Methods for Multiscale Models arising in Physics and Biology*, University of Nantes, France, June 2019.
- N. Crouseilles gave a talk in the Scicade conference, University of Innsbruck, Austria, July 2019.
- A. Debussche gave a talk in the conference *Partial Differential Equations: from theory to applications*, Nancy, France, March 2019.
- A. Debussche gave a talk in the workshop *Numerical Methods for SPDE: 20 Successful Years and Future Challenges*, Reims, France, June 2019.
- A. Debussche gave a mini-course in the workshop *PROPAL : propagation d'ondes en milieux aléatoires*, Mittag-Leffler Institute, Stockholm, Sweden, May 2019
- A. Debussche gave a talk in the workshop *Recents Trends in Stochastic Analysis and SPDEs*, University of Pisa, Italy, July 2019.
- A. Debussche gave a talk in the workshop *Touch down of Stochastic Analysis*, University of Bielefeld, Germany, September 2019.
- A. Debussche gave a talk in the conference *Challenges and New Perspectives in Mathematics*, Hassan II Academy of Sciences and Technology, Morocco, November 2019.
- A. Debussche gave a talk in the conference *Paths between probability, PDEs and physics*, Imperial College, July 2019.
- E. Faou gave a talk in the workshop *The future of structure-preserving algorithms*, ICMS, Edinburgh, UK, October 2019.
- E. Faou gave a talk in the Analysis seminar, CMS University of Cambridge, UK, October 2019.
- E. Faou gave a talk in the *Plasma day* session, at the Isaac Newton Institute, Cambridge, UK, October 2019.
- E. Faou gave a Colloquium talk at the University of Bielefeld, Germany, April 2019.
- E. Faou gave a talk in the workshop *Dynamics of nonlinear dispersive PDEs*, La Thuile, Italy, February 2019.
- E. Faou gave a talk in the workshop *Nonlinear Evolution Equations: Analysis and Numerics*, organized by M. Hochbruck, H. Koch, S.-J. Oh, and A. Ostermann, Oberwolfach, Germany, February 2019.
- Y. Li gave a talk in the Scicade conference, University of Innsbruck, July 2019.
- Y. Li gave a talk in the NumKin conference, Max Planck Institute, Garching, October 2019.
- M. Lemou gave a talk at the Albert Einstein Institute, Golm, Germany, February 2019.
- M. Lemou gave a talk at the workshop *Quantum and Kinetic Transport*, Shanghai, China, April 2019.
- M. Lemou gave a talk at the university of Cardiff seminar, Cardiff, UK, May 2019.

- M. Lemou gave a talk in the ICIAM conference, Valencia, Spain, July 2019.
- M. Lemou gave a talk in the Scicade conference, Innsbruck, Austria, July 2019.
- M. Lemou gave a talk at the university of Wisconsin seminar, Madison, USA, September 2019.
- M. Lemou gave a talk at the Georgia Tech seminar, Atlanta, USA, September 2019.
- M. Lemou gave a talk in the workshop *Recent Progress and Challenge in Quantum and Kinetic Problems*, Singapore, October 2019.
- J. Massot gave a talk in the NumKin conference, Max Planck Institute, Garching, October 2019.
- F. Méhats gave a talk in the workshop *Recent Progress and Challenge in Quantum and Kinetic Problems*, Singapore, October 2019.
- F. Méhats gave a talk in the NumKin conference, Max Planck Institute, Garching, October 2019.
- P. Navaro participated at the conference JuliaCon 2019, Baltimore, USA, August 2019.
- A. Rosello gave a talk in the conference *Paths between probability, PDEs and physics*, Imperial College, July 2019.

9.1.4. Scientific Expertise

- N. Crouseilles was member of the committee of the Blaise Pascal prize (GAMNI-SMAI), 2019.
- N. Crouseilles was member of the committee of the best PhD talk in the Scicade conference, 2019.
- N. Crouseilles was member of the committee of the expert reviewers for the European Doctoral programme of the University of Innsbruck.
- A. Debussche was reviewer for ERC projects.
- E. Faou was member of the committee of the PhD prize SMAI-GAMNI, 2019. 2019).

9.1.5. Research Administration

- F. Castella is member of the UFR mathématiques council, University Rennes 1.
- N. Crouseilles is responsible of Fédération Recherche Fusion for the University of Rennes I.
- N. Crouseilles is member of the IRMAR laboratory council, University Rennes 1.
- N. Crouseilles is member of the scientific council of ENS Rennes (until september 2019).
- A. Debussche is member of the scientific council of the Fédération Denis Poisson.
- A. Debussche is member of the administrative council of ENS Paris-Saclay.
- A. Debussche is scientific vice-deputy and international relations of ENS Rennes.
- A. Debussche is co-director of the Henri Lebesgue Center (Excellence laboratory of the program investissement d'avenir).
- A. Debussche is vice-head of the Lebesgue agency for Mathematic and Innovation.
- E. Faou is co-director of the Henri Lebesgue Center (Excellence laboratory of the program investissement d'avenir).
- E. Faou is member of the Scientific Council of the Pôle Universitaire Léonard de Vinci.
- M. Lemou is the head of the IRMAR team "Analyse numérique" composed of 48 members.
- M. Lemou is member of the scientific council of ENS Rennes.
- M. Lemou is member of the scientific council of the Henri Lebesgue Center.
- P. Navaro is member of the national network "calcul" http://calcul.math.cnrs.fr. This network is well known for interdisciplinarity of CNRS dedicated to technological aspects of scientific computing (programming, optimization, architectures, $\cdots$).

9.2. Teaching - Supervision - Juries

9.2.1. Teaching

Master :

- F. Castella, Numerical methods for ODEs and PDEs, 60 hours, Master 1, University of Rennes.
- N. Crouseilles, Numerical methods for PDEs, 24 hours, Master 1, ENS Rennes.
- E. Faou, Normal forms, 24 hours, Master 2, University of Rennes.
- M. Lemou, Numerical methods for kinetic equations, 18 hours, Master 2, University of Rennes.
- M. Lemou, elliptic PDEs, 36 hours, Master 1, University of Rennes.
- P. Navaro, Python courses, Master 2 Smart Data, ENSAI.
- P. Navaro, Scientific computing tools for big data, Master 2, University of Rennes.

9.2.2. Supervision

PhD : G. Barrué, Approximation diffusion pour des équations dispersives, University of Rennes I, started in september 2019, A. Debussche.

PhD : J. Bernier, Study of some perturbation of equations which involve symmetries: resonancy and stability, University of Rennes I, defended in july 2019, E. Faou and N. Crouseilles.

PhD : Q. Chauleur, Equation de Vlasov singulière et équations reliées, University of Rennes I, started in september 2019, R. Carles (CNRS, Rennes) and E. Faou.

PhD: Y. Li (Chinese Academy of Sciences), Structure preserving methods for Vlasov equations, march 2019-february 2020, Y. Sun (Chinese Academy of Sciences) and N. Crouseilles.

PhD in progress : J. Massot, Exponential methods for hybrid kinetic models, started in october 2018, N. Crouseilles.

PhD in progress : A. Rosello, Approximation-diffusion pour des équations cinétiques pour les modèles de type spray, started in september 2016, A. Debussche and J. Vovelle (CNRS, Lyon).

PhD in progress : L. Trémant, Asymptotic analysis methods and numerical of dissipative multi-scale models: ODE with central manifold and kinetic models, started in october 2018, P. Chartier and M. Lemou.

9.2.3. Juries

- F. Castella was referee for the PhD thesis of H. Moundoyi (Laboratoire de biologie marine de Roscoff, France), supervised by P. Cormier and B. Sarels.
- F. Castella was referee for the PhD thesis of F. Patout (ENS Lyon, France), supervised by V. Calvez and J. Garnier.
- N. Crouseilles was referee for the PhD thesis of B. Fedele (University Toulouse 3, France), supervised by C. Negulescu and M. Ottaviani (CEA).
- A. Debussche was referee for the PhD of E. Altmann (Sorbonne Université, France), supervised by L. Zambotti.
- A. Debussche was member of the defense committee of the PhD of B. Kouegou Kamen (Aix-Marseille Université, France), supervised by E. Pardoux.
- A. Debussche was referee for the PhD of T. Yeo (Aix-Marseille université, France) supervised by E. Pardoux.
- E. Faou was referee for the Habilitation degree of Karolina Kropielnicka (Univ. Gdansk, Poland).
- M. Lemou was referee of the PhD thesis of X. Li (University Paris-Dauphine, France), supervised by J. Dolbeault.
- F. Méhats was referee of the PhD thesis of T. Dolmaire (university Paris Diderot, France), supervised by L. Desvillettes and I. Gallagher.

9.3. Popularization

9.3.1. Internal or external Inria responsibilities

- F. Castella was member of the HCERES committee for the evaluation of the laboratory "Mathématiques et Informatique pour la Complexité et les Systèmes (MICS)", CentraleSupelec, Gif-sur-Yvette.
- P. Chartier was member of the hiring committee CR2-Inria (Bordeaux)
- P. Chartier was member of the hiring committee for the Inria promotion DR1-DR0.
- N. Crouseilles is member of the Inria Evaluation Committe (2019-2023).
- N. Crouseilles was member of the Inria hiring committee for the following Inria promotions: CRHC, DR2-DR1, DR1-DR0, DR0-DR02.
- N. Crouseilles is member of the hiring committee of the professor position, ENS Rennes.
- E. Faou was member of the CNU 26 until the summer 2019.
- E. Faou was member of the HCERES committee for the evaluation of the Mathematics Institute of Toulouse, University Paul Sabatier.
- E. Faou is AMIES correspondent (Agency for Interaction in Mathematics with Business and Society) for Inria Rennes Bretagne atlantique and IRMAR.

9.3.2. Interventions

- N. Crouseilles: participation to high school students internship at IRMAR laboratory (one week), June 2019.
- N. Crouseilles: interview by a first year student of University Rennes I (in order to inform the different ways to become Inria researcher).
- J. Massot: talk at the N. Mandela high school (terminal S classes) about the links between astronomy and mathematics, April 2019.
- J. Massot: participation to high school students internship at IRMAR laboratory (one week), June 2019.
- P. Navaro: participation to the Julia day, Lyon, France, January 2019.
- P. Navaro: participation to the Julia day, Nantes, France, June 2019.
- A. Rosello: participation to "MATHC2+", June 2019, ENS Rennes.
- L. Trémant: participation to "Maths en Jean", June 2019, University of Rennes.

9.3.3. Internal action

- J. Massot: writing of a Python library *ponio* (Python Objects for Numerical IntegratOr) https://pypi.org/project/ponio/0.1/
- P. Navaro: Python training at the IRMAR laboratory.
- P. Navaro: Julia training at the IRMAR laboratory.
- P. Navaro: R training at the Finist'R internal workshop, Roscoff, France, August 2019.
- P. Navaro, Python courses, within the exchange program between ENSAI and Hong Kong university.

10. Bibliography

Publications of the year

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