RESEARCH CENTRE
**Sophia Antipolis - Méditerranée**

**IN PARTNERSHIP WITH:**
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2020
ACTIVITY REPORT

Project-Team
AROMATH

**AlgebRa, geOmetry, Modeling and AlgoriTHms**

**DOMAIN**
**Algorithmics, Programming, Software and Architecture**

**THEME**
**Algorithmics, Computer Algebra and Cryptology**

# Contents

# Project-Team AROMATH

*Creation of the Project-Team: 2016 July 01*

## Keywords

**Computer sciences and digital sciences**

    A5.5.1. – Geometrical modeling

    A6.1. – Methods in mathematical modeling

    A8.3. – Geometry, Topology

    A8.4. – Computer Algebra

**Other research topics and application domains**

    B9.5.1. – Computer science

    B9.5.2. – Mathematics

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# 2 Overall objectives

Our daily life environment is increasingly interacting with digital information. An important amount of this information is of geometric nature. It concerns the representation of our environment, the analysis and understanding of "real" phenomena, the control of physical mechanisms or processes. The interaction between physical and digital worlds is two-way. Sensors are producing digital data related to measurements or observations of our environment. Digital models are also used to "act" on the physical world. Objects that we use at home, at work, to travel, such as furniture, cars, planes, … are nowadays produced by industrial processes which are based on digital representation of shapes. CAD-CAM (Computer Aided Design – Computer Aided Manufacturing) software is used to represent the geometry of these objects and to control the manufacturing processes which create them. The construction capabilities themselves are also expanding, with the development of 3D printers and the possibility to create daily-life objects "at home" from digital models.

The impact of geometry is also important in the analysis and understanding of phenomena. The 3D conformation of a molecule explains its biological interaction with other molecules. The profile of a wing determines its aeronautic behavior, while the shape of a bulbous bow can decrease significantly the wave resistance of a ship. Understanding such a behavior or analyzing a physical phenomenon can nowadays be achieved for many problems by numerical simulation. The precise representation of the geometry and the link between the geometric models and the numerical computation tools are closely related to the quality of these simulations. This also plays an important role in optimisation loops where the numerical simulation results are used to improve the "performance" of a model.

Geometry deals with structured and efficient representations of information and with methods to treat it. Its impact in animation, games and VAMR (Virtual, Augmented and Mixed Reality) is important. It also has a growing influence in e-trade where a consumer can evaluate, test and buy a product from its digital description. Geometric data produced for instance by 3D scanners and reconstructed models are nowadays used to memorize old works in cultural or industrial domains.

Geometry is involved in many domains (manufacturing, simulation, communication, virtual world…), raising many challenging questions related to the representations of shapes, to the analysis of their properties and to the computation with these models. The stakes are multiple: the accuracy in numerical engineering, in simulation, in optimization, the quality in design and manufacturing processes, the capacity of modeling and analysis of physical problems.

# 3 Research program

## 3.1 High order geometric modeling

The accurate description of shapes is a long standing problem in mathematics, with an important impact in many domains, inducing strong interactions between geometry and computation. Developing precise geometric modeling techniques is a critical issue in CAD-CAM. Constructing accurate models, that can be exploited in geometric applications, from digital data produced by cameras, laser scanners, observations or simulations is also a major issue in geometry processing. A main challenge is to construct models that can capture the geometry of complex shapes, using few parameters while being precise.

Our first objective is to develop methods, which are able to describe accurately and in an efficient way, objects or phenomena of geometric nature, using algebraic representations.

The approach followed in CAGD, to describe complex geometry is based on parametric representations called NURBS (Non Uniform Rational B-Spline). The models are constructed by trimming and gluing together high order patches of algebraic surfaces. These models are built from the so-called B-Spline functions that encode a piecewise algebraic function with a prescribed regularity at knots. Although these models have many advantages and have become the standard for designing nowadays CAD models, they also have important drawbacks. Among them, the difficulty to locally refine a NURBS

surface and also the topological rigidity of NURBS patches that imposes to use many such patches with trims for designing complex models, with the consequence of the appearing of cracks at the seams. To overcome these difficulties, an active area of research is to look for new blending functions for the representation of CAD models. Some examples are the so-called T-Splines, LR-Spline blending functions, or hierarchical splines, that have been recently devised in order to perform efficiently local refinement. An important problem is to analyze spline spaces associated to general subdivisions, which is of particular interest in higher order Finite Element Methods. Another challenge in geometric modeling is the efficient representation and/or reconstruction of complex objects, and the description of computational domains in numerical simulation. To construct models that can represent efficiently the geometry of complex shapes, we are interested in developing modeling methods, based on alternative constructions such as skeleton-based representations. The change of representation, in particular between parametric and implicit representations, is of particular interest in geometric computations and in its applications in CAGD.

We also plan to investigate adaptive hierarchical techniques, which can locally improve the approximation of a shape or a function. They shall be exploited to transform digital data produced by cameras, laser scanners, observations or simulations into accurate and structured algebraic models.

The precise and efficient representation of shapes also leads to the problem of extracting and exploiting characteristic properties of shapes such as symmetry, which is very frequent in geometry. Reflecting the symmetry of the intended shape in the representation appears as a natural requirement for visual quality, but also as a possible source of sparsity of the representation. Recognizing, encoding and exploiting symmetry requires new paradigms of representation and further algebraic developments. Algebraic foundations for the exploitation of symmetry in the context of non linear differential and polynomial equations are addressed. The intent is to bring this expertise with symmetry to the geometric models and computations developed by AROMATH.

## 3.2   Robust algebraic-geometric computation

In many problems, digital data are approximated and cannot just be used as if they were exact. In the context of geometric modeling, polynomial equations appear naturally as a way to describe constraints between the unknown variables of a problem. *An important challenge is to take into account the input error in order to develop robust methods for solving these algebraic constraints.* Robustness means that a small perturbation of the input should produce a controlled variation of the output, that is forward stability, when the input-output map is regular. In non-regular cases, robustness also means that the output is an exact solution, or the most coherent solution, of a problem with input data in a given neighborhood, that is backward stability.

Our second long term objective is to develop methods to robustly and efficiently solve algebraic problems that occur in geometric modeling.

Robustness is a major issue in geometric modeling and algebraic computation. Classical methods in computer algebra, based on the paradigm of exact computation, cannot be applied directly in this context. They are not designed for stability against input perturbations. New investigations are needed to develop methods which integrate this additional dimension of the problem. Several approaches are investigated to tackle these difficulties.

One relies on linearization of algebraic problems based on "elimination of variables" or projection into a space of smaller dimension. Resultant theory provides a strong foundation for these methods, connecting the geometric properties of the solutions with explicit linear algebra on polynomial vector spaces, for families of polynomial systems (e.g., homogeneous, multi-homogeneous, sparse). Important progress has been made in the last two decades to extend this theory to new families of problems with specific geometric properties. Additional advances have been achieved more recently to exploit the syzygies between the input equations. This approach provides matrix based representations, which are particularly powerful for approximate geometric computation on parametrized curves and surfaces. They are tuned to certain classes of problems and an important issue is to detect and analyze degeneracies and to adapt them to these cases.

A more adaptive approach involves linear algebra computation in a hierarchy of polynomial vector spaces. It produces a description of quotient algebra structures, from which the solutions of polynomial systems can be recovered. This family of methods includes Gröbner Basis , which provides general

tools for solving polynomial equations. Border Basis is an alternative approach, offering numerically stable methods for solving polynomial equations with approximate coefficients . An important issue is to understand and control the numerical behavior of these methods as well as their complexity and to exploit the structure of the input system.

In order to compute "only" the (real) solutions of a polynomial system in a given domain, duality techniques can also be employed. They consist in analyzing and adding constraints on the space of linear forms which vanish on the polynomial equations. Combined with semi-definite programming techniques, they provide efficient methods to compute the real solutions of algebraic equations or to solve polynomial optimization problems. The main issues are the completness of the approach, their scalability with the degree and dimension and the certification of bounds.

Singular solutions of polynomial systems can be analyzed by computing differentials, which vanish at these points. This leads to efficient deflation techniques, which transform a singular solution of a given problem into a regular solution of the transformed problem. These local methods need to be combined with more global root localisation methods.

Subdivision methods are another type of methods which are interesting for robust geometric computation. They are based on exclusion tests which certify that no solution exists in a domain and inclusion tests, which certify the uniqueness of a solution in a domain. They have shown their strength in addressing many algebraic problems, such as isolating real roots of polynomial equations or computing the topology of algebraic curves and surfaces. The main issues in these approaches is to deal with singularities and degenerate solutions.

# 4    Application domains

## 4.1    Geometric modeling for Design and Manufacturing.

The main domain of applications that we consider for the methods we develop is Computer Aided Design and Manufacturing.

Computer-Aided Design (CAD) involves creating digital models defined by mathematical constructions, from geometric, functional or aesthetic considerations. Computer-aided manufacturing (CAM) uses the geometrical design data to control the tools and processes, which lead to the production of real objects from their numerical descriptions.

CAD-CAM systems provide tools for visualizing, understanding, manipulating, and editing virtual shapes. They are extensively used in many applications, including automotive, shipbuilding, aerospace industries, industrial and architectural design, prosthetics, and many more. They are also widely used to produce computer animation for special effects in movies, advertising and technical manuals, or for digital content creation. Their economic importance is enormous. Their importance in education is also growing, as they are more and more used in schools and educational purposes.

CAD-CAM has been a major driving force for research developments in geometric modeling, which leads to very large software, produced and sold by big companies, capable of assisting engineers in all the steps from design to manufacturing.

Nevertheless, many challenges still need to be addressed. Many problems remain open, related to the use of efficient shape representations, of geometric models specific to some application domains, such as in architecture, naval engineering, mechanical constructions, manufacturing . . . Important questions on the robustness and the certification of geometric computation are not yet answered. The complexity of the models which are used nowadays also appeals for the development of new approaches. The manufacturing environment is also increasingly complex, with new type of machine tools including: turning, 5-axes machining and wire EDM (Electrical Discharge Machining), 3D printer. It cannot be properly used without computer assistance, which raises methodological and algorithmic questions. There is an increasing need to combine design and simulation, for analyzing the physical behavior of a model and for optimal design.

The field has deeply changed over the last decades, with the emergence of new geometric modeling tools built on dedicated packages, which are mixing different scientific areas to address specific applications. It is providing new opportunities to apply new geometric modeling methods, output from research activities.

## 4.2   Geometric modeling for Numerical Simulation and Optimization

A major bottleneck in the CAD-CAM developments is the lack of interoperability of modeling systems and simulation systems. This is strongly influenced by their development history, as they have been following different paths.

The geometric tools have evolved from supporting a limited number of tasks at separate stages in product development and manufacturing, to being essential in all phases from initial design through manufacturing.

Current Finite Element Analysis (FEA) technology was already well established 40 years ago, when CAD-systems just started to appear, and its success stems from using approximations of both the geometry and the analysis model with low order finite elements (most often of degree ≤ 2).

There has been no requirement between CAD and numerical simulation, based on Finite Element Analysis, leading to incompatible mathematical representations in CAD and FEA. This incompatibility makes interoperability of CAD/CAM and FEA very challenging. In the general case today this challenge is addressed by expensive and time-consuming human intervention and software developments.

Improving this interaction by using adequate geometric and functional descriptions should boost the interaction between numerical analysis and geometric modeling, with important implications in shape optimization. In particular, it could provide a better feedback of numerical simulations on the geometric model in a design optimization loop, which incorporates iterative analysis steps.

The situation is evolving. In the past decade, a new paradigm has emerged to replace the traditional Finite Elements by B-Spline basis element of any polynomial degree, thus in principle enabling exact representation of all shapes that can be modeled in CAD. It has been demonstrated that the so-called isogeometric analysis approach can be far more accurate than traditional FEA.

It opens new perspectives for the interoperability between geometric modeling and numerical simulation. The development of numerical methods of high order using a precise description of the shapes raises questions on piecewise polynomial elements, on the description of computational domains and of their interfaces, on the construction of good function spaces to approximate physical solutions. All these problems involve geometric considerations and are closely related to the theory of splines and to the geometric methods we are investigating. We plan to apply our work to the development of new interactions between geometric modeling and numerical solvers.

## 5   Highlights of the year

Fatmanur Yildirim defended her PhD thesis [40] titled "Finite fibers of rational maps by means of matrix representations with applications to distance problem" at University Côte d'Azur on February 3, 2020.

Ahmed Blidia defended his PhD thesis [38] titled "Geometric continuity for surfaces and scalar fields" at University Côte d'Azur on April 8, 2020. He was supervized by Bernard Mourrain and the committee included Ioannis Emiris.

Clément Laroche defended his PhD thesis [39] titled "Compact and efficient implicit representations" ("Représentations implicites compactes et efficaces") at University of Athens on April 30, 2020. He was supervized by Ioannis Emiris and the examination committee included Bernard Mourrain.

Ioannis Emiris was co-general chair of International Symposium on Symbolic and Algebraic Computation (ISSAC 2020), held online on July 20-23, 2020.

## 6   New results

### 6.1   Elimination ideals and Bézout relations

**Participant**    André Galligo.

Let $k$ be an infinite field and $I \subset k[x_1, \ldots, x_n]$ be a non-zero ideal such that $\dim V(I) = q \geq 0$. Denote by $(f_1, \ldots, f_s)$ a set of generators of $I$. One can see that in the set $I \cap k[x_1, \ldots, x_{q+1}]$ there exist non-zero

polynomials, depending only on these $q + 1$ variables. We aim to bound the minimal degree of the polynomials of this type, and of a Bézout (i.e. membership) relation expressing such a polynomial as a combination of the $f_i$. In particular we show in [27] that if $\deg f_i = d_i$ where $d_1 \geq d_2 \geq \cdots \geq d_s$, then there exist a non-zero polynomial $\phi(x) \in I \cap k[x_1, \ldots, x_{q+1}]$, such that $\deg(\phi) \leq d_s \prod_{i=1}^{n-q-1} d_i$. We also give a relative version of this theorem.

This is a joint work with Zbigniew Jelonek, Instytut Matematyczny PAN, Warszawa, Poland.

## 6.2 Separation bounds for polynomial systems

**Participant**    Ioannis Emiris, Bernard Mourrain, Elias Tsigaridas (OURAGAN).

We rely on aggregate separation bounds for univariate polynomials to introduce, in [24], novel worst-case separation bounds for the isolated roots of zero-dimensional, positive-dimensional, and overdetermined polynomial systems. We exploit the structure of the given system, as well as bounds on the height of the sparse (or toric) resultant, by means of mixed volume, thus establishing adaptive bounds. Our bounds improve upon Canny's Gap theorem. Moreover, they exploit sparseness and they apply without any assumptions on the input polynomial system. To evaluate the quality of the bounds, we present polynomial systems whose root separation is asymptotically not far from our bounds. We apply our bounds to three problems. First, we use them to estimate the bitsize of the eigenvalues and eigenvectors of an integer matrix; thus we provide a new proof that the problem has polynomial bit complexity. Second, we bound the value of a positive polynomial over the simplex: we improve by at least one order of magnitude upon all existing bounds. Finally, we asymptotically bound the number of steps of any purely subdivision-based algorithm that isolates all real roots of a polynomial system.

## 6.3 Matrix formulae for Resultants and Discriminants of Bivariate Tensor-product Polynomials

**Participant**    Laurent Busé, Angelos Mantzaflaris, Elias Tsigaridas (OURAGAN).

The construction of optimal resultant formulae for polynomial systems is one of the main areas of research in computational algebraic geometry. However, most of the constructions are restricted to formulae for unmixed polynomial systems, that is, systems of polynomials which all have the same support. Such a condition is restrictive, since mixed systems of equations arise frequently in many problems. Nevertheless, resultant formulae for mixed polynomial systems is a very challenging problem. In [21] we present a square, Koszul-type, matrix, the determinant of which is the resultant of an arbitrary (mixed) bivariate tensor-product polynomial system. The formula generalizes the classical Sylvester matrix of two univariate polynomials, since it expresses a map of degree one, that is, the elements of the corresponding matrix are up to sign the coefficients of the input polynomials. Interestingly, the matrix expresses a primal-dual multiplication map, that is, the tensor product of a univariate multiplication map with a map expressing derivation in a dual space. In addition we prove an impossibility result which states that for tensor-product systems with more than two (affine) variables there are no universal degree-one formulae, unless the system is unmixed. Last but not least, we present applications of the new construction in the efficient computation of discriminants and mixed discriminants.

## 6.4 Fibers of multi-graded rational maps and orthogonal projection onto rational surfaces

**Participant**    Laurent Busé, Fatmanur Yildirim.

In [16] we contribute a new algebraic method for computing the orthogonal projections of a point onto a rational algebraic surface embedded in the three dimensional projective space. This problem is first turned into the computation of the finite fibers of a generically finite dominant rational map: a congruence of normal lines to the rational surface. Then, an in-depth study of certain syzygy modules associated to such a congruence is presented and applied to build elimination matrices that provide universal representations of its finite fibers, under some genericity assumptions. These matrices depend linearly in the variables of the three dimensional space. They can be pre-computed so that the orthogonal projections of points are approximately computed by means of fast and robust numerical linear algebra calculations.

This is a joint work with Nicolas Notbol, the University of Buenos Aires, Argentina and Marc Chardin, CNRS and Institut de Mathématiques de Jussieu, Sorbonne Universités, Paris, France.

## 6.5   Degree and birationality of multi-graded rational maps

**Participant**   Laurent Busé.

In [18] we give formulas and effective sharp bounds for the degree of multi-graded rational maps and provide some effective and computable criteria for birationality in terms of their algebraic and geometric properties. We also extend the Jacobian dual criterion to the multi-graded setting. Our approach is based on the study of blow-up algebras, including syzygies, of the ideal generated by the defining polynomials of the rational map. A key ingredient is a new algebra that we call the saturated special fiber ring, which turns out to be a fundamental tool to analyze the degree of a rational map. We also provide a very effective birationality criterion and a complete description of the equations of the associated Rees algebra of a particular class of plane rational maps.

This is a joint work Carlos D'Andrea and Yairon Cid Ruiz from the University of Barcelona, Spain.

## 6.6   Freeness and invariants of rational plane curves

**Participant**   Laurent Busé.

Given a parameterization $\phi$ of a rational plane curve C, in [20] we study some invariants of C via $\phi$. We first focus on the characterization of rational cuspidal curves, in particular we establish a relation between the discriminant of the pull-back of a line via $\phi$, the dual curve of C and its singular points. Then, by analyzing the pull-backs of the global differential forms via $\phi$, we prove that the (nearly) freeness of a rational curve can be tested by inspecting the Hilbert function of the kernel of a canonical map. As a by product, we also show that the global Tjurina number of a rational curve can be computed directly from one of its parameterization, without relying on the computation of an equation of C.

This is a joint work Alexandru Dimca from the University Côté d'Azur, France, and Gabriel Sticlaru from the Faculty of Mathematics and Informatics of Constanta, Romania.

## 6.7   The geometry of the flex locus of a hypersurface

**Participant**   Laurent Busé.

In [19] we give a formula in terms of multidimensional resultants for an equation for the flex locus of a projective hypersurface, generalizing a classical result of Salmon for surfaces in P3. Using this formula, we compute the dimension of this flex locus, and an upper bound for the degree of its defining equations. We also show that, when the hypersurface is generic, this bound is reached, and that the generic flex line is unique and has the expected order of contact with the hypersurface.

This is a joint work Carlos D'Andrea and Martin Sombra from the University of Barcelona, Spain, and Martin Weimann from the University of Caen, France.

## 6.8 Regularity of bicyclic graphs and their powers

**Participant** Navid Nemati.

In [22] we characterise the Castelnuovo-Mumford regularity of edge ideal $I(G)$ of a bicyclic graph $G$ in terms of the induced matching numbers of $G$. For the base case of this family of graphs, i.e. dumbbell graphs, we explicitly compute the induced matching number. Moreover, we prove that $\mathrm{reg}\,I(G)^q = 2q + \mathrm{reg}\,I(G) - 2$, for all $q \geq 1$, when $G$ is a dumbbell graph with a connecting path having no more than two vertices.

This is a joint work with Yairon Cid-Ruiz from Universitat de Barcelona, Spain, Sepehr Jafari from University Degli Studi di Genova, and Beatrice Picone from University of Catania, Italy.

## 6.9 Lefschetz properties of monomial algebras with almost linear resolution

**Participant** Navid Nemati.

In [22] we study the WLP and SLP of artinian monomial ideals in $S = k[x_1, \ldots, x_n]$ via studying their minimal free resolutions. We study the Lefschetz properties of such ideals where the minimal free resolution of $S/I$ is linear for at least $n - 2$ steps. We give an affirmative answer to a conjecture of Eisenbud, Huneke and Ulrich for artinian monomial ideals with almost linear resolutions.

This is a joint work with Nasrin Altafi from KTH Royal Institute of Technology, Sweden.

## 6.10 Punctual Hilbert Schemes and Certified Approximate Singularities

**Participant** Angelos Mantzaflaris, Bernard Mourrain.

In [35] we provide a new method to certify that a nearby polynomial system has a singular isolated root with a prescribed multiplicity structure. More precisely, given a polynomial system $f = (f_1, \ldots, f_N) \in \mathbb{C}[x_1, \ldots, x_n]^N$, we present a Newton iteration on an extended deflated system that locally converges, under regularity conditions, to a small deformation of $f$ such that this deformed system has an exact singular root. The iteration simultaneously converges to the coordinates of the singular root and the coefficients of the so called inverse system that describes the multiplicity structure at the root. We use $\alpha$-theory test to certify the quadratic convergence, and to give bounds on the size of the deformation and on the approximation error. The approach relies on an analysis of the punctual Hilbert scheme, for which we provide a new description. We show in particular that some of its strata can be rationally parametrized and exploit these parametrizations in the certification. We show in numerical experimentation how the approximate inverse system can be computed as a starting point of the Newton iterations and the fast numerical convergence to the singular root with its multiplicity structure, certified by our criteria.

This is a joint work with Agnes Szanto, Department of mathematics, North Carolina State University, USA.

## 6.11 Ideal Interpolation, H-Bases and Symmetry

**Participant** Evelyne Hubert, Erick Rodriguez Bazan.

Multivariate Lagrange and Hermite interpolation are examples of ideal interpolation. The interpolation problem is defined by a set of linear forms, on the polynomial ring, whose kernels intersect into an ideal. Another class of examples thus arises when the linear forms are defined by the normal forms modulo an ideal. For an ideal interpolation problem with symmetry, we address in [36] the simultaneous computation of a symmetry adapted basis of the least interpolation space and the related symmetry adapted H-basis. The algorithm exploits symmetry at all stages, in addition to preserving it in the output. Defined by de Boor and Ron (1990), the least interpolation space is a canonically defined representation of the quotient of the polynomial ring by the ideal. As a result it has great properties that cannot be achieved by a space spanned by monomials. In particular it is invariant under a group action when the ideal is; Equivalently, when the set of linear forms is. Its natural complement is a H-basis of the ideal, as introduced by Macaulay (1916). As a contrast with Groebner bases, which are tied to a monomial term order and are notorious for not preserving symmetry, we show that symmetry can be naturally preserved and represented in H-bases, as well as exploited in the course of their computation.

## 6.12    Computing Free Non-commutative Groebner Bases over Z with Singular:Letterplace

**Participant**    Tobias Metzlaff.

The extension of Groebner bases concept from polynomial algebras over fields to polynomial rings over rings allows to tackle numerous applications, both of theoretical and of practical importance. Groebner and Groebner-Shirshov bases can be defined for various non-commutative and even non-associative algebraic structures. In [34] we study the case of associative rings and aim at free algebras over principal ideal rings. We concentrate ourselves on the case of commutative coefficient ring without zero divisors (i. e. a domain). Even working over Z allows one to do computations, which can be treated as universal for fields of arbitrary characteristic. By using the systematic approach, we revisit the theory and present the algorithms in the implementable form. We show drastic differences in the behavior of Gröbner bases between free algebras and algebras, close to commutative. Even the formation of critical pairs has to be reengineered, together with the criteria for their quick discarding. We present an implementation of algorithms in the Singular subsystem called Letterplace, which internally uses Letterplace techniques (and Letterplace Gröbner bases), due to La Scala and Levandovskyy. Interesting examples accompany our presentation.

This is joint work with Viktor Levandovskyy and Karim Zeid, RWTH Aachen University, Germany.

## 6.13    On minimal decompositions of low rank symmetric tensors

**Participant**    Bernard Mourrain.

In [30], we use an algebraic approach to construct minimal decompositions of symmetric tensors with low rank. This is done by using Apolarity Theory and by studying minimal sets of reduced points apolar to a given symmetric tensor, namely, whose ideal is contained in the apolar ideal associated to the tensor. In particular, we focus on the structure of the Hilbert function of these ideals of points. We give a procedure which produces a minimal set of points apolar to any symmetric tensor of rank at most 5. This procedure is also implemented in the algebra software Macaulay2.

This is a joint work with Alessandro Oneto, Univ. de Trento, Italy.

## 6.14    Computational geometry in general dimension

**Participant**    Ioannis Emiris, Ioannis Psarros.

In [25] we examine general products of Euclidean metrics and construct a general framework in order to study such metrics and design nearest neighbor (NN) query algorithms for them. Then, we apply our approach to the Frechet distance metric and the Dynamic time warping distance function and show they are both special cases of our framework. Our methods are applied to proximity problems and approximate NN queries among general-dimensional polygonal curves, thus improving upon the existing asymptotic complexity of such operations.

> **Participant**    Ioannis Emiris, Ioannis Psarros.

In [13] we design efficient algorithms for high-dimensional approximate $r$-nets that improve upon the asymptotic complexity of existing approaches. The problem asks to find a subset of a given pointset that shall define the centers of balls of radius $r$ which shall cover the input points. We expect the simplicity of our algorithms to allow for a practical implementation of our techniques.

This is joint work with Zeta Avarikioti (ETH Zurich), and Loukas Kavouras (Athena RC).

> **Participant**    Ioannis Emiris, Evangelos Bartzos.

In [14] we employ the multihomogeneous Bézout (m-Bezout) number in order to derive new asymptotic bounds on the number of Euclidean embeddings of generically minimally rigid graphs. These are distance graphs that admit a finite number of realizations and generalize the class of Laman graphs that are rigid in the plane. Despite a number of studies on this problem, our bounds are the first to improve upon the trivial Bézout bound for dimension larger than 4 as well as 3D graphs that are graph-theoretically planar. The crucial observation is that the m-Bezout number is given by matrix permanents that have a strong combinatorial structure which can be revealed and exploited.

This is joint work with Josef Schicho (U Linz).

## 6.15    Complete Classification and Efficient Determination of Arrangements Formed by Two Ellipsoids

> **Participant**    Bernard Mourrain.

Arrangements of geometric objects refer to the spatial partitions formed by the objects and they serve as an underlining structure of motion design, analysis and planning in CAD/CAM, robotics, molecular modeling, manufacturing and computer-assisted radio-surgery. Arrangements are especially useful to collision detection, which is a key task in various applications such as computer animation , virtual reality, computer games, robotics, CAD/CAM and computational physics. Ellipsoids are commonly used as bounding volumes in approximating complex geometric objects in collision detection. In [28], we present an in-depth study on the arrangements formed by two ellipsoids. Specifically, we present a classification of these arrangements and propose an efficient algorithm for determining the arrangement formed by any particular pair of ellipsoids. A stratification diagram is also established to show the connections among all the arrangements formed by two ellipsoids. Our results for the first time elucidate all possible relative positions between two arbitrary ellipsoids and provides an efficient and robust algorithm for determining the relative position of any two given ellipsoids, therefore providing the necessary foundation for developing practical and trustworthy methods for processing ellipsoids for collision analysis or simulation in various applications.

This is a joint work with Xiaohong Jia, KLMM - Key Laboratory of Mathematics Mechanization, Beijing, China; Changhe Tu, School of Computer Science and Technology, Qingdao, China; Wenping Wang, HKU - The University of Hong Kong, China.

## 6.16   Interpolatory Catmull-Clark volumetric subdivision over unstructured hexahedral meshes for modeling and simulation applications

**Participant**    Bernard Mourrain.

Volumetric modeling is an important topic for material modeling and isogeometric simulation. In [33], two kinds of interpolatory Catmull-Clark volumetric subdivision approaches over unstructured hexahedral meshes are proposed based on the limit point formula of Catmull-Clark subdivision volume. The basic idea of the first method is to construct a new control lattice, whose limit volume by the Catmull-Clark subdivision scheme interpolates vertices of the original hexahedral mesh. The new control lattice is derived by the local push-back operation from one CatmullClark subdivision step with modified geometric rules. This interpolating method is simple and efficient, and several shape parameters are involved in adjusting the shape of the limit volume. The second method is based on progressive-iterative approximation using limit point formula. At each iteration step, we progressively modify vertices of an original hexahedral mesh to generate a new control lattice whose limit volume interpolates all vertices in the original hexahedral mesh. The convergence proof of the iterative process is also given. The interpolatory subdivision volume has $C^2$-smoothness at the regular region except around extraordinary vertices and edges. Furthermore, the proposed interpolatory volumetric subdivision methods can be used not only for geometry interpolation, but also for material attribute interpolation in the field of volumetric material modeling. The application of the proposed volumetric subdivision approaches on isogeometric analysis is also given with several examples.

This is a joint work with Jin Xie, Jinlan Xu, Zhenyu Dong, Gang Xu, Chongyang Deng, HDU - College of Computer Science, Hangzhou, China; Yongjie Jessica Zhang, Department of Mechanics, Carnegie Mellon University, USA.

## 6.17   Geometrically smooth spline bases for data fitting and simulation

**Participant**    Ahmed Blidia, Bernard Mourrain.

Given a topological complex $M$ with glueing data along edges shared by adjacent faces, we study the associated space of geometrically smooth spline functions that satisfy differentiability properties across shared edges. In [15], we present new and efficient constructions of basis functions of the space of $G^1$-spline functions on quadrangular meshes, which are tensor product b-spline functions on each quadrangle and with b-spline transition maps across the shared edges. This new strategy for constructing basis functions is based on a local analysis of the edge functions, and does not depend on the global topology of $M$. We show that the separability of the space of $G^1$ splines across an edge allows to determine the dimension and a basis of the space of $G^1$ splines on $M$. This leads to explicit and effective constructions of basis functions attached to the vertices, edges and faces of $M$. This basis construction has important applications in geometric modeling and simulation. We illustrate it by the fitting of point clouds by $G^1$ splines on quadrangular meshes of complex topology and in Isogeometric Analysis methods for the solution of diffusion equations. The ingredients are detailed and experimentation results showing the behavior of the method are presented.

This is joint work with Gang Xu, HDU - College of Computer Science, Hangzhou, China.

## 6.18   An overview of Geometry plus Simulation Modules

**Participant**    Angelos Mantzaflaris.

In [37] we give an overview of the open-source library "G+Smo". G+Smo is a C++ library that brings together mathematical tools for geometric design and numerical simulation. It implements the relatively

new paradigm of isogeometric analysis, which suggests the use of a unified framework in the design and analysis pipeline. G+Smo is an object-oriented, cross-platform, fully templated library and follows the generic programming principle, with a focus on both efficiency and ease of use. The library aims at providing access to high quality, open-source software to the community of numerical simulation and beyond.

### 6.19 Local (T)HB-spline projectors via restricted hierarchical spline fitting

**Participant** Angelos Mantzaflaris.

The paper [26] is devoted to techniques for adaptive spline projection via quasi-interpolation, enabling the efficient approximation of given functions. We employ local least-squares fitting in restricted hierarchical spline spaces to establish novel projection operators for hierarchical splines of degree p. This leads to efficient spline projectors that require $O(p^d)$ floating point operations and $O(1)$ evaluations of the given function per degree of freedom, while providing essentially the same accuracy as global approximation. Our spline projectors are based on a unifying framework for quasi-interpolation in hierarchical spline spaces. We present a detailed comparison with the scheme of Speleers and Manni (2016).

This is a joint work with Alessandro Giust and Bert Jüttler for the Johannes Kepler University of Linz, Austria.

### 6.20 Efficient Matrix Assembly in Isogeometric Analysis with Hierarchical B-splines

**Participant** Angelos Mantzaflaris.

Hierarchical B-splines that allow local refinement have become a promising tool for developing adaptive isogeometric methods. Unfortunately, similar to tensor-product B-splines, the computational cost required for assembling the system matrices in isogeometric analysis with hierarchical B-splines is also high, particularly if the spline degree is increased. To address this issue, we propose in [31] an efficient matrix assembly approach for bivariate hierarchical B-splines based the previously proposed IIL approach. The new algorithm consists of three stages: approximating the integrals by quasi-interpolation, building three compact look-up tables and assembling the matrices via sum-factorization. A detailed analysis shows that the complexity of our method has the order $O(Np^3)$ under a mild assumption about mesh admissibility, where $N$ and $p$ denote the number of degrees of freedom and spline degree respectively. Finally, several experimental results are demonstrated to verify the theoretical results and to show the performance of the proposed method.

This is a joint work with Maodong Pan and Bert Jüttler for the Johannes Kepler University of Linz, Austria.

## 7 Bilateral contracts and grants with industry

### 7.1 Bilateral contracts with industry

• **NURBSFIX: Repairing the topology of a NURBS model in view of its approximation.** We have a research contract with the industrial partner GeometryFactory, in collaboration with the project-team Titane (Pierre Alliez). The post-doc of Xiao Xiao is funded by this research contract together with a PEPS from the labex AMIES. We continue to develop a robust algorithm which offers the capability to repair 155 while meshing CAD models, without requiring a valid surface mesh as input, and 156 to generate volume meshes which are valid by design and 157 independent of the input NURBS patch layout, without resorting 158 to post-processing steps such as mesh quilting or remeshing.

- **Geometric computing.** Ioannis Emiris coordinates a research contract with the industrial partner ANSYS (Greece), in collaboration with Athena Research Center. PhD candidate A. Chalkis is partially funded.

Electronic design automation (EDA) and simulating Integrated Circuits requires robust geometric operations on thousands of electronic elements (capacitors, resistors, coils etc) represented by polyhedral objects in 2.5 dimensions, not necessarily convex. A special case may concern axis-aligned objects but the real challenge is the general case. The project, extended into 2021, focuses on 3 axes: (1) efficient data structures and prototype implementations for storing the aforementioned polyhedral objects so that nearest neighbor queries are fast in the L-max metric, which is the primary focus of the contract, (2) random sampling of the free space among objects, (3) data-driven algorithmic design for problems concerning data-structures and their construction and initialization.

## 7.2 Bilateral grants with industry

- **Interactive construction of 3D models - Application to the modeling of complex geological structures.**

CIFRE collaboration between Schlumberger Montpellier (A. Azzedine) and Inria Sophia Antipolis (B. Mourrain). The PhD candidate is A. Belhachmi. The objective of the work is the development of a new spline based high quality geomodeler for reconstructing the stratigraphy of geological layers from the adaptive and efficient processing of large terrain information.

# 8 Partnerships and cooperations

## 8.1 International initiatives

### High order methods for computational engineering and data analysis

**Title:** High order methods for computational engineering and data analysis

**Program:** Partenariats Hubert Curien (PHC) Alliance

**Duration:** January 2020 - December 2022

**Other partner:** U. Swansea (UK)

**Inria contact:** Angelos Mantzaflaris

**Summary:** The aim of this project is to develop a mathematical framework for the integration of geometric modeling and simulation using spline-based finite elements of high degree of smoothness. High-order methods are known to provide a robust and efficient methodology to tackle complex challenges in multi-physics simulations, shape optimization, and the analysis of large-scale datasets arising in data-driven engineering and design. However, the analysis and design of high-order methods is a daunting task requiring a concurrent effort from diverse fields such as applied algebraic geometry, approximation theory and splines, topological data analysis, and computational mathematics. Our strategic vision is to create a research team combining uniquely broad research expertise in these areas by establishing a link between the AROMATH and Swansea University.

## 8.2 European initiatives

### 8.2.1 FP7 & H2020 Projects

### ARCADES

**Title:** Algebraic Representations for Computer-Aided Design of Complex Shapes

**Project acronym:** ARCADES

**Program:** Marie Skłodowska-Curie ETN

**Duration:** January 2016 - December 2019

**Coordinator:** I.Z. Emiris, ATHENA Research Innovation Center

**Other partners:**

- U. Barcelona (Spain),
- J. Kepler University, Linz (Austria),
- SINTEF Institute, Oslo (Norway),
- U. Strathclyde, Glascow (UK),
- Technische U. Wien (Austria),
- Evolute GmBH, Vienna (Austria).

**Inria contact:** Laurent Busé

**Webpage:** http://arcades-network.eu/

**Summary:** ARCADES aims at disrupting the traditional paradigm in Computer-Aided Design (CAD) by exploiting cutting-edge research in mathematics and algorithm design. Geometry is now a critical tool in a large number of key applications; somewhat surprisingly, however, several approaches of the CAD industry are outdated, and 3D geometry processing is becoming increasingly the weak link. This is alarming in sectors where CAD faces new challenges arising from fast point acquisition, big data, and mobile computing, but also in robotics, simulation, animation, fabrication and manufacturing, where CAD strives to address crucial societal and market needs. The challenge taken up by ARCADES is to invert the trend of CAD industry lagging behind mathematical breakthroughs and to build the next generation of CAD software based on strong foundations from algebraic geometry, differential geometry, scientific computing, and algorithm design. Our game-changing methods lead to real-time modelers for architectural geometry and visualisation, to isogeometric and design-through-analysis software for shape optimisation, and marine design and hydrodynamics, and to tools for motion design, robot kinematics, path planning, and control of machining tools.

**POEMA**

**Title:** Polynomial Optimization, Efficiency through Moments and Algebra

**Project acronym:** POEMA

**Program:** Marie Skłodowska-Curie ITN

**Duration:** January 2019 - December 2022 (48 months)

**Coordinator:** Bernard Mourrain, Aromath, Inria Sophia Antipolis

**Other partners:**

- LAAS - CNRS, Toulouse (France),
- Sorbonne Université, Paris (France),
- Centrum Wiskunde & Informatica, Amsterdam (The Netherlands),
- Stichting Katholieke Universiteit Brabant, Tilburd (The Netherlands),
- Universität Konstanz (Germany),
- Università degli Studi di Firenze (Italy),
- University of Birmingham (United Kingdom),
- Friedrich Alexander University Erlangen-Nuremberg (Germany),

- Universitet I Tromsø (Norway),
- ARTELYS SAS, Paris (France).

**Inria contact:** Bernard Mourrain

**Webpage:** <http://poema-network.eu/>

**Summary:** Non-linear optimization problems are present in many real-life applications and in scientific areas such as operations research, control engineering, physics, information processing, economy, biology, etc. However, efficient computational procedures, that can provide the guaranteed global optimum, are lacking for them. The project will develop new polynomial optimization methods, combining moment relaxation procedures with computational algebraic tools to address this type of problems. Recent advances in mathematical programming have shown that the polynomial optimization problems can be approximated by sequences of Semi-Definite Programming problems. This approach provides a powerful way to compute global solutions of non-linear optimization problems and to guarantee the quality of computational results. On the other hand, advanced algebraic algorithms to compute all the solutions of polynomial systems, with efficient implementations for exact and approximate solutions, were developed in the past twenty years. The network combines the expertise of active European teams working in these two domains to address important challenges in polynomial optimization and to show the impact of this research on practical applications.

POEMA aims to train scientists at the interplay of algebra, geometry and computer science for polynomial optimization problems and to foster scientific and technological advances, stimulating interdisciplinary and intersectoriality knowledge exchange between algebraists, geometers, computer scientists and industrial actors facing real-life optimization problems.

**GRAPES**

**Title:** learninG, Representing, And oPtimizing shapES.

**Duration:** *2019 - 2023*

**Coordinator:** *ATHENA*

**Partners:**

- *ATHENA,* ATHINA-EREVNITIKO KENTRO KAINOTOMIAS STIS TECHNOLOGIES TIS PLIRO-FORIAS, TON EPIKOINONION KAI TIS GNOSIS (*Greece*)
- *AROMATH,* INRIA SOPHIA-ANTIPOLIS (France)
- *VISUAL COMPUTING,* RHEINISCH-WESTFAELISCHE TECHNISCHE HOCHSCHULE AACHEN (Germany)
- *OSLO,* SINTEF AS (Norway)
- *MATH DEPARTMENT,* UNIVERSITA DEGLI STUDI DI ROMA TOR VERGATA (Italy)
- *INFORMATICS DEPARTMENT),* UNIVERSITA DELLA SVIZZERA ITALIANA (Switzerland)
- *MATH DEPARTMENT,* UNIVERSITAT DE BARCELONA (Spain)
- *MATH Dept,* UNIVERSITAT LINZ (Austria)
- *Naval engineering,* University of Strathclyde (UK)
- *Math DEPARTMENT,* VILNIAUS UNIVERSITETAS (Lithuania)
- GeometryFactory SARL (France)

**Inria contact:** *Laurent Busé*

**Summary:** GRAPES aims at considerably advancing the state of the art in Mathematics, CAD, and Machine Learning in order to promote game changing approaches for generating, optimising, and learning 3D shapes, along with a multisectoral training for young researchers. The scientific goals of GRAPES rely on a multidisciplinary consortium composed of leaders in their respective fields. Top-notch research is also instrumental in forming the new generation of European scientists and engineers. Their disciplines span the spectrum from Computational Mathematics, Numerical Analysis, and Algorithm Design, up to Geometric Modelling, Shape Optimisation, and Deep Learning. This allows the 15 PhD candidates to follow either a theoretical or an applied track and to gain knowledge from both research and innovation through a nexus of intersectoral secondments and Network-wide workshops.

Horizontally, our results lead to open-source, prototype implementations, software integrated into commercial libraries as well as open benchmark datasets. These are indispensable for dissemination and training but also to promote innovation and technology transfer. Innovation relies on the active participation of SMEs, either as a beneficiary hosting an ESR or as associate partners hosting secondments. Concrete applications include simulation and fabrication, hydrodynamics and marine design, manufacturing and 3D printing, retrieval and mining, reconstruction and urban planning.

## 8.3 National initiatives

**GdR EFI and GDM** Evelyne Hubert is part of the Scientific Committee of the GdR *Equations Fonctionnelles et Interactions* (`http://gdrefi.math.cnrs.fr`) and participates to the GdR *Géometrie Differentielle et Mécanique* (`https://gdr-gdm.univ-lr.fr`).

# 9 Dissemination

## 9.1 Promoting scientific activities

### 9.1.1 Scientific events: organisation

**General chair, scientific chair** Ioannis Emiris was general co-chair of the International Symposium on Symbolic and Algebraic Computation (ISSAC) 2020.

Bernard Mourrain was co-chair of the 2020 International conference on Geometric Modeling and Processing (GMP).

**Member of the organizing committees** Laurent Busé was a member of the organizing committee of the Journées Nationales de Calcul Formel (JNCF), Luminy, France, March 2-6.

Angelos Mantzaflaris was proceedings editor of the International Symposium on Symbolic and Algebraic Computation (ISSAC) July 2020, and a co-oorganizer of the 4th G+Smo Developer days Workshop, December 2020.

**Member of the conference program committees** Evelyne Hubert was part of the program committee for the selection of submissions to the International Symposium on Symbolic and Algebraic Computation (ISSAC) 2020.

Evelyne Hubert was part of the committee to select the plenary speakers for the triennal conference Foundation of Computational Mathematics. The conference was canceled but the Board of Directors is organizing a monthly webinar based on the selected pleanry speakers.

Laurent Busé, Angelos Mantzaflaris and Bernard Mourrain were members of the program committee of the 2020 Symposium on Physical and Solid Modeling (SPM), Strasbourg, France, June 2-4.

Angelos Mantzaflaris was a member of the program committee of the 2020 Geometric Modeling and Processing (GMP) Conference, Okinawa, Japan, May 13-15, 2020.

**Reviewer**   Laurent Busé reviewed for Symposium on Physical and Solid Modeling conference and SIGGRAPH conference.

Bernard Mourrain reviewed for SPM conference, GMP conference, ISSAC conference.

### 9.1.2   Journal

**Member of the editorial boards**   Ioannis Emiris is editor of Journal of Symbolic Computation, Elsevier; Mathematics for Computer Science, Springer-Birkhauser; Journal of Applicable Algebra in Engineering, Communication & Computing, Springer.

Evelyne Hubert is associate editor for the journal of Foundations of Computational Mathematics (Springer) and Journal of Symbolic Computation (Elsevier)

Bernard Mourrain is associate editor of the Journal of Symbolic Computation (since 2007) and of the SIAM Journal on Applied Algebra and Geometry (since 2016).

**Reviewer - reviewing activities**   Evelyne Hubert reviewed for the journal Foundations of Computational Mathematics, the Journal of Symbolic Computation, Advances in Applied Mathematics, Mathematics and Computers in Simulation, Symmetry, Computers and Graphics. She also write reviews for MathSciNet.

Laurent Busé reviewed for MS Mathematics of Computations journal, Journal of Symbolic Computation, Israel Journal of Mathematics, Computer Aided-Design, Computer Aided Geometric Design, Journal of Computational and Applied Mathematics, Portugaliae Mathematica, Mathematica Scandinavica and Boletín de la Sociedad Matemática Mexicana.

Bernard Mourrain reviewed for the Journal of Symbolic Computation, for the Journal of Computational and Applied Mathematics, the journal Computer-Aided Desig, the journal Computer Aided Geometric Design, the Journal of Algebra , the journal Foundations of Computational Mathematics, the FWF Austrian Science Fund.

Angelos Mantzaflaris reviewed for Computer Graphics Forum, Journal of Numerical Methods in Engineering (Wiley), Journal on Applied Algebra and Geometry (SIAM), Computer Aided Geometric Design, Computer Methods in Applied Mechanics and Engineering (Elsevier).

### 9.1.3   Invited talks

Evelyne Hubert was invited to give a talk at the workshop *Symmetry, Randomness, and Computations in Real Algebraic Geometry* hosted by the Institute for Computational and Experimental Research in Mathematics (Providence, USA) and at the annual meeting of the GdR *Géométrie Différentielle et Mécanique* held online.

Angelos Mantzaflaris gave an invited talk at the INdAM Workshop on Geometric Challenges in Isogeometric Analysis January 27-31, 2020, Rome, Italy.

### 9.1.4   Leadership within the scientific community

Bernard Mourain is vice chair of the SIAM Algebraic Geometry group.

### 9.1.5   Scientific expertise

Evelyne Hubert was part of the selection committee for a Professor position in Mathematics at Université Versaille Saint Quentin.

Evelyne Hubert was part of the working group around the creation of the Inria Project team Calisto (Mireille Bossy).

### 9.1.6   Research administration

Bernard Mourrain is member of the BCEP (Bureau du Comité des Equipes Projet) of the center Inria-Sophia Antipolis.

Laurent Busé is a member of the admnistrative and scientific committee of the labex AMIES and a member of the executive committee of the MSI of UCA. He is also member of the CDT at Inria Sophia-Antipolis and the CPRH of the Mathematics Laboratory Jean-Alexandre Dieudonné of the University of Nice.

## 9.2 Teaching - Supervision - Juries

### 9.2.1 Teaching

- Licence : Ioannis Emiris, Algorithms and complexity, 52 h (L2), NKU Athens

- Licence : Ioannis Emiris, Software development, 26 h (L3), NKU Athens

- Master : Ioannis Emiris, Geometric data science, 52 h (M2), NKU Athens

- Master : Ioannis Emiris, Structural bioinformatics, 39 h (M2), NKU Athens

- Master : Laurent Busé, Geometric Modeling, 18h (M2), Polytech Nice Sophia - Univ Côte d'Azur.

- Master : Laurent Busé, Computational Algebraic Geometry, 30h (M2), Department of Mathematics of University Côte d'Azur.

### 9.2.2 Supervision

- PhD in progress: Lorenzo Baldi, Structure of moment problems and applications to polynomial optimization. POEMA Marie Skłodowska-Curie ITN, started in October 2019, supervised by Bernard Mourrain.

- PhD in progress: Evangelos Bartzos, Algebraic elimination and Distance graphs. ARCADES Marie Skłodowska-Curie ITN, started in June 2016, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Ayoub Belhachmi, Interactive construction of 3D models - Application to the modeling of complex geological structures. CIFRE, started in August 2020, Inria/Schlumberger, co-supervised by Bernard Mourrain.

- PhD defended on April 8, 2020: Ahmed Blidia, New geometric models for the design and computation of complex shapes. ARCADES Marie Skłodowska-Curie ITN, started in September 2016, supervised by Bernard Mourrain.

- PhD in progress: Apostolos Chalkis, Sampling in high-dimensional convex regions, Google Summer of Code and Pegasus national Project, started in June 2018, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Emmanouil Christoforou, Geometric approximation algorithms for clustering, Structural Bioinformatics national infrastructure "Inspire" and Pegasus national Project, started in Jan. 2018, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Carles Checa, Algebraic computing for geometric predicate operations. GRAPES Marie Skłodowska-Curie ITN, started in December 2020, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Riccardo Di Dio, Building a diagnosis tool to detect broncho-constrictions, BoostUrCAreer Marie Skłodowska-Curie COFUND fellowship. Started on November 2019, co-supervised by Benjamin Mauroy (UCA) and Angelos Mantzaflaris.

- PhD in progress: Edgar Fuentes Figueroa, Polynomial Optimization Techniques for Energy Network Operation and Design, Artelys, POEMA Marie Skłodowska-Curie ITN, started in December 2019, co-supervised by Michael Gabay (Artelys) and Bernard Mourrain.

- PhD in progress: Pablo Gonzalez Mazon; Generation of valid high-order curved meshes. GRAPES Marie Skłodowska-Curie ITN, started in December 2020, Inria, supervised by Laurent Busé.

- PhD in progress: Mehran Hatamzadeh; An innovative gait analysis technology. PhD grant from the EU CoFUND BoostUrCareer program of UCA, co-supervised by Laurent Busé and Raphaël Zory (LAMHESS, UCA).

- PhD in progress: Rima Khouja, Tensor decomposition, best approximations, algorithms and applications. Cotutelle Univ. Liban, started in November 2018, cosupervised by Houssam Khalil and Bernard Mourrain.

- PhD in progress: Thomas Laporte, Towards a 4D model of the respiratory system. Fellowship from ED SFA/UCA. Started on October 2019, co-supervised by Benjamin Mauroy (UCA) and Angelos Mantzaflaris.

- PhD defended on April, 30, 2020: Clément Laroche, Algebraic representations of geometric objects. ARCADES Marie Skłodowska-Curie ITN, started in Nov. 2016, NKUA; supervised by Ioannis Emiris.

- PhD in progress: Tobias Metzlaff. Multivariate orthogonal polynomials and applications to global optimization. POEMA Marie Skłodowska-Curie ITN, started in December 2019, supervised by Evelyne Hubert.

- PhD in progress: Michelangelo Marsala, Modelling and simulation using analysis-suitable subdivision surfaces and solids. GRAPES Marie Skłodowska-Curie ITN, started in November 2020, Inria, supervised by Angelos Mantzaflaris and Bernard Mourrain.

- PhD: Erick Rodriguez Bazan defended on December 2020. Symmetry in Algebraic Computations. Inria. Started in November 2017, supervised by Evelyne Hubert.

- PhD in progress: Konstantinos Tertikas, Machine learning for geometric shapes. GRAPES Marie Skłodowska-Curie ITN, started in December 2020, NKUA, supervised by Ioannis Emiris.

- PhD in progress: Tong Zhao; Learning priors and metrics for 3D reconstruction of large-scale scenes. PhD grant from the 3IA Institut of UCA, co-supervised by Pierre Alliez (TITANE) and Laurent Busé.

### 9.2.3 Juries

Bernard Mourrain was a member of the PhD committee of Clement Laroche, Algebraic representations of geometric objects. NKUA, April, 30, 2020.

Evelyne Hubert was a reviewer for the Habilitation thesis of Georg Regensbuger, Johannes Kepler University (Austria): *Algebraic and algorithmic Approached to Analysis: Integro-differential equations, positive steady states, and wavelets.*

Laurent Busé was a reviewer and member of the committee of the PhD thesis of This Xuan Vu, *Homotopy algorithms for solving structured determinantal systems,* University of Waterloo, Waterloo, Ontario, Canada, December 9th.

Angelos Mantzaflaris was a reviewer of the PhD thesis of Gabriele Loli, Efficient Solvers for Isogeometric Analysis, University of Pavia, Italy.

## 10 Scientific production

### 10.1 Major publications

[1] E. Bartzos, I. Z. Emiris and J. Schicho. 'On the multihomogeneous Bézout bound on the number of embeddings of minimally rigid graphs'. In: *Applicable Algebra in Engineering, Communication and Computing* 31.5-6 (2020), pp. 325–357. URL: https://hal.archives-ouvertes.fr/hal-02696362.

[2] L. Busé, Y. Cid-Ruiz and C. D'Andrea. 'Degree and birationality of multi-graded rational maps'. In: *Proceedings of the London Mathematical Society* 121.4 (2020), pp. 743–787. DOI: 10.1112/plms.12336. URL: https://hal.inria.fr/hal-01793578.

[3] L. Busé and A. Karasoulou. 'Resultant of an equivariant polynomial system with respect to the symmetric group'. In: *Journal of Symbolic Computation* 76 (2016), pp. 142–157. DOI: 10.1016/j.jsc.2015.12.004. URL: https://hal.inria.fr/hal-01022345.

[4] I. Z. Emiris and I. Psarros. 'Products of Euclidean Metrics, Applied to Proximity Problems among Curves'. In: *ACM Transactions on Spatial Algorithms and Systems* 6.4 (Aug. 2020), pp. 1–20. DOI: 10.1145/3397518. URL: https://hal.inria.fr/hal-03045134.

[5] A. J. Fuentes Suárez and E. Hubert. 'Scaffolding skeletons using spherical Voronoi diagrams: feasibility, regularity and symmetry'. In: *Computer-Aided Design* 102 (May 2018), pp. 83–93. DOI: 10.1016/j.cad.2018.04.016. URL: https://hal.inria.fr/hal-01774909.

[6] A. Giust, B. Jüttler and A. Mantzaflaris. 'Local (T)HB-spline projectors via restricted hierarchical spline fitting'. In: *Computer Aided Geometric Design* 80 (June 2020), p. 101865. DOI: 10.1016/j.cagd.2020.101865. URL: https://hal.inria.fr/hal-02985011.

[7] P. Görlach, E. Hubert and T. Papadopoulo. 'Rational invariants of even ternary forms under the orthogonal group'. In: *Foundations of Computational Mathematics* 19 (2019), pp. 1315–1361. DOI: 10.1007/s10208-018-9404-1. URL: https://hal.inria.fr/hal-01570853.

[8] Z. Jelonek and A. Galligo. 'Elimination ideals and Bezout relations'. In: *Journal of Algebra* 562 (2020), pp. 621–626. DOI: 10.1016/j.jalgebra.2020.06.022. URL: https://hal.archives-ouvertes.fr/hal-03138363.

[9] A. Mantzaflaris, B. Jüttler, B. Khoromskij and U. Langer. 'Low Rank Tensor Methods in Galerkin-based Isogeometric Analysis'. In: *Computer Methods in Applied Mechanics and Engineering* 316 (Apr. 2017), pp. 1062–1085. DOI: 10.1016/j.cma.2016.11.013. URL: https://hal.inria.fr/hal-02271847.

[10] B. Mourrain. 'Polynomial-Exponential Decomposition from Moments'. In: *Foundations of Computational Mathematics* 18.6 (Dec. 2018), pp. 1435–1492. DOI: 10.1007/s10208-017-9372-x. URL: https://hal.inria.fr/hal-01367730.

[11] S. Telen, B. Mourrain and M. Van Barel. 'Solving Polynomial Systems via a Stabilized Representation of Quotient Algebras'. In: *SIAM Journal on Matrix Analysis and Applications* 39.3 (Oct. 2018), pp. 1421–1447. DOI: 10.1137/17M1162433. URL: https://hal.inria.fr/hal-01630425.

## 10.2  Publications of the year

**International journals**

[12] N. Altafi and N. Nemati. 'Lefschetz properties of monomial algebras with almost linear resolution'. In: *Communications in Algebra* 48.4 (2nd Apr. 2020), pp. 1499–1509. DOI: 10.1080/00927872.2019.1691568. URL: https://hal.inria.fr/hal-03133849.

[13] Z. Avarikioti, I. Z. Emiris, L. Kavouras and I. Psarros. 'High-Dimensional Approximate r-Nets'. In: *Algorithmica* 82.6 (June 2020), pp. 1675–1702. DOI: 10.1007/s00453-019-00664-8. URL: https://hal.inria.fr/hal-03045138.

[14] E. Bartzos, I. Z. Emiris and J. Schicho. 'On the multihomogeneous Bézout bound on the number of embeddings of minimally rigid graphs'. In: *Applicable Algebra in Engineering, Communication and Computing* 31.5-6 (2020), pp. 325–357. URL: https://hal.archives-ouvertes.fr/hal-02696362.

[15] A. Blidia, B. Mourrain and G. Xu. 'Geometrically smooth spline bases for data fitting and simulation'. In: *Computer Aided Geometric Design* 78 (Mar. 2020), p. 101814. DOI: 10.1016/j.cagd.2020.101814. URL: https://hal.archives-ouvertes.fr/hal-02480959.

[16] N. Botbol, L. Busé, M. Chardin and F. Yildirim. 'Fibers of multi-graded rational maps and orthogonal projection onto rational surfaces'. In: *SIAM Journal on Applied Algebra and Geometry* 4.2 (2020), pp. 322–353. DOI: 10.1137/19M1289522. URL: https://hal.inria.fr/hal-02112357.

[17] L. Busé and F. Chen. 'Determinantal tensor product surfaces and the method of moving quadrics'. In: *Transactions of the American Mathematical Society* (2020). DOI: 10.1090/tran/8358. URL: https://hal.inria.fr/hal-02885789.

[18] L. Busé, Y. Cid-Ruiz and C. D'Andrea. 'Degree and birationality of multi-graded rational maps'. In: *Proceedings of the London Mathematical Society* 121.4 (2020), pp. 743–787. DOI: 10.1112/plms.12 336. URL: https://hal.inria.fr/hal-01793578.

[19] L. Busé, C. D'Andrea, M. Sombra and M. Weimann. 'The geometry of the flex locus of a hypersurface'. In: *Pacific Journal of Mathematics* 304.2 (2020), pp. 419–437. DOI: 10.2140/pjm.2020.304.419. URL: https://hal.archives-ouvertes.fr/hal-01779785.

[20] L. Busé, A. Dimca and G. Sticlaru. 'Freeness and invariants of rational plane curves'. In: *Mathematics of Computation* 89 (2020), pp. 1525–1546. DOI: 10.1090/mcom/3495. URL: https://hal.inria.fr/hal-01767751.

[21] L. Busé, A. Mantzaflaris and E. Tsigaridas. 'Matrix formulae for Resultants and Discriminants of Bivariate Tensor-product Polynomials'. In: *Journal of Symbolic Computation* 98 (June 2020), pp. 65–83. DOI: 10.1016/j.jsc.2019.07.007. URL: https://hal.inria.fr/hal-01654263.

[22] Y. Cid-Ruiz, S. Jafari, N. Nemati and B. Picone. 'Regularity of bicyclic graphs and their powers'. In: *Journal of Algebra and Its Applications* 19.03 (Mar. 2020), p. 2050057. DOI: 10.1142/S0219498820 500577. URL: https://hal.inria.fr/hal-03133852.

[23] I. Z. Emiris, A. Mantzaflaris and E. Tsigaridas. 'Multilinear Polynomial Systems: Root Isolation and Bit Complexity'. In: *Journal of Symbolic Computation*. Special Issue on Milestones in Computer Algebra (MICA 2016) 105 (2021), pp. 145–164. DOI: 10.1016/j.jsc.2020.06.005. URL: https://hal.inria.fr/hal-02099556.

[24] I. Z. Emiris, B. Mourrain and E. Tsigaridas. 'Separation bounds for polynomial systems'. In: *Journal of Symbolic Computation* 101 (2020), pp. 128–151. DOI: 10.1016/j.jsc.2019.07.001. URL: https://hal.inria.fr/hal-01105276.

[25] I. Z. Emiris and I. Psarros. 'Products of Euclidean Metrics, Applied to Proximity Problems among Curves'. In: *ACM Transactions on Spatial Algorithms and Systems* 6.4 (21st Aug. 2020), pp. 1–20. DOI: 10.1145/3397518. URL: https://hal.inria.fr/hal-03045134.

[26] A. Giust, B. Jüttler and A. Mantzaflaris. 'Local (T)HB-spline projectors via restricted hierarchical spline fitting'. In: *Computer Aided Geometric Design* 80 (June 2020), p. 101865. DOI: 10.1016/j.ca gd.2020.101865. URL: https://hal.inria.fr/hal-02985011.

[27] Z. Jelonek and A. Galligo. 'Elimination ideals and Bezout relations'. In: *Journal of Algebra* 562 (2020), pp. 621–626. DOI: 10.1016/j.jalgebra.2020.06.022. URL: https://hal.archives-ouvert es.fr/hal-03138363.

[28] X. Jia, C. Tu, B. Mourrain and W. Wang. 'Complete Classification and Efficient Determination of Arrangements Formed by Two Ellipsoids'. In: *ACM Transactions on Graphics* 39.3 (9th June 2020), pp. 1–12. DOI: 10.1145/3388540. URL: https://hal.inria.fr/hal-02927010.

[29] A. Limkilde, A. Evgrafov, J. Gravesen and A. Mantzaflaris. 'Practical isogeometric shape optimization: Parameterization by means of regularization'. In: *Journal of computational design and engineering* (14th Jan. 2021). DOI: 10.1093/jcde/qwaa093. URL: https://hal.inria.fr/hal-02958668.

[30] B. Mourrain and A. Oneto. 'On minimal decompositions of low rank symmetric tensors'. In: *Linear Algebra and its Applications* 607 (15th Dec. 2020), pp. 347–377. DOI: 10.1016/j.laa.2020.06.029. URL: https://hal.archives-ouvertes.fr/hal-01803571.

[31] M. Pan, B. Jüttler and A. Mantzaflaris. 'Efficient Matrix Assembly in Isogeometric Analysis with Hierarchical B-splines'. In: *Journal of Computational and Applied Mathematics* 390 (2020), p. 113278. DOI: 10.1016/j.cam.2020.113278. URL: https://hal.inria.fr/hal-02953341.

[32] E. Rodriguez Bazan and E. Hubert. 'Multivariate Interpolation: Preserving and Exploiting Symmetry'. In: *Journal of Symbolic Computation* (2021). URL: https://hal.inria.fr/hal-03123418.

[33] J. Xie, J. Xu, Z. Dong, G. Xu, C. Deng, B. Mourrain and Y. J. Zhang. 'Interpolatory Catmull-Clark volumetric subdivision over unstructured hexahedral meshes for modeling and simulation applications'. In: *Computer Aided Geometric Design* 80 (June 2020), p. 101867. DOI: 10.1016/j.cagd.202 0.101867. URL: https://hal.inria.fr/hal-02958501.

**International peer-reviewed conferences**

[34] V. Levandovskyy, T. Metzlaff and K. A. Zeid. 'Computation of free non-commutative Gröbner Bases over Z with Singular:Letterplace'. In: ISSAC 2020 - International Symposium on Symbolic and Algebraic Computation. Kalamata, Greece, 20th July 2020. DOI: 10.1145/3373207.3404052. URL: https://hal.archives-ouvertes.fr/hal-02496535.

[35] A. Mantzaflaris, B. Mourrain and A. Szanto. 'Punctual Hilbert Schemes and Certified Approximate Singularities'. In: ISSAC 2020 - International Symposium on Symbolic and Algebraic Computation. Kalamata, Greece, 20th July 2020. DOI: 10.1145/3373207.3404024. URL: https://hal.inria.fr/hal-02478768.

[36] E. Rodriguez Bazan and E. Hubert. 'Ideal Interpolation, H-Bases and Symmetry'. In: ISSAC 2020 - International Symposium on Symbolic and Algebraic Computation. Kalamata, Greece, 20th July 2020. DOI: 10.1145/3373207.3404057. URL: https://hal.inria.fr/hal-02482098.

**Scientific book chapters**

[37] A. Mantzaflaris. 'An overview of Geometry plus Simulation Modules'. In: *Mathematical Aspects of Computer and Information Sciences. MACIS 2019. Lecture Notes in Computer Science, vol 11989.* 18th Mar. 2020, pp. 453–456. DOI: 10.1007/978-3-030-43120-4_35. URL: https://hal.inria.fr/hal-02958849.

**Doctoral dissertations and habilitation theses**

[38] A. Blidia. 'Geometric continuity for surfaces and scalar fields'. Université Côte d'Azur, 2nd July 2020. URL: https://tel.archives-ouvertes.fr/tel-03135276.

[39] C. Laroche. 'Compact and efficient implicit representations'. Université d'Athènes, 30th Apr. 2020. URL: https://hal.inria.fr/tel-03117752.

[40] F. N. Yildirim. 'Finite fibers of rational maps by means of matrix representations with applications to distance problem'. Université Côte d'Azur, 3rd Feb. 2020. URL: https://tel.archives-ouvertes.fr/tel-03000586.

**Reports & preprints**

[41] L. Baldi and B. Mourrain. *Computing real radicals by moment optimization.* 18th Feb. 2021. URL: https://hal.archives-ouvertes.fr/hal-03145501.

[42] L. Baldi and B. Mourrain. *Exact Moment Representation in Polynomial Optimization.* 9th Feb. 2021. URL: https://hal.archives-ouvertes.fr/hal-03082531.

[43] L. Busé. *Formulas for the eigendiscriminants of ternary and quaternary forms.* 25th June 2020. URL: https://hal.inria.fr/hal-02881339.

[44] L. Busé, A. Dimca, H. Schenck and G. Sticlaru. *The Hessian polynomial and the Jacobian ideal of a reduced surface in* $\mathbb{P}^3$. 2020. URL: https://hal.inria.fr/hal-02337441.

[45] A. Chalkis, I. Emiris, V. Fisikopoulos, P. Repouskos and E. Tsigaridas. *Efficient Sampling from Feasible Sets of SDPs and Volume Approximation.* 8th Oct. 2020. URL: https://hal.inria.fr/hal-02953124.

[46] A. Dickenstein, M. I. Herrero and B. Mourrain. *Curve Valuations and Mixed Volumes in the Implicitization of Rational Varieties.* 5th Oct. 2020. URL: https://hal.inria.fr/hal-02958248.

[47] E. Hubert and M. F. Singer. *Sparse Interpolation in Terms of Multivariate Chebyshev Polynomials.* 24th Jan. 2020. URL: https://hal.inria.fr/hal-02454589.

[48] R. Khouja, H. Khalil and B. Mourrain. *A Riemannian Newton Optimization Framework for the Symmetric Tensor Rank Approximation Problem.* 12th July 2020. URL: https://hal.archives-ouvertes.fr/hal-02494172.

[49]  V. Levandovskyy, T. Metzlaff and K. A. Zeid. *Computing Free Non-commutative Groebner Bases over Z with Singular:Letterplace*. 15th Dec. 2020. URL: https://hal.archives-ouvertes.fr/hal-03085431.

[50]  A. Mantzaflaris, B. Mourrain and A. Szanto. *A certified iterative method for isolated singular roots*. 17th Dec. 2020. URL: https://hal.archives-ouvertes.fr/hal-03079910.

[51]  E. Rodriguez Bazan and E. Hubert. *Symmetry in Multivariate Ideal Interpolation*. 3rd Feb. 2021. URL: https://hal.inria.fr/hal-03129766.

[52]  H. M. Verhelst, M. M. Möller, H. J. Den Besten, A. Mantzaflaris and M. L. Kaminski. *Stretch-Based Hyperelastic Material Formulations for Isogeometric Kirchhoff-Love Shells*. 6th July 2020. URL: https://hal.archives-ouvertes.fr/hal-02890963.

[53]  N. Villamizar, A. Mantzaflaris and B. Jüttler. *Completeness characterization of Type-I box splines*. 5th Nov. 2020. URL: https://hal.inria.fr/hal-02991234.