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ACTIVITY REPORT

Project-Team

CAGE

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team CAGE

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Keywords

Computer sciences and digital sciences

A6. – Modeling, simulation and control

A6.1. – Methods in mathematical modeling

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.4. – Automatic control

A6.4.1. – Deterministic control

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Other research topics and application domains

B1.2. – Neuroscience and cognitive science

B1.2.1. – Understanding and simulation of the brain and the nervous system

B2. – Health

B2.6. – Biological and medical imaging

B5.11. – Quantum systems

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2 Overall objectives

CAGE's activities take place in the field of mathematical control theory, with applications in three main directions: geometric models for vision, control of quantum mechanical systems, and control of systems with uncertain dynamics.

The relations between control theory and geometry of vision rely on the notion of sub-Riemannian structure, a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful control theoretical interpretation. We recall that nonholonomicity refers to the property of a velocity constraint that cannot be recast as a state constraint. In the language of differential geometry, a sub-Riemannian structure is a (possibly rank-varying) Lie bracket generating distribution endowed with a smoothly varying norm.

Sub-Riemannian geometry, and in particular the theory of associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the recent literature (including by members of our team). Our contributions to this field are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

Control theory is one of the components of the forthcoming quantum revolution [1], since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance). The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. Time scales analysis is important for evaluation approaches based on adiabatic approximation theory, which is well-known to improve the robustness of the control strategy. CAGE works for the improvement of evaluation and design tools for efficient quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces.

Simultaneous control of a continuum of systems with slightly different dynamics is a typical problem in quantum mechanics and also a special case of the third applicative axis to which CAGE is contributing: control of systems with uncertain dynamics. The slightly different dynamics can indeed be seen as uncertainties in the system to be controlled, and simultaneous control rephrased in terms of a robustness task. Robustification, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. Our contributions to this research field concern both stabilization (either asymptotic or in finite time) and optimal control, where redundancies and probabilistic tools can be introduced to offset uncertainties.

3 Research program

3.1 Research domain

The activities of CAGE are part of the research in the wide area of control theory. This nowadays mature discipline is still the subject of intensive research because of its crucial role in a vast array of applications.

More specifically, our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential

[1] As anticipated by the recent launch of the FET Flagship on Quantum Technologies

equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [169]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [130]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [151, 131, 160]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [156], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [149] and models for neural activity [118]. Therapy analysis from the point of view of optimal control has also attracted a great attention [153].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [166].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [119].

3.2 Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, observability, optimal control... [79, 123]. The emphasis of such a geometric approach to control theory is put on intrinsic properties of the systems and it is particularly well adapted to study nonlinear and nonholonomic phenomena.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [105]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting from 2009 [108] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [164, 152]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [144, 134], those based on the differentiation of nonlinear flows such as the **return method** [112, 113], and those exploiting the **differential flatness** of the system [117].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [75] or shape optimization [87]. Examples of the second type are inactivation principles in human motricity [90] or neurogeometrical models for image representation of the primary visual cortex in mammals [102].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [142], geometric measure theory [81] and hypoelliptic operator theory [94].

The geometric control approach has historically been related to the development of finite-dimensional control theory. However, its impact in the analysis of distributed parameter control systems and in particular systems of controlled partial differential equations has been growing in the last decades, complementing analytical and numerical approaches, providing dynamical, qualitative and intrinsic insight [111]. CAGE's ambition is to be at the core of this development in the years to come.

4 Application domains

4.1 First axis: Geometry of vision

A suggestive application of sub-Riemannian geometry and in particular of hypoelliptic diffusion comes from a model of geometry of vision describing the functional architecture of the primary visual cortex V1. In 1958, Hubel and Wiesel (Nobel in 1981) observed that the visual cortex V1 is endowed with the so-called **pinwheel structure**, characterized by neurons grouped into orientation columns, that are sensible both to positions and directions [122]. The mathematical rephrasing of this discovery is that the visual cortex lifts an image from $\mathbf{R}^2$ into the bundle of directions of the plane [109, 148, 150, 121].

A simplified version of the model can be described as follows: neurons of V1 are grouped into orientation columns, each of them being sensitive to visual stimuli at a given point of the retina and for a given direction on it. The retina is modeled by the real plane, i.e., each point is represented by a pair $(x, y) \in \mathbb{R}^2$, while the directions at a given point are modeled by the projective line, i.e. an element θ of the projective line P^1. Hence, the primary visual cortex V1 is modeled by the so called projective tangent bundle $\mathrm{PTR}^2 = \mathbf{R}^2 \times \mathbf{P}^1$. From a neurological point of view, orientation columns are in turn grouped into hypercolumns, each of them being sensitive to stimuli at a given point (x, y) with any direction.

Orientation columns are connected between them in two different ways. The first kind of connections are the vertical (inhibitory) ones, which connect orientation columns belonging to the same hypercolumn and sensible to similar directions. The second kind of connections are the horizontal (excitatory) connections, which connect neurons belonging to different (but not too far) hypercolumns and sensible to the same directions. The resulting metric structure is sub-Riemannian and the model obtained in this way provides a convincing explanation in terms of sub-Riemannian geodesics of gestalt phenomena such as Kanizsa illusory contours.

The sub-Riemannian model for image representation of V1 has a great potential of yielding powerful bio-inspired image processing algorithms [116, 102]. Image inpainting, for instance, can be implemented by reconstructing an incomplete image by activating orientation columns in the missing regions in accordance with sub-Riemannian non-isotropic constraints. The process intrinsically defines an hypoelliptic heat equation on PTR^2 which can be integrated numerically using non-commutative Fourier analysis on a suitable semidiscretization of the group of roto-translations of the plane [97].

We have been working on the model and its software implementation since 2012. This work has been supported by several project, as the ERC starting grant GeCoMethods and the ERC Proof of Concept ARTIV1 of U. Boscain, and the ANR GCM.

A parallel approach that we will pursue and combine with this first one is based on **pattern matching in the group of diffeomorphisms**. We want to extend this approach, already explored in the Riemannian setting [165, 139], to the general sub-Riemannian framework. The paradigm of the approach is the following: consider a distortable object, more or less rigid, discretized into a certain number of points. One may track its distortion by considering the paths drawn by these points. One would however like to know how the object itself (and not its discretized version) has been distorted. The study in [165, 139] shed light on the importance of Riemannian geometry in this kind of problem. In particular, they study the Riemannian submersion obtained by making the group of diffeomorphisms act transitively on the manifold formed by the points of the discretization, minimizing a certain energy so as to take into account the whole object. Settled as such, the problem is Riemannian, but if one considers objects involving connections, or submitted to nonholonomic constraints, like in medical imaging where one tracks the motions of organs, then one comes up with a sub-Riemannian problem. The transitive group is then far bigger, and the aim is to lift curves submitted to these nonholonomic constraints into curves in the set of diffeomorphisms satisfying the corresponding constraints, in a unique way and minimizing an energy (giving rise to a sub-Riemannian structure).

4.2 Second axis: Quantum control

The goal of quantum control is to design efficient protocols for tuning the occupation probabilities of the energy levels of a system. This task is crucial in atomic and molecular physics, with applications ranging from photochemistry to nuclear magnetic resonance and quantum computing. A quantum system may be controlled by exciting it with one or several external fields, such as magnetic or electric fields. The goal of quantum control theory is to adapt the tools originally developed by control theory and to develop new specific strategies that tackle and exploit the features of quantum dynamics (probabilistic nature of wavefunctions and density operators, measure and wavefunction collapse, decoherence, ...). A rich variety of relevant models for controlled quantum dynamics exist, encompassing low-dimensional models (e.g., single-spin systems) and PDEs alike, with deterministic and stochastic components, making it a rich and exciting area of research in control theory.

The controllability of quantum system is a well-established topic when the state space is finite-dimensional [114], thanks to general controllability methods for left-invariant control systems on compact Lie groups [104, 124]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [167]. Nevertheless, weaker controllability properties, such as approximate controllability or controllability between eigenstates of the internal Hamiltonian (which are the most relevant physical states), may hold. In certain cases, when the state space is a function space on a 1D manifold, some rather precise description of the set of reachable states has been provided [88]. A similar description for higher-dimensional manifolds seems intractable and at the moment only approximate controllability results are available [140, 146, 125]. The most widely applicable tests for controllability of quantum systems in infinite-dimensional Hilbert spaces are based on the **Lie–Galerkin technique** [108, 96, 99]. They allow, in particular, to show that the controllability property is generic among this class of systems [137].

A family of algorithms which are specific to quantum systems are those based on adiabatic evolution [171, 170, 128]. The basic principle of adiabatic control is that the flow of a slowly varying Hamiltonian can be approximated (up to a phase factor) by a quasi-static evolution, with a precision proportional to the velocity of variation of the Hamiltonian. The advantage of the **adiabatic approach** is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty. The paradigm is based on the adiabatic perturbation theory developed in mathematical physics [95, 145, 163], where it plays an important role for understanding molecular dynamics. Approximation theory by adiabatic perturbation can be used to describe the evolution of the occupation probabilities of the energy levels of a slowly varying Hamiltonian. Results from the last 15 years, including those by members of our team [73, 101], have highlighted the effectiveness of control techniques based on adiabatic path following.

4.3 Third axis: Stability and uncertain dynamics

Switched and hybrid systems constitute a broad framework for the description of the heterogeneous aspects of systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [155], energy management [147] and congestion control [138].

Even if both controllability [159] and observability [126] of switched and hybrid systems have attracted much research efforts, the central role in their study is played by the problem of stability and stabilizability. The goal is to determine whether a dynamical or a control system whose evolution is influenced by a time-dependent signal is uniformly stable or can be uniformly stabilized [131, 160]. Uniformity is considered with respect to all signals in a given class. Stability of switched systems lead to several interesting phenomena. For example, even when all the subsystems corresponding to a constant switching law are exponentially stable, the switched systems may have divergent trajectories for certain switching signals [132]. This fact illustrates the fact that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the class of switching signals which is considered.

The most common class of switching signals which has been considered in the literature is made of all piecewise constant signals.

In this case uniform stability of the system is equivalent to the existence of a common quadratic

Lyapunov function [141]. Moreover, provided that the system has finitely many modes, the Lyapunov function can be taken polyhedral or polynomial [92, 93, 115]. A special role in the switched control literature has been played by common quadratic Lyapunov functions, since their existence can be tested rather efficiently (see the surveys [133, 154] and the references therein). It is known, however, that the existence of a common quadratic Lyapunov function is not necessary for the global uniform exponential stability of a linear switched system with finitely many modes. Moreover, there exists no uniform upper bound on the minimal degree of a common polynomial Lyapunov function [136]. More refined tools rely on multiple and non-monotone Lyapunov functions [103]. Let us also mention linear switched systems technics based on the analysis of the Lie algebra generated by the matrices corresponding to the modes of the system [78].

For systems evolving in the plane, more geometrical tests apply, and yield a complete characterization of the stability [98, 82]. Such a geometric approach also yields sufficient conditions for uniform stability in the linear planar case [100].

In many situations, it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Switching rules can also be imposed, for instance, by a timed automata. When constraints apply, the common Lyapunov function approach becomes conservative and new tools have to be developed to give more detailed characterizations of stable and unstable systems.

Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. One then looks at the corresponding Lyapunov exponents, whose existence is established by the multiplicative ergodic theorem. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [89, 110, 168].

4.4 Joint theoretical core

The theoretical questions raised by the different applicative area will be pooled in a research axis on the transversal aspects of geometric control theory and sub-Riemannian structures.

We recall that sub-Riemannian geometry is a generalization of Riemannian geometry, whose birth dates back to Carathéodory's seminal paper on the foundations of Carnot thermodynamics [106], followed by E. Cartan's address at the International Congress of Mathematicians in Bologna [107]. In the last twenty years, sub-Riemannian geometry has emerged as an independent research domain, with a variety of motivations and ramifications in several parts of pure and applied mathematics. Let us mention geometric analysis, geometric measure theory, stochastic calculus and evolution equations together with applications in mechanics and optimal control (motion planning, robotics, nonholonomic mechanics, quantum control) [83, 84].

One of the main open problems in sub-Riemannian geometry concerns the regularity of length-minimizers [77, 143]. Length-minimizers are solutions to a variational problem with constraints and satisfy a first-order necessary condition resulting from the Pontryagin Maximum Principle (PMP). Solutions of the PMP are either *normal* or *abnormal*. Normal length-minimizer are well-known to be smooth, i.e., C^∞, as it follows by the Hamiltonian nature of the PMP. The question of regularity is then reduced to abnormal length-minimizers. If the sub-Riemannian structure has step 2, then abnormal length-minimizers can be excluded and thus every length-minimizer is smooth. For step 3 structures, the situation is already more complicated and smoothness of length-minimizers is known only for Carnot groups [127, 162]. The question of regularity of length-minimizers is not restricted to the smoothness in the C^∞ sense. A recent result prove that length-minimizers, for sub-Riemannian structures of any step, cannot have corner-like singularities [120]. When the sub-Riemannian structure is analytic, more is known on the size of the set of points where a length-minimizer can lose analyticity [161], regardless of the rank and of the step of the distribution.

An interesting set of recent results in sub-Riemannian geometry concerns the extension to such a setting of the Riemannian notion of sectional curvature. The curvature operator can be introduced in terms of the symplectic invariants of the Jacobi curve [76, 129, 74], a curve in the Lagrange Grassmannian related to the linearization of the Hamiltonian flow. Alternative approaches to curvatures in metric spaces are based either on the associated heat equation and the generalization of the curvature-dimension inequality [85, 86] or on optimal transport and the generalization of Ricci curvature [158, 157, 135, 80].

5 Highlights of the year

5.1 Awards

The PhD thesis “Stabilisation de systèmes hyperboliques non-linéaires en dimension un d’espace” of our former PhD student Amaury Hayat received two prizes

- the 2019 European PhD Award by EECI for the best PhD thesis in Europe in the field of Control for Complex and Heterogeneous Systems;
- Prix Solennel des Chancelleries des universités de Paris : Prix en Sciences “toutes spécialités”.

6 New results

6.1 Geometry of vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In [16] we consider the evolution model proposed in [91] to describe illusory contrast perception phenomena induced by surrounding orientations. Firstly, we highlight its analogies and differences with widely used Wilson-Cowan equations, mainly in terms of efficient representation properties. Then, in order to explicitly encode local directional information, we exploit the model of the primary visual cortex V1 proposed in [109] and largely used over the last years for several image processing problems. The resulting model is capable to describe assimilation and contrast visual bias at the same time, the main novelty being its explicit dependence on local image orientation. We report several numerical tests showing the ability of the model to explain, in particular, orientation-dependent phenomena such as grating induction and a modified version of the Poggendorff illusion. For this latter example, we empirically show the existence of a set of threshold parameters differentiating from inpainting to perception-type reconstructions, describing long-range connectivity between different hypercolumns in the primary visual cortex.
- The article [34] adapts the framework of metamorphosis to the resolution of inverse problems with shape prior. The metamorphosis framework allows to transform an image via a balance between geometrical deformations and changes in intensities (that can for instance correspond to the appearance of a new structure). The idea developed here is to reconstruct an image from noisy and indirect observations by registering, via metamorphosis, a template to the observed data. Unlike a registration with only geometrical changes, this framework gives good results when intensities of the template are poorly chosen. We show that this method is a well-defined regularization method (proving existence, stability and convergence) and present several numerical examples.
- The reconstruction mechanisms built by the human auditory system during sound reconstruction are still a matter of debate. The purpose of [22, 38] is to propose a mathematical model of sound reconstruction based on the functional architecture of the auditory cortex (A1). The model is inspired by the geometrical modelling of vision, which has undergone a great development in the last ten years. The algorithm transforms the degraded sound in an image in the time-frequency domain via a short-time Fourier transform. Such an image is then lifted in the Heisenberg group (i.e., the celebrated Brockett integrator) and it is reconstructed via a Wilson-Cowan differo-integral equation. Numerical experiments are provided.

- Given a surface S in a 3D contact sub-Riemannian manifold M, we investigate in [46] the metric structure induced on S by M, in the sense of length spaces. First, we define a coefficient at characteristic points that determines locally the characteristic foliation of S. Next, we identify some global conditions for the induced distance to be finite. In particular, we prove that the induced distance is finite for surfaces with the topology of a sphere embedded in a tight coorientable distribution, with isolated characteristic points.
- In [54] we establish small-time asymptotic expansions for heat kernels of hypoelliptic Hörmander operators in a neighborhood of the diagonal, generalizing former results obtained in particular by Métivier and by Ben Arous. The coefficients of our expansions are identified in terms of the nilpotentization of the underlying sub-Riemannian structure. Our approach is purely analytic and relies in particular on local and global subelliptic estimates as well as on the local nature of small-time asymptotics of heat kernels. The fact that our expansions are valid not only along the diagonal but in an asymptotic neighborhood of the diagonal is the main novelty, useful in view of deriving Weyl laws for subelliptic Laplacians. In turn, we establish a number of other results on hypoelliptic heat kernels that are interesting in themselves, such as Kac's principle of not feeling the boundary, asymptotic results for singular perturbations of hypoelliptic operators, global smoothing properties for selfadjoint heat semigroups.
- In the survey paper [64] we report on recent works concerning exact observability (and, by duality, exact controllability) properties of subelliptic wave and Schrödinger-type equations. These results illustrate the slowdown of propagation in directions transverse to the horizontal distribution. The proofs combine sub-Riemannian geometry, semi-classical analysis, spectral theory and non-commutative harmonic analysis.
- In [65] we establish two results concerning the Quantum Limits (QLs) of some sub-Laplacians. First, under a commutativity assumption on the vector fields involved in the definition of the sub-Laplacian, we prove that it is possible to split any QL into several pieces which can be studied separately, and which come from well-characterized parts of the associated sequence of eigenfunctions. Secondly, building upon this result, we classify all QLs of a particular family of sub-Laplacians defined on products of compact quotients of Heisenberg groups. We express the QLs through a disintegration of measure result which follows from a natural spectral decomposition of the sub-Laplacian in which harmonic oscillators appear. Both results are based on the construction of an adequate elliptic operator commuting with the sub-Laplacian, and on the associated joint spectral calculus. They illustrate the fact that, because of the possibly high degeneracy of the spectrum, the spectral theory of sub-Laplacians can be very rich.
- In [59] we classify the self-adjoint realisations of the Laplace-Beltrami operator minimally defined on an infinite cylinder equipped with an incomplete Riemannian metric of Grushin type, in the non-trivial class of metrics yielding an infinite deficiency index. Such realisations are naturally interpreted as Hamiltonians governing the geometric confinement of a Schrödinger quantum particle away from the singularity, or the dynamical transmission across the singularity. In particular, we characterise all physically meaningful extensions qualified by explicit local boundary conditions at the singularity. Within our general classification we retrieve those distinguished extensions previously identified in the recent literature, namely the most confining and the most transmitting one.
- In [58] we study the isoperimetric problem for anisotropic left-invariant perimeter measures on $\mathbb{R}^3$, endowed with the Heisenberg group structure. The perimeter is associated with a left-invariant norm ϕ on the horizontal distribution. We first prove a representation formula for the ϕ-perimeter of regular sets and, assuming some regularity on ϕ and on its dual norm ϕ^*, we deduce a foliation property by sub-Finsler geodesics of C^2-smooth surfaces with constant ϕ-curvature. We then prove that the characteristic set of C^2-smooth surfaces that are locally extremal for the isoperimetric problem is made of isolated points and horizontal curves satisfying a suitable differential equation. Based on such a characterization, we characterize C^2-smooth ϕ-isoperimetric sets as the sub-Finsler analogue of Pansu's bubbles. We also show, under suitable regularity properties on ϕ, that such sub-Finsler candidate isoperimetric sets are indeed C^2-smooth. By an approximation

procedure, we finally prove a conditional minimality property for the candidate solutions in the general case (including the case where ϕ is crystalline).

- It is well-known that observability (and, by duality, controllability) of the elliptic wave equation, i.e., with a Riemannian Laplacian, in time T_0 is almost equivalent to the Geometric Control Condition (GCC), which stipulates that any geodesic ray meets the control set within time T_0. We show in [66] that in the subelliptic setting, GCC is never verified, and that subelliptic wave equations are never observable in finite time. More precisely, given any subelliptic Laplacian $\Delta = -\sum_{i=1}^m X_i^* X_i$ on a manifold M such that $\mathrm{Lie}(X_1,\ldots,X_m) = TM$ but $\mathrm{Span}(X_1,\ldots,X_m) \neq TM$, we show that for any $T_0 > 0$ and any measurable subset $\omega \subset M$ such that $M\backslash\omega$ has nonempty interior, the wave equation with subelliptic Laplacian Δ is not observable on ω in time T_0. The proof is based on the construction of sequences of solutions of the wave equation concentrating on spiraling geodesics (for the associated sub-Riemannian distance) spending a long time in $M\backslash\omega$. As a counterpart, we prove a positive result of observability for the wave equation in the Heisenberg group, where the observation set is a well-chosen part of the phase space.

- Because they reduce the search domains for geodesic paths in manifolds, sub-Riemannian methods can be used both as computational and modeling tools in designing distances between complex objects. [43] provides a review of the methods that have been recently introduced in this context to study shapes, with a special focus on shape spaces defined as homogeneous spaces under the action of diffeomorphisms. It describes sub-Riemannian methods that are based on control points, possibly enhanced with geometric information, and their generalization to deformation modules. It also discusses the introduction of implicit constraints on geodesic evolution, and the associated computational challenges. Several examples and numerical results are provided as illustrations.

- In [67], we study the observability (or, equivalently, the controllability) of some subelliptic evolution equations depending on their step. This sheds light on the speed of propagation of these equations, notably in the "degenerated directions" of the subelliptic structure. First, for any $\gamma \geq 1$, we establish a resolvent estimate for the Baouendi-Grushin-type operator $\Delta_\gamma = \partial_x^2 + |x|^{2\gamma}\partial_y^2$, which has step $\gamma+1$. We then derive consequences for the observability of the Schrödinger type equation $i\partial_t u - (-\Delta_\gamma)^s u = 0$ where $s \in N$. We identify three different cases: depending on the value of the ratio $(\gamma+1)/s$, observability may hold in arbitrarily small time, or only for sufficiently large times, or even fail for any time. As a corollary of our resolvent estimate, we also obtain observability for heat-type equations $\partial_t u + (-\Delta_\gamma)^s u = 0$ and establish a decay rate for the damped wave equation associated with Δ_γ.

- In [15] we prove the C^1 regularity for a class of abnormal length-minimizers in rank 2 sub-Riemannian structures. As a consequence of our result, all length-minimizers for rank 2 sub-Riemannian structures of step up to 4 are of class C^1

- In [57] we give necessary and sufficient conditions for the controllability of a Schrödinger equation involving a subelliptic operator on a compact manifold. This subelliptic operator is the sub-Laplacian of the manifold that is obtained by taking the quotient of a group of Heisenberg type by one of its discrete subgroups. This class of nilpotent Lie groups is a major example of stratified Lie groups of step 2. The sub-Laplacian involved in these Schrödinger equations is subelliptic, and, contrarily to what happens for the usual elliptic Schrödinger equation for example on flat tori or on negatively curved manifolds, there exists a minimal time of controllability. The main tools used in the proofs are (operator-valued) semi-classical measures constructed by use of representation theory and a notion of semi-classical wave packets that we introduce here in the context of groups of Heisenberg type.

- In [14] we are concerned with stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds. Employing the Riemannian approximations to the sub-Riemannian manifold which make use of the Reeb vector field, we obtain a second order partial differential operator on the surface arising as the limit of Laplace-Beltrami operators. The stochastic process associated with the limiting operator moves along the characteristic foliation induced on the surface by the contact distribution. We show that for this stochastic process elliptic characteristic points are inaccessible, while hyperbolic characteristic points are accessible from the separatrices. We illustrate the results with examples and we identify canonical surfaces in the Heisenberg group, and in SU(2) and SL(2,ℝ) equipped with the standard sub-Riemannian contact structures as model cases for this setting. Our techniques further allow us to derive an expression for an intrinsic Gaussian curvature of a surface in a general three-dimensional contact sub-Riemannian manifold.

- In [11] we show that, for a sub-Laplacian Δ on a 3-dimensional manifold M, no point interaction centered at a point $q_0 \in M$ exists.

- In [20] we consider a one-parameter family of Grushin-type singularities on surfaces, and discuss the possible diffusions that extend Brownian motion to the singularity. This gives a quick proof and clear intuition for the fact that heat can only cross the singularity for an intermediate range of the parameter. When crossing is possible and the singularity consists of one point, we give a complete description of these diffusions, and we describe a "best" extension, which respects the isometry group of the surface and also realizes the unique symmetric one-point extension of the Brownian motion, in the sense of Chen-Fukushima. This extension, however, does not correspond to the bridging extension, which was introduced by Boscain-Prandi, when they previously considered self-adjoint extensions of the Laplace-Beltrami operator on the Riemannian part for these surfaces. We clarify that several of the extensions they considered induce diffusions that are carried by the Marin compactification at the singularity, which is much larger than the (one-point) metric completion. In the case when the singularity is more than one-point, a complete classification of diffusions extending Brownian motion would be unwieldy. Nonetheless, we again describe a "best" extension which respects the isometry group, and in this case, this diffusion corresponds to the bridging extension. A prominent role is played by Bessel processes (of every real dimension) and the classical theory of one-dimensional diffusions and their boundary conditions.

- Analogous to the characterisation of Brownian motion on a Riemannian manifold as the development of Brownian motion on a Euclidean space, we construct in [50] sub-Riemannian diffusions on equinilpotentisable sub-Riemannian manifolds by developing a canonical stochastic process arising as the lift of Brownian motion to an associated model space. The notion of stochastic development we introduce for equinilpotentisable sub-Riemannian manifolds uses Cartan connections, which take the place of the Levi–Civita connection in Riemannian geometry. We first derive a general expression for the generator of the stochastic process which is the stochastic development with respect to a Cartan connection of the lift of Brownian motion to the model space. We further provide a necessary and sufficient condition for the existence of a Cartan connection which develops the canonical stochastic process to the sub-Riemannian diffusion associated with the sub-Laplacian defined with respect to the Popp volume. We illustrate the construction of a suitable Cartan connection for free sub-Riemannian structures with two generators and we discuss an example where the condition is not satisfied.

- Two-dimension almost-Riemannian structures of step 2 are natural generalizations of the Grushin plane. They are generalized Riemannian structures for which the vectors of a local orthonormal frame can become parallel. Under the 2-step assumption the singular set Z, where the structure is not Riemannian, is a 1D embedded submanifold. While approaching the singular set, all Riemannian quantities diverge. A remarkable property of these structure is that the geodesics can cross the singular set without singularities, but the heat and the solution of the Schrödinger equation (with the Laplace-Beltrami operator Δ) cannot. This is due to the fact that (under a natural compactness hypothesis), the Laplace-Beltrami operator is essentially self-adjoint on a connected component of the manifold without the singular set. In the literature such phenomenon is called quantum confinement. In [49] we study the self-adjointness of the curvature Laplacian, namely $-\frac{1}{2}\Delta + cK$, for $c > 0$ (here K is the Gaussian curvature), which originates in coordinate free quantization procedures (as for instance in path-integral or covariant Weyl quantization). We prove that there is no quantum confinement for this type of operators.

We would also like to mention the monograph [42], which was finally published in 2020.

6.2 Quantum control: new results

Let us list here our new results in quantum control theory.

- In [37] we study the controllability properties of the quantum rotational dynamics of a 3D symmetric molecule, with electric dipole moment not collinear to the symmetry axis of the molecule (that is, an accidentally symmetric-top). We control the dynamics with three orthogonally polarized electric fields. When the dipole has a nonzero component along the symmetry axis, it is known that the dynamics is approximately controllable. We focus here our attention to the case where the dipole moment and the symmetry axis are orthogonal (that is, an orthogonal accidentally symmetric-top), providing a description of the reachable sets.

- In [45] we study up to which extent we can apply adiabatic control strategies to a quantum control model obtained by rotating wave approximation. In particular, we show that, under suitable assumptions on the asymptotic regime between the parameters characterizing the rotating wave and the adiabatic approximations, the induced flow converges to the one obtained by considering the two approximations separately and by combining them formally in cascade. As a consequence, we propose explicit control laws which can be used to induce desired populations transfers, robustly with respect to parameter dispersions in the controlled Hamiltonian.

- In [13] we study one-parametric perturbations of finite dimensional real Hamiltonians depending on two controls, and we show that generically in the space of Hamiltonians, conical intersections of eigenvalues can degenerate into semi-conical intersections of eigenvalues. Then, through the use of normal forms, we study the problem of ensemble controllability between the eigenstates of a generic Hamiltonian.

- In [21] we study the controllability problem for a symmetric-top molecule, both for its classical and quantum rotational dynamics. As controlled fields we consider three orthogonally polarized electric fields which interact with the electric dipole of the molecule. We characterize the controllability in terms of the dipole position: when it lies along the symmetry axis of the molecule nor the classical neither the quantum dynamics are controllable, due to the presence of a conserved quantity, the third component of the total angular momentum; when it lies in the orthogonal plane to the symmetry axis, a quantum symmetry arises, due to the superposition of symmetric states, which as no classical counterpart. If the dipole is neither along the symmetry axis nor orthogonal to it, controllability for the classical dynamics and approximate controllability for the quantum dynamics is proved to hold.

- In the physics literature it is common to see the rotating wave approximation and the adiabatic approximation used “in cascade” to justify the use of chirped pulses for two-level quantum systems driven by one external field, in particular when the resonance frequency of the system is not known precisely. Both approximations need relatively long time and are essentially based on averaging theory of dynamical systems. Unfortunately, the two approximations cannot be done independently since, in a sense, the two time scales interact. The purpose of [70] is to study how the cascade of the two approximations can be justified and how large becomes the final time as the fidelity goes to one, while preserving the robustness of the adiabatic strategy. Our main result gives a precise quantification of the uncertainty interval of the resonance frequency for which the strategy works.

- In [62] we prove complete controllability for rotational states of an asymmetric top molecule belonging to degenerate values of the orientational quantum number M. Based on this insight, we construct a pulse sequence that energetically separates population initially distributed over degenerate M-states, as a precursor for orientational purification. Introducing the concept of enantio-selective controllability, we determine the conditions for complete enantiomer-specific population transfer in chiral molecules and construct pulse sequences realizing this transfer for population initially distributed over degenerate M-states. This degeneracy presently limits enantiomer-selectivity for any initial state except the rotational ground state. Our work thus shows how to overcome an important obstacle towards separating, with electric fields only, left-handed from right-handed molecules in a racemic mixture.

- Optimal Control Theory has become a widely used method to improve process performance in quantum technologies by means of highly efficient control of quantum dynamics. Our review [51] aims at providing an introduction to key concepts of optimal control theory, accessible to physicists and engineers working in quantum control or in related fields. The different mathematical results are introduced intuitively, before being rigorously stated. The review describes modern aspects of optimal control theory, with a particular focus on the Pontryagin Maximum Principle, which is the main tool for determining open-loop control laws without experimental feedback. The different steps to solve an optimal control problem are discussed, before moving on to more advanced topics such as the existence of optimal solutions or the definition of the different types of extremals, namely normal, abnormal, and singular. The review covers various quantum control issues and describes their mathematical formulation suitable for optimal control. The optimal solution of different low-dimensional quantum systems is presented in detail, illustrating how the mathematical tools are applied in a practical way.

6.3 Stability and uncertain dynamics: new results

Let us list here our new results about stability and stabilization of control systems, on the properties of systems with uncertain dynamics.

- In [29] we consider the finite-time stabilization of homogeneous quasilinear hyperbolic systems with one side controls and with nonlinear boundary condition at the other side. We present time-independent feedbacks leading to the finite-time stabilization in any time larger than the optimal time for the null controllability of the linearized system if the initial condition is sufficiently small. One of the key technical points is to establish the local well-posedness of quasilinear hyperbolic systems with nonlinear, non-local boundary conditions.

- In [26] we are concerned with the design of Model Predictive Control (MPC) schemes such that asymptotic stability of the resulting closed loop is guaranteed even if the linearization at the desired set point fails to be stabilizable. Therefore, we propose to construct the stage cost based on the homogeneous approximation and rigorously show that applying MPC yields an asymptotically stable closed-loop behavior if the homogeneous approximation is asymptotically null controllable. To this end, we verify cost controllability – a condition relating the current state, the stage cost, and the growth of the value function w.r.t. time – for this class of systems in order to provide stability and performance guarantees for the proposed MPC scheme without stabilizing terminal costs or constraints.

- In [48] we give sufficient conditions for Input-to-State Stability in C^1 norm of general quasilinear hyperbolic systems with boundary input disturbances. In particular the derivation of explicit Input-to-State Stability conditions is discussed for the special case of 2×2 systems.

- Because they represent physical systems with propagation delays, hyperbolic systems are well suited for feedforward control. This is especially true when the delay between a disturbance and the output is larger than the control delay. In [47] we address the design of feedforward controllers for a general class of 2×2 hyperbolic systems with a single disturbance input located at one boundary and a single control actuation at the other boundary. The goal is to design a feedforward control that makes the system output insensitive to the measured disturbance input. We show that, for this class of systems, there exists an efficient ideal feedforward controller which is causal and stable. The problem is first stated and studied in the frequency domain for a simple linear system. Then, our main contribution is to show how the theory can be extended, in the time domain, to general nonlinear hyperbolic systems. The method is illustrated with an application to the control of an open channel represented by Saint-Venant equations where the objective is to make the output water level insensitive to the variations of the input flow rate. Finally, we address a more complex application to a cascade of pools where a blind application of perfect feedforward control can lead to detrimental oscillations. A pragmatic way of modifying the control law to solve this problem is proposed and validated with a simulation experiment.

- In [27] we are interested in the boundary stabilization in finite time of one-dimensional linear hyperbolic balance laws with coefficients depending on time and space. We extend the so called "backstepping method" by introducing appropriate time-dependent integral transformations in order to map our initial system to a new one which has desired stability properties. The kernels of the integral transformations involved are solutions to nonstandard multi-dimensional hyperbolic PDEs, where the time dependence introduces several new difficulties in the treatment of their well-posedness. This work generalizes previous results of the literature, where only time-independent systems were considered.

- Partial stability characterizes dynamical systems for which only a part of the state variables exhibits a stable behavior. In his book on partial stability, Vorotnikov proposed a sufficient condition to establish this property through a Lyapunov-like function whose total derivative is upper-bounded by a negative definite function involving only the sub-state of interest. In [35], we show with a simple two-dimensional system that this statement is wrong in general. More precisely, we show that the convergence rate of the relevant state variables may not be uniform in the initial state. We also discuss the impact of this lack of uniformity on the connected issue of robustness with respect to exogenous disturbances.

- The paper [68] is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional semilinear wave equation. The control input is selected as the right Neumann trace. The control design goes as follows. First, a preliminary (classical) velocity feedback is applied in order to shift all but a finite number of the eivenvalues of the underlying unbounded operator into the open left half-plane. We then leverage on the projection of the system trajectories into an adequate Riesz basis to obtain a truncated model of the system capturing the remaining unstable modes. Local stability of the resulting closed-loop infinite-dimensional system composed of the semilinear wave equation, the preliminary velocity feedback, and the PI controller, is obtained through the study of an adequate Lyapunov function. Finally, an estimate assessing the set point tracking performance of the left Neumann trace is derived.

- Given a linear control system in a Hilbert space with a bounded control operator, we establish in [36] a characterization of exponential stabilizability in terms of an observability inequality. Such dual characterizations are well known for exact (null) controllability. Our approach exploits classical Fenchel duality arguments and, in turn, leads to characterizations in terms of observability inequalities of approximately null controllability and of α-null controllability. We comment on the relationships between those various concepts, at the light of the observability inequalities that characterize them.

- In [18] we propose an extension of the theory of control sets to the case of inputs satisfying a dwell-time constraint. Although the class of such inputs is not closed under concatenation, we propose a suitably modified definition of control sets that allows to recover some important properties known in the concatenable case. In particular we apply the control set construction to dwell-time linear switched systems, characterizing their maximal Lyapunov exponent looking only at trajectories whose angular component is periodic. We also use such a construction to characterize supports of invariant measures for random switched systems with dwell-time constraints.

- Given a discrete-time linear switched system associated with a finite set of matrices, we consider the measures of its asymptotic behavior given by, on the one hand, its deterministic joint spectral radius and, on the other hand, its probabilistic joint spectral radius for Markov random switching signals with given transition matrix and corresponding invariant probability. In [25], we investigate the cases of equality between the two measures.

- The general context of [32] is the feedback control of an infinite-dimensional system so that the closed-loop system satisfies a fading-memory property and achieves the setpoint tracking of a given reference signal. More specifically , this paper is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional reaction-diffusion equation with a delayed right Dirichlet boundary control. In this setting, the studied reaction-diffusion equation might be either open-loop stable or unstable. The proposed control strategy goes as follows. First, a finite-dimensional truncated model that captures the unstable dynamics of the original infinite-dimensional system is obtained via spectral decomposition. The truncated model is then augmented by an integral component on the tracking error of the left Neumann trace. After resorting to the Artstein transformation to handle the control input delay, the PI controller is designed by pole shifting. Stability of the resulting closed-loop infinite-dimensional system, consisting of the original reaction-diffusion equation with the PI controller, is then established thanks to an adequate Lyapunov function. In the case of a time-varying reference input and a time-varying distributed disturbance, our stability result takes the form of an exponential Input-to-State Stability (ISS) estimate with fading memory. Finally, another exponential ISS estimate with fading memory is established for the tracking performance of the reference signal by the system output. In particular, these results assess the setpoint regulation of the left Neumann trace in the presence of distributed perturbations that converge to a steady-state value and with a time-derivative that converges to zero. Numerical simulations are carried out to illustrate the efficiency of our control strategy.

- In [72] we use the backstepping method to study the stabilization of a 1-D linear transport equation on the interval $(0, L)$, by controlling the scalar amplitude of a piecewise regular function of the space variable in the source term. We prove that if the system is controllable in a periodic Sobolev space of order greater than 1, then the system can be stabilized exponentially in that space and, for any given decay rate, we give an explicit feedback law that achieves that decay rate.
- In [39], we discuss our recent works on the null-controllability, the exact controllability, and the stabilization of linear hyperbolic systems in one dimensional space using boundary controls on one side for the optimal time. Under precise and generic assumptions on the boundary conditions on the other side, we first obtain the optimal time for the null and the exact controllability for these systems for a generic source term. We then prove the null-controllability and the exact controllability for any time greater than the optimal time and for any source term. Finally, for homogeneous systems, we design feedbacks which stabilize the systems and bring them to the zero state at the optimal time. Extensions for the non-linear homogeneous system are also discussed.
- Hyperbolic systems in one dimensional space are frequently used in modeling of many physical systems. In our recent works, we introduced time independent feedbacks leading to the finite stabilization for the optimal time of homogeneous linear and quasilinear hyperbolic systems. In [56] we present Lyapunov's functions for these feedbacks and use estimates for Lyapunov's functions to rediscover the finite stabilization results.
- In [60] we deal with infinite-dimensional nonlinear forward complete dynamical systems which are subject to external disturbances. We first extend the well-known Datko lemma to the framework of the considered class of systems. Thanks to this generalization, we provide characterizations of the uniform (with respect to disturbances) local, semi-global, and global exponential stability, through the existence of coercive and non-coercive Lyapunov functionals. The importance of the obtained results is underlined through some applications concerning 1) exponential stability of nonlinear retarded systems with piecewise constant delays, 2) exponential stability preservation under sampling for semilinear control switching systems, and 3) the link between input-to-state stability and exponential stability of semilinear switching systems.

6.4 Controllability: new results

Let us list here our new results on controllability beyond the quantum control framework.

- The paper [55] is devoted to the local null-controllability of the nonlinear KdV equation equipped the Dirichlet boundary conditions using the Neumann boundary control on the right. Rosier proved that this KdV system is small-time locally controllable for all non-critical lengths and that the uncontrollable space of the linearized system is of finite dimension when the length is critical. Concerning critical lengths, Coron and Crépeau showed that the same result holds when the uncontrollable space of the linearized system is of dimension 1, and later Cerpa, and then Cerpa and Crépeau established that the local controllability holds at a finite time for all other critical lengths. In this paper, we prove that, for a class of critical lengths, the nonlinear KdV system is *not* small-time locally controllable.

- Under a regularity assumption we prove in [52] that reachability in fixed time for nonlinear control systems is robust under control sampling.

- In [28], we investigate the small-time global exact controllability of the Navier-Stokes equation , both towards the null equilibrium state and towards weak trajectories. We consider a viscous incompressible fluid evolving within a smooth bounded domain, either in 2D or in 3D. The controls are only located on a small part of the boundary, intersecting all its connected components. On the remaining parts of the boundary, the fluid obeys a Navier slip-with-friction boundary condition. Even though viscous boundary layers appear near these uncontrolled boundaries, we prove that small-time global exact controllability holds. Our analysis relies on the controllability of the Euler equation combined with asymptotic boundary layer expansions. Choosing the boundary controls with care enables us to guarantee good dissipation properties for the residual boundary layers, which can then be exactly canceled using local techniques.

- In [24], we study approximate and exact controllability of linear difference equations using as a basic tool a representation formula for its solution in terms of the initial condition, the control, and some suitable matrix coefficients. When the delays are commensurable, approximate and exact controllability are equivalent and can be characterized by a Kalman criterion. The paper focuses on providing characterizations of approximate and exact controllability without the commensurability assumption. In the case of two-dimensional systems with two delays, we obtain an explicit characterization of approximate and exact controllability in terms of the parameters of the problem. In the general setting, we prove that approximate controllability from zero to constant states is equivalent to approximate controllability in L^2. The corresponding result for exact controllability is true at least for two-dimensional systems with two delays.

- It has been proved by Zuazua in the nineties that the internally controlled semilinear 1D wave equation $\partial_{tt} y - \partial_{xx} y + g(y) = f 1_\omega$, with Dirichlet boundary conditions, is exactly controllable in $H_0^1(0,1) \cap L^2(0,1)$ with controls $f \in L^2((0,1)\times(0,T))$, for any $T > 0$ and any nonempty open subset ω of $(0,1)$, assuming that $g \in C^1(\mathbb{R})$ does not grow faster than $\beta|x|\ln^2|x|$ at infinity for some $\beta > 0$ small enough. The proof, based on the Leray-Schauder fixed point theorem, is however not constructive. In [69], we design a constructive proof and algorithm for the exact controllability of semilinear 1D wave equations.

- Our goal in [63] is to relate the observation (or control) of the wave equation on observation domains which evolve in time with some dynamical properties of the geodesic flow. In comparison to the case of static domains of observation, we show that the observability of the wave equation in any dimension of space can be improved by allowing the domain of observation to move.

- In [33] we consider the controllability problem for finite-dimensional linear autonomous control systems with nonnegative controls. Despite the Kalman condition, the unilateral nonnegativity control constraint may cause a positive minimal controllability time. When this happens, we prove that, if the matrix of the system has a real eigenvalue, then there is a minimal time control in the space of Radon measures, which consists of a finite sum of Dirac impulses. When all eigenvalues are real, this control is unique and the number of impulses is less than half the dimension of the space. We also focus on the control system corresponding to a finite-difference spatial discretization of the one-dimensional heat equation with Dirichlet boundary controls, and we provide numerical simulations.

6.5 Optimal control: new results

Let us list here our new results in optimal control theory beyond quantum control and the sub-Riemannian framework.

- In [12] we introduce a new optimal control model which encompasses pace optimization and motor control effort for a runner on a fixed distance. The system couples mechanics, energetics, neural drive to an economic decision theory of cost and benefit. We find how effort is minimized to produce the best running strategy, in particular in the bend. This allows us to discriminate between different types of tracks and estimate the discrepancy between lanes. Relating this model to the optimal path problem called the Dubins path, we are able to determine the geometry of the optimal track and estimate record times.

- Our aim in [44] is to present a new model which encompasses pace optimization and motor control effort for a runner on a fixed distance. We see that for long races, the long term behaviour is well approximated by a turnpike problem. We provide numerical simulations quite consistent with this approximation which leads to a simplified problem. We are also able to estimate the effect of slopes and ramps.

- In [30] we study a two-sided space-time L^1 optimization problem and show how to reformulate the problem within the framework of optimal control theory for polynomial systems. This yields insight on the structure of the optimal solution. We prove existence and uniqueness of the optimal solution, and we characterize it by means of the Pontryagin maximum principle. The cost function and the control converge when the polynomial degree tends to $+\infty$. We illustrate the theory with numerical simulations, which show that our optimal control interpretation leads to efficient algorithms.

- In [17] we study how bad can be the singularities of a time-optimal trajectory of a generic control affine system. In the case where the control is scalar and belongs to a closed interval it was recently shown that singularities cannot be, generically, worse than finite order accumulations of Fuller points, with order of accumulation lower than a bound depending only on the dimension of the manifold where the system is set. We extend here such a result to the case where the control has an even number of scalar components and belongs to a closed ball.

- In [19] we develop a geometric analysis and a numerical algorithm, based on indirect methods, to solve optimal guidance of endo-atmospheric launch vehicle systems under mixed control-state constraints. Two main difficulties are addressed. First, we tackle the presence of Euler singularities by introducing a representation of the configuration manifold in appropriate local charts. In these local coordinates, not only the problem is free from Euler singularities but also it can be recast as an optimal control problem with only pure control constraints. The second issue concerns the initialization of the shooting method. We introduce a strategy which combines indirect methods with homotopies, thus providing high accuracy. We illustrate the efficiency of our approach by numerical simulations on missile interception problems under challenging scenarios.

- We introduce and study in [31] the turnpike property for time-varying shapes, within the viewpoint of optimal control. We focus here on second-order linear parabolic equations where the shape acts as a source term and we seek the optimal time-varying shape that minimizes a quadratic criterion. We first establish existence of optimal solutions under some appropriate sufficient conditions. We then provide necessary conditions for optimality in terms of adjoint equations and, using the concept of strict dissipativity, we prove that state and adjoint satisfy the measure-turnpike property, meaning that the extremal time-varying solution remains essentially close to the optimal solution of an associated static problem. We show that the optimal shape enjoys the exponential turnpike property in term of Hausdorff distance for a Mayer quadratic cost. We illustrate the turnpike phenomenon in optimal shape design with several numerical simulations.

- The work [61] proposes a new approach to optimize the consumption of a hybrid electric vehicle taking into account the traffic conditions. The method is based on a bi-level decomposition in order to make the implementation suitable for online use. The offline lower level computes cost maps thanks to a stochastic optimization that considers the influence of traffic, in terms of speed/acceleration probability distributions. At the online upper level, a deterministic optimization computes the ideal state of charge at the end of each road segment, using the computed cost maps. Since the high computational cost due to the uncertainty of traffic conditions has been managed at the lower level, the upper level is fast enough to be used online in the vehicle. Errors due to discretization and computation in the proposed algorithm have been studied. Finally, we present numerical simulations using actual traffic data, and compare the proposed bi-level method to a deterministic optimization with perfect information about traffic conditions. The solutions show a reasonable over-consumption compared with deterministic optimization, and manageable computational times for both the offline and online parts.

- In [23] we revisit and extend the Riccati theory, unifying continuous-time linear-quadratic optimal permanent and sampled-data control problems, in finite and infinite time horizons. In a nutshell, we prove that: – when the time horizon T tends to $+\infty$, one passes from the Sampled-Data Difference Riccati Equation (SD-DRE) to the Sampled-Data Algebraic Riccati Equation (SD-ARE), and from the Permanent Differential Riccati Equation (P-DRE) to the Permanent Algebraic Riccati Equation (P-ARE); – when the maximal step of the time partition Δ tends to 0, one passes from (SD-DRE) to (P-DRE), and from (SD-ARE) to (P-ARE). Our notations and analysis provide a unified framework in order to settle all corresponding results.

- The turnpike phenomenon stipulates that the solution of an optimal control problem in large time, remains essentially close to a steady-state of the dynamics, itself being the optimal solution of an associated static optimal control problem. Under general assumptions, it is known that not only the optimal state and the optimal control, but also the adjoint state coming from the application of the Pontryagin maximum principle, are exponentially close to a steady-state, except at the beginning and at the end of the time frame. In such results, the turnpike set is a singleton, which is a steady-state. In [71] we establish a turnpike result for finite-dimensional optimal control problems in which some of the coordinates evolve in a monotone way, and some others are partial steady-states of the dynamics. We prove that the discrepancy between the optimal trajectory and the turnpike set is then linear, but not exponential: we thus speak of a linear turnpike theorem.

- An extension of the bi-level optimization for the energy management of hybrid electric vehicles (HEVs) proposed in [61] to the eco-routing problem is presented in [40]. Using the knowledge of traffic conditions over the entire road network, we search both the optimal path and state of charge trajectory. This problem results in finding the shortest path on a weighted graph whose nodes are (position, state of charge) pairs for the vehicle, the edge cost being evaluated thanks to the cost maps from optimization at the 'micro' level of a bi-level decomposition. The error due to the discretization of the state of charge is proven to be linear if the cost maps are Lipschitz. The classical A^* algorithm is used to solve the problem, with a heuristic based on a lower bound of the energy needed to complete the travel. The eco-routing method is validated by numerical simulations and compared to the fastest path on a synthetic road network.

- In [41] we study a driftless system on a three-dimensional manifold driven by two scalar controls. We assume that each scalar control has an independent bound on its modulus and we prove that, locally around every point where the controlled vector fields satisfy some suitable nondegeneracy Lie bracket condition, every time-optimal trajectory has at most five bang or singular arcs. The result is obtained using first-and second-order necessary conditions for optimality.

- In [53] we deal with the problem of approximating a scalar conservation law by a conservation law with nonlocal flux. As convolution kernel in the nonlocal flux, we consider an exponential-type approximation of the Dirac distribution. This enables us to obtain a total variation bound on the nonlocal term. By using this, we prove that the (unique) weak solution of the nonlocal problem converges strongly in $C(L^1_{\mathrm{loc}})$ to the entropy solution of the local conservation law. We conclude with several numerical illustrations which underline the main results and, in particular, the difference between the solution and the nonlocal term.

7 Bilateral contracts and grants with industry

7.1 Bilateral contracts with industry

Contract CIFRE with ArianeGroup (les Mureaux), 2019–2021, funding the thesis of A. Nayet. Participants : M. Cerf (ArianeGroup), E. Trélat (coordinator).

A new contract is being signed by E. Trélat with MBDA.

7.2 Bilateral grants with industry

New grant by AFOSR (Air Force Office of Scientific Research), 2020–2023. Participants : Mohab Safey El Din (LIP6), E. Trélat.

8 Partnerships and cooperations

8.1 International research visitors

Andrei Agrachev (SISSA, Italy) was expected to start his Inria International Chair in 2020. However, due to Covid-related restrictions, he had to cancel his visit scheduled for the Fall. He will start his Chair spending two months in Paris in Fall 2021.

8.1.1 Visits of international scientists

Hoai-Minh Nguyen spend two months visitng CAGE (October and December), working in particular with Jean-Michel Coron.

8.1.2 Visits to international teams

Ugo Boscain visited SISSA in January 2020.

8.2 European initiatives

8.2.1 FP7 & H2020 Projects

Program: H2020-EU.1.3.1. - Fostering new skills by means of excellent initial training of researchers

Call for proposal: MSCA-ITN-2017 - Innovative Training Networks

Project acronym: QUSCO

Project title: Quantum-enhanced Sensing via Quantum Control

Duration: From November 2017 to October 2021.

Coordinator: Christiane Koch

Coordinator for the participant Inria: Ugo Boscain

Abstract: Quantum technologies aim to exploit quantum coherence and entanglement, the two essential elements of quantum physics. Successful implementation of quantum technologies faces the challenge to preserve the relevant nonclassical features at the level of device operation. It is thus deeply linked to the ability to control open quantum systems. The currently closest to market quantum technologies are quantum communication and quantum sensing. The latter holds the promise of reaching unprecedented sensitivity, with the potential to revolutionize medical imaging or structure determination in biology or the controlled construction of novel quantum materials. Quantum control manipulates dynamical processes at the atomic or molecular scale by means of specially tailored external electromagnetic fields. The purpose of QuSCo is to demonstrate the enabling capability of quantum control for quantum sensing and quantum measurement, advancing this field by systematic use of quantum control methods. QuSCo will establish quantum control as a vital part for progress in quantum technologies. QuSCo will expose its students, at the same time, to fundamental questions of quantum mechanics and practical issues of specific applications. Albeit challenging, this reflects our view of the best possible training that the field of quantum technologies can offer. Training in scientific skills is based on the demonstrated tradition of excellence in research of the consortium. It will be complemented by training in communication and commercialization. The latter builds on strong industry participation whereas the former existing expertise on visualization and gamification and combines it with more traditional means of outreach to realize target audience specific public engagement strategies.

8.3 National initiatives

The Inria Exploratory Action “StellaCage” is supporting since Spring 2020 a collaboration between CAGE, Yannick Privat (Inria team TONUS), and the startup Renaissance Fusion, based in Grenoble (Francesco Volpe, CEO & Chris Smiet, CSO).

StellaCage approaches the problem of designing better stellarators (yielding better confinement, with simpler coils, capable of higher fields) by combining geometrical properties of magnetic field lines from the control perspective with shape optimization techniques.

8.3.1 ANR

- ANR SRGI, for *Sub-Riemannian Geometry and Interactions*, coordinated by **Emmanuel Trélat**, started in 2015 and runs until early 2021, thanks to a 6 months extension related to the Covid pandemic. Other partners: Toulon University and Grenoble University. SRGI deals with sub-Riemannian geometry, hypoelliptic diffusion and geometric control.
- ANR TRECOS, for *New Trends in Control and Stabilization: Constraints and non-local terms*, coordinated by Sylvain Ervedoza, University of Bordeaux. The ANR started in 2021 and runs up to 2024. TRECOS’ focus is on control theory for partial differential equations, and in particular models from ecology and biology.
- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and runs until 2021. Other partners: Burgundy University. QUACO aims at contributing to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improve the efficiency of control algorithms for MRI.

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

Member of the organizing committees Emmanuel Trélat was the main organizer of the *SRGI conference: Sub-Riemannian Geometry and Interactions* which was held in Paris in September 2020. Ugo Boscain was also in the Organizing Committe.

9.1.2 Journal

Member of the editorial boards

- Ugo Boscain is Associate editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Member of the editorial board of Journal of Evolution Equations
- Jean-Michel Coron is Member of the editorial board of Asymptotic Analysis
- Jean-Michel Coron is Member of the editorial board of ESAIM : Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Member of the editorial board of Applied Mathematics Research Express
- Jean-Michel Coron is Member of the editorial board of Advances in Differential Equations
- Jean-Michel Coron is Member of the editorial board of Math. Control Signals Systems

- Jean-Michel Coron is Member of the editorial board of Annales de l'IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of ESAIM : Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Editor-in-chief of ESAIM : Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Syst. Cont. Letters
- Emmanuel Trélat is Associate editor of J. Dynam. Cont. Syst.
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM Math. Modelling Num. Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of J. Optim. Theory Appl.
- Emmanuel Trélat is Associate editor of Math. Control Related Fields
- Emmanuel Trélat is Associate editor of Mathematics of Control, Signals, and Systems (MCSS)
- Emmanuel Trélat is Associate editor of Optimal Control Applications and Methods (OCAM)

9.1.3 Invited talks

- Ugo Boscain was invited speaker at the Max-Planck Institute for Quantum Optics. Munich (Germany), February.
- Ugo Boscain was invited speaker at the Workshop "Analysis, Control, and Learning of Dynamic Ensemble and Population Systems" at the 21st IFAC World Congress 2020, Berlin (virtual), July.
- Mario Sigalotti was invited speaker at the SRGI conference: Sub-Riemannian Geometry and Interactions, September.
- Mario Sigalotti was invited speaker at the Moscow seminar on optimal control geometric theory (virtual), June.
- Emmanuel Trélat was invited speaker at Colloquium of the University of Erlangen, Germany, January.
- Emmanuel Trélat was invited speaker at Conference "Spectral theory and geometry", Chaleès, September.
- Emmanuel Trélat was invited speaker at the Opening Workshop of the GdR Sport, January.
- Emmanuel Trélat was invited speaker at the "Séminaire Analyse numérique – Equations aux dérivées partielles", Lille, September.
- Emmanuel Trélat was invited speaker at the India PDE webinar (virtual), September.
- Emmanuel Trélat was invited speaker at SMU, Texas (virtual), May.

9.1.4 Research administration

Emmanuel Trélat is Head of the Laboratoire Jacques-Louis Lions (LJLL).

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Ugo Boscain thought “Automatic Control” (with Mazyar Mirrahimi) at Ecole Polytechnique
- Ugo Boscain thought “MODAL of applied mathematics. Contrôle de modèles dynamiques” at Ecole Polytechnique
- Ugo Boscain thought “Geometric Control theory and sub-Riemannian geometry” to PhD students at SISSA, Trieste, Italy
- Emmanuel Trélat thought “Contrôle en dimension finie et infinie” to M2 students at Sorbonne Université
- Emmanuel Trélat thought “Optimisation numeérique et sciences des donneées” to M1 students at Sorbonne Université
- Emmanuel Trélat thought “Matheématiques pour les eétudes scientifiques II” to L1 students at Sorbonne Université

9.2.2 Supervision

- PhD in progress: Daniele Cannarsa, “Contact Geometry, Direction Fields, and Applications to the Geometry of Vision”, started in October 2018. Supervisors: Davide Barilari (Padova, Italy) and Ugo Boscain.
- PhD in progress: Justine Dorsz, “Controôle de fluides et limites champ moyen”, started in September 2020. Supervisors: Olivier Glass (Dauphine) and Emmanuel Trélat.
- PhD in progress: Gontran Lance, started in September 2018, supervisors: Emmanuel Trélat and Enrique Zuazua.
- PhD in progress: Cyril Letrouit, “Équation des ondes sous-riemanniennes”, started in September 2019, supervisor Emmanuel Trélat.
- PhD in progress: Emilio Molina, “Application of optimal control techniques to natural resources management”, started in September 2018, supervisors: Pierre Martinon, Héctor Ramírez, and Mario Sigalotti.
- PhD in progress: Aymeric Nayet, “Améliorations d’un solveur de contrôle optimal pour de nouvelles missions Ariane”, started in September 2019. Supervisor: Emmanuel Trélat.
- PhD in progress: Eugenio Pozzoli, “Adiabatic Control of Open Quantum Systems”, started in September 2018, supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Rémi Robin, “Orbit spaces of Lie groups and applications to quantum control”, started in September 2019, supervisors: Ugo Boscain and Mario Sigalotti.

9.2.3 Juries

- Ugo Boscain was member of the jury of the HDR of Alain Sarlette.
- Mario Sigalotti was president of the jury of the PhD thesis of Cristina Urbani, GSSI, L’Aquila, Italy.
- Emmanuel Trélat was president of the jury of the HDR of L. Bourdin, Université de Limoges.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of I. Djebour, Université de Lorraine.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of C. Moreau, Inria Nice.
- Emmanuel Trélat was referee and member of the jury of the PhD thesis of D. Pighin, Univ. Madrid.
- Emmanuel Trélat was member of the jury of the PhD thesis of C. Zammali, CNAM.

9.3 Popularization

9.3.1 Interventions

- Emmanel Trélat was a speaker at the event “40 ans du CRAN”, Nancy.
- Emmanel Trélat was a speaker at the event “Mois de l’Optimisation”, Limoges.

10 Scientific production

10.1 Major publications

[1] D. Barilari, U. Boscain, D. Cannarsa and K. Habermann. ‘Stochastic processes on surfaces in three-dimensional contact sub-Riemannian manifolds’. In: *Annales de l’Institut Henri Poincaré (B) Probabilités et Statistiques* (2021). 25 pages, 2 figures. DOI: 10.1214/20-AIHP1124. URL: https://hal.archives-ouvertes.fr/hal-02557862.

[2] D. Barilari, Y. Chitour, F. Jean, D. Prandi and M. Sigalotti. ‘On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures’. In: *Journal de Mathématiques Pures et Appliquées* 133 (2020), pp. 118–138. DOI: 10.1016/j.matpur.2019.04.008. URL: https://hal.archives-ouvertes.fr/hal-01757343.

[3] M. Bertalmio, L. Calatroni, V. Franceschi, B. Franceschiello and D. Prandi. ‘Cortical-inspired Wilson-Cowan-type equations for orientation-dependent contrast perception modelling’. In: *Journal of Mathematical Imaging and Vision* (June 2020). DOI: 10.1007/s10851-020-00960-x. URL: https://hal.archives-ouvertes.fr/hal-02316989.

[4] R. Bonalli, B. Hérissé and E. Trélat. ‘Optimal Control of Endo-Atmospheric Launch Vehicle Systems: Geometric and Computational Issues’. In: *IEEE Transactions on Automatic Control* 65.6 (2020), pp. 2418–2433. DOI: 10.1109/tac.2019.2929099. URL: https://hal.archives-ouvertes.fr/hal-01626869.

[5] U. Boscain, E. Pozzoli and M. Sigalotti. ‘Classical and quantum controllability of a rotating 3D symmetric molecule’. In: *SIAM Journal on Control and Optimization* (2020). URL: https://hal.inria.fr/hal-02421593.

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10.2 Publications of the year

International journals

[11] R. Adami, U. Boscain, V. Franceschi and D. Prandi. 'Point interactions for 3D sub-Laplacians'. In: *Annales de l'Institut Henri Poincaré (C) Non Linear Analysis* (27th Nov. 2020). DOI: 10.1016/j.anihpc.2020.10.007. URL: https://hal.archives-ouvertes.fr/hal-02020844.

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