RESEARCH CENTRE

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IN PARTNERSHIP WITH:

CNRS, Université Côte d'Azur, Université de Bourgogne

2020
ACTIVITY REPORT

Project-Team

MCTAO

Mathematics for Control, Transport and Applications

IN COLLABORATION WITH: Institut Mathématique de Bourgogne, Laboratoire Jean-Alexandre Dieudonné (JAD)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team MCTAO

Creation of the Team: 2012 January 01, updated into Project-Team: 2013 January 01

Keywords

Computer sciences and digital sciences

A5.10.3. – Planning

A5.10.4. – Robot control

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.5. – Multiphysics modeling

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.6. – Optimization

A6.4. – Automatic control

A6.4.1. – Deterministic control

A6.4.3. – Observability and Controlability

A6.4.4. – Stability and Stabilization

A6.4.6. – Optimal control

A6.5. – Mathematical modeling for physical sciences

A8.2.3. – Calculus of variations

A8.12. – Optimal transport

Other research topics and application domains

B2.6. – Biological and medical imaging

B2.7.2. – Health monitoring systems

B5.2.3. – Aviation

B5.2.4. – Aerospace

B5.6. – Robotic systems

1 Team members, visitors, external collaborators

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- Jean-Baptiste Pomet [Team leader, Inria, Senior Researcher, HDR]
- Lamberto Dell'Elce [Inria, Researcher]

Faculty Members

- Bernard Bonnard [Université de Bourgogne, Professor, HDR]
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- Sandrine Gayrard [Segula Technologies, CIFRE, from Sep 2020]
- Alesia Herasimenka [Université Côte d'Azur, from Oct 2020]
- Clément Moreau [Ecole normale supérieure Paris-Saclay, until Aug 2020]
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Interns and Apprentices

- Hao Deng [Inria, from Apr 2020 until Sep 2020]
- Giulio Gargantini [Inria, until Mar 2020]
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External Collaborators

- Olivier Cots [Université de Toulouse / ENSEEIHT, from Oct 2020]
- Thierry Dargent [Thales Alenia Space]
- Joseph Gergaud [Université de Toulouse / ENSEEIHT]
- Jérémy Rouot [ISEN, Brest]

2 Overall objectives

2.1 Control, Transport and Dynamics

VVV Our goal is to develop methods in geometric control theory for finite-dimensional nonlinear systems, and to transfer our expertise through real applications of these techniques.

Our primary domain of industrial applications in the past years is space engineering, namely designing trajectories in space mechanics using optimal control and stabilization techniques: transfer of a satellite between two Keplerian orbits, rendez-vous problem, transfer of a satellite from the Earth to the Moon or more complicated space missions. A second field of applications is quantum control with applications to Nuclear Magnetic Resonance and medical image processing. A third and more recent one is the control of micro-swimmers, i.e. swimming robots where the fluid-structure coupling has a very low Reynolds number.

There is also a form of transfer to other mathematical fields: some problems in dynamical systems are being solved thanks to control theory techniques.

3 Research program

3.1 Control Problems

McTAO's major field of expertise is control theory in the large sense. Let us give an overview of this field.

Modelling. Our effort is directed toward efficient methods for the control of real (physical) *systems*, based on a *model* of the system to be controlled. Choosing accurate models yet simple enough to allow control design is in itself a key issue. The typical continuous-time model is of the form $\mathrm{d}x/\mathrm{d}t = f(x,u)$ where x is the *state*, ideally finite dimensional, and u the *control*; the control is left free to be a function of time, or a function of the state, or obtained as the solution of another dynamical system that takes x as an input. Modelling amounts to deciding the nature and dimension of x, as well as the dynamics (roughly speaking the function f). Connected to modeling is identification of parameters when a finite number of parameters are left free in "f".

Controllability, path planning. Controllability is a property of a control system (in fact of a model) that two states in the state space can be connected by a trajectory generated by some control, here taken as an explicit function of time. Deciding on local or global controllability is still a difficult open question in general. In most cases, controllability can be decided by linear approximation, or non-controllability by "physical" first integrals that the control does not affect. For some critically actuated systems, it is still difficult to decide local or global controllability, and the general problem is anyway still open. Path planning is the problem of constructing the control that actually steers one state to another.

Optimal control. In optimal control, one wants to find, among the controls that satisfy some contraints at initial and final time (for instance given initial and final state as in path planning), the ones that minimize some criterion. This is important in many control engineering problems, because minimizing a cost is often very relevant. Mathematically speaking, optimal control is the modern branch of the calculus of variations, rather well established and mature [59, 36, 25], but with a lot of hard open questions. In the end, in order to actually compute these controls, ad-hoc numerical schemes have to be derived for effective computations of the optimal solutions. See more about our research program in optimal control in section 3.2.

Feedback control. In the above two paragraphs, the control is an explicit function of time. To address in particular the stability issues (sensitivity to errors in the model or the initial conditions for example), the control has to be taken as a function of the (measured) state, or part of it. This is known as closed-loop control; it must be combined with optimal control in many real problems. On the problem of stabilization, there is longstanding research record from members of the team, in particular on the construction of "Control Lyapunov Functions", see [50, 61].

Classification of control systems One may perform various classes of transformations acting on systems, or rather on models... The simpler ones come from point-to-point transformations (changes of variables) on the state and control, and more intricate ones consist in embedding an extraneous dynamical system into the model, these are dynamic feedback transformations, they change the dimension of the state. In most problems, choosing the proper coordinates, or the right quantities that describe a phenomenon, sheds light on a path to the solution; these proper choices may sometimes be found from an understanding of the modelled phenomenons, or it can come from the study of the geometry of the equations and the transformation acting on them. This justifies the investigations of these transformations on models for themselves. These topics are central in control theory; they are present in the team, see for instance the classification aspect in [41] or —although this research has not been active very recently— the study [58] of dynamic feedback and the so-called "flatness" property [53].

3.2 Optimal Control and its Geometry

Let us detail our research program concerning optimal control. Relying on Hamiltonian dynamics is now prevalent, instead of the Lagrangian formalism in classical calculus of variations. The two points of view run parallel when computing geodesics and shortest path in Riemannian Geometry for instance, in that there is a clear one-to-one correspondance between the solutions of the geodesic equation in the tangent bundle and the solution of the Pontryagin Maximum Principle in the cotangent bundle. In most optimal control problems, on the contrary, due to the differential constraints (velocities of feasible trajectories do not cover all directions in the state space), the Lagrangian formalism becomes more involved, while the Pontryagin Maximum Principle keeps the same form, its solutions still live in the cotangent bundle, their projections are the extremals, and a minimizing curve must be the projection of such a solution.

Cut and conjugate loci. The cut locus —made of the points where the extremals lose optimality— is obviously crucial in optimal control, but usually out of reach (even in low dimensions), and anyway does not have an analytic characterization because it is a non-local object. Fortunately, conjugate points —where the extremals lose *local* optimality— can be effectively computed with high accuracy for many control systems. Elaborating on the seminal work of the Russian and French schools (see [63, 24, 26] and [42] among others), efficient algorithms were designed to treat the smooth case. This was the starting point of a series of papers of members of the team culminating in the outcome of the *cotcot* software [35], followed by the *Hampath* [44] code. Over the years, these codes have allowed for the computation of conjugate loci in a wealth of situations including applications to space mechanics, quantum control, and more recently swimming at low Reynolds number. With in mind the two-dimensional analytic Riemannian framework, a heuristic approach to the global issue of determining cut points is to search for singularities of the conjugate loci; this line is however very delicate to follow on problems stemming from applications in three or more dimensions (see *e.g.* [45] and [32]). In all these situations, the fundamental object underlying the analysis is the curvature tensor. In Hamiltonian terms, one considers the dynamics of subspaces (spanned by Jacobi fields) in the Lagrangian Grassmannian [23]. This point of view withstands generalizations far beyond the smooth case: In L^1-minimization, for instance, discontinuous curves in the Grassmannian have to be considered (instantaneous rotations of Lagrangian subspaces still obeying symplectic rules [49]). The cut locus is a central object in Riemannian geometry, control and optimal transport. This is the motivation for a series of conferences on "The cut locus: A bridge over differential geometry, optimal control, and transport", co-organized by team members and Japanese colleagues.

Riemann and Finsler geometry. Studying the distance and minimising geodesics in Riemannian Geometry or Finsler Geometry is a particular case of optimal control, simpler because there are no differential constraints; it is studied in the team for the following two reasons. On the one hand, after some tranformations, like averaging or reduction, some more difficult optimal control problems lead to a Riemann or Finsler geometry problem. On the other hand, optimal control, mostly the Hamiltonian setting, brings a fresh viewpoint on problems in Riemann and Finsler geometry. On Riemannian ellipsoids of revolution, the optimal control approach allowed to decide on the convexity of the injectivity domain, which, associated with non-negativity of the Ma-Trudinger-Wang curvature tensor, ensures continuity of the optimal transport on the ambient Riemannian manifold [51, 52]. The analysis in the oblate geometry [33] was completed in [48] in the prolate one, including a preliminary analysis of non-focal domains associated with conjugate loci. Averaging in systems coming from space mechanics control with L^2-minimization yields a Riemannian metric, thoroughly computed in [31] together with its geodesic flow; in reduced dimension, its conjugate and cut loci were computed in [34] with Japanese Riemannian geometers. Averaging the same systems for minimum time yields a Finsler Metric, as noted in [30]. In [40], the geodesic convexity properties of these two types of metrics were compared. When perturbations (other than the control) are considered, they introduce a "drift", *i.e.* the Finsler metric is no longer symmetric.

Sub-Riemannian Geometry. Optimal control problems that pertain to sub-Riemannian Geometry bear all the difficulties of optimal control, like the role of singular/abnormal trajectories, while having some useful structure. They lead to many open problems, like smoothness of minimisers, see the recent monograph [57] for an introduction. Let us detail one open question related to these singular trajectories: the Sard conjecture in sub-Riemannian geometry. Given a totally non-holonomic distribution on a smooth manifold, the Sard Conjecture is concerned with the size of the set of points that can be reached by singular horizontal paths starting from a given point. In the setting of rank-two distributions in dimension three, the Sard conjecture is that this set should be a subset of the so-called Martinet surface, indeed small both in measure and in dimension. In [28], it has been proved that the conjecture holds in the case where the Martinet surface is smooth. Moreover, the case of singular real-analytic Martinet surfaces was also addressed. In this case, it was shown that the Sard Conjecture holds true under an assumption of non-transversality of the distribution on the singular set of the Martinet surface. It is, of course, very intersting to get rid of the remaining technical assumption, or to go to higher dimension. Note that any Sard-type result has strong consequences on the regularity of sub-Riemannian distance functions and in turn on optimal transport problems in the sub-Riemannian setting.

Small controls and conservative systems, averaging. Using averaging techniques to study small perturbations of integrable Hamiltonian systems is as old an idea as celestial mechanics. It is very subtle in the case of multiple periods but more elementary in the single period case, here it boils down to taking the average of the perturbation along each periodic orbit [27, 62]. This line of research stemmed out of applications to space engineering (see section 4.1): the control of the super-integrable Keplerian motion of a spacecraft orbiting around the Earth is an example of a slow-fast controlled system. Since weak propulsion is used, the control itself acts as a perturbation, among other perturbations of similar magnitudes: higher order terms of the Earth potential (including J_2 effect, first), potential of more distant celestial bodies (such as the Sun and the Moon), atmospheric drag, or even radiation pressure. Properly qualifying the convergence properties (when the small parameter goes to zero) is important and is made difficult by the presence of control. In [30], convergence is seen as convergence to a differential inclusion; this applies to minimum time; a contribution of this work is to put forward the metric character of the averaged system by yielding a Finsler metric (see section 3.2). Proving convergence of the extremals (solutions of the Pontryagin Maximum Principle) is more intricate. In [47], standard averaging ([27, 62]) is performed on the minimum time extremal flow after carefully identifying slow variables of the system thanks to a symplectic reduction. This alternative approach allows to retrieve the previous metric approximation, and to partly address the question of convergence. Under suitable assumptions on a given geodesic of the averaged system (disconjugacy conditions, namely), one proves existence of a family of quasi-extremals for the original system that converge towards the geodesic when the small perturbation parameter goes to zero. This needs to be improved, but convergence of all extremals to extremals of an "averaged Pontryagin Maximum Principle" certainly fails. In particular, one cannot hope for C^1-regularity on the value function when the small parameter goes to zero as swallowtail-like singularities due to the structure of local minima in the problem are expected. (A preliminary analysis has been made in [46].)

Optimality of periodic solutions/periodic controls. When seeking to minimize a cost with the constraint that the controls and/or part of the states are periodic (and with other initial and final conditions), the notion of conjugate points is more difficult than with straightforward fixed initial point. In [37], for the problem of optimizing the efficiency of the displacement of some micro-swimmers (see section 4.3) with periodic deformations, we used the sufficient optimality conditions established by R. Vinter's group [67, 54] for systems with non unique minimizers due to the existence of a group of symmetry (always present with a periodic minimizer-candidate control). This takes place in a long term collaboration with P.

Bettiol (Univ. Bretagne Ouest) on second order sufficient optimality conditions for periodic solutions, or in the presence of higher dimensional symmetry groups, following [67, 54]. Another question relevant to locomotion is the following. Observing animals (or humans), or numerically solving the optimal control problem associated with driftless micro-swimmers for various initial and final conditions, we remark that the optimal strategies of deformation seem to be periodic, at least asymptotically for large distances. This observation is the starting point for characterizing dynamics for which some optimal solutions are periodic, and asymptotically attract other solutions as the final time grows large; this is reminiscent of the "turnpike theorem" (classical, recently applied to nonlinear situations in [66]).

Software. Optimal control applications (but also the development of theory where numerical experiments can be very enlightening) require many algorithmic and numerical developments that are an important side of the team activity. The software *Hampath* is maintained by former members of the team in close collaboration with McTAO. We also use direct discretization approaches (such as the Bocop solver developed by COMMANDS) in parallel. We are developing, on the basis of these two softwares the control toolbox *ct* (see Section 6.1.1 for news).

Apart from this, we develop on-demand algorithms and pieces of software, for instance we have to interact with a production software developed by Thales Alenia Space. A strong asset of the team is the interplay of its expertise in geometric control theory with applications and algorithms.

4 Application domains

4.1 Aerospace Engineering

Participants Bernard Bonnard, Jean-Baptiste Caillau, Thierry Dargent, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet, Jérémy Rouot.

Space engineering is very demanding in terms of safe and high-performance control laws. It is therefore prone to fruitful industrial collaborations. McTAO now has an established expertise in space and celestial mechanics. Our collaborations with industry are mostly on orbit transfer problems with low-thrust propulsion. It can be orbit transfer to put a commercial satellite on station, in which case the dynamics are a Newtonian force field plus perturbations and the small control. There is also, currently, a renewed interest in low-thrust missions such as Lisa Pathfinder (ESA mission towards a Lagrange point of the Sun-Earth system) or BepiColombo (joint ESA-JAXA mission towards Mercury). Such missions look more like a controlled multibody system. In all cases the problem involves long orbit transfers, typically with many revolutions around the primary celestial body. When minimizing time, averaging techniques provide a good approximation. Another important criterion in practice is fuel consumption minimization (crucial because only a finite amount of fuel is onboard a satellite for all its "life"), which amounts to L^1-minimization. Both topics are studied by the team. We have a steady relationships with CNES and Thales Alenia Space (Cannes), that have financed or co-financed 3 PhDs and 2 post-docs in the Sophia location of the team in the decade and are a source of inspiration even at the methodological level. Team members also have close connections with Airbus-Safran (Les Mureaux) on launchers.

Some of the authoritative papers in the field were written by team members, with an emphasis on the geometric analysis and on algorithms (coupling of shooting and continuation methods). There are also connections with peers more on the applied side, like D. Scheeres (Colorado Center for Astrodynamics Research at Boulder), the group of F. Bernelli (Politecnico Milano), and colleagues from U. Barcelona (A. Farrès, A. Jorba).

A new action has started in Sep. 2020 with the Phd thesis of Alesia Herasimenka (2020-2023) on the control of solar sails. Solar sailing has been actively studied for two decades and recent missions have demonstrated its interest for "zero-fuel" missions. A lot has to be done to understand even the basic properties of such systems, for instance regarding controllability. Depending on the model used for the sail, not all directions of control are available. Some preliminary studies obtain controllability results by analysing the flow around equilibria of the system, these equilibria being changed when the orientation of the sail is updated. We want to provide a comprehensive understanding of controllability thanks to a systematic use of geometric control theory (orbit theorem, Lie algebraic approach) combined with effective numerical computations to check local controllability properties. The PhD thesis has been selected by ESA for a three-year research co-sponsorship.

4.2 Magnetic resonance imaging (MRI)

Participants Bernard Bonnard, Jérémy Rouot, Joseph Gergaud, Olivier Cots *(ENSEEIHT, Toulouse)*.

The starting point of our interest in optimal control for quantum systems was a collaboration with physicist from ICB, University of Burgundy (Dominique Sugny), motivated by an ANR project where we worked on the control of molecular orientation in a dissipative environment using a laser field, and developed optimal control tools, combined with numerical simulations, to analyze the problem for Qubits. This was related to quantum computing rather than MRI. Using this expertise and under the impulse of Prof. S. Glaser and his group (Chemistry, TU München), we investigated Nuclear Magnetic resonance (NMR) for medical imaging (MRI), where the model is the Bloch equation describing the evolution of the Magnetization vector controlled by a magnetic field, but in fine is a specific Qubit model without decoherence. We worked on, and brought strong contributions to, the contrast problem: typically, given two chemical substances that have an importance in medicine, like oxygenated and de-oxygenated blood, find the (time-dependent) magnetic field that will produce the highest difference in brightness between these two species on the image resulting from Nuclear Magnetic Resonance. This has immediate and important industrial applications in medical imaging. Our contacts are with the above mentioned physics academic labs, who are themselves in contact with major companies. The team has produced and is producing important work on this problem. One may find a good overview in [39], a reference book has been published on the topic [43], a very complete numerical study comparing different optimization techniques was performed in [38]. We conduct this project in parallel with S. Glaser team, which validated experimentally the pertinence of the methods, the main achievement being the in vivo experiments realized at the Creatis team of Insa Lyon showing the interest to use optimal control methods implemented in modern softwares in MRI in order to produce a better image in a shorter time. A goal is to arrive to a cartography of the optimal contrast with respect to the relaxation parameters using LMI techniques and numerical simulations with the Hamapth and Bocop code; note that the theoretical study is connected to the problem of understanding the behavior of the extremal solutions of a controlled pair of Bloch equations, and this is an ambitious task. Also, one of the difficulties to go from the obtained results, checkable on experiments, to practical control laws for production is to deal with magnetic field space inhomogeneities.

4.3 Swimming at low-Reynolds number

Participants Bernard Bonnard, Yacine El Alaoui-Faris, Laetitia Giraldi (CALISTO project team since January, 2021), Clément Moreau, Jean-Baptiste Pomet, Jérémy Rouot.

Following the historical reference for low Reynolds number locomotion [60], the study of the swimming strategies of micro-organisms is attracting increasing attention in the recent literature. This is both because of the intrinsic biological interest, and for the possible implications these studies may have on the design of bio-inspired artificial replicas reproducing the functionalities of biological systems. In the case of micro-swimmers, the surrounding fluid is dominated by the viscosity effects of the water and becomes reversible. In this regime, it turns out that the infinite dimensional dynamics of the fluid do not have to be retained as state variables, so that the dynamics of a micro-swimmer can be expressed by ordinary differential equations if its shape has a finite number of degrees of freedom. Assuming this finite dimension, and if the control is the rate of deformation, one obtains a control system that is linear (affine without drift) with respect to the controls, *i.e.* the optimal control problem with a quadratic cost defines a sub-Riemannian structure (see section 3.2). This is the case where the shape is "fully actuated", *i.e.* if all the variables describing the shape are angles, there is an actuator on each of these angles. For artificial micro-swimmers, this is usually unrealistic. For example, (artificial) magneto-elastic micro-swimmers are deformed by an external magnetic field. In this case, the control functions are the external magnetic field. In both cases, questions are controllability (straightforward in the fully actuated case), optimal control, possibly path planning. We collaborate with teams that have physical experiments for both.

- In collaboration with D. Takagi and M. Chyba (University of Hawaii), this approach is currently at the experimental level for copepod-like swimmer at the university of Hawaii: on the one hand, this zooplankton and its locomotion can be observed, and a robot micro swimmer mimicking a copepod has been constructed. The robot is large enough for direct actuation to be possible, and the low Reynolds number is achieved by using a very viscous fluid. This gives possibilities, through an inverse optimization problem, to determine what cost can be optimised by these crustaceans, see [29, 65], and to validate models on the robot.
- For magneto-elastic micro-robots, Y. El-Alaoui's PhD is co-advised with Stéphane Régnier from the robotics lab ISIR, Sorbonne Université. Magneto-elastic micro-robots and their magnetic actuation are actually built at ISIR and the aim of the collaboration is to validate models and improve the existing control laws both in performance and in energy; of course, the micro scale does make things difficult.

The questions about optimality of periodic controls raised in section 3.2 are related to these applications for periodic deformations, or strokes, playing important role in locomotion.

4.4 Stability of high frequency active circuits

Participants Sébastien Fueyo, Jean-Baptiste Pomet, Laurent Baratchart *(FACTAS project-team)*.

Let us focus on amplifiers, as active circuits. Nonlinear hyper-frequency amplifiers are ubiquitous in cell phone relays and many other devices. They must be as compact as possible, yielding a more complicated design. Computer Assisted Design tools are extensively used; for a given amplifier design, they provide frequency responses but fail to provide information of the stability of the response for each frequency. This stability is crucial for an unstable response will not be observed in practice; the actual device should not be built before stability is asserted. Predicting stability/instability from "simulations" in the Computer Assisted Design tool is of utmost importance (simulation between quotation marks because these simulations are in fact computations in the frequency domain). Potential transfer to industry is important.

Some techniques do exist, see [64], based on creating some virtual perturbations and treating them as the input of a (linearized) control system to be "simulated" using the same tools. In an ongoing collaboration between McTAO and the project-team FACTAS, we work on the mathematical ground of these methods and in particular of the relation between stability and the property of the identified time-varying infinite dimensional systems. See recent developments in Section 7.6.

4.5 Optimal control of microbial cells

Participants Jean-Baptiste Caillau, Walid Djema *(BIOCORE project-team)*, Laetitia Giraldi *(CALISTO project-team since January, 2021)*, Jean-Luc Gouzé *(BIOCORE project-team)*, Sofya Maslovskaya, Jean-Baptiste Pomet, Agustín Yabo.

The growth of microorganisms is fundamentally an optimization problem which consists in dynamically allocating resources to cellular functions so as to maximize growth rate or another fitness criterion.

Simple ordinary differential equation models, called self-replicators, have been used to formulate this problem in the framework of optimal and feedback control theory, allowing observations in microbial physiology to be explained. The resulting control problems are very challenging due to the nonlinearity of the models, parameter uncertainty, the coexistence of different time-scales, a dynamically changing environment, and various other physical and chemical constraints. In the framework of the ANR Maximic (PI Hidde de Jong, Inria Grenoble Rhône-Alpes) we aim at developing novel theoretical approaches for addressing these challenges in order to (i) study natural resource allocation strategies in microorganisms and (ii) propose new synthetic control strategies for biotechnological applications. In order to address (i), we develop extended self-replicator models accounting for the cost of regulation and energy metabolism in bacterial cells. We study these models by a combination of analytical and numerical approaches to derive optimal control solutions and a control synthesis, dealing with the bang-bang-singular structure of the solutions. Moreover, we define quasi-optimal feedback control strategies inspired by known regulatory mechanisms in the cell. To test whether bacteria follow the predicted optimal strategies, we quantify dynamic resource allocation in the bacterium Escherichia coli by monitoring, by means of time-lapse fluorescent microscopy, the expression of selected genes in single cells growing in a microfluidics device. In order to address (ii), we build self-replicator models that include a pathway for the production of a metabolite of interest. We also add a mechanism to turn off microbial growth by means of an external input signal, at the profit of the production of the metabolite. We formulate the maximization of the amount of metabolite produced as an optimal control problem, and derive optimal solutions and a control synthesis, as well as quasi-optimal feedback strategies satisfying chemical and physical design constraints. The proposed synthetic control strategies are being tested experimentally by growing E. coli strains capable of producing glycerol from glucose in a mini-bioreactor system. We aim at quantifying the amount of glucose consumed and glycerol produced, in the case of a predefined input signal (open-loop control) and the adaptive regulation of the input signal based on on-line measurements of the growth rate and the expression of fluorescent reporters of selected genes (closed-loop control). Currently, one PhD (A. Yabo) and one postdoc (S. Maslovskaya) are involved in these tasks and jointly supervised by colleagues from McTAO and Biocore teams at Sophia. Preliminary results concern the definition on extended (higher dimensional) models for the bacteria dynamics, check of second order optimality conditions on the resulting optimal control problem, and study of the turnpike phenomenon for these optimization problems.

5 Highlights of the year

- A. Herasimenka received a three-year research co-sponsorship for her PhD from the European Space Agency (ESA).
- The gallery of the `ct` (Control Toolbox) project is online: ct.gitlabpages.inria.fr/gallery

6 New software and platforms

The software *Hampath* was developed and is maintained by former members of the team in close collaboration with McTAO. We also use direct discretization approaches (such as the Bocop solver developed by COMMANDS) in parallel. In 2019, we obtained an “ADT”, where the Experimentation and Development for Research Department (SED) provides strong support for two years, in order to help some reseachers from McTAO, together with colleagues from the COMMANDS and CAGE project-teams and from the APO team (CNRS Toulouse), put together the efforts on BOCOP and HamPath to go towards a reference toolbox in optimal control, under the name `ct` (Control Toolbox).

6.1 New software

6.1.1 ct

Name: control toolbox

Keywords: Optimal control, Ordinary differential equations, Mathematical Optimization, Differential homotopy, Automatic differentiation

Scientific Description: Numerical resolution of optimal control problems

Functional Description: BOCOP implements a direct method for solving optimal control problems. This approach can be easily chained with a single / multiple shooting solving coupled with differential continuation (nutopy library, evolution of Hampath).

Release Contributions: - bocop refactoring - nutopy library - project gallery

URL: `http://ct.gitlabpages.inria.fr/gallery`

Contacts: Jean-Baptiste Caillau, Pierre Martinon, Olivier Cots, Thibaud Kloczko

Participants: Jean-Baptiste Caillau, Pierre Martinon, Olivier Cots, Thibaud Kloczko, Tristan Cabel, Jean-Luc Szpyrka, Erwan Demairy, Julien Wintz, Carlos Zubiaga Pena

Partners: Université de Toulouse, CNRS, IRIT, ENSEEIHT

7 New results

7.1 Study of the convergence of time-optimal trajectories of fast oscillating control systems

Participants Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet.

For a control system with one fast periodic variable, with a small parameter measuring the ratio between time derivatives of fast and slow variables, we considered the Hamiltonian equation resulting from applying Pontryagin Maximum Principle for the minimum time problem with fixed initial and final slow variables and free fast variable. One may perform averaging at least under normalization of the adjoint vectors and define a "limit" average system.

This study was devoted to the convergence properties of this problem as the small parameter approaches 0. We showed that using the right transformations between initial/final conditions in the "real" and average systems led to a reconstruction of the fast variable on interval of times of order $1/\varepsilon$ where ε is the small parameter. This was only evidenced numerically at this stage of the analysis. Relying on this, we proposed a procedure to reconstruct very efficiently the solution of the two point boundary problem for nonzero ε using only the solution of the averaged optimal control problem. This result will be presented at the MTNS conference.

7.2 A necessary condition for the local controllability of non-ideal solar sails

Participants Jean-Baptiste Caillau, Lamberto Dell'Elce, Alesia Herasimenka, Jean-Baptiste Pomet.

Solar sails offer a propellant-less solution to achieve interplanetary transfers, planet escapes, and deorbiting maneuvers by leveraging on solar radiation pressure (SRP). Despite very few solar sail missions were launched, the possibility to use SRP as an inexhaustible source of propulsion attracted the interest of researchers since decades, and several contributions on the guidance and control of solar sails are available. Most often, solutions of two-point boundary value problems were offered without investigating whether the targeted point is within the reachable set of the control system. Surprisingly, a thorough analysis of the controllability of solar sails was not yet available.

A major difficulty in assessing the controllability of an SRP-actuated system is that the sail cannot generate a force with a positive component toward the direction of the Sun, so that classical tools of geometric control theory cannot be used. For example, Lie algebra of the system is full rank (unless a fully absorptive model of the sail is considered), but this result, which should indicate that the system is weakly controllable, requires that the interior of the control set includes the origin, so that it is not sufficient to analyze the sailing problem. This aspect is particularly critical when considering a non-ideal sail because the angle of the convex cone associated to the control set is strictly smaller than 90 deg.

In this context, we proposed a necessary condition for the local controllability of non-ideal solar sails both in the proximity of a planet (e.g., escape trajectory) and in heliocentric orbit (e.g., interplanetary transfer). This requirement was aimed at assessing if the sail at hand were capable of decreasing all possible functions of the Keplerian integrals of motion over an orbital period. The necessary condition was posed as a worst-case optimization problem characterized by a finite number of design variables and a one-parameter family of inequality constraints. The formulation relied on the convexification of the control set by defining the minimal cone containing all possible directions of the force vector.

7.3 Feasible set of initial conditions for the collision avoidance of two satellites

Participants Jean-Baptiste Caillau, Lamberto Dell'Elce, Jean-Baptiste Pomet, Simeon Vom Dahl.

Fast and broadband communications are the core of modern society. Nonetheless, access to the web is poor or even not possible in several regions of the world. So-called "mega constellations", i.e., constellations of thousands of small satellites, offer an innovative solution to this problem and are capable of guaranteeing a fast, global and reliable coverage for telecommunications. The simultaneous maneuvering of several agents in a possibly-clustered initial configuration is the key challenge of the deployment of the constellation. Hence, the definition of a reliable and computationally-efficient control strategy that is capable of preventing collisions between satellites of the constellation is mandatory. However, because of the large magnitude of orbital perturbations to thrust ratio (i.e., the control authority is not sufficient to fully compensate orbital drift), unavoidable collisions may occur for some initial configuration of the constellation.

In this context, our aim is to characterize the feasible set of initial conditions such that all collisions can be avoided. The classical approach to collision avoidance starts with the intuitive idea of a safety volume which shall not be penetrated by any other spacecraft. The mathematical formulation of the safety and collision regions is related to the viability analysis of a dynamical system associated to the safety volume. Hamilton-Jacobi equations are generally used in the literature to tackle this problem. We proposed a different approach based on the possibility to use minimum-time trajectories of a control system as a means to identify the boundary of the reachable set. The proposed approach is less computationally intensive as it requires the integration of several ordinary differential equations instead of solving the Hamilton Jacobi partial differential equations.

7.4 Modeling of attitude determination and control systems of nanosatellites

Participants Lamberto Dell'Elce, Hao Deng, Jean-Baptiste Pomet.

Université Côte d'Azur is planning to build its first single-unit CubeSat (i.e., a recent standard for satellites with mass of the order of few kilograms and volume of few dm^3), named NiceCube. The objective of the mission is to establish an data optical link between a ground station in Nice and the satellite by enlightning it with a laser and by moduling the reflected signal with an occulter onboard NiceCube. To this end, the face of the satellite equipped with retroreflectors should be oriented towards the ground station so that the reflected signal can be received. McTAO part of UCA and is in charge of the attitude determination and control system of NiceCube.

The mission is currently in phase A (i.e., feasibility study). We developed a numerical simulator of the orbital and attitude motion of a satellite in low-Earth orbit. The simulator includes models of attitude actuators (reaction wheels and magnetotorquers) and sensors (Sun, horizon, magnetometer and gyroscopes). A continuous-time extended Kalman filter was also implemented to carry out the estimation process.

7.5 Optimal allocation of ressources in bacteria

Participants Agustín Yabo, Jean-Baptiste Caillau, Jean-Luc Gouzé.

In the framework of the ANR Maximic, we carry on the study of self-replicator models. These models describe the allocation of resources inside the bacteria: the substrate is used to produce precursors that, in turn, can be employed either to produce genetic machinery (and increase the biomass) or metabolic machinery (that will further catalyse the transformation of substrate into precursors). To this internal control and external action that aims, after some genetic engineering on the bacteria (to create a strain that reacts to light stimuli), at producing a new metabolite of interest. Then, while the behaviour of the untouched bacteria tends to be very well mimicked by biomass maximization strategies, maximizing the production of the metabolite of interest induces new biological strategies. This kind of model (and refinements) are studied in Yabo *et al*, Math. Biosci. Eng. (2020). Key properties of the system are: (i) the Fuller phenomenon as connection between bang and singular arcs requires an infinite number of switchings in finite time; (ii) the turnpike phenomenon. Indeed, for large fixed final times, trajectories of the system are essentially singular and close to the optimal (wrt. a constant static control) equilibrium which is a hyperbolic fixed point of the singular flow. See ct gallery for an example.

7.6 Stability of linear time-varying time-delay systems, application to nonlinear high frequency amplifiers or other active circuits

Participants Laurent Baratchart *(FACTAS project-team)*, Sébastien Fueyo, Gilles Lebeau *(Université Côte d'Azur)*, Jean-Baptiste Pomet.

These amplifiers contain on the one hand nonlinear active components and on the other hand lines, that induce some sort of delays and make the system infinite-dimensional: for each choice of a periodic input (to be amplified), the model is a nonlinear delay dynamical system, infinite dimensional because of the delays. The Computer Aided Design tools mentioned in Section 4.4 provide a periodic solution under this periodic forcing and may also give the frequency response of the linearized system along this trajectory with some artificial "small" excitation. The goal is to deduce stability from these data.

It is an opportunity to build theoretical basis and justification to a stability analysis through harmonic identification; the latter is one of the specialties of FACTAS, we collaborate on the infinite-dimensional non-linear stability analysis for periodic solutions and how it works with the results of harmonic identification. All results can be found in Sébastien Fueyo's PhD thesis [16].

- Stability of time-delay time-varying linear delay difference systems had to be re-visited. A new sufficient condition can be found in [4], that has a passivity interpretation. A more general result is in the thesis and is the purpose of a publication to come, it is a powerful generalisation of a well known necessary and sufficient condition for stability of time-invariant systems due to Hale and Henry [56, 55]. These result are important to the domain of linear time-delay systems because the time-varying case has seldom been touched.

- The full process of detection of stability from the frequency response of the time-varying linear approximation along a periodic solution of a nonliner delay system of the class that amplifier or active circuits belong to is also given, in which the previous result is instrumental. Via a Fourier development, stability is studied through the singularities of the harmonic transfer function (HTF) which is an infinite matrix depending on a complex variable with Banach value. Under high frequency dissipativity assumption, which are always verified for amplifiers, the HTF has at most poles in a complex right half-plane containing strictly the imaginary axis. These poles are in particular the logarithms of a finite family of complex numbers, and under an assumption of controllability and observability, the periodic solution is locally stable if and only if the HTF has no poles in the complex right half-plane.

Two publications on the basis of this second point are under preparation.

7.7 Modeling and experimental identification and control of a micro swimmer

Participants Yacine El Alaoui-Faris, Laetitia Giraldi *(CALISTO project-team since January, 2021)*, Jean-Baptiste Pomet, Stéphane Régnier *(Sorbonne Université, Paris)*.

Y. El Alaoui-Faris's PhD [15] was a joint project between McTAO and *Institut des Systèmes Intelligents et Robotiques* (ISIR) in *Sorbonne Université* on controlling a magnetized flexible micro-swimmer —by this we mean a small robotic device that evolves in a fluid in such condition that the Reynold number, from the fluid mechanics point of view, is very small, implying dynamics where viscosity dominates inertia; it is the case of microscopic devices in water, the present robot is a few milimeters long but “swimms” in a very viscous fluid, proving the same low Reynolds number conditions— actuated by an external magnetic field. A simplified 3D dynamical model of a flexible swimmer was developed, based on the approximation of hydrodynamic forces (Resistive Force Theory) and the discretization of the curvature and elasticity of the tail of the swimmer, namely modelling the magnetized head and flexible continuous tail by some “N-link” device made of a finite number of straight rods with elastic torques localised at the joints.

Fitting the hydrodynamic and elastic parameters of our model to experiments was done through propulsion characteristics (mainly the frequency response of the swimmer in terms of its mean speed) that can be experimentally measured in a reliable manner. The optimal control problem of finding the actuating magnetic fields that maximize the propulsion speed of the experimental swimmer under constraints on the control that reflect the constraints physically imposed on the magnetic field, although finite dimensionnal is still too large and implicit to be tackled other than numerically. Direct numerical methods were used. The optimal magnetic fields found via numerical optimization have implemented in the ISIR experimental setup in order to benchmark the experimental performance of the computed controls and the ability of the model to predict the trajectories of the experimental swimmer. The obtained controls, translated as “strokes” that replace the usual sinusoidal ones, have proved to bring significant improvement in speed and to perform well in a feedback loop, although feedback was not a concern of the PhD per se.

This is presented in [9] from the experimental point of view and the model is further discussed in [13]. The thesis [15] contains valuable information that is not published otherwise.

7.8 Controllability in finite and infinite dimension and applications to bio-inspired nonlinear systems

Participants Laetitia Giraldi *(CALISTO project-team since January, 2021)*, Pierre Lissy *(Université Paris Dauphine, Paris)*, Clément Moreau, Jean-Baptiste Pomet.

One part of C. Moreau's PhD [17] was motivated by studying controllability properties of planar N-link models of magnically actuated swimmers, the same kind of control systems as in section 7.7 although in small dimensions. “Strong” local controllability at straught configuration fails for these systems, except for very specific values of parameters. This motivated some research on necessary conditions for local controllability of a particular class of systems with two controls, technical results based on the study of the Chen-Fliess series associated to these systems were obtained.

Another part of this PhD was concerned with linear coupled parabolic systems of controlled partial differential equations with state and control constraints. The literature contains many (rather recent) results on the scalar case, and this can be viewed as a non trivial generalization. It is shown that one may control these systems while making sure that the state remains approximately nonnegative when the diffusion matrix is diagonalizable, and that it remains nonnegative in the particular case where it is equal to the identity matrix.

The last part is about the use of N-link models as a natural numerical discretisation of elastic rods, and testing it on elastic micro-filaments motility at low Reynolds number, establishing its efficy, robustness and versatility. It was used to conduct various numerical experiment, in particular on filament buckling.

7.9 Optimal control techniques for 2-D Zermelo navigation problems in the case of revolution symmetry

Participants Bernard Bonnard, Joseph Gergaud, Boris Wembe *(ENSEEIHT, Toulouse)*, Olivier Cots.

In this preliminary study we consider the problem of navigation with a vortex singularity of a ship identified to a passive tracer [7]. We prove the existence of Reeb foliations in the optimal dynamics. It is extended to more general situations for a Zermelo problem, in the case of revolution. This study can be applied to hydrodynamics and to the case of controlled n-body models .

7.10 Muscular Electrostimulation Project

Participants Bernard Bonnard, Sandrine Gayrard, Jérémy Rouot, Toufik Bakir *(Univ. de Bourgogne Franche Conté)*, Jérémy Rouot.

This project started in 2017 under the impulse of Toufik Bakir. This projects aims to use recent models to predict and optimize the muscular response to pulses trains of electrical stimulations, in the isometric case. This project was supported by a Peps1 Amies (2019-2020) and a PGMO project (2019-2021). We start in September the conception of a smart muscular electro-stimulator in the framework of the Cifre thesis (Sandrine Gayrard) in collaboration with *Segula Technologies* [3]. This study is extended in [8] in the non isometric case in collaboration with J. Rouot. Those studies combined geometric analysis and numerical simulations with direct and indirect schemes.

8 Bilateral contracts and grants with industry

- A grant “PEPS AMIES”, title: “Conception d’un électrostimulateur intelligent”, was obtained, co-financed by AMIES and Segula Technologies.
  PI: Bernard Bonnard.
  Start: December 2018. Duration: 2 years.

- A grant CIFRE co-financed by Segula Technologies, title: “Réalisation d’un prototype d’électrostimulateur intelligent”, was obtained.
  PI: Bernard Bonnard and T. Bakir (IMvia).
  Start: January 2020. Duration: 3 years.

9 Partnerships and cooperations

9.1 European initiatives

The project around PhD thesis of A. Herasimenka will be partially supported by the **European Space Agency** (ESA). Contract to be signed in 2021.

9.2 National initiatives

9.2.1 ANR

Maximic: optimal control of microbial cells by natural and synthetic strategies. Started 2017, duration: 4 years. J.-B. Caillau, L. Giraldi, J.-B. Pomet, S. Maslovkaya and A. Yabo are participants. More information and news on the site of this project.

9.2.2 Others

PGMO grant (2019-2021) on "Sampled-Data Control Systems and Applications" (PI B. Bonnard).

PGMO grant (2020-2022) on "Extremal determinants". Participants are Y. Chitour (Université Paris Saclay), J.-B. Caillau, P. Freitas (University of Lisbon), Y. Privat (Université de Strasbourg)

The McTAO team participates in the GdR MOA, a CNRS network on Mathematics of Optimization and Applications.

J.-B. Caillau is associate researcher of the CNRS team Parallel Algorithms & Optimization at ENSEEIHT, Univ. Toulouse.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

J.-B. Caillau was member of PGMO days 2020 organising committee

10.1.2 Scientific events: selection

J.-B. Caillau was member of the SMAI-MODE conference scientific committee

10.1.3 Journal

Member of the editorial boards J.-B. Caillau was member of the Editorial board of ESAIM M2AN

Reviewer - reviewing activities Lamberto Dell'Elce reviewed papers submitted to:

- Journal of Guidance, Control, and Dynamics, AIAA
- Mathematics and Computers in Simulation, Elsevier

10.1.4 Research administration

Jean-Baptiste Pomet was, in 2020,

- a member of the scientific council of «Complex systems» Academy of Excellence Université Côte d'Azur,
- an elected member of Inria's *Commission d'Évaluation*.

J.-B. Caillau was member of

- 3IA Scientific council
- PGMO Scientific council
- GdR Calcul Scientific council

J.B. Caillau is head of the department of Applied Math. of Polytech Nice Sophia - Université Côte d'Azur since September 2018.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- Master in Astrophysics Université Côte Azur (MAUCA): Lamberto Dell'Elce, Build a Nanosatellite (Attitude Determination and Control System), 6 hours TH, niveau M1, Université Côte Azur, France.
- Engineering school: J.-B. Caillau has a full teaching duty of Professor at L3, M1 and M2 level of the Applied Math. department of Polytech Nice Sophia - Université Côte d'Azur.

10.2.2 Supervision

- PhD: Sébastien Fueyo, "Time-varying delay systems and 1-D hyperbolic equations, Harmonic transfer function and nonlinear electric circuits" (Systèmes à retard instationnaires et EDP hyperboliques 1-D instationnaires, fonctions de transfert harmoniques et circuits électriques non-linéaire) [16], defended October 30, 2020. Co-supervised by J.-B. Pomet and L. Baratchart (FACTAS team).
- PhD: Yacine El Alaoui-Faris, "Modeling and Optimal Control of Magnetic Micro-Swimmers" [15], defended December 21, 2020. Co-supervised by L. Giraldi, J.-B. Pomet and S. Régnier (Univ. Paris Sorbonne).
- PhD: Clément Moreau, "Controllability in finite and infinite dimension and applications to bio-inspired nonlinear systems" [17], defended June 17, 2020. Co-supervised by L. Giraldi, Pierre Lissy (Univ. Paris Dauphine) and J.-B. Pomet.
- PhD in progress : Agustín Yabo, "Optimal control of microbial cells", started October, 2018, co-supervised by J.-L. Gouzé (Biocore team) and J.-B. Caillau.
- PhD in progress : Alesia Herasimenka, "Optimal control of solar sails", started October, 2020, co-supervised by J.-B. Caillau and L. Dell'Elce.

10.2.3 Juries

- J.-B. Caillau sat in the PhD jury of Clément Moreau (Nice), HDR jury of Loïc Bourdin (Limoges). He is member of the jury of agrégation de mathématiques.

10.3 Popularization

10.3.1 Articles and contents

J.B. Caillau authored the *Brève* "De la Terre à la Lune" (republished).

10.3.2 Interventions

J.-B. Caillau gave conferences for high-school / college audiences at

- Collège Yves Klein, La Colle sur Loup
- Lycée Rouvière, Toulon
- Collège international, Valbonne
- Journée formation du Rectorat, Nice
- Remise du prix des Olympiades, Nice

Lamberto Dell'Elce was involved in the PoBot challenge promoted by MEDITES. Specifically he supervised a class in the College Emile Roux in Cannes.

11 Scientific production

11.1 Publications of the year

International journals

[1] C. Aldana, J.-B. Caillau and P. Freitas. 'Maximal determinants of Schrödinger operators on bounded intervals'. In: *Journal de l'École polytechnique — Mathématiques* 7 (2020), pp. 803–829. URL: https://hal.inria.fr/hal-01406270.

[2] T. Bakir, B. Bonnard, L. Bourdin and J. Rouot. 'Direct and Indirect Methods to Optimize the Muscular Force Response to a Pulse Train of Electrical Stimulation'. In: *ESAIM: Proceedings and Surveys* (2020). URL: https://hal.inria.fr/hal-02053566.

[3] T. Bakir, B. Bonnard, L. Bourdin and J. Rouot. 'Pontryagin-Type Conditions for Optimal Muscular Force Response to Functional Electric Stimulations'. In: *Journal of Optimization Theory and Applications* 184.2 (Feb. 2020), pp. 581–602. DOI: 10.1007/s10957-019-01599-4. URL: https://hal.inria.fr/hal-01854551.

[4] L. Baratchart, S. Fueyo, G. Lebeau and J.-B. Pomet. 'Sufficient Stability Conditions for Time-varying Networks of Telegrapher's Equations or Difference Delay Equations'. In: *SIAM Journal on Mathematical Analysis* 53 (1st Apr. 2021), pp. 1831–1856. DOI: 10.1137/19M1301795. URL: https://hal.inria.fr/hal-02385548.

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International peer-reviewed conferences

[11] A. Gabriel Yabo and J.-L. Gouzé. 'Optimizing bacterial resource allocation: metabolite production in continuous bioreactors'. In: IFAC 2020 - 1st Virtual World Congress by the International Federation of Automatic Control. Berlin / Virtual, Germany, 12th July 2020. URL: https://hal.archives-ouvertes.fr/hal-02980429.

Conferences without proceedings

[12] J.-B. Caillau, W. Djema, L. Giraldi, J.-L. Gouzé, S. Maslovskaya and J.-B. Pomet. 'The turnpike property in maximization of microbial metabolite production'. In: IFAC 2020 - 21rst IFAC World Congress. Berlin / Virtual, Germany, 12th July 2020. URL: https://hal.archives-ouvertes.fr/hal-02916081.

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Scientific book chapters

[14] B. Bonnard and J. Rouot. 'Towards Geometric Time Minimal Control without Legendre Condition and with Multiple Singular Extremals for Chemical Networks'. In: *Advances in Nonlinear Biological Systems: Modeling and Optimal Control.* Applied Mathematics. American Institute of Mathematical Sciences, 2020, pp. 1–34. URL: https://hal.inria.fr/hal-02431684.

Doctoral dissertations and habilitation theses

[15] Y. El Alaoui-Faris. 'Modeling and Optimal Control of Magnetic Micro-Swimmers'. Université Côte D'Azur, 21st Dec. 2020. URL: https://tel.archives-ouvertes.fr/tel-03129238.

[16] S. Fueyo. 'Time-varying delay systems and 1-D hyperbolic equations, Harmonic transfer function and nonlinear electric circuits'. Université Cote d'Azur, 30th Oct. 2020. URL: https://hal.archives-ouvertes.fr/tel-03105344.

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Reports & preprints

[18] T. Bakir, B. Bonnard, S. Gayrard and J. Rouot. *Finite Dimensional Approximation to Muscular Response in Force-Fatigue Dynamics using Functional Electrical Stimulation.* 12th Mar. 2021. URL: https://hal.inria.fr/hal-03154450.

[19] B. Bonnard, O. Cots, J. Gergaud and B. Wembe. *Abnormal Geodesics in 2D-Zermelo Navigation Problems in the Case of Revolution and the Fan Shape of the Small Time Balls.* 2021. URL: https://hal.archives-ouvertes.fr/hal-02437507.

[20] A. Herasimenka, J.-B. Caillau, L. Dell'Elce and J.-B. Pomet. *Control of a Solar Sail: An augmented version of "Effective Controllability Test for Fast Oscillating Control Systems. Application to Solar Sailing".* 14th Apr. 2021. URL: https://hal.inria.fr/hal-03185532.

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Other scientific publications

[22] B. Bonnard, T. Bakir, P. Bettiol, L. Bourdin, J. Rouot and S. Gayrard. *Optimal Control Techniques for Sampled-Data Control Systems with Medical Applications.* Paris, France, 1st Dec. 2020. URL: https://hal.inria.fr/hal-03021332.

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