RESEARCH CENTRE

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ACTIVITY REPORT

Project-Team

PARADYSE

PARticles And DYnamical SystEms

IN COLLABORATION WITH: Laboratoire Paul Painlevé (LPP)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Project-Team PARADYSE

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Keywords

Computer sciences and digital sciences

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Other research topics and application domains

B5.3. – Nanotechnology

B5.11. – Quantum systems

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1 Team members, visitors, external collaborators

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- Guillaume Dujardin [Team leader, Inria, Researcher, HDR]
- Clément Erignoux [Inria, from Oct 2020, Starting Faculty Position]
- Marielle Simon [Inria, Researcher, HDR]

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- Stephan De Bièvre [Université de Lille, Professor, HDR]
- André De Laire [Université de Lille, Associate Professor, HDR]
- Olivier Goubet [Université de Lille, Professor, from Sep 2020, HDR]
- Adrien Hardy [Université de Lille, Associate Professor, until Aug 2020]

Post-Doctoral Fellows

- Salvador López Martinez [Inria, from Oct 2020]
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- Karine Lewandowski [Inria]

2 Overall objectives

The PARADYSE team gathers mathematicians from different communities with the same motivation: to provide a better understanding of dynamical phenomena involving particles. These phenomena are described by fundamental models arising from several fields of physics. We shall focus on model derivation, study of stationary states and asymptotic behaviors, as well as links between different levels of description (from microscopic to macroscopic) and numerical methods to simulate such models. Applications include nonlinear optics, thermodynamics and ferromagnetism. Research in this direction has a long history, that we shall only partially describe in the sequel. We are confident that the fact that we come from different mathematical communities (PDE theory, mathematical physics, probability theory and numerical analysis), as well as the fact that we have strong and effective collaborations with physicists, will bring new and efficient scientific approaches to the problems we plan to tackle and will make our team strong and unique in the scientific landscape. Our goal is to obtain original and important results on a restricted yet ambitious set of problems that we develop in this document.

3 Research program

3.1 Time asymptotics: Stationary states, solitons, and stability issues

The team investigates the existence of *solitons* and their link with the global dynamical behavior for nonlocal problems such as the Gross–Pitaevskii (GP) equation which arises in models of dipolar gases. These models, in general, also introduce nonzero boundary conditions which constitute an additional theoretical and numerical challenge. Numerous results are proved for local problems, and numerical simulations allow to verify and illustrate them, as well as making a link with physics. However, most fundamental questions are still open at the moment for nonlocal problems.

The nonlinear Schrödinger (NLS) equation finds applications in numerous fields of physics. We concentrate, in a continued collaboration with our colleagues from the physics department (PhLAM) of the Université de Lille (UdL), in the framework of the Laboratoire d'Excellence CEMPI, on its applications in nonlinear optics and cold atom physics. Issues of orbital stability and modulational instability are central here.

Another typical example of problems that the team wishes to address concerns the Landau–Lifshitz (LL) equation, which describes the dynamics of the spin in ferromagnetic materials. This equation is a fundamental model in the magnetic recording industry [37] and solitons in magnetic media are of particular interest as a mechanism for data storage or information transfer [39]. It is a quasilinear PDE involving a function that takes values on the unit sphere $\mathbb{S}^2$ of $\mathbb{R}^3$. Using the stereographic projection, it can be seen as a quasilinear Schrödinger equation and the questions about the solitons, their dynamics and potential blow-up of solutions evoked above are also relevant in this context. This equation is less understood than the NLS equation: even the Cauchy theory is not completely understood [33, 36]. In particular, the geometry of the target sphere imposes nonvanishing boundary conditions; even in dimension one, there are kink-type solitons having different limits at $\pm\infty$.

3.2 Derivation of macroscopic laws from microscopic dynamics

The team investigates, from a microscopic viewpoint, the dynamical mechanism at play in the phenomenon of relaxation towards thermal equilibrium for large systems of interacting particles. For instance, a first step consists in giving a rigorous proof of the fact that a particle repeatedly scattered by random obstacles through a Hamiltonian scattering process will eventually reach thermal equilibrium, thereby completing previous works in this direction by the team. As a second step, similar models as the ones considered classically will be defined and analyzed in the quantum mechanical setting, and more particularly in the setting of quantum optics.

Another challenging problem is to understand the interaction of large systems with the boundaries, which is responsible for most energy exchanges (forcing and dissipation), even though it is concentrated in very thin layers. The presence of boundary conditions to evolution equations sometimes lacks understanding from a physical and mathematical point of view. In order to legitimate the choice done at the macroscopic level of the mathematical definition of the boundary conditions, we investigate systems of atoms (precisely chains of oscillators) with different local microscopic defects. We apply our recent techniques to understand how anomalous (in particular fractional) diffusive systems interact with the boundaries. For instance, the powerful tool given by Wigner functions that we already used has been successfully applied to the derivation of anomalous behaviors in open systems (for instance in [35]). The next step consists in developing an extension of that tool to deal with bounded systems provided with fixed boundaries. We also intend to derive anomalous diffusion by adding long-range interactions to diffusive models. There are very few rigorous results in this direction.

Finally, we aim at obtaining from a microscopic description the fractional porous medium equation (FPME), a nonlinear variation of the fractional diffusion equation, involving the fractional Laplacian instead of the usual one. Its rigorous study carries out many mathematical difficulties in treating at the same time the nonlinearity and fractional diffusion. We want to make PDE theorists and probabilists work together, in order to take advantage of the analytical results which went further ahead and are more advanced than the statistical physics theory.

3.3 Numerical methods: analysis and simulations

The team addresses both questions of precision and numerical cost of the schemes for the numerical integration of nonlinear evolution PDEs, such as the NLS equation. In particular, we aim at developing, studying and implementing numerical schemes with high order that are more efficient for these problems. We also want to contribute to the design and analysis of schemes with appropriate qualitative properties. These properties may as well be "asymptotic-preserving" properties, energy-preserving properties, or convergence to an equilibrium properties. Other numerical goals of the team include the numerical simulation of standing waves of nonlinear nonlocal GP equations. We also keep on developing numerical methods to efficiently simulate and illustrate theoretical results on instability, in particular in the context of the modulational instability in optical fibers, where we study the influence of randomness in the physical parameters of the fibers.

The team also designs simulation methods to estimate the accuracy of the physical description via microscopic systems, by computing precisely the rate of convergence as the system size goes to infinity. One method under investigation is related to cloning algorithms, which were introduced very recently and turn out to be essential in molecular simulation.

4 Application domains

4.1 Optical fibers

In the propagation of light in optical fibers, the combined effect of nonlinearity and group velocity dispersion (GVD) may lead to the destabilization of the stationary states (plane or continuous waves). This phenomenon, known under the name of modulational instability (MI), consists in the exponential growth of small harmonic perturbations of a continuous wave. MI has been pioneered in the 60s in the context of fluid mechanics, electromagnetic waves as well as in plasmas, and it has been observed in nonlinear fiber optics in the 80s. In uniform fibers, MI arises for anomalous (negative) GVD, but it may also appear for normal GVD if polarization, higher order modes or higher order dispersion are considered. A different kind of MI related to a parametric resonance mechanism emerges when the dispersion or the nonlinearity of the fiber are periodically modulated.

As a follow-up of our work on MI in periodically modulated optical fibers, we investigate the effect of random modulations in the diameter of the fiber on its dynamics. It is expected on theoretical grounds that such random fluctuations can lead to MI and this has already been illustrated for some models of the randomness. We investigate precisely the conditions under which this phenomenon can be strong enough to be experimentally verified. For that purpose, we investigate different kinds of random processes describing the modulations, taking into account the manner in which such modulations can be created experimentally by our partners of the fiber facility of the PhLAM. This necessitates careful modeling of the fiber and a precise numerical simulation of its behavior as well as a theoretical analysis of the statistics of the fiber dynamics.

This application domain involves in particular S. De Bièvre and G. Dujardin.

4.2 Ferromagnetism

The Landau–Lifshitz equation describes the dynamics of the spin in ferromagnetic materials. This equation is a fundamental model in the magnetic recording industry and solitons in magnetic media are of particular interest as a mechanism for data storage or information transfer. Depending on the properties of the material, the LL equation can include a dissipation term (the so-called Gilbert damping) and different types of anisotropic terms. The (N-dimensional) LL equation belongs to a larger class of nonlinear PDEs which are often referred to as geometric PDEs, and some related models are the Schrödinger map equation and the harmonic heat flow. The main mathematical difficulties lie in the facts that the spin function m takes values on the unit sphere $\mathbb{S}^2$ of $\mathbb{R}^3$, i.e. $m : [0, T[\times\mathbb{R}^N \to \mathbb{S}^2$ and that there is the term $m|\nabla m|^2$ in the equation, which is critical for the elliptic regularity theory in the energy space. By using the stereographic projection, the LL equation can be seen as a quasilinear Schrödinger equation and the questions about the solitons, their dynamics and potential blow-up of solutions evoked above are also relevant in this context. However, this equation is less understood than the NLS equation: even the Cauchy theory is not completely understood.

We focus on the following aspects of the LL equation.

Solitons In the absence of Gilbert damping, the LL equation is Hamiltonian. Moreover, it is integrable in the one-dimensional case and explicit formulas for solitons can be given. In the easy-plane case, the orbital and asymptotic stability of these solitons have been established. However, the stability in other cases, such as in biaxial ferromagnets, remains an open problem. In higher dimensional cases, the existence of solitons is more involved. In a previous work, a branch of semitopological solitons with different speeds has been obtained numerically in planar ferromagnets. A rigorous proof of the existence of such solitons is established using perturbation arguments, provided that the speed is small enough. However, the proof does not give information about their stability. We would like to propose a variational approach to study the existence of the branch of solitons, that would lead to the existence and stability of the whole branch of ground-state solitons as predicted. We also investigate numerically the existence of other types of localized solutions for the LL equation, such as excited states or vortices in rotation.

Approximative models An important physical conjecture is that the LL model is to a certain extent universal, so that the nonlinear Schrödinger and Sine-Gordon equations can be obtained as its various limiting cases. In a previous work, A. de Laire has proved a result in this direction and established an error estimate in Sobolev norms, in any dimension. A next step is to produce numerical simulations that will enlighten the situation and allow to prove further developments in this direction.

Self-similar behavior Self-similar solutions have brought a lot of attention in the study on nonlinear PDEs because they can provide some important information about the dynamics of the equation. While self-similar expanders are related to nonuniqueness and long time description of solutions, self-similar shrinkers are related to a possible singularity formation. However, there is not much known about the self-similar solutions for the LL equation. A. de Laire and S. Gutierrez have studied expander solutions and proved their existence and stability in the presence of Gilbert damping. We will investigate further results about these solutions, as well as the the existence and properties of self-similar shrinkers.

This application domain involves in particular A. de Laire and G. Dujardin.

4.3 Cold atoms

The cold atom team of the PhLAM Laboratory is reputed for having realized experimentally the so-called Quantum Kicked Rotor, which provides a model for the phenomenon of Anderson localization. The latter was predicted by Anderson in 1958, who received in 1977 a Nobel Prize for this work. Anderson localization is the absence of diffusion of quantum mechanical wave functions (and of waves in general) due to the presence of randomness in the medium in which they propagate. Its transposition to the Quantum Kicked Rotor goes as follows: a freely moving quantum particle periodically subjected to a "kick" will see its energy saturate at long times. In this sense, it "localizes" in momentum space since its momenta do not grow indefinitely, as one would expect on classical grounds. In its original form, Anderson localization applies to non-interacting quantum particles and the same is true for the saturation effect observed in the Quantum Kicked Rotor.

The challenge is now to understand the effects of interactions between the atoms on the localization phenomenon. Transposing this problem to the Quantum Kicked Rotor, this means describing the interactions between the particles with a Gross–Pitaevskii equation, which is a NLS equation with a local (typically cubic) nonlinearity. So the particle's wave function evolves between kicks following the Gross–Pitaevskii equation and not the linear Schrödinger equation, as is the case in the Quantum Kicked Rotor. Preliminary studies for the Anderson model have concluded that in that case the localization phenomenon gives way to a slow subdiffusive growth of the particle's kinetic energy. A similar phenomenon is expected in the nonlinear Quantum Kicked Rotor, but a precise understanding of the dynamical mechanisms at work, of the time scale at which the subdiffusive growth will manifest itself and of the subdiffusive growth exponent is lacking. It is crucial to design and calibrate the experimental setup intended to observe the phenomenon. The analysis of these questions poses considerable theoretical and numerical challenges due to the difficulties involved in understanding and simulating the long term dynamics of the nonlinear system. A collaboration of the team members with the PhLAM cold atoms group is currently under way.

This application domain involves in particular S. De Bièvre and G. Dujardin.

4.4 Qualitative and quantitative properties of numerical methods

Numerical simulation of multimode fibers The use of multimode fibers is a possible way to overcome the bandwidth crisis to come in our worldwide communication network consisting in singlemode fibers. Moreover, multimode fibers have applications in several other domains, such as high power fiber lasers and femtosecond-pulse fiber lasers which are useful for clinical applications of nonlinear optical microscopy and precision materials processing. From the modeling point of view, the envelope equations are a system of nonlinear nonlocal coupled Schrödinger equations. For a better understanding of several physical phenomena in multimode fibers (e.g. continuum generation, condensation) as well as for the design of physical experiments, numerical simulations are an adapted tool. However, the huge number of equations, the coupled nonlinearities and the nonlocal effects are very difficult to handle numerically. Some attempts have been made to build and provide efficient numerical codes for such simulations. However, there is room for improvement: one may want to go beyond MATLAB test codes, and to develop an alternative parallelization to the existing ones, which could use the linearly implicit methods that we plan to develop and analyze. In link with the application domain 4.1, we develop in particular a code for the numerical simulation of the propagation of light in multimode fibers, using high-order efficient methodss, that is to be used by the physics community.

This application domain involves in particular G. Dujardin and A. Roget.

Qualitative and long-time behavior of numerical methods We contribute to the design and analysis of schemes with good qualitative properties. These properties may as well be “asymptotic-preserving” properties, energy-preserving properties, decay properties, or convergence to an equilibrium properties. In particular, we contribute to the design and analysis of numerically hypocoercive methods for Fokker–Planck equations [14], as well as energy-preserving methods for hamiltonian problems [9].

This application domain involves in particular G. Dujardin.

High-order methods We contribute to the design of efficient numerical methods for the simulation of evolution problems. In particular, we focus on a class of linearly implicit high-order methods, that have been introduced for ODEs [29]. We wish both to extend their analysis to PDE contexts, and to analyze their qualitative properties in such contexts.

This application domain involves in particular G. Dujardin.

4.5 Modeling of the liquid-solid transition and interface propagation

Analogously to so-called Kinetically Constrained Models (KCM) that have served as toy models for glassy transitions, stochastic particle systems on a lattice can be used as toy models for a variety of physical phenomena. Among them, the kinetically constrained lattice gases (KCLG) are models in which particles jump randomly on a lattice, but are only allowed to jump if a local constraint is satisfied by the system.

Because of the hard constraint, the typical local behavior of KCLGs will differ significantly depending on the value of local conserved fields (e.g. particle density), because the constraint will either be typically satisfied, in which case the system is locally diffusive (liquid phase), or not, in which case the system quickly freezes out (solid phase).

Such a toy model for liquid-solid transition is investigated by M. Simon, C. Erignoux and their co-authors in [4] and [11]. The focus of these articles is the so-called facilitated exclusion process, which is a terminology coined by physicists for a specific KCLG, in which particles can only jump on an empty neighbor if another neighboring site is occupied. They derive the macroscopic behavior of the model, and show that in dimension 1 the hydrodynamic limit displays a phase separated behavior where the liquid phase progressively invades the solid phase.

Both from a physics and mathematics point of view, much remains to be done regarding theses challenging models: in particular, they present significant mathematical difficulties because of the way the local physical constraints put on the system distort the equilibrium and steady-states of the model. For this reason, M. Simon, C. Erignoux and A. Roget are currently trying to work with A. Shapira to generate numerical results on generalizations of the facilitated exclusion process, in order to shine some light on the microscopic and macroscopic behavior of these difficult models.

This application domain involves in particular C. Erignoux, A. Roget and M. Simon.

4.6 Mathematical modeling for ecology

This application domain is at the interface of mathematical modeling and numerics. Its object of study is a set of concrete problems in ecology. The landscape of the south of Hauts-de-France is made of agricultural land, encompassing forest patches and ecological corridors such as hedges. The issues are

- the study of the invasive dynamics and the control of a population of beetles which damages the oaks and beeches of our forests;
- the study of native protected species (the purple wireworm and the pike-plum) which find refuge in certain forest species.

Running numerics on models co-constructed with ecologists is also at the heart of the project. The timescales of animals and plants are no different; the beetle larvae spend a few years in the earth before moving. As a by-product, the mathematical model may tackle other major issues such as the interplay between Heterogeneity, Diversity and Invasibility.

The models use Markov chains at a mesoscopic scale and evolution advection-diffusion equations at a macroscopic scale.

This application domain involves O. Goubet. Interactions with Paradyse members concerned with particle models and hydrodynamic limits are planned.

5 New results

5.1 Traveling waves for some nonlocal 1D Gross–Pitaevskii equations with nonzero conditions at infinity

The nonlocal Gross–Pitaevskii equation is a model that appears naturally in several areas of quantum physics, for instance in the description of superfluids and in optics when dealing with thermo-optic materials because the thermal nonlinearity is usually highly nonlocal. A. de Laire and P. Mennuni have considered a nonlocal family of Gross–Pitaevskii equations in dimension one, and they have provided in [22] conditions on the nonlocal interaction such that there is a branch of traveling waves solutions with non-vanishing conditions at infinity. Moreover, they showed that the branch is orbitally stable. In this manner, this result generalizes known properties for the contact interaction given by a Dirac delta function. Their proof relies on the minimization of the energy at fixed momentum.

5.2 The cubic Schrödinger regime of the Landau–Lifshitz equation with a strong easy-axis anisotropy

It is well-known that the dynamics of biaxial ferromagnets with a strong easy-axis anisotropy is essentially governed by the cubic Schrödinger equation. A. de Laire and P. Gravejat provided in [21] a rigorous justification to this observation, continuing with the work started in [8]. More precisely, they showed the convergence of the solutions to the Landau-Lifshitz equation for biaxial ferromagnets towards the solutions to the cubic Schrödinger equation in the regime of an easy-axis anisotropy. This result holds for solutions to the Landau–Lifshitz equation in high-order Sobolev spaces. By introducing high-order energy quantities with good symmetrization properties, they derived the convergence from the consistency of the Landau–Lifshitz equation with the Sine-Gordon equation by using well-tailored energy estimates.

In this regime, they additionally classified the one-dimensional solitons of the Landau-Lifshitz equation and quantified their convergence towards the solitons of the one-dimensional cubic Schrödinger equation.

5.3 Self-similar shrinkers of the one-dimensional Landau–Lifshitz–Gilbert equation

A. de Laire and S. Gutierrez continue their investigation begun in in [6] concerning the existence and properties of self-similar solutions to the Landau–Lifshitz–Gilbert (LLG) equation, which is a PDE describing the dynamics for the magnetization in ferromagnetic materials. In [17], they performed an analytical study of self-shrinker solutions of the one-dimensional LLG equation and showed that there is a unique smooth family of backward self-similar solutions, up to symmetries. In addition, they obtained their asymptotics and proved that the trajectories of the self-similar profiles converge to great circles on the sphere $\mathbb{S}^2$, at an exponential rate.

In particular, their results provide examples of blow-up in finite time, where the singularity develops due to rapid oscillations forming limit circles.

Let us mention that the results in paragraphs 5.2 and 5.3 are presented in A. de Laire's "Habilitation à diriger des recherches" [25], defended in October 2020.

5.4 Quantum optics

In previous works, S. De Bièvre and his co-authors introduced a new measure of the nonclassicality of the quantum states of an optical field, the so-called "ordering sensitivity" of the state, that measures the fluctuations of its Wigner function. In [7], S. De Bièvre and his postdoc A. Hertz, re-interpret the ordering sensitivity in terms of another physical property of the quantum states of an optical field, namely their quadrature coherence scale. It is shown in particular that a large such coherence scale is responsible for very fast environmental decoherence of the state. In [18], S. De Bièvre, A. Hertz and N. Cerf, have further explored the link between this new notion of nonclassicality and measures of entanglement, providing upper bounds on the latter by the first. In [23], S. De Bièvre and his co-authors studied the effect of interactions on the kicked rotor, providing a new approximate model to study the time-asymptotic nature of the system.

5.5 Exponential time-decay for discrete Fokker–Planck equations

In the research direction exposed in Section 3.3, G. Dujardin and his co-authors proposed and studied in [14] several discrete versions of homogeneous and inhomogeneous one-dimensional Fokker–Planck equations. They proved in particular, for these discretizations of velocity and space, the exponential convergence to the equilibrium of the solutions, for time-continuous equations as well as for time-discrete equations. Their method uses new types of discrete Poincaré inequalities for a "two-direction" discretization of the derivative in velocity. For the inhomogeneous problem, they adapted for the very first time hypocoercive methods to the discrete level.

5.6 Numerical integration of the stochastic Manakov system

The stochastic Manakov system is a dispersive nonlinear system of PDEs that models the propagation of light in an optical fiber with randomly varying birefringence.

In [27], G. Dujardin and his collaborators introduced a linearly implicit scheme for the time integration of the stochastic Manakov system, that they analyzed and compared to the existing methods from the literature. In particular, they proved that the strong order of the numerical approximation is $1/2$ if the nonlinear term in the system is globally Lipschitz-continuous. They also proved this numerical method converges with order $1/2$ in probability and with order $1/2^-$ almost surely, in the case of the cubic nonlinear coupling which is relevant in optical fibers. They also proposed a modification of their method to obtain a mass-preserving scheme.

In [28], G. Dujardin and his collaborators developed, analyzed and implemented a numerical method based on the Lie–Trotter formula for the integration of the stochastic Manakov system. In particular, they proved that the strong order of the numerical approximation is 1/2 if the nonlinear term in the system is globally Lipschitz. They also proved that this splitting scheme converges with order 1/2 in probability, and converges almost surely with order $1/2^-$ as well. They provided numerical experiments to compare the efficiency of this scheme with existing methods from the literature, and they investigated numerically the possible blow-up in finite time of solutions to this SPDE system.

These two results enter the research axis of Section 3.3.

5.7 Linearly implicit high-order numerical methods for evolution problems

G. Dujardin and his collaborator derived in [29] a new class of numerical methods for the time integration of evolution equations set as Cauchy problems of ODEs or PDEs, in the research direction detailed in Section 3.3. The systematic design of these methods mixes the Runge–Kutta collocation formalism with collocation techniques, in such a way that the methods are linearly implicit and have high order. The fact that these methods are implicit allows to avoid CFL conditions when the large systems to integrate come from the space discretization of evolution PDEs. Moreover, these methods are expected to be efficient since they only require to solve one linear system of equations at each time step, and efficient techniques from the literature can be used to do so.

5.8 Energy-preserving methods for nonlinear Schrödinger equations

G. Dujardin and his co-authors have revisited and extended relaxation methods for nonlinear Schrödinger equations (NLS). The classical relaxation method for NLS is an energy-preserving method and a mass-preserving method. Moreover, it is only linearly implicit. A first proof of the second-order accuracy was achieved in [9]. Moreover, the method was extended to enable to treat noncubic nonlinearities, nonlocal nonlinearities, as well as rotation terms. The resulting methods are still energy-preserving and mass-preserving. Moreover, they are shown to have second-order accuracy numerically. These new methods are compared with fully implicit, mass-and energy-preserving methods of Crank and Nicolson.

The results presented in this paper follow the research direction of Section 3.3.

5.9 CLT for circular beta-ensembles at high temperature

In [34], A. Hardy and G. Lambert have obtained a central limit theorem for the 2D Coulomb gas particle system constrained on a circle in the high temperature regime. An interesting feature is that the limiting variance interpolates between the Lebesgue L^2 norm, corresponding to the infinite temperature setting, and the Sobolev $H^{1/2}$ semi-norm, corresponding to the zero temperature regime.

5.10 DLR equations and rigidity for the Sine-beta process

The work [13] by A. Hardy and his collaborators, recently accepted for publication in Communications on Pure and Applied Mathematics, provides a “statistical physics” description of the sine-β process by means of Dobroshin–Lanford–Ruelle (DLR) equations. This basically allows to give a meaning to the natural infinite configurations process on the real line in the 2D Coulomb interaction, provided there is a unique solution to the DLR equation which turns out to be true in this setting.

5.11 Decay of solutions to one-dimensional nonlinear Schrödinger equations with white noise dispersion

Together with S. Dumont and Y. Mammeri [15], O. Goubet investigated the asymptotic behavior of the solution to the one-dimensional Schrödinger equations with stochastic modulation and polynomial nonlinearity. They showed that if the initial data is small enough and the degree of nonlinearity large enough, then the expectation of the solution converges to 0 when time tends to infinity with the same speed as that of the solution to the linearized equation. It should be noted that this speed of convergence is half as fast as that of the corresponding deterministic equation.

5.12 Uniform approximation of the 2d Navier-Stokes equation by stochastic interacting particle systems

M. Simon and co-authors considered in [16] an interacting particle system modeled as a system of N stochastic differential equations driven by Brownian motions. They proved that the (mollified) empirical process converges, uniformly in time and space variables, to the solution of the two-dimensional Navier–Stokes equations written in vorticity form. The proofs follow a semigroup approach.

5.13 Review on various dynamics of interacting particle systems

M. Simon and co-authors presented in the ESAIM Proceedings a collection of recent results covering various aspects of the dynamical properties of interacting particle systems [10], namely:

- the hydrodynamic limit of a facilitated exclusion process;
- a cut-off phenomenon for the mixing time of the weakly asymmetric exclusion process;
- a study of the infection time in the Duarte model;
- the study of a front propagation in the FA-1f model.

5.14 Hydrodynamic limit for a chain with thermal and mechanical boundary forces

In a collaboration with S. Olla and T. Komorowski, [20], M. Simon proved the hydrodynamic limit for an harmonic chain with a random exchange of momentum that conserves the kinetic energy but not the momentum. The system is open and subject to two thermostats at the boundaries and to external tension. Under a diffusive scaling of space-time, the authors proved that the empirical profiles of the two locally conserved quantities, the volume stretch and the energy, converge to the solution of a nonlinear diffusive system of conservative partial differential equations.

5.15 Stefan problem for a non-ergodic facilitated exclusion process

In [4], M. Simon, O. Blondel and C. Erignoux investigated the general hydrodynamics for the *facilitated exclusion process* whose supercritical phase's hydrodynamics has been previously investigated in [11]. This process is similar to the celebrated *symmetric simple exclusion process*, except that a particle is only allowed to jump to a neighboring site if its other neighbor is occupied by a particle. This hard constraint on the particle's motion has a number of consequences on the microscopic and macroscopic behavior of the system. In particular, under the critical density $\rho_c = 1/2$, the system quickly freezes out and particles stop moving.

The purpose of this work is to investigate the macroscopic invasion of the frozen phase by the ergodic phase, and the authors were able to prove that starting from a profile with both supercritical and subcritical regions, the hydrodynamics for the facilitated exclusion process is given by a Stefan problem: the diffusive supercritical phase progressively invades the subcritical phase via flat interfaces, until either one of the phases disappears.

5.16 Non-equilibrium fluctuations of the weakly asymmetric normalized binary contact path process

In [32], X. Xue and L. Zhao further investigated the problem studied in [38], where the authors proved a law of large numbers for the empirical measure of the weakly asymmetric normalized binary contact path process on $\mathbb{Z}^d$, $d \geq 3$, and then conjectured that a central limit theorem should hold under a non-equilibrium initial condition. They proved that the aforesaid conjecture is true when the dimension d of the underlying lattice and the infection rate λ of the process are sufficiently large.

5.17 Non-existence results for semilinear elliptic problems

Establishing nonexistence results of nontrivial solutions to partial differential equations is of fundamental importance, either from the theoretical point of view or also for the applications. In the context of semilinear Dirichlet boundary value problems (posed in bounded domains of the Euclidean space), the classical identity by Pohozaev provides a nonexistence result if the following three conditions hold: the dimension of the Euclidean space is greater or equal than three, the domain is star-shaped and the nonlinearity has non-positive primitive. In the work [24], S. López-Martínez and A. Molino proved with a different technique that the first two conditions may be removed, so they obtained a nonexistence result for any smooth bounded domain and in any dimension assuming only the condition on the nonlinearity. As an application, they proved that the unique solution to the dissipative Sine-Gordon equation tends to zero in the energy norm as the time diverges.

5.18 Lane–Emden problems with quadratic gradient terms

Semilinear Dirichlet boundary value problems with a power-like nonlinearity with superlinear growth are called Lane–Emden problems in the literature. These problems are classical and have been widely studied since the pioneering work by Ambrosetti and Rabinowitz, in which existence of nontrivial solutions is established via the Mountain Pass Theorem. In the work [30], S. López-Martínez considered a Lane-Emden problem perturbed with a lower order term with natural growth in the gradient of the solution. For these modified Lane–Emden problems, it has been previously observed in the literature that the structure of the solutions may differ or not from the one of the classical semilinear problem, depending such differences depending on on the parameters of the equation. The main achievement in [30] is the fact that the new approach allows to consider lower order terms with non-constant coefficients and singularities. The techniques are based on Topological Degree theory and the a priori estimates are obtained via a blow-up method.

6 Partnerships and cooperations

6.1 Action de développement technologique

The team has a 2-year ADT project named "SIMPAPH" (2019–2021), the aim of which is the production of a numerical code for multimode optical fibers and another code for the simulation of large systems of particles. A. Roget is funded on this project.

6.2 ANR

ANR ODA project

Participants André de Laire.

- Title: Dispersive and random waves
- ANR reference: ANR-18-CE40-0020
- Coordinator: N. Tzvetkov, Université de Cergy-Pontoise
- Setpember 2018 – September 2022

ANR BoB project

Participants Adrien Hardy.

- Title: Bayes on a Budget — big data and expensive models
- ANR reference: ANR-16-CE23-0003
- Coordinator: R. Bardenet, CNRS & Université de Lille
- Duration: October 2016 – October 2020

ANR JCJC project MICMOV

Participants Marielle Simon *(project coordinator)*.

- Title: MICroscopic description of MOVing interfaces
- Type: Mathématiques (CE40) 2019
- ANR reference: ANR-19-CE40-0012
- Duration: February 2020 – January 2024

MATMOVIN project cofunded by the European Union together with the "fonds de développement régional"

Participants Marielle Simon *(project coordinator)*.

- Title: Description microscopique des transitions de phase et interfaces mobiles : avancées mathématiques
- Type: Post-doc grant of 2 years
- Duration: September 2020 – August 2022

ANR PRC MAMBO

Participants Olivier Goubet *(coordinator)*.

- Title: MAthematical Modelling of Biological invasiOn
- O. Goubet has applied as Principal Investigator to an ANR PRC project related to mathematical modeling in Ecology. This project gathers mathematicians from Lille and Amiens and ecologists from Amiens.
- Project accepted for Phase 2 — to be submitted on Apr 26, 2021

6.3 LabEx CEMPI

Through its affiliation to the Laboratoire Paul Painlevé (LPP), the PARADYSE team is involved in the LabEx CEMPI, a common project between the LPP and the *laboratoire de Physique des Lasers, Atomes et Molécules* (PhLAM). In particular, S. De Bièvre is a member of the executive committee of the LabEx.

- Title: Centre Européen pour les Mathématiques la Physique et leurs Interactions
- ANR reference: 11-LABX-007
- Coordinator: E. Fricain (LPP, Université de Lille)
- Duration: February 2012 – December 2024 (project renewed in 2019)
- Partners: Laboratoire Paul Painlevé (LPP) and Laser Physics department (PhLAM), université de Lille
- Budget: 6 960 395 euros

7 Dissemination

7.1 Promoting scientific activities

7.1.1 Scientific events: organisation

A. de Laire co-organized the "Journée des Doctorants en Mathématiques du Nord-Pas-de-Calais 2020", on Friday September 11 2020, at Auditorium du Musée des Beaux-Arts (Musée des Beaux-Arts de Calais). Link to the event's webpage: https://indico.math.cnrs.fr/event/5629/timetable.

O. Goubet will organize the scientific meeting of the Fédération de Recherche in Mathematics in Hauts-de-France. Due to the pandemic situation this is not confirmed at this date.

7.1.2 Member of the organizing committees

O. Goubet is member of the organizing committee of two international conferences:

- "Analyse appliquée et modélisation", Monastir, Tunisia, scheduled in October 2021
  http://dimenza.perso.math.cnrs.fr/zahrouni.html
- a conference in the honor of S. Nicaise, Valenciennes, scheduled in November 2021
  https://nicaise2021.sciencesconf.org/

7.1.3 Member of the Editorial Boards

- S. De Bièvre is Associate Editor of the Journal of Mathematical Physics
- O. Goubet is Associate Editor of the Journal of Mathematical Studies
- O. Goubet is Associate Editor of Advances in Nonlinear Analysis
- O. Goubet is Associate Editor for the DCDS-S Special Issue in memory of E. Zahrouni. The Special Editorial Board is A. Miranville, F. Ben Nasr, O. Goubet and P. Poullet.

7.1.4 Reviewer – reviewing activities

All permanent members of the PARADYSE team work as referees for many of the main scientific publications in analysis, probability and statistical physics.

7.1.5 Invited talks

The paradyse team members take active part in numerous scientific conferences, workshops and seminars, and in particular give frequent talks both in France and abroad.

7.1.6 Leadership within the scientific community

O. Goubet is the president of Société de Mathématiques Appliquées et Industrielles.

7.1.7 Scientific expertise

A. de Laire served as reviewer for the evaluation of research at Charles University (Prague, Czech republic).

7.1.8 Research administration

A. de Laire and S. De Bièvre are both members of "Conseil de Laboratoire Paul Painlevé" at Université de Lille.

S. De Bièvre is member of the executive committee of the LabEx CEMPI.

7.2 Teaching - Supervision - Juries

7.2.1 Teaching

The Paradyse team teaches various undergraduate level courses in several partner universities and *Grandes Écoles*. We only make explicit mention here of the Master courses (level M1-M2) and the doctoral courses.

- Master: O. Goubet and A. de Laire, "Modélisation et Approximation par Différences Finies", M1, Université de Lille.
- Master: M. Simon, "Introduction à la physique statistique", M2, Université de Lille
- Master: M. Simon, "Markov Chains and Applications", Université de Lille and École Centrale Lille (before 2019-2020)
- Doctoral School: S. De Bièvre, "Quantum information", 12h
- Master: O. Goubet, "Markov Chains and Applications", Université de Lille and École Centrale Lille (2020-2021)
- Master: O. Goubet "Etude de problèmes elliptiques et paraboliques", M1, Université de Lille

In addition, A. de Laire was in charge of the Master 2 of Applied Mathematics at Université de Lille until August 2020. S. De Bièvre represents (since 2018) the department of Mathematics in the organization of the newly created Master of Data Science of EC Lille, Université de Lille and IMT.

7.2.2 Supervision

- A. de Laire is the post-doc advisor of S. López Martinez, on the study of nonlocal Gross-Pitaevski equations. This post-doctoral research visit is funded by Inria from October 2020 until April 2022.
- M. Simon and C. Erignoux are the post-doc advisors of L. Zhao. This post-doctoral research visit is funded by the Tremplin ERC project MATMOVIN (2020–2022).
- G. Dujardin is co-advising the PhD dissertation of A. Nahas, on the numerical simulation of multi-species Bose-Einstein condensates (2019–2022, funded by the Region Hauts-de-France and the LabEx CEMPI).
- G. Dujardin co-advised the M1 internship of Esther Rubinowitz (June – July 2020), which was funded by the PARADYSE team.
- C. Erignoux, G. Dujardin and M. Simon supervize the work of A. Roget in the context of the ADT SIMPAPH.
- O. Goubet is co-advising the PhD dissertation of G. Delvoye on mathematical modeling for ecology. This PhD was supported by the Région Hauts-de-France (2017–2020)

- O. Goubet is co-advising the PhD dissertation of M. Abidi on logarithmic Schrödinger equations. This PhD is supported (1998–2021) by the Région Hauts-de-France.
- O. Goubet is co-advising the PhD dissertation of A. Masset on the shallow water equations with Coriolis term. This PhD is supported (2019–2022) by Université Picardie Jules Verne.

7.2.3 Juries

- G. Dujardin served as a reviewer in the jury of the PhD thesis of Sami Siraj-Dine, at École des Ponts ParisTech, co-advised by Éric Cancès, Clotilde Fermanian-Kammerer and Antoine Levitt, defended on December 17, 2020, entitled "Dynamics of electrons in 2D materials".

7.3 Popularization

G. Dujardin collaborated in a comic strip popularizing the work of the team in optical fibers for the "magazine du centre Inria de Lille" in 2020.

8 Scientific production

8.1 Major publications

[1] C. Beltrán and A. Hardy. 'Energy of the Coulomb gas on the sphere at low temperature'. In: *Archive for Rational Mechanics and Analysis* 231 (2019), pp. 2007–2017. DOI: 10.1007/s00205-018-1316-3. URL: https://arxiv.org/abs/1803.11018.

[2] C. Bernardin, P. Gonçalves, M. Jara and M. Simon. 'Interpolation process between standard diffusion and fractional diffusion'. In: *Annales de l'IHP, Probabilités et Statistiques* 54.3 (2018), pp. 1731–1757.

[3] C. Besse, G. Dujardin and I. Lacroix-Violet. 'High Order Exponential Integrators for Nonlinear Schrödinger Equations with Application to Rotating Bose–Einstein Condensates'. In: *SIAM Journal on Numerical Analysis* 55.3 (2017), pp. 1387–1411. DOI: 10.1137/15M1029047. eprint: \url{https://dx.doi.org/10.1137/15M1029047}. URL: https://dx.doi.org/10.1137/15M1029047.

[4] O. Blondel, C. Erignoux and M. Simon. 'Stefan problem for a non-ergodic facilitated exclusion process'. to appear in: Probability and Mathematical Physics. 2020. URL: https://hal.inria.fr/hal-02482922.

[5] D. Chafaï, A. Hardy and M. Maïda. 'Concentration for Coulomb gases and Coulomb transport inequalities'. In: *Journal of Functional Analysis* 275.6 (2018), pp. 1447–1483.

[6] S. Gutiérrez and A. de Laire. 'The Cauchy problem for the Landau-Lifshitz-Gilbert equation in BMO and self-similar solutions'. In: *Nonlinearity* 32.7 (2019), pp. 2522–2563. DOI: 10.1088/1361-6544/ab1296. URL: https://hal.archives-ouvertes.fr/hal-01948679.

[7] A. Hertz and S. De Bièvre. 'Quadrature coherence scale driven fast decoherence of bosonic quantum field states'. In: *Physical Review Letters* (Mar. 2020). DOI: 10.1103/PhysRevLett.124.090402. URL: https://hal.archives-ouvertes.fr/hal-02394344.

[8] A. de Laire and P. Gravejat. 'The Sine–Gordon regime of the Landau–Lifshitz equation with a strong easy-plane anisotropy'. In: *Annales de l'Institut Henri Poincaré C, Analyse non linéaire* (2018). DOI: 10.1016/j.anihpc.2018.03.005. URL: http://www.sciencedirect.com/science/article/pii/S029414491830026X.

8.2 Publications of the year

International journals

[9] C. Besse, S. Descombes, G. Dujardin and I. Lacroix-Violet. 'Energy preserving methods for nonlinear Schrödinger equations'. In: *IMA Journal of Numerical Analysis* 41.1 (Jan. 2021), pp. 618–653. DOI: 10.1093/imanum/drz067. URL: https://hal.archives-ouvertes.fr/hal-01951527.

[10] O. Blondel, A. Deshayes, C. Labbé, L. Marêché and M. Simon. 'Dynamics of interacting particle systems'. In: *ESAIM: Proceedings and Surveys* 68 (2020), pp. 52–72. DOI: 10.1051/proc/202068004. URL: https://hal.inria.fr/hal-02877315.

[11] O. Blondel, C. Erignoux, M. Sasada and M. Simon. 'Hydrodynamic limit for a facilitated exclusion process'. In: *Annales de l'Institut Henri Poincaré (B) Probabilités et Statistiques* 56.1 (2020), pp. 667–714. DOI: 10.1214/19-AIHP977. URL: https://hal.archives-ouvertes.fr/hal-02008606.

[12] O. Blondel, C. Erignoux and M. Simon. 'Stefan problem for a non-ergodic facilitated exclusion process'. In: *Probability and Mathematical Physics* (2021). URL: https://hal.inria.fr/hal-02482922.

[13] D. Dereudre, A. Hardy, T. Leblé and M. Maïda. 'DLR equations and rigidity for the Sine-beta process'. In: *Communications on Pure and Applied Mathematics* (18th Nov. 2020). DOI: 10.1002/cpa.21963. URL: https://hal.archives-ouvertes.fr/hal-01954367.

[14] G. Dujardin, F. Hérau and P. Lafitte-Godillon. 'Coercivity, hypocoercivity, exponential time decay and simulations for discrete Fokker- Planck equations'. In: *Numerische Mathematik* 144 (2020). DOI: 10.1007/s00211-019-01094-y. URL: https://hal.archives-ouvertes.fr/hal-01702545.

[15] S. Dumont, O. Goubet and Y. Mammeri. 'Decay of solutions to one dimensional nonlinear Schrödinger equations with white noise dispersion'. In: *Discrete and Continuous Dynamical Systems - Series S* (2020). URL: https://hal.archives-ouvertes.fr/hal-02944262.

[16] F. Flandoli, C. Olivera and M. Simon. 'Uniform approximation of 2d Navier-Stokes equation by stochastic interacting particle systems'. In: *SIAM Journal on Mathematical Analysis* 52.6 (2020). DOI: 10.1137/20M1328993. URL: https://hal.inria.fr/hal-02529632.

[17] S. Gutiérrez and A. de Laire. 'Self-similar shrinkers of the one-dimensional Landau-Lifshitz-Gilbert equation'. In: *Journal of Evolution Equations* (11th June 2020). DOI: 10.1007/s00028-020-00589-8. URL: https://hal.archives-ouvertes.fr/hal-02480999.

[18] A. Hertz, N. Cerf and S. D. Bièvre. 'Relating the Entanglement and Optical Nonclassicality of Multimode States of a Bosonic Quantum Field'. In: *Physical Review A* (21st Sept. 2020). DOI: 10.1103/PhysRevA.102.032413. URL: https://hal.inria.fr/hal-02557297.

[19] A. Hertz and S. De Bièvre. 'Quadrature coherence scale driven fast decoherence of bosonic quantum field states'. In: *Physical Review Letters* (5th Mar. 2020). DOI: 10.1103/PhysRevLett.124.090402. URL: https://hal.archives-ouvertes.fr/hal-02394344.

[20] T. Komorowski, S. Olla and M. Simon. 'Hydrodynamic limit for a chain with thermal and mechanical boundary forces'. In: *Electronic Journal of Probability* (2021). URL: https://hal.inria.fr/hal-02538469.

[21] A. de Laire and P. Gravejat. 'The cubic Schrödinger regime of the Landau-Lifshitz equation with a strong easy-axis anisotropy'. In: *Revista Matemática Iberoamericana* 37.1 (2021), pp. 95–128. DOI: 10.4171/rmi/1202. URL: https://hal.archives-ouvertes.fr/hal-01954762.

[22] A. de Laire and P. Mennuni. 'Traveling waves for some nonlocal 1D Gross-Pitaevskii equations with nonzero conditions at infinity'. In: *Discrete and Continuous Dynamical Systems - Series A* 40.1 (Jan. 2020), pp. 635–682. DOI: 10.3934/dcds.2020026. URL: https://hal.archives-ouvertes.fr/hal-01962779.

[23] S. Lellouch, A. Rançon, S. D. Bièvre, D. Delande and J. C. Garreau. 'Dynamics of the mean-field-interacting quantum kicked rotor'. In: *Physical Review A* 101.4 (Apr. 2020). DOI: 10.1103/PhysRevA.101.043624. URL: https://hal.archives-ouvertes.fr/hal-02567094.

[24] S. López Martínez and A. Molino. 'Non existence result of nontrivial solutions to the equation -$\Delta u = f(u)$'. In: *Complex Variables and Elliptic Equations* (21st Dec. 2020). DOI: 10.1080/17476933.2020.1825397. URL: https://hal.inria.fr/hal-02969790.

Doctoral dissertations and habilitation theses

[25] A. de Laire. 'The Landau-Lifshitz equation and related models'. Université de Lille, 28th Oct. 2020. URL: https://hal.archives-ouvertes.fr/tel-02985356.

Reports & preprints

[26] N. Bedjaoui, V. Desveaux, O. Goubet and A. Masset. *Initial value problem for one-dimensional rotating shallow water equations*. 8th Mar. 2021. URL: https://hal.archives-ouvertes.fr/hal-03162689.

[27] A. Berg, D. Cohen and G. Dujardin. *Exponential integrators for the stochastic Manakov equation*. 15th May 2020. URL: https://hal.inria.fr/hal-02586778.

[28] A. Berg, D. Cohen and G. Dujardin. *Lie-Trotter Splitting for the Nonlinear Stochastic Manakov System*. 27th Oct. 2020. URL: https://hal.inria.fr/hal-02975684.

[29] G. Dujardin and I. Lacroix-Violet. *High order linearly implicit methods for evolution equations: How to solve an ODE by inverting only linear systems*. 22nd Oct. 2020. URL: https://hal.inria.fr/hal-02361814.

[30] S. López Martínez. *A blow-up approach for singular elliptic problems with natural growth in the gradient*. 12th Nov. 2020. URL: https://hal.archives-ouvertes.fr/hal-03001143.

[31] X. Xue and L. Zhao. *Moderate Deviations for the SSEP with a Slow Bond*. 18th Feb. 2021. URL: https://hal.archives-ouvertes.fr/hal-03145320.

[32] X. Xue and L. Zhao. *Non-equilibrium Fluctuations of the Weakly Asymmetric Normalized Binary Contact Path Process*. 8th Jan. 2021. URL: https://hal.archives-ouvertes.fr/hal-03103489.

8.3 Cited publications

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[35] M. Jara, T. Komorowski and S. Olla. 'Superdiffusion of energy in a chain of harmonic oscillators with noise'. working paper or preprint. 2014. URL: https://hal.archives-ouvertes.fr/hal-00997642.

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[39] S. Zhang, A. A. Baker, S. Komineas and T. Hesjedal. 'Topological computation based on direct magnetic logic communication'. In: *Scientific Reports* 5 (2015). URL: http://dx.doi.org/10.1038/srep15773.