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2021 ACTIVITY REPORT

Project-Team ANGE

Numerical Analysis, Geophysics and Environment

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Digital Health, Biology and Earth

THEME

Earth, Environmental and Energy Sciences

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Project-Team ANGE

Creation of the Project-Team: 2014 January 01

Keywords

Computer sciences and digital sciences

A6. - Modeling, simulation and control

A6.1. – Methods in mathematical modeling

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.1.4. - Multiscale modeling

A6.1.5. - Multiphysics modeling

A6.2. - Scientific computing, Numerical Analysis & Optimization

A6.2.1. - Numerical analysis of PDE and ODE

A6.2.6. - Optimization

A6.3. - Computation-data interaction

A6.3.2. – Data assimilation

A6.3.4. - Model reduction

A6.3.5. - Uncertainty Quantification

Other research topics and application domains

B3. - Environment and planet

B3.3. - Geosciences

B3.3.2. - Water: sea & ocean, lake & river

B3.3.3. - Nearshore

B3.4. – Risks

B3.4.1. - Natural risks

B3.4.3. - Pollution

B4. – Energy

B4.3. - Renewable energy production

B4.3.1. - Biofuels

B4.3.2. - Hydro-energy

1 Team members, visitors, external collaborators

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2 Overall objectives

2.1 Presentation

Among all aspects of geosciences, we mainly focus on gravity driven flows arising in many situations such as

- hazardous flows (flooding, rogue waves, landslides...),
- sustainable energies (hydrodynamics-biology coupling, biofuel production, marine energies...),
- risk management and land-use planning (morphodynamic evolutions, early warning systems...)

There exists a strong demand from scientists and engineers in fluid mechanics for models and numerical tools able to simulate not only the water depth and the velocity field but also the distribution and evolution of external quantities such as pollutants or biological species and the interaction between flows and structures (seashores, erosion processes...). The key point of the researches carried out within ANGE is to answer this demand by the development of efficient, robust and validated models and numerical tools.

2.2 Scientific challenges

Due to the variety of applications with a wide range of spatial scales, reduced-size models like the shallow water equations are generally required. From the modelling point of view, the main issue is to describe the behaviour of the flow with a reduced-size model taking into account several physical processes such as non-hydrostatic terms, biological species evolution, topography and structure interactions within the flow. The mathematical analysis of the resulting model do not enter the field of hyperbolic equations anymore and new strategies have to be proposed. Moreover, efficient numerical resolutions of reduced-size models require particular attention due to the different time scales of the processes and in order to recover physical properties such as positivity, conservativity, entropy dissipation and equilibria.

The models can remain subject to uncertainties that originate from incomplete description of the physical processes and from uncertain parameters. Further development of the models may rely on the assimilation of observational data and the uncertainty quantification of the resulting analyses or forecasts.

3 Research program

3.1 Overview

The research activities carried out within the ANGE team strongly couple the development of methodological tools with applications to real–life problems and the transfer of numerical codes. The main purpose is to obtain new models adapted to the physical phenomena at stake, identify the main properties that reflect the physical meaning of the models (uniqueness, conservativity, entropy dissipation, ...), propose effective numerical methods to approximate their solution in complex configurations (multi-dimensional,

unstructured meshes, well-balanced, ...) and to assess the results with data in the purpose of potentially correcting the models.

The difficulties arising in gravity driven flow studies are threefold.

- Models and equations encountered in fluid mechanics (typically the free surface Navier-Stokes
 equations) are complex to analyze and solve.
- The underlying phenomena often take place over large domains with very heterogeneous length scales (size of the domain, mean depth, wave length, ...) and distinct time scales, *e.g.* coastal erosion, propagation of a tsunami, ...
- These problems are multi-physics with strong couplings and nonlinearities.

3.2 Modelling and analysis

Hazardous flows are complex physical phenomena that can hardly be represented by shallow water type systems of partial differential equations (PDEs). In this domain, the research program is devoted to the derivation and analysis of reduced complexity models compared to the Navier-Stokes equations, but relaxing the shallow water assumptions. The main purpose is then to obtain models well-adapted to the physical phenomena at stake.

Even if the resulting models do not strictly belong to the family of hyperbolic systems, they exhibit hyperbolic features: the analysis and discretisation techniques we intend to develop have connections with those used for hyperbolic conservation laws. It is worth noticing that the need for robust and efficient numerical procedures is reinforced by the smallness of dissipative effects in geophysical models which therefore generate singular solutions and instabilities.

On the one hand, the derivation of the Saint-Venant system from the Navier-Stokes equations is based on two approximations (the so-called shallow water assumptions), namely

- the horisontal fluid velocity is well approximated by its mean value along the vertical direction,
- the pressure is hydrostatic or equivalently the vertical acceleration of the fluid can be neglected compared to the gravitational effects.

As a consequence the objective is to get rid of these two assumptions, one after the other, in order to obtain models accurately approximating the incompressible Euler or Navier-Stokes equations.

On the other hand, many applications require the coupling with non-hydrodynamic equations, as in the case of micro-algae production or erosion processes. These new equations comprise non-hyperbolic features and a special analysis is needed.

Multilayer approach As for the first shallow water assumption, *multi-layer* systems were proposed to describe the flow as a superposition of Saint-Venant type systems [39, 42, 43]. Even if this approach has provided interesting results, layers are considered separate and non-miscible fluids, which implies strong limitations. That is why we proposed a slightly different approach [40, 41] based on a Galerkin type decomposition along the vertical axis of all variables and leading, both for the model and its discretisation, to more accurate results.

A kinetic representation of our multilayer model allows to derive robust numerical schemes endowed with crucial properties such as: consistency, conservativity, positivity, preservation of equilibria, ... It is one of the major achievements of the team but it needs to be analyzed and extended in several directions namely:

- The convergence of the multilayer system towards the hydrostatic Euler system as the number of layers goes to infinity is a critical point. It is not fully satisfactory to have only formal estimates of the convergence and sharp estimates would provide an optimal number of layers.
- The introduction of several source terms due for instance to the Coriolis force or extra terms from changes of coordinates seems necessary. Their inclusion should lead to substantial modifications of the numerical scheme.

 Its hyperbolicity has not yet been proven and conversely the possible loss of hyperbolicity cannot be characterised. Similarly, the hyperbolic feature is essential in the propagation and generation of waves

Non-hydrostatic models The hydrostatic assumption consists in neglecting the vertical acceleration of the fluid. It is considered valid for a large class of geophysical flows but is restrictive in various situations where the dispersive effects (like wave propagation) cannot be neglected. For instance, when a wave reaches the coast, bathymetry variations give a vertical acceleration to the fluid that strongly modifies the wave characteristics and especially its height.

Processing an asymptotic expansion (w.r.t. the aspect ratio for shallow water flows) into the Navier-Stokes equations, we obtain at the leading order the Saint-Venant system. Going one step further leads to a vertically averaged version of the Euler/Navier-Stokes equations involving some non-hydrostatic terms. This model has several advantages:

- it admits an energy balance law (that is not the case for most dispersive models available in the literature),
- it reduces to the Saint-Venant system when the non-hydrostatic pressure term vanishes,
- it consists in a set of conservation laws with source terms,
- · it does not contain high order derivatives.

Multi-physics modelling The coupling of hydrodynamic equations with other equations in order to model interactions between complex systems represents an important part of the team research. More precisely, three multi-physics systems are investigated. More details about the industrial impact of these studies are presented in the following section.

- To estimate the risk for infrastructures in coastal zones or close to a river, the resolution of the shallow water equations with moving bathymetry is necessary. The first step consisted in the study of an additional equation largely used in engineering science: The Exner equation. The analysis enabled to exhibit drawbacks of the coupled model such as the lack of energy conservation or the strong variations of the solution from small perturbations. A new formulation is proposed to avoid these drawbacks. The new model consists in a coupling between conservation laws and an elliptic equation, like the Euler/Poisson system, suggesting to use well-known strategies for the analysis and the numerical resolution. In addition, the new formulation is derived from classical complex rheology models and allowed physical phenomena like threshold laws.
- Interaction between flows and floating structures is the challenge at the scale of the shallow water equations. This study requires a better understanding of the energy exchanges between the flow and the structure. The mathematical model of floating structures is very hard to solve numerically due to the non-penetration condition at the interface between the flow and the structure. It leads to infinite potential wave speeds that could not be solved with classical free surface numerical schemes. A relaxation model was derived to overcome this difficulty. It represents the interaction with the floating structure with a free surface model-type.
- If the interactions between hydrodynamics and biology phenomena are known through laboratory experiments, it is more difficult to predict the evolution, especially for the biological quantities, in a real and heterogeneous system. The objective is to model and reproduce the hydrodynamics modifications due to forcing term variations (in time and space). We are typically interested in phenomena such as eutrophication, development of harmful bacteria (cyanobacteria) and upwelling phenomena.

Data assimilation and inverse modelling In environmental applications, the most accurate numerical models remain subject to uncertainties that originate from their parameters and shortcomings in their physical formulations. It is often desirable to quantify the resulting uncertainties in a model forecast. The propagation of the uncertainties may require the generation of ensembles of simulations that ideally

sample from the probability density function of the forecast variables. Classical approaches rely on multiple models and on Monte Carlo simulations. The applied perturbations need to be calibrated for the ensemble of simulations to properly sample the uncertainties. Calibrations involve ensemble scores that compare the consistency between the ensemble simulations and the observational data. The computational requirements are so high that designing fast surrogate models or metamodels is often required.

In order to reduce the uncertainties, the fixed or mobile observations of various origins and accuracies can be merged with the simulation results. The uncertainties in the observations and their representativeness also need to be quantified in the process. The assimilation strategy can be formulated in terms of state estimation or parameter estimation (also called inverse modelling). Different algorithms are employed for static and dynamic models, for analyses and forecasts. A challenging question lies in the optimization of the observational network for the assimilation to be the most efficient at a given observational cost.

3.3 Numerical analysis

Non-hydrostatic scheme The main challenge in the study of the non-hydrostatic model is to design a robust and efficient numerical scheme endowed with properties such as: positivity, wet/dry interfaces treatment, consistency. It must be noticed that even if the non-hydrostatic model looks like an extension of the Saint-Venant system, most of the known techniques used in the hydrostatic case are not efficient as we recover strong difficulties encountered in incompressible fluid mechanics due to the extra pressure term. These difficulties are reinforced by the absence of viscous/dissipative terms.

Space decomposition and adaptive scheme In the quest for a better balance between accuracy and efficiency, a strategy consists in the adaptation of models. Indeed, the systems of partial differential equations we consider result from a hierarchy of simplifying assumptions. However, some of these hypotheses may turn out to be irrelevant locally. The adaptation of models thus consists in determining areas where a simplified model (*e.g.* shallow water type) is valid and where it is not. In the latter case, we may go back to the "parent" model (*e.g.* Euler) in the corresponding area. This implies to know how to handle the coupling between the aforementioned models from both theoretical and numerical points of view. In particular, the numerical treatment of transmission conditions is a key point. It requires the estimation of characteristic values (Riemann invariant) which have to be determined according to the regime (torrential or fluvial).

Asymptotic-Preserving scheme for source terms Hydrodynamic models comprise advection and sources terms. The conservation of the balance between source terms, typically viscosity and friction, has a significant impact since the overall flow is generally a perturbation around an equilibrium. The design of numerical schemes able to preserve such balances is a challenge from both theoretical and industrial points of view. The concept of Asymptotic-Preserving (AP) methods is of great interest in order to overcome these issues.

Another difficulty occurs when a term, typically related to the pressure, becomes very large compared to the order of magnitude of the velocity. At this regime, namely the so-called *low Froude* (shallow water) or *low Mach* (Euler) regimes, the difference between the speed of the gravity waves and the physical velocity makes classical numerical schemes inefficient: firstly because of the error of truncation which is inversely proportional to the small parameters, secondly because of the time step governed by the largest speed of the gravity wave. AP methods made a breakthrough in the numerical resolution of asymptotic perturbations of partial-differential equations concerning the first point. The second one can be fixed using partially implicit scheme.

Multi-physics models Coupling problems also arise within the fluid when it contains pollutants, density variations or biological species. For most situations, the interactions are small enough to use a splitting strategy and the classical numerical scheme for each sub-model, whether it be hydrodynamic or non-hydrodynamic.

The sediment transport raises interesting issues from a numerical aspect. This is an example of coupling between the flow and another phenomenon, namely the deformation of the bottom of the basin that can be carried out either by bed load where the sediment has its own velocity or suspended load in which the particles are mostly driven by the flow. This phenomenon involves different time scales and nonlinear retroactions; hence the need for accurate mechanical models and very robust numerical methods. In collaboration with industrial partners (EDF–LNHE), the team already works on the improvement of numerical methods for existing (mostly empirical) models but our aim is also to propose new (quite) simple models that contain important features and satisfy some basic mechanical requirements. The extension of our 3D models to the transport of weighted particles can also be here of great interest.

Optimisation Numerical simulations are a very useful tool for the design of new processes, for instance in renewable energy or water decontamination. The optimisation of the process according to a well-defined objective such as the production of energy or the evaluation of a pollutant concentration is the logical upcoming challenge in order to propose competitive solutions in industrial context. First of all, the set of parameters that have a significant impact on the result and on which we can act in practice is identified. Then the optimal parameters can be obtained using the numerical codes produced by the team to estimate the performance for a given set of parameters with an additional loop such as gradient descent or Monte Carlo method. The optimisation is used in practice to determine the best profile for turbine pales, the best location for water turbine implantation, in particular for a farm.

4 Application domains

4.1 Overview

Sustainable development and environment preservation have a growing importance and scientists have to address difficult issues such as: management of water resources, renewable energy production, bio/geo-chemistry of oceans, resilience of society w.r.t. hazardous flows, urban pollutions, ...

As mentioned above, the main issue is to propose models of reduced complexity, suitable for scientific computing and endowed with stability properties (continuous and/or discrete). In addition, models and their numerical approximations have to be confronted with experimental data, as analytical solutions are hardly accessible for these problems/models. A. Mangeney (IPGP) and N. Goutal (EDF) may provide useful data.

4.2 Geophysical flows

Reduced models like the shallow water equations are particularly well-adapted to the modelling of geophysical flows since there are characterized by large time or/and space scales. For long time simulations, the preservation of equilibria is essential as global solutions are a perturbation around them. The analysis and the numerical preservation of non-trivial equilibria, more precisely when the velocity does not vanish, are still a challenge. In the fields of oceanography and meteorology, the numerical preservation of the so-called geostrophic state, which is the balance between the gravity field and the Coriolis force, can significantly improve the forecasts. In addition, data assimilation is required to improve the simulations and correct the dissipative effect of the numerical scheme.

The sediment transport modelling is of major interest in terms of applications, in particular to estimate the sustainability of facilities with silt or scour, such as canals and bridges. Dredging or filling-up operations are expensive and generally not efficient in the long term. The objective is to determine a configuration almost stable for the facilities. In addition, it is also important to determine the impact of major events like emptying dam which is aimed at evacuating the sediments in the dam reservoir and requires a large discharge. However, the downstream impact should be measured in terms of turbidity, river morphology and flood.

4.3 Hydrological disasters

It is a violent, sudden and destructive flow. Between 1996 and 2005, nearly 80% of natural disasters in the world have meteorological or hydrological origines. The main interest of their study is to predict the areas in which they may occur most probably and to prevent damages by means of suitable amenities. In France, floods are the most recurring natural disasters and produce the worst damages. For example, it can be a cause or a consequence of a dam break. The large surface they cover and the long period they can last require the use of reduced models like the shallow water equations. In urban areas, the flow can be largely impacted by the debris, in particular cars, and this requires fluid/structure interactions be well understood. Moreover, underground flows, in particular in sewers, can accelerate and amplify the flow. To take them into account, the model and the numerical resolution should be able to treat the transition between free surface and underground flows.

Tsunamis are another hydrological disaster largely studied. Even if the propagation of the wave is globally well described by the shallow water model in oceans, it is no longer the case close to the epicenter and in the coastal zone where the bathymetry leads to vertical accretions and produces substantial dispersive effects. The non-hydrostatic terms have to be considered and an efficient numerical resolution should be induced.

While viscous effects can often be neglected in water flows, they have to be taken into account in situations such as avalanches, debris flows, pyroclastic flows, erosion processes, ... *i.e.* when the fluid rheology becomes more complex. Gravity driven granular flows consist of solid particles commonly mixed with an interstitial lighter fluid (liquid or gas) that may interact with the grains and decrease the intensity of their contacts, thus reducing energy dissipation and favoring propagation. Examples include subaerial or subaqueous rock avalanches (e.g. landslides).

4.4 Biodiversity and culture

Nowadays, simulations of the hydrodynamic regime of a river, a lake or an estuary, are not restricted to the determination of the water depth and the fluid velocity. They have to predict the distribution and evolution of external quantities such as pollutants, biological species or sediment concentration.

The potential of micro-algae as a source of biofuel and as a technological solution for CO2 fixation is the subject of intense academic and industrial research. Large-scale production of micro-algae has potential for biofuel applications owing to the high productivity that can be attained in high-rate raceway ponds. One of the key challenges in the production of micro-algae is to maximize algae growth with respect to the exogenous energy that must be used (paddlewheel, pumps, ...). There is a large number of parameters that need to be optimized (characteristics of the biological species, raceway shape, stirring provided by the paddlewheel). Consequently our strategy is to develop efficient models and numerical tools to reproduce the flow induced by the paddlewheel and the evolution of the biological species within this flow. Here, mathematical models can greatly help us reduce experimental costs. Owing to the high heterogeneity of raceways due to gradients of temperature, light intensity and nutrient availability through water height, we cannot use depth-averaged models. We adopt instead more accurate multilayer models that have recently been proposed. However, it is clear that many complex physical phenomena have to be added to our model, such as the effect of sunlight on water temperature and density, evaporation and external forcing.

Many problems previously mentioned also arise in larger scale systems like lakes. Hydrodynamics of lakes is mainly governed by geophysical forcing terms: wind, temperature variations, . . .

4.5 Sustainable energy

One of the booming lines of business is the field of renewable and decarbonated energies. In particular in the marine realm, several processes have been proposed in order to produce electricity thanks to the recovering of wave, tidal and current energies. We may mention water-turbines, buoys turning variations of the water height into electricity or turbines motioned by currents. Although these processes produce an amount of energy which is less substantial than in thermal or nuclear power plants, they have smaller dimensions and can be set up more easily.

The fluid energy has kinetic and potential parts. The buoys use the potential energy whereas the water-turbines are activated by currents. To become economically relevant, these systems need to be optimized in order to improve their productivity. While for the construction of a harbour, the goal is to minimize swell, in our framework we intend to maximize the wave energy.

This is a complex and original issue which requires a fine model of energy exchanges and efficient numerical tools. In a second step, the optimisation of parameters that can be changed in real-life, such as bottom bathymetry and buoy shape, must be studied. Eventually, physical experiments will be necessary for the validation.

4.6 Urban environment

The urban environment is essentially studied for air and noise pollutions. Air pollution levels and noise pollution levels vary a lot from one street to next. The simulations are therefore carried out at street resolution and take into account the city geometry. The associated numerical models are subject to large uncertainties. Their input parameters, e.g. pollution emissions from road traffic, are also uncertain. Quantifying the simulation uncertainties is challenging because of the high computational costs of the numerical models. An appealing approach in this context is the use of metamodels, from which ensembles of simulations can be generated for uncertainty quantification.

The simulation uncertainties can be reduced by the assimilation of fixed and mobile sensors. High-quality fixed monitoring sensors are deployed in cities, and an increasing number of mobile sensors are added to the observational networks. Even smartphones can be used as noise sensors and dramatically increase the spatial coverage of the observations. The processing and assimilation of the observations raises many questions regarding the quality of the measurements and the design of the network of sensors.

4.7 SmartCity

There is a growing interest for environmental problems at city scale, where a large part of the population is concentrated and where major pollutions can occur. Numerical simulation is well established to study the urban environment, *e.g.* for road traffic modelling. As part of the smartcity movement, an increasing number of sensors collect measurements, at traditional fixed observation stations, but also on mobile devices, like smartphones. They must properly be taken into account given their number but also their potential low quality.

Pratical applications include air pollution and noise pollution. These directly relate to road traffic. Data assimilation and uncertainty propagation are key topics in these applications.

5 Social and environmental responsibility

5.1 Footprint of research activities

Only few travels were done last year (including one flight), due to the sanitary crisis but also because of a will of the team to avoid this type of transportation.

5.2 Impact of research results

Part of ANGE activity is devoted to research on renewable energy. In this way, J. Salomon (with J. Ledoux and S. Riffo, former members of the team) submitted a paper about turbine design [21].

6 Highlights of the year

Two ANR projects have been accepted by the team this year (ANR DEEPNUM-16161 and ANR-SAPHIR-16213).

7 New results

7.1 Numerical methods

7.1.1 Simulation of Complex Free Surface Flows

Participants: Yohan Penel.

The purpose of [10] is to model incompressible shallow free-surface flows, while taking into account the effects of non-hydrostatic pressure and dispersive phenomena that play an important role especially in coastal oceanography. Several numerical schemes are presented, based on a splitting method, separating the hyperbolic part (prediction) and the non-hydrostatic part (correction). The algorithms are compared in computation time through simulations with solitary waves.

7.1.2 Numerical approximation of the shallow water equations with Coriolis source term

Participants: Emmanuel Audusse, Virgile Dubos, Yohan Penel.

We investigate in [8] a class of numerical schemes dedicated to the non-linear Shallow Water equations with topography and Coriolis force. The proposed algorithms rely on Finite Volume approximations formulated on collocated and staggered meshes, involving appropriate diffusion terms in the numerical fluxes, expressed as discrete versions of the linear geostrophic balance. It follows that, contrary to standard Finite-Volume approaches, the linear versions of the proposed schemes provide a relevant approximation of the geostrophic equilibrium. We also show that the resulting methods ensure semi-discrete energy estimates. Numerical experiments exhibit the efficiency of the approach in the presence of Coriolis force close to the geostrophic balance, especially at low Froude number regimes.

7.1.3 On the method of reflections

Participants: Julien Salomon.

Reference [19] aims at reviewing and analysing the method of reflections. The latter is an iterative procedure designed to linear boundary value problems set in multiply connected domains. Being based on a decomposition of the domain boundary, this method is particularly well-suited to numerical solvers relying on integral representation formulas. For the parallel and sequential forms of the method appearing in the literature, we propose a general abstract formulation in a given Hilbert setting and interpret the procedure in terms of subspace corrections. We then prove the unconditional convergence of the sequential form and propose a modification of the parallel one that makes it unconditionally converging. An alternative proof of convergence is provided in a case which does not fit into the previous framework. We finally present some numerical tests.

7.2 Modelling

7.2.1 Explicit solutions to a free interface model for the static/flowing transition in thin granular flows

Participants: Anne Mangeney.

The reference [22] is devoted to an analytical description of the dynamics of the static/flowing interface in thin dry granular flows. Our starting point is the asymptotic model derived by Bouchut et al. (2016) from a free surface incompressible model with viscoplastic rheology including a Drucker–Prager yield stress. This asymptotic model is based on the thin-layer approximation (the flow is thin in the direction normal to the topography compared to its down-slope extension), but the equations are not depth-averaged. In addition to the velocity, the model includes a free surface at the top of the flow and a free time-dependent static/flowing interface at the bottom. In the present work, we simplify this asymptotic model by decoupling the space coordinates, and keeping only the dependence on time and on the normal space coordinate z. We introduce a time- and z-dependent source term, assumed here to be given, which represents the opposite of the net force acting on the flowing material, including gravity, pressure gradient, and internal friction. We prove several properties of the resulting simplified model that has a time- and Z-dependent velocity and a time-dependent static/flowing interface as unknowns. The crucial advantage of this simplified model is that it can provide explicit solutions in the inviscid case, for different shapes of the source term. These explicit inviscid solutions exhibit a rich behaviour and qualitatively reproduce some physical features observed in granular flows.

7.2.2 Analysis of the Blade Element Momentum Theory

Participants: Julien Salomon.

The blade element momentum (BEM) theory introduced by Lock et al. and formulated in its modern form by Glauert provides a framework to model the aerodynamic interaction between a turbine and a fluid flow. This theory is used either to estimate turbine efficiency or as a design aid. However, a lack of mathematical interpretation limits the understanding of some of its issues. The aim of [21] is to propose an analysis of BEM equations. Our approach is based on a reformulation of Glauert's model which enables us to identify criteria to guarantee the existence of solutions, analyze the convergence of usual and new (and more efficient) solution algorithms, and study turbine design procedures. The mathematical analysis is completed by numerical experiments.

7.2.3 Congested shallow water model: on floating body

Participants: Jacques Sainte-Marie, Edwige Godlewski.

In [17], we are interested in the numerical modeling of body floating freely on the water such as icebergs or wave energy converters. The fluid-solid interaction is formulated using a congested shallow water model for the fluid and Newton's second law of motion for the solid. We make a particular focus on the energy transfer between the solid and the water since it is of major interest for energy production. A numerical approximation based on the coupling of a nite volume scheme for the fluid and a Newmark scheme for the solid is presented. An entropy correction based on an adapted choice of discretization for the coupling terms is made in order to ensure a dissipation law at the discrete level. Simulations are presented to verify the method and to show the feasibility of extending it to more complex cases

7.2.4 A homogeneous relaxation low Mach number model

Participants: Yohan Penel.

In [16], we study a new homogeneous relaxation model that describes the behavior of a two-phase fluid in the low Mach number regime. This model can be obtained as the asymptotic limit at low Mach number of the HRM model. For this model, we define an equation of state describing the thermodynamics of the two-phase fluid. We show some theoretical properties verified by the solutions of the model, and

introduce an equilibrium scheme. Then, we focus on the instantaneous relaxation regime, and show the formal convergence of this model to the low Mach number approximation of the HEM model. We introduce an asymptotic preserving scheme allowing numerical simulations of the spatial coupling between two regions with different characteristic relaxation times.

7.2.5 Optimization of Bathymetry for Long Waves with Small Amplitude

Participants: Julien Salomon.

Reference [15] deals with bathymetry-oriented optimization in the case of long waves with small amplitude. Under these two assumptions, the free-surface incompressible Navier–Stokes system can be written as a wave equation, where the bathymetry appears as a parameter in the spatial operator. Looking then for time-harmonic fields and writing the bathymetry, i.e., the bottom topography, as a perturbation of a flat bottom, we end up with a heterogeneous Helmholtz equation with an impedance boundary condition. In this way, we study a PDE-constrained optimization problem for a Helmholtz equation in heterogeneous media whose coefficients are only bounded with bounded variation. We provide necessary condition for a general cost function to have at least one optimal solution. We also prove the convergence of a finite element approximation of the solution to the considered Helmholtz equation as well as the convergence of the discrete optimum toward the continuous ones. We end this paper with some numerical experiments to illustrate the theoretical results and show that some of their assumptions are necessary.

7.2.6 Some analytical solutions for validation of free surface flow computational codes

Participants: Jacques Sainte-Marie, Bernard Di Martino, Marie-Odile Bristeau,

Anne Mangeney.

In [12], we present several time dependent analytical solutions for the incompressible Euler system with free surface. These analytical solutions give quantitative descriptions of some physical phenomena, such as water motion or waves on space variable topography, and can be used as reference solutions when validating numerical simulation codes. They concern fluid flows governed by Euler equations with or without hydrostatic hypothesis including wet/dry interface, variable density and wide variety of boundary conditions.

7.2.7 Pseudo-compressibility, dispersive model and acoustic waves in shallow water flows

Participants: Marie-Odile Bristeau, Edwige Godlewski, Anne Mangeney,

Jacques Sainte-Marie.

In [11], we study a dispersive shallow water type model derived from the free surface compressible Navier-Stokes system. The compressible effects allow to capture the acoustic-like waves propagation and can be seen as a relaxation of an underlying incompressible model. The first interest of such a model is thus to capture both acoustic and water waves. The second interest lies in its numerical approximation. Indeed, at the discrete level, the pseudo-compressibility terms circumvent the resolution of an elliptic equation to capture the non-hydrostatic part of the pressure. This drastically reduces the cost of the numerical resolution of dispersive models especially in 2d and 3d.

7.2.8 Asymptotic derivation and simulations of a non-local Exner model in large viscosity regime

Participants: Emmanuel Audusse.

The paper [7] deals with the modeling and numerical approximation of bed load transport under the action of water. A new shallow water type model is derived from the stratified two-fluid Navier–Stokes equations. Its novelty lies in the magnitude of a viscosity term that leads to a momentum equation of elliptic type. The full model, sediment and water, verifies a dissipative energy balance for smooth solutions. The numerical resolution of the sediment layer is not trivial since the viscosity introduces a non-local term in the model. Adding a transport threshold makes the resolution even more challenging. A scheme based on a staggered discretization is proposed for the full model, sediment and water.

7.3 Assessments of models by means of experimental data and assimilation

7.3.1 Analysis of a greedy reconstruction algorithm

Participants: Julien Salomon.

In [13], a novel and detailed convergence analysis is presented for a greedy algorithm that was introduced in [Y. Maday and J. Salomon, Joint Proceedings of the 48th IEEE Conference on Decision and Control and the 28th Chinese Control Conference, 2009, pp. 375–379] for operator reconstruction problems in the field of quantum mechanics. This algorithm is based on an offline/online decomposition of the reconstruction process and on an ansatz for the unknown operator obtained by an a priori chosen set of linearly independent matrices. The presented convergence analysis focuses on linear-quadratic (optimization) problems governed by linear differential systems and reveals the strong dependence of the performance of the greedy algorithm on the observability properties of the system and on the ansatz of the basis elements. Moreover, the analysis allows us to use a precise (and in some sense optimal) choice of basis elements for the linear case and led to the introduction of a new and more robust optimized greedy reconstruction algorithm. This optimized approach also applies to nonlinear Hamiltonian reconstruction problems, and its efficiency is demonstrated by numerical experiments.

7.3.2 Novel method for a posteriori uncertainty quantification in wildland fire spread simulation

Participants: Frédéric Allaire, Vivien Mallet.

In [6], simulation is used to predict the spread of a wildland fire across land in real-time. Nevertheless, the large uncertainties in these simulations must be quantified in order to provide better information to fire managers. Ensemble forecasts are usually applied for this purpose, with an input parameter distribution that is defined based on expert knowledge. A novel approach is proposed in order to generate calibrated ensembles whose input distribution is defined by a posterior PDF with a pseudo-likelihood function that involves the Wasserstein distance between simulated and observed burned surfaces of several fire cases. Due to the high dimension and the computational requirements of the pseudo-likelihood function, a Gaussian process emulator is built to obtain a sample of the calibrated input distribution with a MCMC algorithm in about one day of computation on 8 computing cores. The calibrated ensembles lead to better overall accuracy than the uncalibrated ensembles. The a posteriori probability distribution of the inputs favors lower values of rate of spread and lower uncertainty in wind direction. This strongly limits overprediction, while keeping the ability of the ensemble to cover the observed burned area.

7.3.3 Global sensitivity analysis for road traffic noise modelling

Participants: Vivien Mallet.

Regulatory road traffic noise maps are based on input data that are sometimes incomplete, erroneous or non-existent. When designing them, it is therefore necessary to label and qualify these data by giving priority to certain sources of information and certain parameters over others. Beforehand, a sensitivity analysis of the sound prediction model to these input parameters should be carried out to concentrate efforts on the most influential inputs (either physical or configurational parameters). In [9], an overall sensitivity analysis of the CNOSSOS-EU model is proposed using the Morris screening method. It is conducted using the open source software Noise Modelling, on a case study of a French city. The analysis is performed on 15 of its input parameters at 14343 receivers. The selection of the parameters and their ranges of variation were chosen to mimic those faced by an operator when producing monthly noise maps. Whether or not to consider diffraction at the horizontal edges appears to be the most influential parameter when estimating the number of people exposed to sound levels above 65 dB(A). However, a finer analysis by receiver shows how the influence of each parameter strongly depends on the source-receiver configuration.

7.3.4 Uncertainty study on atmospheric dispersion simulations using meteorological ensembles with a Monte Carlo approach, applied to the Fukushima nuclear accident

Participants: Vivien Mallet.

In emergency cases, when nuclear accidental releases take place, numerical models, developed by IRSN (French Institute of Radiation Protection and Nuclear Safety), are used to forecast the atmospheric dispersion of radionuclides. These models compute the quantity of radionuclides in the atmosphere, their deposited amount on the ground, and the subsequent gamma dose rate. Their results are used to make recommendations to protect the population in case of nuclear accident. However, the simulations are subject to considerable uncertainties. These uncertainties originate from different sources: input variables (weather forecasting, source term), physical parameters used in the models (turbulent diffusion, scavenging coefficient, deposition velocity, etc.) and model approximations (representativeness and numerical errors). Our work [20] presents the propagation of input uncertainties through an Eulerian radionuclide transport model, ldX, applied to the Fukushima nuclear disaster. This uncertainty propagation involves perturbing the input variables and making numerous calls to the model. The perturbations should be broad enough to cover the possible range of variation of uncertain variables. Weather forecast ensembles are used to take into account meteorological uncertainties, and several source terms from the literature are included. The following step is to evaluate the spread of the outputs in order to draw insights about the subsequent uncertainties. In order to assess the quality of the ensemble of simulations, comparisons with radiological observations were carried out, using statistical indicators, both deterministic such as RMSE (Root Mean Square Error) or FMS (Figure of Merit in Space), and probabilistic indicators such as rank histograms, Brier score and DRPS.

7.4 Software Devlopments

Participants: Jacques Sainte-Marie, Apolline El Baz.

Members: A. El Baz, J. Sainte-Marie

Several improvements of FreshKiss3D software have been made:

1. upgrade installation for all versions of python 3 on mac and linux system;

- work on Coriolis force, currently under development;
- 3. inclusion of binary files as outputs / accurate the writting and the reading of outputs;

8 Bilateral contracts and grants with industry

Participants: Yohan Penel.

Yohan Penel supervises the PhD thesis of Giuseppe Parasiliti about the Physical, mathematical and numerical modelling of a gas flow for the transportation of liquified natural gas. This work is the result of a close collaboration with the corporation GTT, which has already collaborated with ANGE in the last years, through the Carnot institute SMILE.

Participants: Jacques Sainte-Marie.

Jacques Sainte-Marie has a contract with Eaux de Paris about Hydraulic modeling, calibration and diagnosis. (2020-2023, with S. Labbé, Laboratoire D'Alembert and LPSM)

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Associate Teams in the framework of an Inria International Lab or in the framework of an Inria International Program

OCEANIA

Title: Intelligence Artificielle, Données et Modèles pour Comprendre les Océans et le Changement Climatique

Partner Institution(s): • ANGE, BIOCORE, TAU Inria teams,

- Universidad de Chile (Center of Mathematical Modeling), Chile
- Pontificia Universidad Catól ica de Chile, Chile
- Fondation TARA Océan, France,
- GO-SEE CNRS Federation, France
- Université de Nantes (ComBi team)

Date/Duration: 11.2020–10.2024

Additionnal info/keywords: Artificial Intelligence and Modeling for Understanding Oceans and Climate Change

There is strong scientific evidence of the negative effects of climate change on the ocean. These changes will have a drastic impact on nearly all life forms in the ocean, as well as additional consequences for food security and ecosystems in coastal communities as well as inland. Despite these impacts, scientific data and infrastructure are still lacking to better understand and quantify the consequences of these disturbances on the marine ecosystem. There is a need not only to collect more data, but also to develop and apply state-of-the-art mechanisms capable of transforming this data into real knowledge, policy, and action. This is where artificial intelligence, machine learning and modeling tools are needed. OceanIA, this ambitious interdisciplinary Inria Challenge, aims to develop new artificial intelligence and mathematical modeling tools to contribute to the understanding of the structure, functioning, and underlying mechanisms and dynamics of the Ocean and its role in regulating and sustaining the biosphere and fighting climate change. OceanIA is also an opportunity to structure Inria's contributions around a global scientific challenge in the convergence of artificial intelligence, biodiversity and climate change.

9.1.2 Visits to international teams

Research stays abroad

Yohan Penel

Visited institution: University of Sevilla

Country: Spain

Dates: 6-17 december

Context of the visit: collaboration with E. Fernandez-Nieto

Mobility program/type of mobility: research stay

9.2 National initiatives

Geosophy (2020-2021)

Participants: Cindy Guichard.

• Project acronym: Geosophy

• Project title: Geosophy

• Coordinator: Karine Laurent (Institut Carnot SMILES)

• Funding: 35 00 euros.

Simulation and Code development for geonergy

Projet Emergence ALARM (2018-2022)

Participants: Jacques Sainte-Marie, Apolline El-Baz.

· Project acronym: ALARM

• Project title: Alboran sea submarine landslides

• Coordinator: Sara Lafuerza (ISTeP - UMR 7193 Institut des Sciences de la Terre de Paris)

• Funding: 55 00 euros.

Simulation et étude de glissements de terrain et tsunami dans la mer d'Alboran

équipe junior ISCD (2019-2022)

Participants: Jacques Sainte-Marie, Nina Aguillon, Sybille Téchène, Julien Guillod.

• Project acronym: Andiamo

• Project title: Andiamo

• Coordinator: N. Aguillon, S. Téchène, J. Deshayes (SU)

• Funding: 70 000 euros.

The ANDIAMO project brings together mathematicians and oceanographers from SU around long term global ocean models. The main challenge is that the mesh size and time step are large, yielding non-negligible truncation errors and schemes dominated by numerical diffusion. Importantly, solutions for regional simulations do not transfer to our needs, as small errors in water mass characteristics have large impact on long term simulations. Thus "non-converged" methods and numerical analysis on coarse mesh are needed.

Another important part of the project is to establish a dialogue and to build further collaborations between mathematicians and oceanographers, around questions arising in climatology that require new numerical methods (interactions with continental ice, quantification of uncertainties...).

Projet Emergence (2021-2023)

Participants: Julien Guillod.

· Project acronym: Emergence

• Project title: Etudes numériques d'équations fluides

• Coordinator: Julien Guillod (SU)

• Funding: 28 000 euros.

ANR MFG (2016-2021)

Participants: Julien Salomon.

• Project acronym: MFG

• Project title: Mean Field Games

• Coordinator: Pierre Cardaliaguet (CEREMADE - Université Paris-Dauphine)

• Funding: 299 160 euros.

Mean field game theory (MFG) is a new and active field of mathematics, which analyses the dynamics of a very large number of agents. Introduced about ten years ago, MFG models have been used in different fields: economics, finance, social sciences, engineering,... MFG theory is at the intersection of mean field theory, mathematical game theory, optimal control, stochastic analysis, variation calculation, partial differential equations and scientific calculation. Drawing on an internationally recognized French team on the subject, the project seeks to obtain major contributions in 4 main directions: the "medium field" aspect (i.e., how to obtain macroscopic models from microscopic models); the analysis of new MFG systems; their numerical analysis; the development of new applications. In this period of rapid expansion of MFG models, the project seeks to foster French leadership in the field and attract new researchers from related fields.

ANR ALLOWAPP (2019-2023)

Participants: Julien Salomon.

• Project acronym: ALLOWAPP

- Project title: Algorithmes pour l'optimisation à grande échelle de problèmes de propagation d'ondes
- Coordinator: Laurence Halpern (Université Paris-Nord)
- Funding: 317 891 euros.

The goal of the ALLOWAPP project is the design of space-time parallel algorithms for large-scale optimization problems associated with wave propagation phenomena. Such problems appear in seismology, geophysics, but also in various applications from data assimilation. The large amount of data and the volume of computations required for the accurate numerical solution of wave propagation problems, within an optimization loop, requires the use of massively parallel computers. Time-parallel methods have experienced a great development in the last ten years, and for parabolic problems an almost perfect efficiency for a large number of processors has been achieved (scalability). It is quite different for wave propagation problems. In this project, we propose to develop robust, efficient and scalable methods for space-time parallelization of these optimization problems.

ANR GeoFun (2020-2024)

Participants: Nina Aguillon.

- · Project acronym: GeoFun
- Project title: Ecoulements géophysiques avec des modèles unifiés
- Coordinator: Martin Parisot (INRIA Bordeaux Sud-Ouest)
- Funding: 524 880 euros.

The GeoFun project aims to improve the modeling and simulation of geophysical flows involving at least two different processes. Numerical simulation of watersheds and estimation of water resources is the main application of the project's achievements. In this context, a free surface flow (rivers, lakes) is the upper part of a groundwater flow (water table). Our vision of river transport is often naive, because we think first of rivers, lakes and floods, but in reality, 80 % of the water of the continents is underground. Sometimes, the porous substratum is covered by an impermeable rock stratum, which confines the flow as in pipes, except for some points where springs and resurgences appear.

ANR SingFlows (2019-2023)

Participants: Julien Guillod.

- Project acronym: SingFlows
- Project title: Ecoulements avec singularités : couches limites, filaments de vortex, interaction vague-structure
- Coordinator: David Gerard-Varet (Institut de mathématiques de Jussieu Paris Rive Gauche)
- Funding: 263 628 euros.

The objective of SingFlows is to develop mathematical and numerical tools for the analysis of three problems in fluid dynamics: the behaviour of anisotropic flows (boundary layers, shallow water flows), the dynamics of vortical structures, and the evolution of fixed or floating structures in water waves. Our will to unify these different problems is natural, because they share many mathematical features. The underlying keypoint is that they are described by singular solutions of Euler or Navier-Stokes equations.

The word singular refers here: - either to a lack of smoothness: it applies for instance to vortex filaments, which are Dirac masses along curves, or to the contact line between water and the floating structure, - or to a singular dependence of the solution with respect to a parameter, typically the Reynolds number (like in boundary layers). The connection between the two points of view is usually made by viscous regularization of the non-smooth structure, or conversely by taking the vanishing limit of the parameter. More generally, the three problems considered in SingFlows involve flows with very small scales. A relevant description then requires the derivation of reduced models.

ANR Top-up (2021-2024)

Participants: Cindy Guichard.

- Project acronym: Top-up
- · Project title: High-resolution topography upscaling for overland flows
- Coordinator: Konstantin Brenner (UNIVERSITE COTE D'AZUR Laboratoire Jean-Alexandre Dieudonné)
- Funding: 248 335 euros.

The objective of the project is to design efficient DD and Ms methods adapted to multi-scale free-surface flow problems, to implement them in the form of an HPC code, and finally to validate them on a set of tests based on realistic high-resolution topographic data. The last objective will be achieved through a close collaboration with the Nice Côte d'Azur Metropolis

GdR EGRIN (2017-2021)

Participants: Emmanuel Audusse, , Bernard di Martino, , Nicole Goutal, ,

Cindy Guichard, , Anne Mangeney, , Martin Parisot, Jacques Sainte-

Marie.

EGRIN stands for Gravity-driven flows and natural hazards. J. Sainte-Marie is the head of the scientific committee of this CNRS research group and A. Mangeney is a member of the committee. Other members of the team involved in the project are local correspondents. The scientific goals of this project are the modelling, analysis and simulation of complex fluids by means of reduced-complexity models in the framework of geophysical flows.

GdR EOL-EMR (2021-2026)

Participants: Julien Salomon, Jacques Sainte-Marie.

OBJECTIVES: To promote the dissemination of existing knowledge and expertise within and across disciplines. The GDR EMR is a forum for the exchange of expertise and know-how within and across disciplines. To promote the implementation of collaborations, between partners of the GDR and with the industrial fabric. The GDR is an entry and orientation point. It provides a forum for the exchange of information concerning industrial needs and the skills of the academic community; and enables the bringing together of players. Valuing the national scientific community The GDR EMR gives visibility to the community, in particular through the development of a mapping of the actors and themes available on the web platform

ANR FireCaster (2017-2021)

Participants: Frédéric Allaire, Vivien Mallet.

• ANR project call: DS0104

· Project acronym: FireCaster

• Project title: Plateforme de prévision incendie et de réponse d'urgence

• Coordinator: Jean-Baptiste Filippi (Univ. Corse)

• Funding: 442k euros

The goal of the FireCaster project is to prototype a fire decision support system at the national scale to estimate upcoming fire risk (H+24 to H+48) and in case of crisis, to predict fire front position and local pollution (H+1 to H+12).

ANR CENSE (2017-2021)

Participants: Antoine Lesieur, Vivien Mallet.

ANR project call: DS0601

• Project acronym: CENSE

• Project title: Caractérisation des environnements sonores urbains : vers une approche globale associant données libres, mesures et modélisations

• Coordinator: Judicaël Picaut (IFSTTAR)

• Funding: 856k euros

The CENSE project aims at proposing a new methodology for the production of more realistic noise maps, based on an assimilation of simulated and measured data through a dense network of low-cost sensors.

ANR RAVEX (2016-2021)

Participants: Anne Mangeney.

• ANR project call: DS0106

· Project acronym: RAVEX

• Project title: Développement d'une approche intégrée pour la réduction des Risques Associés au Volcanisme EXplosif, de la recherche sur l'aléa aux outils de gestion de crise : le cas de la Martinique

• Coordinator: Olivier Roche (IRD)

• Funding: 619k euros

PGMO Project ORACLE (2019-2021)

Participants: Julien Salomon.

• PGMO Call

• Project acronym: Oracle

• Project title: Optimal Resource Allocation in micro-organisms under Changing Environment

• Coordinator: Térence Bayen

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Member of the conference program committees Edwige Godlewski belongs to the scientific comittee of 2020 KES 50 (fifty years of Kruzhkov entropy solutions, and beyond)

Seminar organization

- Julien Salomon Co-organizes the seminar "Rencontres INRIA-JLL en analyse numérique et calcul scientifique" (INRIA-SU)
- Julien Guillod Co-organizes the seminar "Analyse non-linéaire et EDP" (ENS-Paris)
- Julien Guillod Co-organizes the seminar "Infomath" (ENS-Paris)
- Nina Aguillon Co-organizes "Journée interne du LJLL 2021" (SU)

10.1.2 Journal

Member of the editorial boards Julien Salomon is editor-in-chief of MATAPLI (Journal of the french Applied Mathematics Community)

Reviewer - reviewing activities Members of ANGE had review activities in SIAM SISC, AIMS Maths, JSSC Philosophical Transactions of the Royal Society A, Communications in Mathematical Sciences, Zeitschrift für Angewandte Mathematik und Mechanik, Communications in Nonlinear Science and Numerical Simulation Communication in Mathematical Sciences, Journal of Fluid Mechanics Computational Methods in Applied Mathematics (CMAM), Journal of Scientific Computing (JOMP), SN Partial Differential Equations and Applications (PDEA).

10.1.3 Invited talks

- 2021 CRM Summer School Quebec (visio) 31.05.21-04.06.21 (JS)
- Summer school on advanced DD methods Milan 24.11.21-26.11.21 (JS)
- Fudan International Seminar on Analysis, PDEs, and Fluid mechanics Shanghai (visio) 20.05.21 (IG)
- Infomath Création de sites web Paris 04.11.21 (JG)
- Séminaire Modélisation, Analyse et Calculs Toulouse (visio) 18/05/21 (NA,LL)

- ACC2021 New Orleans (visio) 25.05.21-28.05.21 (LL)
- 8ème EGRIN Ecole Visio 28/05/21 (LL)
- ADCHEM2021 Venice (visio) 13.06.21-18.06.21 (LL)
- Séminaire d'analyse numérique Rennes (visio) 02/12/2021 (CG)
- SMAI 2021 Toulouse 24/06 (NB)
- PDEFM2021 Liban (visio) 16/06 (NB)
- Séminaire d'EDP Strasbourg 09/11 (MR)
- PDEFM2021 Liban (visio) 16/06 (CEH)
- Waves and/versus advection Asymptotic expansion, analytical solutions & hyperbolicity Oxford univ. 15/01 (JSM)
- Journée "Parallélisme et Contrôle" Paris 13 10/09 (LP)
- Journée "Parallélisme et Contrôle" Paris 13 10/09 (NT)

Legend: JS=Julien Salomon, JG=Julien Guillod, NA=Nina Aguillon, LL=Liu-di Lu, CG=Cindy Guichard, NB=Nelly Boulos, MR=Mathieu Rigal, CEH=Chourouk El Hassanieh, JSM=Jacques Sainte-Marie, LP=Lucas Perrin, NT=Norbert Tognon.

10.1.4 Research administration

- Julien Salomon belongs to the board of AMIES (Industry-Maths promoting association)
- Cindy Guichard belongs to Commission ATER section 26 Sorbonne Université
- Cindy Guichard has been member of "Comité de sélection MdC section 60" Sorbonne Université
- Julien Salomon is a member of INRIA Scientific positions council
- Jacques Sainte-Marie is INRIA CEO's deputy
- Julien Guillod is a member of the administration council of the Institut Henri Poincaré
- Jacques Sainte-Marie belongs to the external advisory board ERC Synergy

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

Teaching activities of ANGE are summarized in the following.

- Julien Salomon
 - Méthodes numériques pour des modèles incluants des EDP, 45,M2,Université d'Abomey-Calavi, Bénin, CM
 - Méthodes numériques pour des modèles incluants des EDP, 45H, M2, Univ. Paris-Dauphine,
 CM
- · Cindy Guichard
 - Analyse numérique 48H, L3, Sorbonne Université TD+TP
 - Méthodes numériques 31H, M2, Sorbonne Université CM+TD
 - Co responsable de la majeure Ingénierie Mathématiques pour l'Entreprise M2 Sorbonne Université

· Jacques Sainte-Marie

- Modélisation des écoulements gravitaires, 40 H, M1, Univ. Paris-Diderot et IPGP
- Méthodes numériques en géosciences, 50 H, M2, Univ. Paris-Diderot et IPGP,
- Hyperbolic models for complex flows, 25 H, M2, Sorbonne Université

· Nelly Boulos Al Makary

- Analyse2, 36 H, L1, Université Sorbonne Paris Nord, TD
- Mathématiques pour les études scientifiques, 36H, L1, Sorbonne Université

• Nathalie Ayi

- Probabilités, 38H, L3, Sorbonne Université, TD
- Approximation des EDPs, 36H, M1, Sorbonne Université CM
- Algèbre linéaire, 60H, L2, Sorbonne Université CM-TD

· Edwige Godlewski

 Modèles hyperboliques d'écoulements complexes dans le domaine de l'énergie, 10 H, M2, SU, CM

· Virgile Dubos

- Mathématiques approfondies, 52H, L1, Sorbonne Université

• Bernard Di Martino

- Calcul différentiel, 54H, L3, Université de Corse TP
- Analyse Numérique Matricielle, 54H, L3, Université de Corse CM-TD-TP
- Outils Mathématiques, 81H, L1, Université de Corse
- Pratique d'analyse, 42H, L2, Université de Corse TP

• Emmanuel Audusse

- EDO,30 H, ING1, USPN, TD-TP
- Optimisation, 30 H, ING2, USPN, TD-TP
- Calcul scientifique, 30 H, L2, USPN, CM-TD-TP

· Léon Migus

- Informatique 2 (fortran), 40 H, M1, Polytech Sorbonne, TP
- Informatique générale 1 + 2, Python, 36H, M1/M2, Polytech Sorbonne TP

· Julien Guillod

- Méthodes numériques pour les EDP instationnaires 18 M2 Sorbonne Université TD/TP
- Programmation Python pour les mathématiques 47 L2 Sorbonne Université TP/TD

· Nina Aguillon

- Mathématiques pour les études scientifiques 2, 42H, L1, Sorbonne Université CM-TD
- Co responsable de la double licence maths informatique, 12H, L2/L3, Sorbonne Université responsabilité
- Topologie et calcul différentiel 1, 36H, L2, Sorbonne Université TD

• Mathieu Rigal

- Topologie, analyse hilbertienne et intégration 36 L3 (ING1) Polytech Sorbonne
- TP d'introduction à Matlab 4 L1 Polytech Sorbonne
- · Chourouk El Hassanieh
 - Mathématiques pour les études scientifiques 1 36 L1 Sorbonne Université
- · Antoine Leblond
 - Analyse numérique, 72H, L3, Sorbonne Université, TP, TD+TP
- · Juliette Dubois
 - Structures mathématiques, 22H, L3, Polytech Sorbonne, TD

10.2.2 Supervision

- JS, PhD, Lucas Perrin, SU, 2021-2024, Parallélisation en temps et assimilation de données
- JS, VM, PhD, Antoine Lesieur, Inria, 2017-2021, Estimation d'état et modélisation inverse appliquées à la pollution sonore en milieu urbain
- JS, PhD, Léon Migus, SU, 2020-2023, Deep Neural Networks and Differential Equations
- JS, JSM, PhD, Liudi Lu, SU, 2018-2021, Approches Lagrangiennes pour la modélisation et l'optimisation du couplage hydrodynamique-photosynthèse
- JS, Stage M2, Lucas Perrin, U. Paris-Dauphine, 03.2021-07.2021, Parallélisation en temps et assimilation de données
- JS, Stage M2, Norbert Tognon, U. Abomey-Calavi (Bénin), 03.2021-07.2021, Analyse de l'algorithme ParaOpt
- JS, Stage L3, Dylan Machado, INRIA 03.2021-07.2021, Etude de la méthode BEM pour l'analyse et la conception d'hélice
- JS, Stage M2, Ignacio Calderon, INRIA Chile, 09.2021-12.2021, The BEM methode, implementation aspects
- JSM, VM, PhD, Frédéric Allaire, Inria, 2017-2021, Quantification du risque incendie par métamodélisation de la propagation de feux de forêt
- YP, CG, JSM, EA, PhD, Virgile Dubos, SU, 2017-2021, Numerical methods around shallow water flows: dispersive effects, Coriolis force
- NA, EA, Martin Parisot, PhD, Nelly BOULOS, Paris 13, 2018-2021, Modélisation et simulation numérique de la dynamique d'un acquifère érodable
- BDM, JSM, Samer Israwi (Libanese university), PhD, Chourouk El Hassanieh, Inria, 2019-2022, Mathematical and numerical analysis of some dispersive models in fluids mechanics
- NA, JSM, Nayi, PhD, Mathieu Rigal, SU, 2019-2022, Low Froude regime and dispersive effects in kinetic formulations
- JSM, Sébastien Impériale, PhD, Juliette Dubois, Inria, 2020-2023, Modélisation et approximation numérique de la propagation des ondes acoustique et des ondes de gravité dans les fluides à surface libre
- YP, Nora Aissiouene, Pierre-Yves Lagrée, PhD, Giuseppe Parasiliti, SU 2020-2023, Physical, mathematical and numerical modelling of a gas flow for the transportation of liquified natural gas
- JG, Anne-Laure Dalibard, PhD, Antoine Leblond, SU, 09.2020, Evolution de patches de densité dans des fluides incompressibles

• JG, Tutorat FSMP, Sagbo Marcel Zodji, FSMP/SU, 09.2020-06.2021, Tutorat d'un étudiant de M2 étranger lauréat d'une bourse PGSM de la FSMP

- JG, Tutorat FSMP, Kala Agbo Bidi, FSMP/SU, 09.2021-06.2022, Tutorat d'un étudiant de M2 étranger lauréat d'une bourse PGSM de la FSMP
- NA, Stage M2, Yri Amandine Kambiri, SU, 04.2021-09.2021, Quantification a posteriori de la diffusion numérique
- JSM, Etienne Mémin, Post-doc, Pierre-Marie Boulvard, Inria 2021-2022, Location uncertainties in free surface flows models Numerical analysis and implementation in Freshkiss3d

Legend: JS=Julien Salomon, JG=Julien Guillod, NA=Nina Aguillon, CG=Cindy Guichard, JSM=Jacques Sainte-Marie, VM=Vivien Mallet, Nayi=Nathalie Ayi, EA=Emmanuel Audusse, YP=Yohan Penel.

10.2.3 Juries

- Julien Salomon
 - 29.09.2021, PhD comittee (dir. de thèse), Liudi Lu, SU, "Approaches Lagrangiennes pour la modélisation et l'optimisation du couplage hydrodynamique-photosynthèse"
 - 09.09.2021, PhD comittee (dir. de thèse), Antoine Lesieur, SU, "Estimation d'état et modélisation inverse appliquées à la pollution sonore en milieu urbain"
 - 28.06.2021, PhD Reviewer, Duc Quang Bui, U. Sorbonne Paris Nord, New space-time domain decomposition algorithms combined with the Parareal algorithm
- · Julien Guillod
 - 07.2021, IGE Member, SU, Administrateur-trice systèmes et réseaux
- Jacques Sainte-Marie, Yohan Penel, Cindy Guichard
 - 12.2021 PhD comittee (dir. de thèse), Virgile Dubos SU Numerical methods for the elliptic/parabolic parts of non-hydrostatic fluid models

10.2.4 Interventions

- Julien Salomon gave a conference in the framework of "Cycle SMAI & Musée des arts et métiers" (public Lycéen, october 2021)
- Julien Salomon took part of pupils visit board (Accueil de Collégiens, december 2021)
- Nina Aguillon co-organises de Mathematic Park (licence and classe préparatoires, 2018-)
- Apolline El Baz took part of "Atelier RJMI Inria" (November 2021)

11 Scientific production

11.1 Major publications

- [1] E. Audusse, M.-O. Bristeau, M. Pelanti and J. Sainte-Marie. 'Approximation of the hydrostatic Navier-Stokes system for density stratified flows by a multilayer model. Kinetic interpretation and numerical validation'. In: *J. Comput. Phys.* 230 (2011), pp. 3453–3478. DOI: 10.1016/j.jcp.2011.01.042. URL: http://dx.doi.org/10.1016/j.jcp.2011.01.042.
- [2] E. Audusse, M.-O. Bristeau, B. Perthame and J. Sainte-Marie. 'A multilayer Saint-Venant system with mass exchanges for Shallow Water flows. Derivation and numerical validation'. In: *ESAIM Math. Model. Numer. Anal.* 45 (2011), pp. 169–200. DOI: 10.1051/m2an/2010036. URL: http://dx.doi.org/10.1051/m2an/2010036.

- [3] M.-O. Bristeau, A. Mangeney, J. Sainte-Marie and N. Seguin. 'An energy-consistent depth-averaged Euler system: derivation and properties'. In: *Discrete and Continuous Dynamical Systems Series B* 20.4 (2015), p. 28.
- [4] J. Ledoux, S. Riffo and J. Salomon. 'Analysis of the Blade Element Momentum Theory'. working paper or preprint. Apr. 2020. URL: https://hal.archives-ouvertes.fr/hal-02550763.
- [5] J. Sainte-Marie. 'Vertically averaged models for the free surface Euler system. Derivation and kinetic interpretation'. In: *Math. Models Methods Appl. Sci. (M3AS)* 21.3 (2011), pp. 459–490. DOI: 10.1142/S0218202511005118. URL: http://dx.doi.org/10.1142/S0218202511005118.

11.2 Publications of the year

International journals

- [6] F. Allaire, V. Mallet and J.-B. Filippi. 'Novel method for a posteriori uncertainty quantification in wildland fire spread simulation'. In: *Applied Mathematical Modelling* 90 (Feb. 2021), pp. 527–546. DOI: 10.1016/j.apm.2020.08.040. URL: https://hal.inria.fr/hal-02957983.
- [7] E. Audusse, L. Boittin and M. Parisot. 'Asymptotic derivation and simulations of a non-local Exner model in large viscosity regime'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 55.4 (July 2021), pp. 1635–1668. DOI: 10.1051/m2an/2021031. URL: https://hal.archives-ouvertes.fr/hal-03312836.
- [8] E. Audusse, V. Dubos, A. Duran, N. Gaveau, Y. Nasseri and Y. Penel. 'Numerical approximation of the shallow water equations with Coriolis source term'. In: *ESAIM: Proceedings* 70 (1st June 2021), pp. 31–44. URL: https://hal.archives-ouvertes.fr/hal-03182659.
- [9] P. Aumond, A. Can, V. Mallet, B. Gauvreau and G. Guillaume. 'Global sensitivity analysis for road traffic noise modelling'. In: *Applied Acoustics* 176 (1st Jan. 2021), 9 p. DOI: 10.1016/j.apacoust.2020.107899. URL: https://hal.archives-ouvertes.fr/hal-03122955.
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- [11] A.-S. Bonnet-Ben Dhia, M.-O. Bristeau, E. Godlewski, S. Imperiale, A. Mangeney and J. Sainte-Marie. 'Pseudo-compressibility, dispersive model and acoustic waves in shallow water flows'. In: *SEMA SIMAI Springer Series* (21st May 2021), pp. 209–250. URL: https://hal.inria.fr/hal-02493518.
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- [23] O. Bernard, L.-d. Lu, J. Sainte-Marie and J. Salomon. 'Controlling the bottom topography of a microalgal pond to optimize productivity'. In: 2021 American Control Conference (ACC). New Orleans, United States, 2020, pp. 634–639. DOI: 10.23919/ACC50511.2021.9483235. URL: https://hal.archives-ouvertes.fr/hal-02970776.
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Scientific books

[26] N. Sanchez-Pi, L. Marti, J. Salomon, J. Sainte-Marie, O. Bernard, M. Sebag, M. Schoenauer, A. Maass, D. Eveillard, A. Abreu, C. de Vargas and P. A. Marquet. *OcéanIA: AI, Data, and Models for Understanding the Ocean and Climate Change*. 1st July 2021, pp. 1–64. URL: https://hal.archives-ouvertes.fr/hal-03274323.

Scientific book chapters

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Reports & preprints

- [30] F. Allaire, V. Mallet and J. B. Filippi. *Emulation of wildland fire spread simulation using deep learning*. 15th Feb. 2021. URL: https://hal.inria.fr/hal-03142281.
- [31] F. Allaire, J.-B. Filippi, V. Mallet and F. Vaysse. *Simulation-based high resolution fire danger mapping using deep learning*. 5th Apr. 2021. URL: https://hal.inria.fr/hal-03189847.
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- [34] O. Bernard and L.-d. Lu. *Optimal optical conditions for Microalgal production in photobioreactors*. 20th Aug. 2021. URL: https://hal.archives-ouvertes.fr/hal-03323094.
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