2021
ACTIVITY REPORT

Project-Team

FACTAS

Functional Analysis for ConcepTion and Assessment of Systems

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

Contents

Project-Team FACTAS

Creation of the Project-Team: 2019 July 01

Keywords

Computer sciences and digital sciences

A6.1.1. – Continuous Modeling (PDE, ODE)

A6.2.1. – Numerical analysis of PDE and ODE

A6.2.5. – Numerical Linear Algebra

A6.2.6. – Optimization

A6.3.1. – Inverse problems

A6.3.4. – Model reduction

A6.4.3. – Observability and Controlability

A6.4.4. – Stability and Stabilization

A6.4.5. – Control of distributed parameter systems

A6.5.4. – Waves

A8.2. – Optimization

A8.3. – Geometry, Topology

A8.4. – Computer Algebra

Other research topics and application domains

B1.2.3. – Computational neurosciences

B2.6.1. – Brain imaging

B3.1. – Sustainable development

B3.3. – Geosciences

B4.5. – Energy consumption

1   Team members, visitors, external collaborators

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- Fabien Seyfert [Team leader, Inria, Researcher, until Sep 2021, HDR]

- Juliette Leblond [Team leader, Inria, Senior Researcher, HDR]

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- Paul Asensio [Inria]

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- Fabien Seyfert [HighFSolutions, from Oct 2021, Nice]

2   Overall objectives

2.1   Research Themes

The team develops constructive, function-theoretic approaches to inverse problems arising in modeling and design, in particular for electro-magnetic systems as well as in the analysis of certain classes of signals.

Data typically consist of measurements or desired behaviors. The general thread is to approximate them by families of solutions to the equations governing the underlying system. This leads us to consider

various interpolation and approximation problems in classes of rational and meromorphic functions, harmonic gradients, or solutions to more general elliptic partial differential equations (PDE), in connection with inverse potential problems. A recurring difficulty is to control the singularities of the approximants.

The mathematical tools pertain to complex and harmonic analysis, approximation theory, potential theory, system theory, differential topology, optimization and computer algebra. Targeted applications include:

- identification and synthesis of analog microwave devices (filters, amplifiers),

- non-destructive control from field measurements in medical engineering (source recovery in magneto/electro encephalography), and paleomagnetism (determining the magnetization of rock samples).

In each case, the endeavor is to develop algorithms resulting in dedicated software.

Factas undergo the departure of its team leader Fabien Seyfert at the end of September. It substantially reduces the manpower of the team, and essentially freezes[1] our research in areas where Fabien was the main driving force. Among our targeted applications, those generating the most intensive technology transfer pertain to the first item and were led by him. He is now running his own consulting company.

3   Research program

3.1   Introduction

Within the extensive field of inverse problems, much of the research by Factas deals with reconstructing solutions of classical elliptic PDEs from their boundary behavior. Perhaps the simplest example lies with harmonic identification of a stable linear dynamical system: the transfer-function f can be evaluated at a point $i\omega$ of the imaginary axis from the response to a periodic input at frequency ω. Since f is holomorphic in the right half-plane, it satisfies there the Cauchy-Riemann equation $\bar{\partial} f = 0$, and recovering f amounts to solve a Dirichlet problem which can be done in principle using, *e.g.* the Cauchy formula.

Practice is not nearly as simple, for f is only measured pointwise in the pass-band of the system which makes the problem ill-posed [80]. Moreover, the transfer function is usually sought in specific form, displaying the necessary physical parameters for control and design. For instance if f is rational of degree n, then $\bar{\partial} f = \sum_1^n a_j \delta_{z_j}$ where the z_j are its poles and δ_{z_j} is a Dirac unit mass at z_j. Thus, to find the domain of holomorphy (*i.e.* to locate the z_j) amounts to solve a (degenerate) free-boundary inverse problem, this time on the left half-plane. To address such questions, the team has developed a two-step approach as follows.

> **Step 1:** To determine a complete model, that is, one which is defined at every frequency, in a sufficiently versatile function class (*e.g.* Hardy spaces). This ill-posed issue requires regularization, for instance constraints on the behavior at non-measured frequencies.

> **Step 2:** To compute a reduced order model. This typically consists of rational approximation of the complete model obtained in step 1, or phase-shift thereof to account for delays. We emphasize that deriving a complete model in step 1 is crucial to achieve stability of the reduced model in step 2.

Step 1 relates to extremal problems and analytic operator theory, see Section 3.3.1. Step 2 involves optimization, and some Schur analysis to parametrize transfer matrices of given Mc-Millan degree when dealing with systems having several inputs and outputs, see Section 3.3.2. It also makes contact with the topology of rational functions, in particular to count critical points and to derive bounds, see Section 3.3.2. Step 2 raises further issues in approximation theory regarding the rate of convergence and the extent to which singularities of the approximant (*i.e.* its poles) tend to singularities of the approximated function; this is where logarithmic potential theory becomes instrumental, see Section 3.3.3.

Applying a realization procedure to the result of step 2 yields an identification procedure from incomplete frequency data which was first demonstrated in [86] to tune resonant microwave filters.

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[1]Until his hypothetical return from leave, or appropriate recruitment by Inria of a new team member.

Harmonic identification of nonlinear systems around a stable equilibrium can also be envisaged by combining the previous steps with exact linearization techniques from [41].

A similar path can be taken to approach design problems in the frequency domain, replacing the measured behavior by some desired behavior. However, describing achievable responses in terms of the design parameters is often cumbersome, and most constructive techniques rely on specific criteria adapted to the physics of the problem. This is especially true of filters, the design of which traditionally appeals to polynomial extremal problems [82, 65]. To this area, we contributed the use of Zolotarev-like problems for multi-band synthesis, although we presently favor interpolation techniques in which parameters arise in a more transparent manner, as well as convex relaxation of hyperbolic approximation problems, see Sections 3.2.2 and 7.2.

The previous example of harmonic identification quickly suggests a generalization of itself. Indeed, on identifying $\mathbb{C}$ with $\mathbb{R}^2$, holomorphic functions become conjugate-gradients of harmonic functions, so that harmonic identification is, after all, a special case of a classical issue: to recover a harmonic function on a domain from partial knowledge of the Dirichlet-Neumann data; when the portion of boundary where data are not available is itself unknown, we meet a free boundary problem. This framework for 2-D non-destructive control was first advocated in [70] and subsequently received considerable attention. It makes clear how to state similar problems in higher dimensions and for more general operators than the Laplacian, provided solutions are essentially determined by the trace of their gradient on part of the boundary which is the case for elliptic equations [2] [39], [90]. Such questions are particular instances of the so-called inverse potential problem, where a measure μ has to be recovered from the knowledge of the gradient of its potential (*i.e.*, the field) on part of a hypersurface (a curve in 2-D) encompassing the support of μ. For Laplace's operator, potentials are logarithmic in 2-D and Newtonian in higher dimensions. For elliptic operators with non constant coefficients, the potential depends on the form of fundamental solutions and is less manageable because it is no longer of convolution type. Nevertheless it is a useful concept bringing perspective on how problems could be raised and solved, using tools from harmonic analysis.

Inverse potential problems are severely indeterminate because infinitely many measures within an open set of $\mathbb{R}^n$ produce the same field outside this set; this phenomenon is called *balayage* [79]. In the two steps approach previously described, we implicitly removed this indeterminacy by requiring in step 1 that the measure be supported on the boundary (because we seek a function holomorphic throughout the right half-space), and by requiring in step 2 that the measure be discrete in the left half-plane (in fact: a finite sum of point masses $\sum_1^N a_j \delta_{z_j}$). The discreteness assumption also prevails in 3-D inverse source problems, see Section 4.3. Conditions that ensure uniqueness of the solution to the inverse potential problem are part of the so-called regularizing assumptions which are needed in each case to derive efficient algorithms.

To recap, the gist of our approach is to approximate boundary data by (boundary traces of) fields arising from potentials of measures with specific support. This differs from standard approaches to inverse problems, where descent algorithms are applied to integration schemes of the direct problem; in such methods, it is the equation which gets approximated (in fact: discretized).

Along these lines, Factas advocates the use of steps 1 and 2 above, along with some singularity analysis, to approach issues of nondestructive control in 2-D and 3-D [2], [48, 52]. The team is currently engaged in the generalization to inverse source problems for the Laplace equation in 3-D, to be described further in Section 3.2.1. There, holomorphic functions are replaced by harmonic gradients; applications are to inverse source problems in neurosciences (in particular in EEG/MEG) and inverse problems in geosciences.

The approximation-theoretic tools developed by Factas to handle issues mentioned so far are outlined in Section 3.3. In Section 3.2 to come, we describe in more detail which problems are considered and which applications are targeted.

We also began to investigate inverse scattering problems of plane waves by obstacles (playing here the role of a source term), with partners at Leat. Such problems are again governed by Maxwell's equations and, in the time-harmonic regime, these reduce to Helmholtz equations depending on the frequency of

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[2]There is a subtle difference here between dimension 2 and higher. Indeed, a function holomorphic on a plane domain is defined by its non-tangential limit on a boundary subset of positive linear measure, but there are non-constant harmonic functions in the 3-D ball, C^1 up to the boundary sphere, yet having vanishing gradient on a subset of positive measure of the sphere. Such a "bad" subset, however, cannot have interior points on the sphere.

the plane wave. Such issues have applications to detection and identification of metal objects, and this is part of Leat research program, but at this early stage our study has remained academic (see Section 7.7). Once again, our approach combines steps 1 and 2 above.

3.2 Range of inverse problems

3.2.1 Elliptic partial differential equations (PDE)

Participants: Paul Asensio, Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Masimba Nemaire.

By standard properties of conjugate differentials, reconstructing Dirichlet-Neumann boundary conditions for a function harmonic in a plane domain, when these conditions are already known on a subset E of the boundary, is equivalent to recover a holomorphic function in the domain from its boundary values on E. This is the problem raised on the half-plane in step 1 of Section 3.1. It makes good sense in holomorphic Hardy spaces where functions are entirely determined by their values on boundary subsets of positive linear measure, which is the framework for Problem (P) that we set up in Section 3.3.1. Such issues naturally arise in nondestructive testing of 2-D (or 3-D cylindrical) materials from partial electrical measurements on the boundary. For instance, the ratio between the tangential and the normal currents (the so-called Robin coefficient) tells one about corrosion of the material. Thus, solving Problem (P) where ψ is chosen to be the response of some uncorroded piece with identical shape yields non destructive testing of a potentially corroded piece of material, part of which is inaccessible to measurements. This was an initial application of holomorphic extremal problems to non-destructive control [63, 66].

Another application by the team deals with non-constant conductivity over a doubly connected domain, the set E being now the outer boundary. Measuring Dirichlet-Neumann data on E, one wants to recover level lines of the solution to a conductivity equation, which is a so-called free boundary inverse problem. For this, given a closed curve inside the domain, we first quantify how constant the solution on this curve. To this effect, we state and solve an analog of Problem (P), where the constraint bears on the real part of the function on the curve (it should be close to a constant there), in a Hardy space of a conjugate Beltrami equation, of which the considered conductivity equation is the compatibility condition (just like the Laplace equation is the compatibility condition of the Cauchy-Riemann system). Subsequently, a descent algorithm on the curve leads one to improve the initial guess. For example, when the domain is regarded as separating the edge of a tokamak's vessel from the plasma (rotational symmetry makes this a 2-D situation), this method can be used to estimate the shape of a plasma subject to magnetic confinement.

This was actually carried out in collaboration with CEA (French nuclear agency) and the Univ. Côte d'Azur (JAD Lab.), to data from *Tore Supra* in [69]. The procedure is fast because no numerical integration of the underlying PDE is needed, as an explicit basis of solutions to the conjugate Beltrami equation in terms of Bessel functions was found in this case. Generalizing this approach in a more systematic manner to free boundary problems of Bernoulli type, using descent algorithms based on shape-gradient for such approximation-theoretic criteria, is an interesting prospect to the team.

The piece of work we just mentioned requires defining and studying Hardy spaces of conjugate Beltrami equations, which is another interesting topic. For Sobolev-smooth coefficients of exponent greater than 2, they were investigated in [4], [42]. The case of the critical exponent 2 is treated in [38], which apparently provides the first example of well-posed Dirichlet problem in the non-strictly elliptic case: the conductivity may be unbounded or zero on sets of zero capacity and, accordingly, solutions need not be locally bounded. More importantly perhaps, the exponent 2 is also the key to a corresponding theory on very general (still rectifiable) domains in the plane, as coefficients of pseudo-holomorphic functions obtained by conformal transformation onto a disk are merely of L^2-class in general, even if the initial problem deals with coefficients of L^r-class for some $r > 2$. Such generalizations are now under study within the team, in collaboration with E. Pozzi (Saint Louis Univ., Missouri, USA) and E. Russ (Univ. J. Fourier, Grenoble), and fairly deep connections between the regularity of the conformal parametrization of the domain and the range of exponents p for which the Dirichlet problem is solvable in L^p were brought to light.

Generalized Hardy classes as above are used in [39] where we address the uniqueness issue in the classical Robin inverse problem on a Lipschitz domain of $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with uniformly bounded Robin coefficient, L^2 Neumann data and conductivity of Sobolev class $W^{1,r}(\Omega)$, $r > n$. We show that uniqueness of the Robin coefficient on a subset of the boundary, given Cauchy data on the complementary part, does hold in dimension $n = 2$, thanks to a unique continuation result, but needs not hold in higher dimension. In higher dimension, this raises an open issue on harmonic gradients, namely whether the positivity of the Robin coefficient is compatible with identical vanishing of the boundary gradient on a subset of positive measure.

The 3-D version of step 1 in Section 3.1 is another subject investigated by Factas: to recover a harmonic function (up to an additive constant) in a ball or a half-space from partial knowledge of its gradient. This prototypical inverse problem (*i.e.* inverse to the Cauchy problem for the Laplace equation) often recurs in electromagnetism. At present, Factas is involved with solving instances of this inverse problem arising in two fields, namely medical imaging *e.g.* for electroencephalography (EEG) or magneto-encephalography (MEG), and paleomagnetism (recovery of rocks magnetization) [2], [44], see Section 7.1. The question is considerably more difficult than its 2-D counterpart, due mainly to the lack of multiplicative structure for harmonic gradients. Still, substantial progress has been made over the last years using methods of harmonic analysis and operator theory.

The team is further concerned with 3-D generalizations and applications to non-destructive control of step 2 in Section 3.1. A typical problem is here to localize inhomogeneities or defaults such as cracks, sources or occlusions in a planar or 3-dimensional object, knowing thermal, electrical, or magnetic measurements on the boundary. These defaults can be expressed as a lack of harmonicity of the solution to the associated Dirichlet-Neumann problem, thereby posing an inverse potential problem in order to recover them. In 2-D, finding an optimal discretization of the potential in Sobolev norm amounts to solve a best rational approximation problem, and the question arises as to how the location of the singularities of the approximant (*i.e.* its poles) reflects the location of the singularities of the potential (*i.e.* the defaults we seek). This is a fairly deep issue in approximation theory, to which the project Apics (predecessor of Factas[3]) contributed convergence results for certain classes of fields (expressed as Cauchy integrals over extremal contours for the logarithmic potential [6], [45, 62]). Initial schemes to locate cracks or sources *via* rational approximation on planar domains were obtained this way [48, 52, 63]. It is remarkable that finite inverse source problems in 3-D balls, or more general algebraic surfaces, can be approached using these 2-D techniques upon slicing the domain into planar sections [8], [49]. More precisely, each section cuts out a planar domain, the boundary of which carries data which can be proved to match an algebraic function. The singularities of this algebraic function are not located at the 3-D sources, but are related to them: the section contains a source if and only if some function of the singularities in that section meets a relative extremum. Using bisection it is thus possible to determine an extremal place along all sections parallel to a given plane direction, up to some threshold which has to be chosen small enough that one does not miss a source. This way, we reduce the original source problem in 3-D to a sequence of inverse poles and branchpoints problems in 2-D. This bottom line generates a steady research activity within Factas, and again applications are sought to medical imaging and geosciences, see Sections 4.3, 4.2 and 7.1.

Conjectures may be raised on the behavior of optimal potential discretization in 3-D, but answering them is an ambitious program still in its infancy.

3.2.2   Systems, transfer and scattering

Participants: Laurent Baratchart, Sylvain Chevillard, Adam Cooman, Martine Olivi, Fabien Seyfert.

Through contacts with CNES (French space agency), members of the team became involved in identification and tuning of microwave electromagnetic filters used in space telecommunications, see Section 4.4. The initial problem was to recover, from band-limited frequency measurements, physical parameters of the device under examination. The latter consists of interconnected dual-mode resonant

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[3]Actually, Factas started as a team from January 2018, and became a project-team (EPI) from July 2019.

cavities with negligible loss, hence its scattering matrix is modeled by a 2×2 unitary-valued matrix function on the frequency line, say the imaginary axis to fix ideas. In the bandwidth around the resonant frequency, a modal approximation of the Helmholtz equation in the cavities shows that this matrix is approximately rational, of Mc-Millan degree twice the number of cavities.

This is where system theory comes into play, through the so-called *realization* process mapping a rational transfer function in the frequency domain to a state-space representation of the underlying system of linear differential equations in the time domain. Specifically, realizing the scattering matrix allows one to construct a virtual electrical network, equivalent to the filter, the parameters of which mediate in between the frequency response and the geometric characteristics of the cavities (*i.e.* the tuning parameters).

Hardy spaces provide a framework to transform this ill-posed issue into a series of regularized analytic and meromorphic approximation problems. More precisely, the procedure sketched in Section 3.1 goes as follows:

1. infer from the pointwise boundary data in the bandwidth a stable transfer function (*i.e.* one which is holomorphic in the right half-plane), that may be infinite dimensional (numerically: of high degree). This is done by solving a problem analogous to (*P*) in Section 3.3.1, while taking into account prior knowledge on the decay of the response outside the bandwidth, see [11] for details.

2. A stable rational approximation of appropriate degree to the model obtained in the previous step is performed. For this, a descent method on the compact manifold of inner matrices of given size and degree is used, based on an original parametrization of stable transfer functions developed within the team [32], [11].

3. Realizations of this rational approximant are computed. To be useful, they must satisfy certain constraints imposed by the geometry of the device. These constraints typically come from the coupling topology of the equivalent electrical network used to model the filter. This network is composed of resonators, coupled according to some specific graph. This realization step can be recast, under appropriate compatibility conditions [64], as solving a zero-dimensional multivariate polynomial system. To tackle this problem in practice, we use Gröbner basis techniques and continuation methods which team up in the Dedale-HF software (see Section 3.4.2).

We actively continue our collaboration with the Chinese Hong Kong University on the topic of frequency depending couplings appearing in the equivalent circuits we compute continuing our work [1] on wide-band design and dispersive coupling, that led to a major publication [12].

Factas also investigates issues pertaining to design rather than identification. Given the topology of the filter, a basic problem in this connection is to find the optimal response subject to specifications that bear on rejection, transmission and group delay of the scattering parameters. Generalizing the classical approach based on Chebyshev polynomials for single band filters, we recast the problem of multi-band response synthesis as a generalization of the classical Zolotarev min-max problem for rational functions [31, 30]. Thanks to quasi-convexity, the latter can be solved efficiently using iterative methods relying on linear programming. These were implemented in the software easy-FF). Currently, the team is engaged in the synthesis of more complex microwave devices like multiplexers and routers, which connect several filters through wave guides. Schur analysis plays an important role here, because scattering matrices of passive systems are of Schur type (*i.e.* contractive in the stability region). The theory originates with the work of I. Schur [85], who devised a recursive test to check for contractivity of a holomorphic function in the disk. The so-called Schur parameters of a function may be viewed as Taylor coefficients for the hyperbolic metric of the disk, and the fact that Schur functions are contractions for that metric lies at the root of Schur's test. Generalizations thereof turn out to be efficient to parametrize solutions to contractive interpolation problems [35]. Dwelling on this, Factas contributed differential parametrizations (atlases of charts) of lossless matrix functions [32, 81, 75] which are fundamental to our rational approximation software RARL2 (see Section 3.4.5). Schur analysis is also instrumental to approach de-embedding issues, and provides one with considerable insight into the so-called matching problem. The latter consists in maximizing the power a multiport can pass to a given load, and for reasons of efficiency it is all-pervasive in microwave and electric network design, *e.g.* of antennas, multiplexers, wifi cards and more. It can be viewed as a rational approximation problem in the hyperbolic metric, and

the team presently deals with this hot topic using contractive interpolation with constraints on boundary peak points, within the framework of the (defense funded) ANR Cocoram, see Sections 7.2.

In recent years, our attention was driven by CNES and UPV (Bilbao) to questions about stability of high-frequency amplifiers. Contrary to previously discussed devices, these are *active* components. The response of an amplifier can be linearized around a set of primary current and voltages, and then admittances of the corresponding electrical network can be computed at various frequencies, using the so-called harmonic balance method. The initial goal is to check for stability of the linearized model, so as to ascertain existence of a well-defined working state. The network is composed of lumped electrical elements namely inductors, capacitors, negative *and* positive resistors, transmission lines, and controlled current sources. Our research so far has focused on describing the algebraic structure of admittance functions, so as to set up a function-theoretic framework where the two-steps approach outlined in Section 3.1 can be put to work. The main discovery is that the unstable part of each partial transfer function is rational and can be computed by analytic projection, see Section 7.5. We now start investigating the linearized harmonic transfer-function around a periodic cycle, to check for stability under non necessarily small inputs.

3.3 Approximation

Participants:	Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Martine Olivi, Fabien Seyfert.

3.3.1 Best analytic approximation

In dimension 2, the prototypical problem to be solved in step 1 of Section 3.1 may be described as: given a domain $D \subset \mathbb{R}^2$, to recover a holomorphic function from its values on a subset K of the boundary of D. For the discussion it is convenient to normalize D, which can be done by conformal mapping. So, in the simply connected case, we fix D to be the unit disk with boundary unit circle T. We denote by H^p the Hardy space of exponent p, which is the closure of polynomials in $L^p(T)$-norm if $1 \le p < \infty$ and the space of bounded holomorphic functions in D if $p = \infty$. Functions in H^p have well-defined boundary values in $L^p(T)$, which makes it possible to speak of (traces of) analytic functions on the boundary.

To find an analytic function g in D matching some measured values f approximately on a sub-arc K of T, we formulate a constrained best approximation problem as follows.

> (P) Let $1 \le p \le \infty$, K a sub-arc of T, $f \in L^p(K)$, $\psi \in L^p(T \setminus K)$ and $M > 0$; find a function $g \in H^p$ such that $\|g - \psi\|_{L^p(T \setminus K)} \le M$ and $g - f$ is of minimal norm in $L^p(K)$ under this constraint.

Here ψ is a reference behavior capturing *a priori* assumptions on the behavior of the model off K, while M is some admissible deviation thereof. The value of p reflects the type of stability which is sought and how much one wants to smooth out the data. The choice of L^p classes is suited to handle pointwise measurements.

To fix terminology, we refer to (P) as a *bounded extremal problem*. As shown in [47, 50], [57], the solution to this convex infinite-dimensional optimization problem can be obtained when $p \ne 1$ upon iterating with respect to a Lagrange parameter the solution to spectral equations for appropriate Hankel and Toeplitz operators. These spectral equations involve the solution to the special case $K = T$ of (P), which is a standard extremal problem [73]:

> (P_0) Let $1 \le p \le \infty$ and $\varphi \in L^p(T)$; find a function $g \in H^p$ such that $g - \varphi$ is of minimal norm in $L^p(T)$.

In the case $p = 1$, partial results are known but computational issues remain open.

Various modifications of (P) can be tailored to meet specific needs. For instance when dealing with lossless transfer functions (see Section 4.4), one may want to express the constraint on $T \setminus K$ in a pointwise manner: $|g - \psi| \le M$ a.e. on $T \setminus K$, see [51]. In this form, the problem comes close to (but still is different

from) H^∞ frequency optimization used in control [76, 84]. One can also impose bounds on the real or imaginary part of $g - \psi$ on $T \setminus K$, which is useful when considering Dirichlet-Neumann problems.

In view of our current research on stability of active devices via analyticity of the harmonic transfer function, on inverse magnetization issues, and on inverse scattering via identification of the frequency response, bounded extremal problems for analytic functions are receiving renewed interest by the team. In such issues, a function on an interval of the real line (or an arc of the circle) must be approximated by the trace of a function holomorphic in the half-plane (or the disk), that meets suitable size constraints. In the case of least square approximation and constraints on the complementary subset, formulas were recently found to express the solution in important cases, and we currently develop this aspect towards applications of various type, in collaboration with J. Mashreghi (Laval Univ., Canada). We mention related work in collaboration with D. Ponomarev (TU Vienna, Austria), and with E. Pozzi (Saint Louis Univ., Missouri, USA), associated with formulations of inverse magnetization issues.

The analog of Problem (P) on an annulus, K being now the outer boundary, can be seen as a means to regularize a classical inverse problem occurring in nondestructive control, namely to recover a harmonic function on the inner boundary from Dirichlet-Neumann data on the outer boundary (see Sections 3.2.1, 4.3, 7.1.2). It may serve as a tool to approach Bernoulli type problems, where we are given data on the outer boundary and we *seek the inner boundary*, knowing it is a level curve of the solution. In this case, the Lagrange parameter indicates how to deform the inner contour in order to improve data fitting. Similar topics are discussed in Section 3.2.1 for more general equations than the Laplacian, namely isotropic conductivity equations of the form $\mathrm{div}(\sigma \nabla u) = 0$ where σ is no longer constant (*i.e.*, varies in the space). Then, the Hardy spaces in Problem (P) are those of a so-called conjugate Beltrami equation: $\bar{\partial} f = \nu \overline{\partial f}$ [77], which are studied for $1 < p < \infty$ in [4], [38], [42] and [67]. Expansions of solutions needed to constructively handle such issues in the specific case of linear fractional conductivities (occurring for instance in plasma shaping) have been expounded in [69].

Though originally considered in dimension 2, Problem (P) carries over naturally to higher dimensions where analytic functions get replaced by gradients of harmonic functions. Namely, given some open set $\Omega \subset \mathbb{R}^n$ and some $\mathbb{R}^n$-valued vector field V on an open subset O of the boundary of Ω, we seek a harmonic function in Ω whose gradient is close to V on O.

When Ω is a ball or a half-space, a substitute for holomorphic Hardy spaces is provided by the Stein-Weiss Hardy spaces of harmonic gradients [88]. Conformal maps are no longer available when $n > 2$, so that Ω can no longer be normalized. More general geometries than spheres and half-spaces have not been much studied so far.

On the ball, the analog of Problem (P) is

(P_1) Let $1 \le p \le \infty$ and $B \subset \mathbb{R}^n$ the unit ball. Fix O an open subset of the unit sphere $S \subset \mathbb{R}^n$. Let further $V \in L^p(O)$ and $W \in L^p(S \setminus O)$ be $\mathbb{R}^n$-valued vector fields. Given $M > 0$, find a harmonic gradient $G \in H^p(B)$ such that $\|G - W\|_{L^p(S \setminus O)} \le M$ and $G - V$ is of minimal norm in $L^p(O)$ under this constraint.

When $p = 2$, Problem (P_1) was solved in [2] as well as its analog on a shell, when the tangent component of V is a gradient (when O is Lipschitz the general case follows easily from this). The solution extends the work in [47] to the 3-D case, using a generalization of Toeplitz operators. The case of the shell was motivated by applications to the processing of EEG data. An important ingredient is a refinement of the Hodge decomposition, that we call the *Hardy-Hodge* decomposition, allowing us to express a $\mathbb{R}^n$-valued vector field in $L^p(S)$, $1 < p < \infty$, as the sum of a vector field in $H^p(B)$, a vector field in $H^p(\mathbb{R}^n \setminus \overline{B})$, and a tangential divergence free vector field on S; the space of such divergence-free fields is denoted by $D(S)$. If $p = 1$ or $p = \infty$, L^p must be replaced by the real Hardy space or the space of functions with bounded mean oscillation. More generally this decomposition, which is valid on any sufficiently smooth surface (see Section 7.1), seems to play a fundamental role in inverse potential problems. In fact, it was first introduced formally on the plane to describe silent magnetizations supported in $\mathbb{R}^2$ (*i.e.* those generating no field in the upper half space) [44].

Just like solving problem (P) appeals to the solution of problem (P_0), our ability to solve problem (P_1) will depend on the possibility to tackle the special case where $O = S$:

(P_2) Let $1 \le p \le \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find a harmonic gradient $G \in H^p(B)$ such that $\|G - V\|_{L^p(S)}$ is minimum.

Problem (P_2) is simple when $p = 2$ by virtue of the Hardy-Hodge decomposition together with orthogonality of $H^2(B)$ and $H^2(\mathbb{R}^n \setminus \overline{B})$, which is the reason why we were able to solve (P_1) in this case. Other values of p cannot be treated as easily and are still under investigation, especially the case $p = \infty$ which is of particular interest and presents itself as a 3-D analog to the Nehari problem [83].

Companion to problem (P_2) is problem (P_3) below.

(P_3) Let $1 \le p \le \infty$ and $V \in L^p(S)$ be a $\mathbb{R}^n$-valued vector field. Find $G \in H^p(B)$ and $D \in D(S)$ such that $\|G + D - V\|_{L^p(S)}$ is minimum.

Note that (P_2) and (P_3) are identical in 2-D, since no non-constant tangential divergence-free vector field exists on T. It is no longer so in higher dimension, where both (P_2) and (P_3) arise in connection with inverse potential problems in divergence form, like source recovery in electro/magneto encephalography and paleomagnetism, see Sections 3.2.1 and 4.3.

3.3.2   Best meromorphic and rational approximation

The techniques set forth in this section are used to solve step 2 in Section 3.2 and they are instrumental to approach inverse boundary value problems for the Poisson equation $\Delta u = \mu$, where μ is some (unknown) measure.

Scalar meromorphic and rational approximation   We put R_N for the set of rational functions with at most N poles in D. By definition, meromorphic functions in $L^p(T)$ are (traces of) functions in $H^p + R_N$.

A natural generalization of problem (P_0) is:

(P_N) Let $1 \le p \le \infty$, $N \ge 0$ an integer, and $f \in L^p(T)$; find a function $g_N \in H^p + R_N$ such that $g_N - f$ is of minimal norm in $L^p(T)$.

Only for $p = \infty$ and f continuous is it known how to solve (P_N) in semi-closed form. The unique solution is given by AAK theory (named after Adamjan, Arov and Krein), which connects the spectral decomposition of Hankel operators with best approximation [83].

The case where $p = 2$ is of special importance for it reduces to rational approximation. Indeed, if we write the Hardy decomposition $f = f^+ + f^-$ where $f^+ \in H^2$ and $f^- \in H^2(\mathbb{C} \setminus \overline{D})$, then $g_N = f^+ + r_N$ where r_N is a best approximant to f^- from R_N in $L^2(T)$. Moreover, r_N has no pole outside D, hence it is a *stable* rational approximant to f^-. However, in contrast to the case where $p = \infty$, this best approximant may *not* be unique.

The Miaou project (predecessor of Apics) already designed a dedicated steepest-descent algorithm for the case $p = 2$ whose convergence to a *local minimum* is guaranteed; the algorithm has evolved over years and still now, it seems to be the only procedure meeting this property. This gradient algorithm proceeds recursively with respect to N on a compactification of the parameter space [40]. Although it has proved to be effective in all applications carried out so far (see Sections 4.3, 4.4), it is still unknown whether the absolute minimum can always be obtained by choosing initial conditions corresponding to *critical points* of lower degree (as is done by the RARL2 software, Section 3.4.5).

In order to establish global convergence results, the team has undertaken a deeper study of the number and nature of critical points (local minima, saddle points, . . .), in which tools from differential topology and operator theory team up with classical interpolation theory [54, 56]. Based on this work, uniqueness or asymptotic uniqueness of the approximant was proved for certain classes of functions like transfer functions of relaxation systems (*i.e.* Markov functions) [58] and more generally Cauchy integrals over hyperbolic geodesic arcs [59]. These are the only results of this kind. Research on this topic remained dormant for a while by reasons of opportunity, but revisiting the work [36] in higher dimension is a worthy and timely endeavor today. Meanwhile, an analog to AAK theory was carried out for $2 \le p < \infty$ in [57]. Although not as effective computationally, it was recently used to derive lower bounds [3]. When $1 \le p < 2$, problem (P_N) is still quite open.

A common feature to the above-mentioned problems is that critical point equations yield non-Hermitian orthogonality relations for the denominator of the approximant. This stresses connections with interpolation, which is a standard way to build approximants, and in many respects best or near-best rational approximation may be regarded as a clever manner to pick interpolation points. This

was exploited in [61, 60], and is used in an essential manner to assess the behavior of poles of best approximants to functions with branched singularities, which is of particular interest for inverse source problems (*cf.* Sections 3.4.3 and 7.1).

In higher dimensions, the analog of Problem (P_N) is best approximation of a vector field by gradients of discrete potentials generated by N point masses. This basic issue is by no means fully understood, and it is an exciting field of research. It is connected with certain generalizations of Toeplitz or Hankel operators, and with constructive approaches to so-called weak factorizations for real Hardy functions [68].

Besides, certain constrained rational approximation problems, of special interest in identification and design of passive systems, arise when putting additional requirements on the approximant, for instance that it should be smaller than 1 in modulus (*i.e.* a Schur function). In particular, Schur interpolation lately received renewed attention from the team, in connection with matching problems. There, interpolation data are subject to a well-known compatibility condition (positive definiteness of the so-called Pick matrix), and the main difficulty is to put interpolation points on the boundary of D while controlling both the degree and the extremal points (peak points for the modulus) of the interpolant. Results obtained by the team in this direction generalize a variant of contractive interpolation with degree constraint as studied in [74]. We mention that contractive interpolation with nodes approaching the boundary has been a subsidiary research topic by the team in the past, which plays an interesting role in the spectral representation of certain non-stationary stochastic processes [43, 46].

Matrix-valued rational approximation   Matrix-valued approximation is necessary to handle systems with several inputs and outputs but it generates additional difficulties as compared to scalar-valued approximation, both theoretically and algorithmically. In the matrix case, the McMillan degree (*i.e.* the degree of a minimal realization in the System-Theoretic sense) generalizes the usual notion of degree for rational functions. For instance when poles are simple, the McMillan degree is the sum of the ranks of the residues.

The basic problem that we consider now goes as follows: *let $\mathscr{F} \in (H^2)^{m \times l}$ and n an integer; find a rational matrix of size $m \times l$ without poles in the unit disk and of McMillan degree at most n which is nearest possible to $\mathscr{F}$ in $(H^2)^{m \times l}$.* Here the L^2 norm of a matrix is the square root of the sum of the squares of the norms of its entries.

The scalar approximation algorithm derived in [40] and mentioned in Section 3.3.2 generalizes to the matrix-valued situation [72]. The first difficulty here is to parametrize inner matrices (*i.e.* matrix-valued functions analytic in the unit disk and unitary on the unit circle) of given McMillan degree degree n. Indeed, inner matrices play the role of denominators in fractional representations of transfer matrices (using the so-called Douglas-Shapiro-Shields factorization). The set of inner matrices of given degree is a smooth manifold that allows one to use differential tools as in the scalar case. In practice, one has to produce an atlas of charts (local parametrizations) and to handle changes of charts in the course of the algorithm. Such parametrization can be obtained using interpolation theory and Schur-type algorithms, the parameters of which are vectors or matrices ([32, 75, 81]). Some of these parametrizations are also interesting to compute realizations and achieve filter synthesis ([75, 81]). The rational approximation software "RARL2" developed by the team is described in Section 3.4.5.

Difficulties relative to multiple local minima of course arise in the matrix-valued case as well, and deriving criteria that guarantee uniqueness is even more difficult than in the scalar case. The case of rational functions of degree n or small perturbations thereof (the consistency problem) was solved in [53]. Matrix-valued Markov functions are the only known example beyond this one [37].

Let us stress that RARL2 seems the only algorithm handling rational approximation in the matrix case that demonstrably converges to a local minimum while meeting stability constraints on the approximant. It is still a working pin of many developments by Factas on frequency optimization and design.

3.3.3   Behavior of poles of meromorphic approximants

Participants: Laurent Baratchart.

We refer here to the behavior of poles of best meromorphic approximants, in the L^p-sense on a closed curve, to functions f defined as Cauchy integrals of complex measures whose support lies inside the curve. Normalizing the contour to be the unit circle T, we are back to Problem (P_N) in Section 3.3.2; invariance of the latter under conformal mapping was established in [52]. Research so far has focused on functions whose singular set inside the contour is polar, meaning that the function can be continued analytically (possibly in a multiple-valued manner) except over a set of logarithmic capacity zero.

Generally speaking in approximation theory, assessing the behavior of poles of rational approximants is essential to obtain error rates as the degree goes large, and to tackle constructive issues like uniqueness. However, as explained in Section 3.2.1, the original twist by Apics, now Factas, is to consider this issue also as a means to extract information on singularities of the solution to a Dirichlet-Neumann problem. The general theme is thus: *how do the singularities of the approximant reflect those of the approximated function?* This approach to inverse problem for the 2-D Laplacian turns out to be attractive when singularities are zero- or one-dimensional (see Section 4.3). It can be used as a computationally cheap initial condition for more precise but much heavier numerical optimizations which often do not even converge unless properly initialized. As regards crack detection or source recovery, this approach boils down to analyzing the behavior of best meromorphic approximants of given pole cardinality to a function with branch points, which is the prototype of a polar singular set. For piecewise analytic cracks, or in the case of sources, we were able to prove ([6], [52, 45]), that the poles of the approximants accumulate, when the degree goes large, to some extremal cut of minimum weighted logarithmic capacity connecting the singular points of the crack, or the sources [48]. Moreover, the asymptotic density of the poles turns out to be the Green equilibrium distribution on this cut in D, therefore it charges the singular points if one is able to approximate in sufficiently high degree (this is where the method could fail, because high-order approximation requires rather precise data).

The case of two-dimensional singularities is still an outstanding open problem.

It is remarkable that inverse source problems inside a sphere or an ellipsoid in 3-D can be approached with such 2-D techniques, as applied to planar sections, see Section 7.1. The technique is implemented in the software FindSources3D, see Section 3.4.3.

3.4   Software tools of the team

In addition to the above-mentioned research activities, Factas develops and maintains a number of long-term software tools that either implement and illustrate effectiveness of the algorithms theoretically developed by the team or serve as tools to help further research by team members. We present briefly the most important of them.

3.4.1   pisa

Name: pisa

Keywords: Electrical circuit, Stability

Functional Description: To minimise prototyping costs, the design of analog circuits is performed using computer-aided design tools which simulate the circuit's response as accurately as possible.

Some commonly used simulation tools do not impose stability, which can result in costly errors when the prototype turns out to be unstable. A thorough stability analysis is therefore a very important step in circuit design. This is where pisa is used.

pisa is a Matlab toolbox that allows designers of analog electronic circuits to determine the stability of their circuits in the simulator. It analyses the impedance presented by a circuit to determine the circuit's stability. When an instability is detected, pisa can estimate location of the unstable poles to help designers fix their stability issue.

Release Contributions: First version

URL: https://project.inria.fr/pisa

Publications: hal-01381731, hal-01098616

Authors: Adam Cooman, David Martinez Martinez, Fabien Seyfert, Martine Olivi

Contact: Fabien Seyfert

3.4.2 DEDALE-HF

Keyword: Microwave filter

Scientific Description: Dedale-HF consists in two parts: a database of coupling topologies as well as a dedicated predictor-corrector code. Roughly speaking each reference file of the database contains, for a given coupling topology, the complete solution to the C.M. problem associated to particular filtering characteristics. The latter is then used as a starting point for a predictor-corrector integration method that computes the solution to the C.M. corresponding to the user-specified filter characteristics. The reference files are computed off-line using Gröbner basis techniques or numerical techniques based on the exploration of a monodromy group. The use of such continuation techniques, combined with an efficient implementation of the integrator, drastically reduces the computational time.

Dedale-HF has been licensed to, and is currently used by TAS-Espana.

Functional Description: Dedale-HF is a software dedicated to solve exhaustively the coupling matrix synthesis problem in reasonable time for the filtering community. Given a coupling topology, the coupling matrix synthesis problem (C.M. problem for short) consists in finding all possible electromagnetic coupling values between resonators that yield a realization of given filter characteristics. Solving the latter problem is crucial during the design step of a filter in order to derive its physical dimensions as well as during the tuning process where coupling values need to be extracted from frequency measurements.

URL: http://www-sop.inria.fr/apics/Dedale/

Contact: Fabien Seyfert

Participant: Fabien Seyfert

3.4.3 FindSources3D

Keywords: Health, Neuroimaging, Visualization, Compilers, Medical, Image, Processing

Scientific Description: Though synthetic data could be static, actual signal recordings are dynamical. The time dependency is either neglected and the data processed instant by instant, or separated from the space behavior using a singular value decomposition (SVD). This preliminary step allows to estimate the number of independent activities (uncorrelated sources) and to select the corresponding quantity of principal static components. After a first data transmission ("cortical mapping") step of the static data, using the harmonicity property of the potential in the outermost layers (solving BEP problems on spherical harmonics bases), FS3D makes use of best rational approximation on families of 2-D planar cross-sections and of the software RARL2 in order to locate singularities and to determine the expected quantity of sources. From those planar singularities, the 3-D sources are finally estimated, together with their moment, in a last clustering step. Through this process, FS3D is able to recover time correlated sources, which is an important advantage. When simultaneously available, EEG and MEG data can now be processed together, and this also improves the recovery performance. In case of dynamical data, a recent additional step is to find the linear combination of the preliminary selected static components (change of basis) that produces source estimates which minimize the error w.r.t. data, an original criterion, which allows to improve the recovery quality.

Functional Description: FindSources3D (FS3D) is a software program written in Matlab dedicated to the resolution of inverse source problems in brain imaging, electroencephalography (EEG) and magnetoencephalography (MEG). From data consisting in pointwise measurements of the electrical potential taken by electrodes on the scalp (EEG), or of a component of the magnetic field taken on

a helmet (MEG), FS3D estimates pointwise dipolar current sources within the brain in a spherical layered model. Each layer (brain, skull, scalp) is assumed to have a constant conductivity.

URL: http://www-sop.inria.fr/apics/FindSources3D/en/index.html

Contact: Juliette Leblond

Participants: Jean-Paul Marmorat, Juliette Leblond, Maureen Clerc, Nicolas Schnitzler, Théodore Papadopoulo

3.4.4 PRESTO-HF

Keywords: CAO, Telecommunications, Microwave filter

Scientific Description: For the matrix-valued rational approximation step, Presto-HF relies on RARL2. Constrained realizations are computed using the Dedale-HF software. As a toolbox, Presto-HF has a modular structure, which allows one for example to include some building blocks in an already existing software.

The delay compensation algorithm is based on the following assumption: far off the pass-band, one can reasonably expect a good approximation of the rational components of S11 and S22 by the first few terms of their Taylor expansion at infinity, a small degree polynomial in $1/s$. Using this idea, a sequence of quadratic convex optimization problems are solved, in order to obtain appropriate compensations. In order to check the previous assumption, one has to measure the filter on a larger band, typically three times the pass band.

This toolbox has been licensed to (and is currently used by) Thales Alenia Space in Toulouse and Madrid, Thales airborne systems and Flextronics (two licenses). Xlim (University of Limoges) is a heavy user of Presto-HF among the academic filtering community and some free license agreements have been granted to the microwave department of the University of Erlangen (Germany) and the Royal Military College (Kingston, Canada).

Functional Description: Presto-HF is a toolbox dedicated to low-pass parameter identification for microwave filters. In order to allow the industrial transfer of our methods, a Matlab-based toolbox has been developed, dedicated to the problem of identification of low-pass microwave filter parameters. It allows one to run the following algorithmic steps, either individually or in a single stroke:

- Determination of delay components caused by the access devices (automatic reference plane adjustment),
- Automatic determination of an analytic completion, bounded in modulus for each channel,
- Rational approximation of fixed McMillan degree,
- Determination of a constrained realization.

URL: https://project.inria.fr/presto-hf/

Contact: Fabien Seyfert

Participants: Fabien Seyfert, Jean-Paul Marmorat, Martine Olivi

3.4.5 RARL2

Name: Réalisation interne et Approximation Rationnelle L2

Keyword: Approximation

Scientific Description: The method is a steepest-descent algorithm. A parametrization of MIMO systems is used, which ensures that the stability constraint on the approximant is met. The implementation, in Matlab, is based on state-space representations.

RARL2 performs the rational approximation step in the software tools PRESTO-HF and Find-Sources3D. It is distributed under a particular license, allowing unlimited usage for academic research purposes. It was released to the universities of Delft and Maastricht (the Netherlands), Cork (Ireland), Brussels (Belgium), Macao (China) and BITS-Pilani Hyderabad Campus (India).

Functional Description: RARL2 is a software for rational approximation. It computes a stable rational L2-approximation of specified order to a given L2-stable (L2 on the unit circle, analytic in the complement of the unit disk) matrix-valued function. This can be the transfer function of a multivariable discrete-time stable system. RARL2 takes as input either:

- its internal realization,
- its first N Fourier coefficients,
- discretized (uniformly distributed) values on the circle. In this case, a least-square criterion is used instead of the L2 norm.

It thus performs model reduction in the first or the second case, and leans on frequency data identification in the third. For band-limited frequency data, it could be necessary to infer the behavior of the system outside the bandwidth before performing rational approximation.

An appropriate Möbius transformation allows to use the software for continuous-time systems as well.

URL: http://www-sop.inria.fr/apics/RARL2/rarl2.html

Contact: Martine Olivi

Participants: Jean-Paul Marmorat, Martine Olivi

3.4.6 Sollya

Keywords: Computer algebra system (CAS), Supremum norm, Proof synthesis, Code generator, Remez algorithm, Curve plotting, Numerical algorithm

Functional Description: Sollya is an interactive tool where the developers of mathematical floating-point libraries (libm) can experiment before actually developing code. The environment is safe with respect to floating-point errors, i.e. the user precisely knows when rounding errors or approximation errors happen, and rigorous bounds are always provided for these errors.

Among other features, it offers a fast Remez algorithm for computing polynomial approximations of real functions and also an algorithm for finding good polynomial approximants with floating-point coefficients to any real function. As well, it provides algorithms for the certification of numerical codes, such as Taylor Models, interval arithmetic or certified supremum norms.

It is available as a free software under the CeCILL-C license.

URL: https://sollya.org/

Contact: Sylvain Chevillard

Participants: Christoph Lauter, Jérôme Benoit, Marc Mezzarobba, Mioara Joldes, Nicolas Jourdan, Sylvain Chevillard

Partners: CNRS, UPMC, ENS Lyon, LIP6, UCBL Lyon 1, Loria

4 Application domains

4.1 Introduction

Application domains are naturally linked to the problems described in Sections 3.2.1 and 3.2.2. By and large, they split into a systems-and-circuits part and an inverse-source-and-boundary-problems part, united under a common umbrella of function-theoretic techniques as described in Section 3.3.

4.2   Inverse magnetization problems

Participants: Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Anass Yousfi.

Generally speaking, inverse potential problems, similar to the one appearing in Section 4.3, occur naturally in connection with systems governed by Maxwell's equation in the quasi-static approximation regime. In particular, they arise in magnetic reconstruction issues. A specific application is to geophysics, which led us to form the Inria Associate Team IMPINGE (Inverse Magnetization Problems IN GEosciences) together with MIT and Vanderbilt University that reached the end of its term in 2018.

To set up the context, recall that the Earth's geomagnetic field is generated by convection of the liquid metallic core (geodynamo) and that rocks become magnetized by the ambient field as they are formed or after subsequent alteration. Their remanent magnetization provides records of past variations of the geodynamo, which is used to study important processes in Earth sciences like motion of tectonic plates and geomagnetic reversals. Rocks from Mars, the Moon, and asteroids also contain remanent magnetization which indicates the past presence of core dynamos. Magnetization in meteorites may even record fields produced by the young sun and the protoplanetary disk which may have played a key role in solar system formation.

For a long time, paleomagnetic techniques were only capable of analyzing bulk samples and compute their net magnetic moment. The development of SQUID microscopes has recently extended the spatial resolution to sub-millimeter scales, raising new physical and algorithmic challenges. The associate team IMPINGE aims at tackling them, experimenting with the SQUID microscope set up in the Paleomagnetism Laboratory of the department of Earth, Atmospheric and Planetary Sciences at MIT. Typically, pieces of rock are sanded down to a thin slab, and the magnetization has to be recovered from the field measured on a planar region at small distance from the slab.

Mathematically speaking, both inverse source problems for EEG from Section 4.3 and inverse magnetization problems described presently amount to recover the (3-D valued) quantity m (primary current density in case of the brain or magnetization in case of a thin slab of rock) from measurements of the potential:

$$V(x) = \int_\Omega \frac{\operatorname{div} m(x')\, dx'}{|x - x'|}, \tag{1}$$

outside the volume Ω of the object. Depending on the geometry of models, the magnetization distribution m may lie in a volume or spread out on a surface. This results in quite different identifiability properties, see [44] and Section 7.1.1, but the two situations share a substantial mathematical common core.

Another timely instance of inverse magnetization problems lies with geomagnetism. Satellites orbiting around the Earth measure the magnetic field at many points, and nowadays it is a challenge to extract global information from those measurements. In collaboration with C. Gerhards (Geomathematics and Geoinformatics Group, Technische Universität Bergakademie Freiberg, Germany), we started to work on the problem of separating the magnetic field due to the magnetization of the globe's crust from the magnetic field due to convection in the liquid metallic core. The techniques involved are variants, in a spherical context, from those developed within the IMPINGE associate team for paleomagnetism, see Section 7.1.1.

4.3   Inverse source problems in EEG

Participants: Paul Asensio, Laurent Baratchart, Juliette Leblond, Jean-Paul Marmorat, Masimba Nemaire.

Solving over-determined Cauchy problems for the Laplace equation on a spherical layer (in 3-D) in order to extrapolate incomplete data (see Section 3.2.1) is a necessary ingredient of the team's approach to inverse source problems, in particular for applications to EEG, see [8]. Indeed, the latter involves propagating the initial conditions through several layers of different conductivities, from the boundary shell down to the center of the domain where the singularities (*i.e.* the sources) lie. Once propagated to

the innermost sphere, it turns out that traces of the boundary data on 2-D cross sections coincide with analytic functions with branched singularities in the slicing plane [6], [49]. The singularities are related to the actual location of the sources, namely their moduli reach in turn a maximum when the plane contains one of the sources. Hence we are back to the 2-D framework of Section 3.3.3, and recovering these singularities can be performed *via* best rational approximation. The goal is to produce a fast and sufficiently accurate initial guess on the number and location of the sources in order to run heavier descent algorithms on the direct problem, which are more precise but computationally costly and often fail to converge if not properly initialized. Our belief is that such a localization process can add a geometric, valuable piece of information to the standard temporal analysis of EEG signal records.

Numerical experiments obtained with our software FindSources3D give very good results on simulated data and we are now engaged in the process of handling real experimental data, simultaneously recorded by EEG and MEG devices, in collaboration with our partners at INS, hospital la Timone, Marseille (see Section 7.1.2).

Furthermore, another approach is being studied for EEG, that consists in regularizing the inverse source problem by a total variation constraint on the source term (a measure), added to the quadratic data approximation criterion. It is similar to the path that is taken for inverse magnetization problems (see Sections 4.2 and 7.1.1), and it presently focuses on surface-distributed models.

4.4   Identification and design of microwave devices

Participants: Laurent Baratchart, Sylvain Chevillard, Jean-Paul Marmorat, Martine Olivi, Fabien Seyfert.

This is joint work with Stéphane Bila (Xlim, Limoges).

One of the best training grounds for function-theoretic applications by the team is the identification and design of physical systems whose performance is assessed frequency-wise. This is the case of electromagnetic resonant systems which are of common use in telecommunications.

In space telecommunications (satellite transmissions), constraints specific to on-board technology lead to the use of filters with resonant cavities in the microwave range. These filters serve multiplexing purposes (before or after amplification), and consist of a sequence of cylindrical hollow bodies, magnetically coupled by irises (orthogonal double slits). The electromagnetic wave that traverses the cavities satisfies the Maxwell equations, forcing the tangent electrical field along the body of the cavity to be zero. A deeper study of the Helmholtz equation states that an essentially discrete set of wave vectors is selected. In the considered range of frequency, the electrical field in each cavity can be decomposed along two orthogonal modes, perpendicular to the axis of the cavity (other modes are far off in the frequency domain, and their influence can be neglected). Each cavity (see Figure 1) has three screws, horizontal, vertical and midway (horizontal and vertical are two arbitrary directions, the third direction makes an angle of 45 or 135 degrees, the easy case is when all cavities show the same orientation, and when the directions of the irises are the same, as well as the input and output slits). Since screws are conductors, they behave as capacitors; besides, the electrical field on the surface has to be zero, which modifies the boundary conditions of one of the two modes (for the other mode, the electrical field is zero hence it is not influenced by the screw), the third screw acts as a coupling between the two modes. The effect of an iris is opposite to that of a screw: no condition is imposed on a hole, which results in a coupling between two horizontal (or two vertical) modes of adjacent cavities (in fact the iris is the union of two rectangles, the important parameter being their width). The design of a filter consists in finding the size of each cavity, and the width of each iris. Subsequently, the filter can be constructed and tuned by adjusting the screws. Finally, the screws are glued once a satisfactory response has been obtained. In what follows, we shall consider a typical example, a filter designed by the CNES in Toulouse, with four cavities near 11 GHz.

Near the resonance frequency, a good approximation to the Helmholtz equations is given by a second order differential equation. Thus, one obtains an electrical model of the filter as a sequence of electrically-coupled resonant circuits, each circuit being modeled by two resonators, one per mode, the resonance frequency of which represents the frequency of a mode, and whose resistance accounts for electric losses (surface currents) in the cavities.
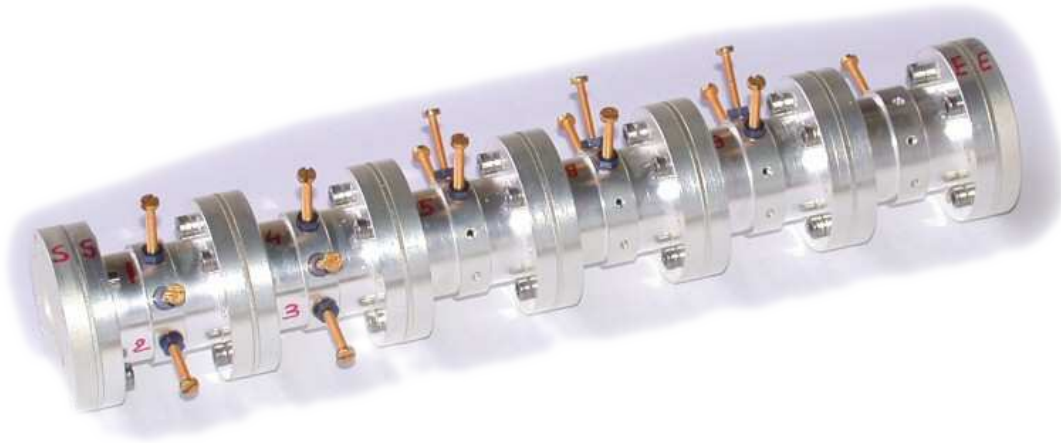
Figure 1: Picture of a 6-cavities dual mode filter. Each cavity (except the last one) has 3 screws to couple the modes within the cavity, so that 16 quantities must be optimized. Quantities such as the diameter and length of the cavities, or the width of the 11 slits are fixed during the design phase.

This way, the filter can be seen as a quadripole, with two ports, when plugged onto a resistor at one end and fed with some potential at the other end. One is now interested in the power which is transmitted and reflected. This leads one to define a scattering matrix S, which may be considered as the transfer function of a stable causal linear dynamical system, with two inputs and two outputs. Its diagonal terms $S_{1,1}$, $S_{2,2}$ correspond to reflections at each port, while $S_{1,2}$, $S_{2,1}$ correspond to transmission. These functions can be measured at certain frequencies (on the imaginary axis). The matrix S is approximately rational of order 4 times the number of cavities (that is 16 in the example on Figure 2), and the key step consists in expressing the components of the equivalent electrical circuit as functions of the S_{ij} (since there are no formulas expressing the lengths of the screws in terms of parameters of this electrical model). This representation is also useful to analyze the numerical simulations of the Maxwell equations, and to check the quality of a design, in particular the absence of higher resonant modes.

In fact, resonance is not studied via the electrical model, but via a low-pass equivalent circuit obtained upon linearizing near the central frequency, which is no longer conjugate symmetric (*i.e.* the underlying system may no longer have real coefficients) but whose degree is divided by 2 (8 in the example).

In short, the strategy for identification is as follows:

- measuring the scattering matrix of the filter near the optimal frequency over twice the pass band (which is 80MHz in the example).

- Solving bounded extremal problems for the transmission and the reflection (the modulus of he response being respectively close to 0 and 1 outside the interval measurement, cf. Section 3.3.1) in order to get a models for the scattering matrix as an analytic matrix-valued function. This provides us with a scattering matrix known to be close to a rational matrix of order roughly 1/4 of the number of data points.

- Approximating this scattering matrix by a true rational transfer-function of appropriate degree (8 in this example) via the Endymion or RARL2 software (cf. Section 3.3.2).

- A state space realization of S, viewed as a transfer function, can then be obtained, where additional symmetry constraints coming from the reciprocity law and possibly other physical features of the device have to be imposed.

- Finally one builds a realization of the approximant and looks for a change of variables that eliminates non-physical couplings. This is obtained by using algebraic-solvers and continuation algorithms on the group of orthogonal complex matrices (symmetry forces this type of transformation).
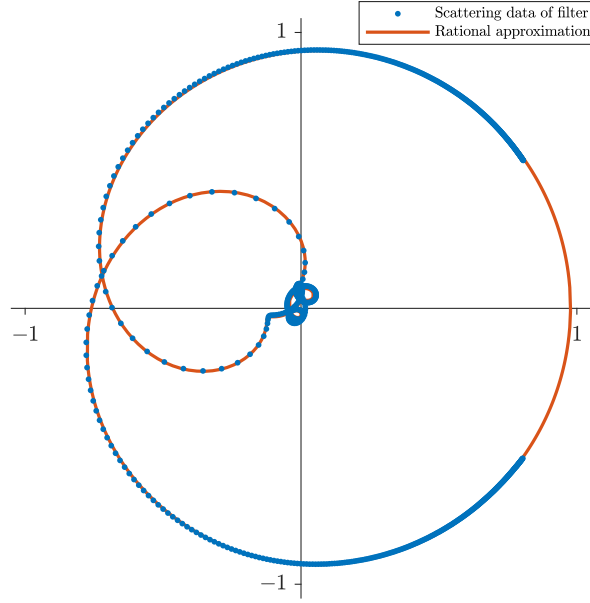
Figure 2: Nyquist Diagram of S_{22}. The rational approximation is of degree 8.

The final approximation is of high quality. This can be interpreted as a confirmation of the linearity assumption on the system: the relative L^2 error is less than 10^{-3}. This is illustrated by a reflection diagram (Figure 2).

The above considerations are valid for a large class of filters. These developments have also been used for the design of non-symmetric filters, which are useful for the synthesis of repeating devices.

The team further investigates problems relative to the design of optimal responses for microwave devices. The resolution of a quasi-convex Zolotarev problems was proposed, in order to derive guaranteed optimal multi-band filter responses subject to modulus constraints [30]. This generalizes the classical single band design techniques based on Chebyshev polynomials and elliptic functions. The approach relies on the fact that the modulus of the scattering parameter $|S_{1,2}|$ admits a simple expression in terms of the filtering function $D = |S_{1,1}|/|S_{1,2}|$, namely

$$|S_{1,2}|^2 = \frac{1}{1+D^2}.$$

The filtering function appears to be the ratio of two polynomials p_1/p_2, the numerator of the reflection and transmission scattering factors, that may be chosen freely. The denominator q is then obtained as the unique stable unitary polynomial solving the classical Feldtkeller spectral equation:

$$qq^* = p_1 p_1^* + p_2 p_2^*.$$

The relative simplicity of the derivation of a filter's response, under modulus constraints, owes much to the possibility of forgetting about Feldtkeller's equation and express all design constraints in terms of the filtering function. This no longer the case when considering the synthesis N-port devices for $N > 3$, like multiplexers, routers and power dividers, or when considering the synthesis of filters under matching conditions. The efficient derivation of multiplexers responses is the subject of recent investigation by Factas, using techniques based on constrained Nevanlinna-Pick interpolation (see Section 7.2).

Through contacts with CNES (Toulouse) and UPV (Bilbao), the team got additionally involved in the design of amplifiers which, unlike filters, are active devices. A prominent issue here is stability. A twenty years back, it was not possible to simulate unstable responses, and only after building a device could one detect instability. The advent of so-called *harmonic balance* techniques, which compute steady state responses of linear elements in the frequency domain and look for a periodic state in the time domain of a network connecting these linear elements *via* static non-linearities made it possible to compute the

harmonic response of a (possibly nonlinear and unstable) device [89]. This has had tremendous impact on design, and there is a growing demand for software analyzers. The team is also becoming active in this area.

In this connection, there are two types of stability involved. The first is stability of a fixed point around which the linearized transfer function accounts for small signal amplification. The second is stability of a limit cycle which is reached when the input signal is no longer small and truly nonlinear amplification is attained (*e.g.* because of saturation). Applications by the team so far have been concerned with the first type of stability, and emphasis is put on defining and extracting the "unstable part" of the response, see Section 7.5. The stability check for limit cycles has made important theoretical advances (see [14], [28]), and numerical algorithms are now under investigation.

4.5 Imaging and modeling ancient materials

Participants: Vanna Lisa Coli, Juliette Leblond.

This is a recent activity of the team, linked to image classification in archaeology in the framework of the projects ToMaT and Arch-AI-Story, see Section 9.3; it is pursued in collaboration with L. Blanc-Féraud (project-team Morpheme, I3S-CNRS/Inria Sophia/iBV), D. Binder (CEPAM-CNRS, Nice), in particular.

The pottery style is classically used as the main cultural marker within Neolithic studies. Archaeological analyses focus on pottery technology, and particularly on the first stages of pottery manufacturing processes. These stages are the most demonstrative for identifying the technical traditions, as they are considered as crucial in apprenticeship processes. Until now, the identification of pottery manufacturing methods was based on macro-traces analysis, i.e. surface topography, breaks and discontinuities indicating the type of elements (coils, slabs, . . .) and the way they were put together for building the pots. Overcoming the limitations inherent to the macroscopic pottery examination requires a complete access to the internal structure of the pots. Micro-computed tomography (μCT) has recently been used for exploring ancient materials micro-structure. This non-invasive method provides quantitative data for a big set of proxies and is perfectly adapted to the analysis of Cultural heritage materials.

The main challenge of our current analyses aims to overcome the lack of existing protocols to apply in order to quantify observations. In order to characterize the manufacturing sequences, the mapping of the paste variability (distribution and composition of temper) and the discontinuities linked to different classes of pores, fabrics and/or organic inclusions appears promising. The totality of the acquired images composes a set of 2-D and 3-D surface and volume data at different resolutions and with specific physical characteristics related to each acquisition modality (multimodal and multi-scale data). Specific shape recognition methods need to be developed by application of robust imaging techniques and 3-D-shapes recognition algorithms.

In a first step, we devised a method to isolate pores from the 3-D data volumes in binary 3-D images, to which we apply a process named Hough transform (derived from Radon transform). This method, of which the generalization from 2-D to 3-D is quite recent, allows us to evaluate the presence of parallel lines going through the pores. The quantity of such lines and their parallelism furnish good indicators of the "coiling" manufacturing, that they allow to distinguish from the other "spiral patchwork" technique, in particular. These progresses are described in [18].

Other possibilities of investigation are being analyzed as well, such as machine learning and deep learning techniques.

5 Social and environmental responsibility

5.1 Footprint of research activities

Participants: Sylvain Chevillard, Martine Olivi.

We continued in 2021 the work engaged in 2020 in coordination with Céline Serrano (in charge of "Sustainable development" at the national level) on setting up methods to evaluate the carbon footprint of research activities at Inria.

First, we discovered and corrected a few minor mistakes in the evaluation that we had made of the 2019 carbon footprint of team Factas. Together with people in charge of monitoring the electricity consumption in the research center, we spotted inconsistencies in the meter readings and improved their reliability. Questions still remain, that cannot be answered yet, due to the very particular case of 2020 (two lock-downs) and of the corresponding data. We hope that 2021 data (coming soon) will be more useful on this respect.

Second, we wrote a complete report, explaining in details both the methodology that we used, and the figures obtained for the carbon footprint of the team Factas in 2019, see [29]. We improved the software tool that we developed for that purpose (see 6.1.1), and wrote the necessary documentation to make it usable by other teams.

Third, we volunteered to do the carbon footprint of the research center with the GES1p5 methodology. It accounts for the greenhouse gases emissions due to mobility (commute travels and missions), electricity and gas consumption in the buildings. We are still analyzing the results to understand where the efforts should be put to reduce the emissions of the research center in the future. Some preliminary figures are available on the website of the local "sustainable development" commission (access restricted to the Inria internal network). Overall, more than 80% of the emissions are due to the mobility.

6   New software and platforms

Let us describe new/updated software.

6.1   New software

6.1.1   FootprintTools

Name: FootprintTools

Keywords: CO2, Carbon footprint

Functional Description: The tool contains python scripts that allow one to extract the information about missions paid by the team during a given year and electronic equipment bought by the team in the past years, from the Inria information system. Convenient functions are proposed to explore, filter and format the obtained data, so as to use them to compute the carbon footprint as accurately as possible.

A HTML survey is also provided to easily query the members of a team about their habits in terms of meals and commute travels.

URL: `https://gitlab.inria.fr/chevilla/FootprintTools/-/tree/master/doc`

Contact: Sylvain Chevillard

7   New results

7.1   Inverse problems for Poisson-Laplace equations

Participants: Paul Asensio, Laurent Baratchart, Sylvain Chevillard, Juliette Leblond, Jean-Paul Marmorat, Masimba Nemaire, Cristóbal Villalobos Guillén, Anass Yousfi.

7.1.1   Inverse magnetization issues for planar samples

The goal is to invert magnetizations carried by a planar set in Euclidean space from measurements of the magnetic field nearby. A typical application is to paleomagnetism, to determine magnetic properties of rock samples, shaped into thin slabs, with measurements taken by a superconducting quantum interference device (SQUID). Figure 3 sketches the corresponding experimental set up, brought up to our knowledge by collaborators from the Earth and Planetary Sciences Laboratory at MIT.
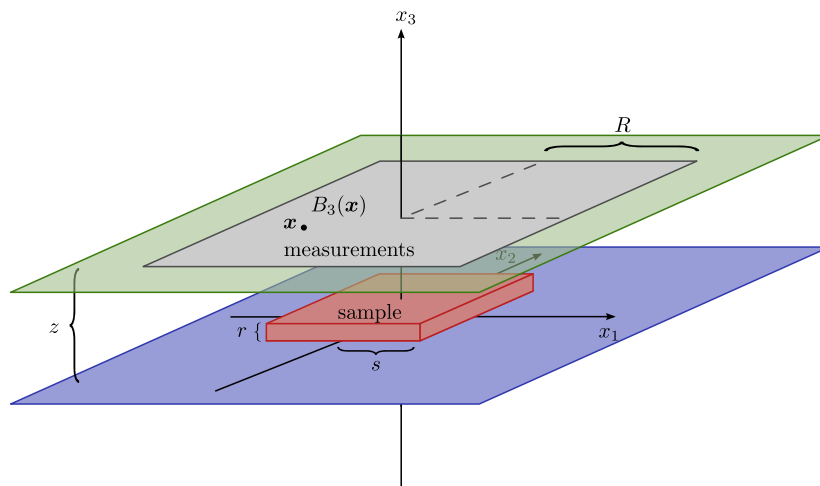

Figure 3: Schematic view of the SQUID experimental setup. The sample lies on a horizontal plane at height 0 and its support is included in a parallelepiped (in red). The vertical component B_3 of the field produced by the sample is measured in points of a horizontal square at height z (in gray).

We pursued our research program on the recovery of magnetizations modeled by signed measures on thin samples, which is an instance of Poisson inverse problem with right hand side in divergence form (the divergence of a $\mathbb{R}^3$-valued measure supported on a planar set in space). We showed in previous years that consistent recovery is possible, in the Morozov discrepancy limit, by penalizing the total variation, when either the support of the magnetization is purely 1-unrectifiable (which holds in particular for dipolar models) or when the magnetization is unidirectional (an assumption of physical interest: indeed, when igneous rocks cool down in some ambient field, they acquire a magnetization whose intensity may vary internally depending on the composition of the rock, but that is mainly oriented in the same direction as the ambient field), or else when it is supported on sufficiently separated line segments. These notions play a role similar to sparsity in this infinite-dimensional context. We also established that the argument of the minimum of the regularized criterion is unique, by taking advantage of the particular form of the critical point equation for the criterion, together with a loop decomposition of planar divergence free measures which sharpens in the 2-D setting the structure theorem of [87]. An article has been published on these results [17].

We focused this year on consistent discretization schemes, meaning discrete versions of the continuous *regularized* problem whose solutions converge to the solution of this continuous regularized problem. We were able to show that, for a fairly large family of discretizations based on weak-star dense collections of mutually singular measures, the solutions to the regularized discretized problem are asymptotically supported on a system of analytic arcs that contains the solution to the continuous problem, regularized with the same value of the regularizing parameter. We are currently analyzing the significance of this system of arcs, in connection with the maximizing function of a certain extremal problem for the measure generating the data when it exists; *i.e.*, when we assume that the data are exact. This extremal problem, that recurs in inverse magnetization problems for planar samples, consists in best approximation by a divergence-free measure; such measures form the kernel of the forward operator. An implementation of a variant of the FISTA algorithm is used to solve the discrete regularized problem when the discrete model consists of dipoles or else of uniformly magnetized squares, in order to substantiate our results in this case. This is collaborative research with D. Hardin from Vanderbilt university in the course of the postdoctoral program of C. Villalobos Guillén.

7.1.2   Inverse problems in medical imaging

In 3-D, functional or clinically active regions in the cortex are often modeled by pointwise sources that have to be localized from measurements, taken by electrodes on the scalp, of an electrical potential satisfying a Laplace equation (EEG, electroencephalography). In the works [6], [49] on the behavior of poles in best rational approximants of fixed degree to functions with branch points, it was shown how to proceed via best rational approximation on a sequence of 2-D disks cut along the inner sphere, for the case where there are finitely many sources (see Section 4.3).

In this connection, a dedicated software FindSources3D (FS3D, see Section 3.4.3) is being developed, in collaboration with the Inria team Athena and the CMA - Mines ParisTech. Its Matlab version now incorporates the treatment of MEG data, the aim being to handle simultaneous EEG–MEG recordings available from our partners at INS, hospital la Timone, Marseille. Indeed, it is now possible to use simultaneously EEG and MEG measurement devices, in order to measure both the electrical potential and a component of the magnetic field (its normal component on the MEG helmet, that can be assumed to be spherical). Solving the inverse source problem from joint EEG and MEG data actually improves accuracy of the source estimation.

From synthetic data simulated with MNE, that consist in two asynchronous source patches (in the visual cortex), FS3D furnishes the results shown in Figure 4 where they are mapped in a realistic head.
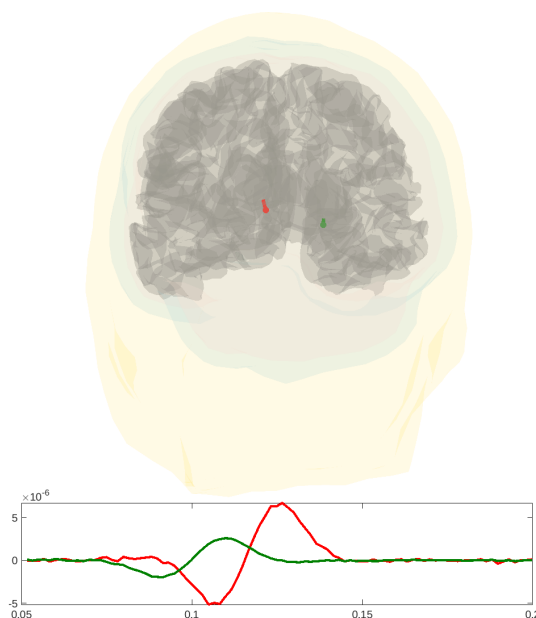

Figure 4: Above: estimated sources mapped in the head referential; below: potentials generated by each of the two sources w.r.t. time.

Note that FS3D takes as inputs actual EEG and MEG measurements, like time signals, and performs a suitable singular value decomposition in order to separate independent sources.

It appears that, in the rational approximation step, *multiple* poles possess a nice behavior with respect to branched singularities. This is due to the very physical assumptions on the model from dipolar current sources: for EEG data that correspond to measurements of the electrical potential, one should consider *triple* poles; this will also be the case for MEG – magneto-encephalography – data. However, for (magnetic) field data produced by magnetic dipolar sources within rocks, one should consider poles of order five. Though numerically observed in [8], there is no mathematical justification so far why multiple poles generate such strong accumulation of the poles of the approximants. This intriguing property, however, is definitely helping source recovery and will be the topic of further study. It is used in order to automatically estimate the "most plausible" number of sources (numerically: up to 3, at the moment).

FS3D is now used by some of our collaborators. Together with M. Darbas (LAGA, Univ. Sorbonne Paris Nord) and P.-H. Tournier (labo. JLL, Sorbonne Univ), we recently began to handle the EEG inverse problem with a variable conductivity in the intermediate skull layer, in order to model hard / spongy bones, especially for neonates. Coupled with FS3D, the related transmission step is performed using a mixed variational regularization and finite elements (FreeFem++) on tetrahedral meshes, and furnishes very promising results.

Another aspect is the influence of the time dependency within the PDE on the behavior of the solution, which we study with I. Stratis and A. Yannacopoulos from National & Kapodistrian Univ., Athens. It seems that the time derivative of the electric field is not so small within the brain, compared to that of the magnetic field in particular, whence the quasi-static assumption may not be valid.

We also studied the uniqueness of the critical point of the quadratic criterion in the electroencephalography problem for a single dipole situation (PhD of P. Asensio). This issue is essential for the use of descent algorithms. This leads to the study of the following criterion:

$$J(X, p) = \left\| \frac{p \cdot (X - Y)}{|X - Y|^3} - \frac{p_0 \cdot (X_0 - Y)}{|X_0 - Y|^3} \right\|_{L^2(Y, \partial\Omega)}^2 ,$$

where X and X_0 (actual dipole position) are in a smooth open set Ω included in $\mathbb{R}^3$ and p, p_0 (actual dipole moment) belong to $\mathbb{R}^3$. In the half-space case where $\Omega = \{(x, y, z) \in \mathbb{R}^3, z < 0\}$, we have proven that there is a unique critical point; *i.e.*, a unique point X where the gradient vanishes, which is but X_0 (the absolute minimum). We currently try to obtain an analogous result on the ball, which is considerably more difficult because no translation invariance holds anymore.

We started considering a different class of models, not necessarily dipolar, and related estimation algorithms. Such models may be supported on the surface of the cortex or in the volume of the encephalon. We represent sources by vector-valued measures, and in order to favor sparsity in this infinite-dimensional setting we use a TV (i.e. total variation) regularization term as in Section 7.1.1. The approach follows that of [7] and is implemented through two different algorithms, whose convergence properties are currently being studied. Tests on synthetic data from a few dipolar sources provide results of different qualities that need to be better understood. In particular, a weight is being added in the TV term in order to better identify deep sources. This is the topic of the PhD researches of P. Asensio and M. Nemaire. Ultimately, the results will be compared to those of FS3D and other available software tools. Progresses were made
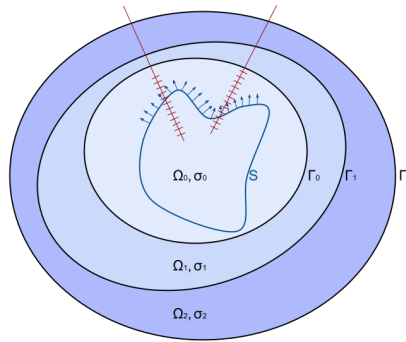

Figure 5: Schematic view of a 3 layered head model. Deep electrodes with sensors along them (red) in Ω_0, current source term μ distributed on $S \subset \Omega_0$ with normally oriented dipoles.

on the inverse problem of "Stereo" EEG (SEEG), where the potential is measured by deep electrodes and sensors within the brain as in the scheme of Figure 5. Assuming that the current source term μ is a $\mathbb{R}^3$-valued vector field (or a measure, or a dipolar distribution) supported on a surface S and normally oriented to S, makes it possible to describe the potential $\phi(\mu)$ generated by μ as a double layer potential:

$$\phi(\mu)(x) = -\frac{1}{4\pi} \int_S \mu \cdot \frac{x - y}{|x - y|^3} \, dy, \ x \notin S.$$

The associated forward and inverse problems were solved for both an infinite medium conductor and a more realistic single model of the brain Ω_0 (as first step towards nested domains for the head). Concerning the latter, it is preliminary assumed that $S \subset \partial\Omega_0$, whence μ consists in patches on the cortex surface.
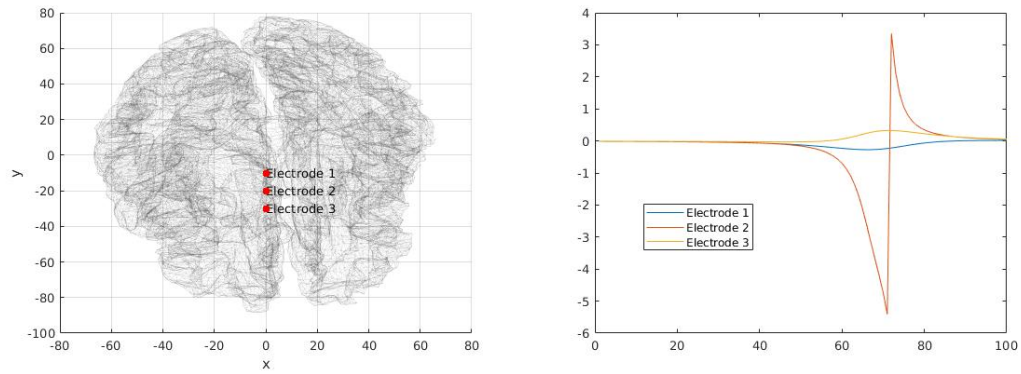
Figure 6: Realistic brain mesh. Electrode positions (left); potential interpolated along the 100 sensors on each electrode (right).
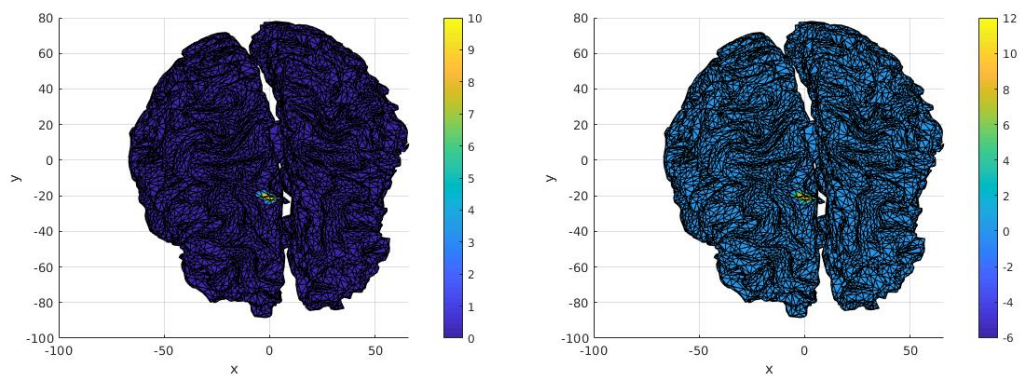

Figure 7: Realistic brain mesh. Actual (left) and estimated (right) source supports.

The numerical implementation was done by approximating the density μ of the double layer potential with linear shape functions on triangulations of the involved surface S. For the discretized version, μ is then approximated on every triangle by a piecewise linear function, and the associated moment is taken to be located at the center of the triangle, perpendicular to the triangle. Note that explicit expressions of the double layer potential are also available at points located on S (that supports μ).

The inverse problem for SEEG is ill-posed and a Tychonov regularization is used in order to solve the problem: find a distribution μ defined on the prescribed surface S such that $\|\phi(\mu) - \phi^d\|_{L^2(\Omega_0)} + \lambda R(\mu)$ is minimized, where ϕ^d is the given data in Ω_0, $\phi(\mu)$ is the "forward model" (measurements of the double layer potential), $R(\mu)$ is an appropriate regularization term that results in μ having some desired properties (being bounded in L^2 norm or TV or L^1 on S), and λ a (Lagrange) parameter. Because ϕ^d is only furnished at sensor positions, the L^2 term in the above criterion becomes an l^2 term. Figures 6 and 7 show the obtained results.

We now consider $S \subset \Omega_0$ within the brain, at the gray / white matters interface and we handle outer layers Ω_1 and Ω_2 for the skull and the scalp, as illustrated in Figure 5. Such configurations involve a data transmission / "cortical mapping" step, that relates the potential and normal current values at the boundaries Γ_i, that the present double layer formulation allows to solve with the source localization issue.

We are now able to handle MEG, EEG, SEEG modalities, simultaneously or not.

For more general source terms (vector valued measures, L^p vector-fields), an alternating minimization procedure between refining the cortical mapping and the source localization is being studied and implemented. It takes advantage of the fact that both the source and the surface potentials are assumed to be elements of Banach spaces that have certain smoothness properties which enable the convergence to a solution as soon as the problem is convex. This alleviates problems that may be encountered when the regularizing term is non-smooth. This opens up the possibility to consider sources that may exhibit properties usually associated with distributions rather than functions.

7.2   Matching problems and their applications: Finite degree bounds

Participants: Laurent Baratchart, Martine Olivi, Gibin Bose, Fabien Seyfert.

On the topic of uniform matching, Gibin Bose defended his thesis «Approximation H^∞, Interpolation Analytique et Optimisation Convexe : Application à l'Adaptation d'Impédance Large Bande» on January 8th 2021. Triggered by remarks of one of the reviewers the PUMA code was extended to treat the presence of one transmission zero at infinity in the antenna's response and compare results obtained by our code with the classical Fano bound.

The Fano bound is very popular in the antenna community because of its simplicity. Under the realistic hypothesis that the load is totally reflective at high frequency, it states that: if L_{11} is the reflection coefficient of the load, and S_{11} the reflection coefficient of the global system (i.e. Matching Circuit + Load), then:

$$\left| \int_{-\infty}^{+\infty} \log(|S_{11}(j\omega)|) \, \mathrm{d}\omega \right| \le K \, Ang_\infty(L_{11})$$

holds. Here $Ang_\infty(.)$ stays for the angular derivative at infinity and $K = 20\pi/\log(10)$ is a constant. In other words the log modulus integral of the global system is bounded by the angular derivative of the load. Eventually, if we assume ideally that the global system's response is a step function centered on the pass-band B (which is only possible with an infinite dimensional matching circuit), we get the following bound on the maximal matching log-level R_L of the global system:

$$R_L \le K \, Ang_\infty(L_{11})/|B| = F_B$$

where $|B|$ stands for the length of the pass-band. F_B is called the Fano bound, called after the name of its inventor, Fano, a PhD student of Bode. For a single resonance load, this band is sharp, provided an infinite dimensional matching circuit is at hand. For more complex loads, the Fano bound might be utterly optimistic.
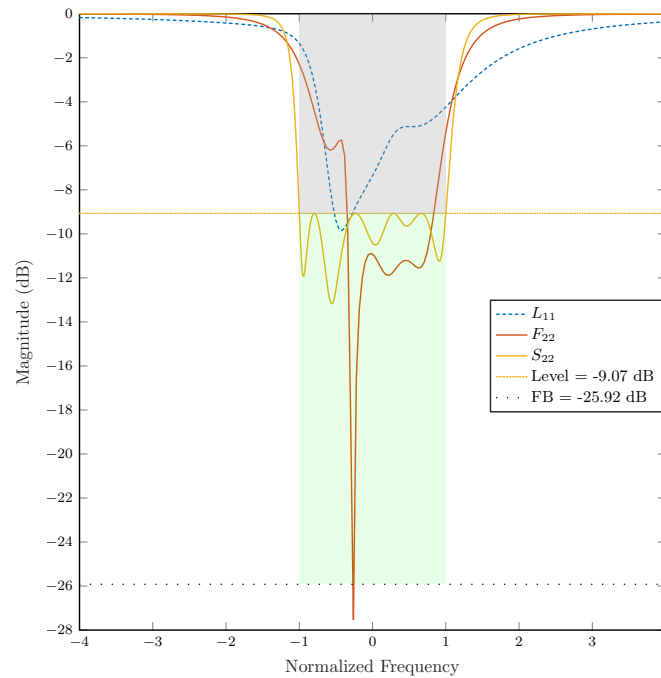
Figure 8: Reflection coefficients: L_{11} antenna, S_{22} global system, F_{22} matching circuit. FB is the Fano bound.

We treated extensively the case of an antenna with a reflection coefficient of degree 3. On Figure 8 an absolute bound for a degree 2 matching circuit has been computed, leading to the depicted reflection coefficient S_{22} yielding a matching level of 9.07 dB in the prescribed band. This must be compared here to the 25.92 dB of the Fano bound, which appears to be largely over optimistic. This discrepancy between the theoretical Fano bound and the bound computed by our method has two reasons. The antenna is of degree 3 and has in addition to its zero at infinity two additional finite transmission zeros. This deviation from a single resonance behavior is not taken into account by the classical Fano bound. Secondly as already mentioned the Fano bound supposes implicitly a step function shape of the system's response that can hardly be achieved in practice with a finite degree matching circuit. This underlines the necessity to obtain accurate finite degree bounds, as the ones obtained by the Puma software. These render effective a convex relaxation of the finite degree matching problem based on non-linear semi-definite programming and the notion of Pick matrix. A publication on this topic is under way.

7.3 Modular filter synthesis with dispersive elements

Participants: Smain Amari, Fabien Seyfert.

This work was pursued in collaboration with Ke-L. Wu and Yan Zhang of the Chinese University of Hong-Kong.

In microwave filter synthesis the dispersive nature of couplings between resonators is usually neglected, due to the narrow band hypothesis. We however showed with S.Amari in 2008 [34, 33] that a linear dependency in frequency of the couplings could be added to the usual state space model up to the addition of descriptor form, and that filter synthesis was still possible for simple inline topologies. We showed at that time that, if properly controlled, dispersion could be used to enrich the designer
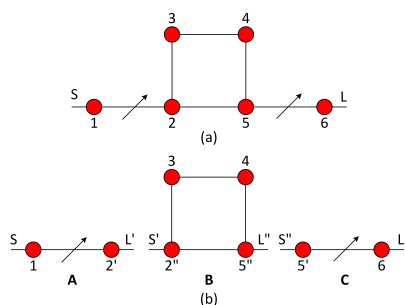
Figure 9: a: Topology combining a quadruplet and two sections with dispersive couplings - b: circuital blocs associated to the functional decomposition
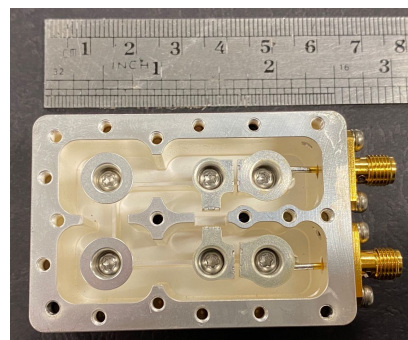

Figure 10: Hardware realization of an order 6 filter in combline technology

possibilities and lead to more selective filter responses by generation of additional transmission zeros.

Last year we developed a completely new filter synthesis for cascaded topologies [12]. The method starts with a factorization of the scattering matrix into blocks, according to a partition of the transmission zeros (see Figure 9). In functional analytic terms, this step corresponds to a Potapov factorization. In a second step, each block gets realized as a circuit, and when cascaded these will constitute the whole filter. The procedure stands therefore at the cross-roads between Darlington synthesis and Kalman realization theory, unifying extracted pole techniques and coupling matrix synthesis.

Dwelling on this procedure, we showed that the circuital synthesis of elementary building blocks containing dispersive couplings is equivalent to a generalized Gram-Schmidt orthogonalization process with two scalar products involved. The method allows one for a very flexible, modular synthesis, where each functional block can be implemented according to a different technique. In particular, blocks including dispersive elements (even complex ones [26]) can be combined with classical triplets, quadruplets as well as with sections involving non-resonating nodes [24]. The manufacturing took place at the Hong-Kong lab (see Figure 10).

Recently, we studied in collaboration with P. Macchiarella from Politecnico Milano and S. Tamiazzo from Commscope, coupling topologies with an inline footprint that, unexpectedly, allow one to realize transmission zeros. We proved [20, 21, 25] that although these topologies are defective (design parameters are missing to accommodate all possible responses), they can realize Chebyshev characteristics with a particular return loss or variations thereof. Such topologies are particularly suitable for filters to be stacked in recent base stations, that need to line up like sardines in order to save space.

7.4   Automatic tuning of microwave devices

Participants: Sylvain Chevillard, Martine Olivi, Fabien Seyfert.

This work was supported by the Contract Inria-Inoveos.

The team has considerable experience in this area, that goes back to the end of the 90's when, in collaboration with CNES and later Xlim, they were first to propose a computer-aided tuning technique for microwave filters (see [86, 64]). This year, we participated to a consortium funded by BPI France, led by Inoveos and involving robotic experts (Cisteme) and millimeter wave filter engineers from Xlim. The objective was to built a prototypical robot dedicated to the automatic tuning of microwave devices. Our software was adapted to cope with real time constraints and the massive measurement flow that is possible to perform in such a fully automated scenario. Circuit extractions need now less than 1 second to be performed. This version of Presto-HF (see section 3.4.4) was transferred and gave rise to the first prototypical tuning robot of Inoveos, see Figure 11.

Figure 11: Tuning Robot at the company Inoveos

7.5   Stability assessment of microwave circuits

Participants: Laurent Baratchart, Sylvain Chevillard, Martine Olivi, Fabien Seyfert,
Sébastien Fueyo, Adam Cooman.

The goal is here to help design amplifiers and oscillators, in particular to detect instability at an early stage of the design. This is the subject of a joint research effort with J.-B. Pomet (from the McTao Inria project-team). Application to oscillator design methodologies is also studied in collaboration with Smain Amari from the Royal Military College of Canada (Kingston, Canada).

As opposed to Filters and Antennas, Amplifiers and Oscillators are active components that intrinsically entail non-linear functioning. The latter is due to the use of transistors governed by electric laws exhibiting saturation effects, and therefore inducing input/output characteristics that are no longer proportional to the magnitude of the input signal. Hence, they typically produce non-linear distortions. A central issue arising in the design of amplifiers is to assess stability. The latter may be understood around a functioning point when no input but noise is considered, or else around a periodic trajectory when an input signal at a specific frequency is applied. For oscillators, a precise estimation of their oscillating frequency is crucial during the design process. For devices operating at relatively low frequencies, time domain simulations perform satisfactorily to check stability. For complex microwave amplifiers and oscillators, the situation is however drastically different: the time step necessary to integrate the dynamical equations of the transmission lines (which behave like a simple electrical wire at low frequency) becomes so small that simulations are intractable in reasonable time. Moreover, most linear components of such circuits are known through their frequency response, and prior to any time domain simulation a preliminary, numerically delicate step is needed to obtain their impulse response.

For these reasons, the analysis of such systems is carried out in the frequency domain. To study stability around a functioning point, small input signals are considered and the stability of the linearized system can be investigated, using a first order approximation of each non-linear component, *via* the transfer impedance functions computed at certain ports of the circuit. In recent years, we showed that under realistic dissipativity assumptions at high frequency for the building blocks of the circuit, these transfer functions are meromorphic in the complex frequency variable *s*, with at most finitely many unstable poles in the right half-plane [13]. Dwelling on the unstable/stable decomposition in Hardy Spaces, we developed a procedure to assess the stability or instability of the transfer functions at hand, from their evaluation on a finite frequency grid [10], that was further improved in [9] to address the design of oscillators, in collaboration with Smain Amari. This has resulted in the development of a software library called Pisa (see Section 3.4.1), aiming at making these techniques available to practitioners. Research in this direction now focuses on the links between the width of the measurement band, the density of the measurement points, and the precision with which an unstable pole, located within a certain depth into the complex plane, can be identified.

Extending this methodology to the strong signal case, where linearisation is considered around a

periodic trajectory, is considerably more difficult and has received much attention by the team in recent years. When stability is understood around a periodic trajectory, computed in practice by Harmonic Balance algorithms, linearization yields a linear time varying dynamical system with periodic coefficients and a periodic trajectory thereof. While in finite dimension the stability of such systems is well understood via the Floquet theory, this is no longer the case in the present setting which is infinite dimensional, due to the presence of delays. Dwelling on known constructions for delay systems and adapting the resolvent approach for Volterra equations to the context of functions of bounded variations, it was shown in the PhD thesis of our former student S. Fueyo's [71] that, for general circuits, the monodromy operator of the linearized system along its periodic trajectory is a compact perturbation of a "high frequency" circuit where coils are replaced by short circuits and condensers by wires, so that there is no dynamics anymore but only transmission lines (modeled by delays) and transistors / resistors. We showed in [14] that, under a (realistic) assumption of passivity of components of the circuit at arbitrary high frequency, such a circuit is exponentially stable, in any L^p sense, $1 \le p \le \infty$. Consequently, since compact perturbation preserve the essential spectrum, the linearized system along the periodic trajectory has at most finitely many unstable points in the spectrum of its monodromy operator. Now, using tools from function theory (notably on almost periodic analytic functions), realization theory and periodic evolution families, we proved in the recent manuscript [28] that the spectrum of the monodromy operator is the exponential of the singularities of the so-called harmonic transfer function, viewed as a holomorphic function with values in periodic L^2 functions. This result may be seen as a generalization to the case of periodic coefficients of the well-known Henry-Hale theorem characterizing the exponential stability of delay systems with constant coefficients.

Altogether, what precedes shows that the system is unstable if and only if the harmonic transfer function has poles in the right half plane, and these must lie equally spaced on finitely many vertical lines in that half plane. In principle, this warrants the approach to instability detection set up in the linear case, by chasing unstable poles of the harmonic transfer function in the right half-plane. It should be remarked that, in general, singularities of the harmonic transfer function need not necessarily be singularities of its Fourier coefficients (the Fourier series can diverge even if all coefficients are very smooth), which is a completely new phenomenon never occurring for finite-dimensional systems. However, in the present case, we know we are dealing with polar singularities (in the right half-plane), and at least one of these Fourier coefficients must have a pole. But since harmonic balance techniques can only estimate finitely many Fourier coefficients, one is not guaranteed to get a singular holomorphic function among them in the neighborhood of the pole under examination. This issue was apparently never considered by practitioners and is now under consideration by the team.

7.6 Hardy-Hodge decomposition with applications to silent sources

Participants: Laurent Baratchart, Juliette Leblond, Masimba Nemaire.

In a joint work with T. Qian and P. Dang from the university of Macao [55], we proved that on a compact hypersurface Σ embedded in $\mathbb{R}^n$, a $\mathbb{R}^n$-valued vector field of L^p class decomposes as the sum of a harmonic gradient from inside Σ, a harmonic gradient from outside Σ, and a tangent divergence-free field, provided that $2 - \varepsilon < p < 2 + \varepsilon'$, where ε and ε' depend on the Lipschitz constant of the surface. We also showed that the decomposition is valid for $1 < p < \infty$ when Σ is VMO-smooth (*i.e.* Σ is locally the graph of Lipschitz function with derivatives in VMO). By projection onto the tangent space, this gives us a Helmholtz-Hodge decomposition for vector fields on a compact Lipschitz hypersurface, which is apparently new since existing results deal with smooth surfaces. In fact, the Helmholtz-Hodge decomposition holds on Lipschitz surfaces (not just hypersurfaces),

In [15], we have showed that L^2-magnetizations (i.e. $\mathbb{R}^n$-valued vector fields) on a compact connected Lipschitz surface embedded in $\mathbb{R}^n$ that are silent outside (i.e. that produce no magnetic field in the unbounded component of the complement of the surface) form the orthogonal space to harmonic gradients inside the surface. This result teams up with the Hardy-Hodge decomposition to produce a description of silent magnetizations distributions of L^p-class on a surface as solutions to a certain spectral equation for the double layer potential (for restricted range of p when the surface is only Lipschitz). This

last piece of research has been a collaboration with C. Gerhards and A. Kegeles from TU Freiberg and can be used use to characterize, *via* balayage, silent magnetizations in a volume rather than on a surface.

We have also been working on a direct characterization of silent $[L^p(\Omega)]^n$, ($n \geq 3$) vector-fields. We showed that a vector field carried by some finitely connected bounded open set Ω is silent outside Ω if and only if, in its Helmholtz decomposition as the sum of a L^p gradient and a divergence-free L^p field over $\mathbb{R}^n$, both summands vanish a.e. outside of Ω. When Ω is Lipschitz, this entails that silent vector-fields $M \in [L^p(\Omega)]^n$ (i.e. vector fields representing magnetizations that produce the zero magnetic field outside Ω) are of the form $M = \nabla \psi + D$ with $\psi \in W_0^{1,p}(\Omega)$ (the familiar space of Sobolev functions with zero trace) and D divergence-free with zero normal component on the boundary $\partial\Omega$. In fact, the result extends to very general open sets if one uses a potential-theoretic definition of the trace (the mean at Lebesgue points, defined Bessel-$(1, p)$ quasi everywhere). It should be noticed that this holds for $1 < p < \infty$, even though the Helmholtz decomposition on Ω may not hold for every L^p-field (in general, for Lipschitz open sets, it only holds for p in a neighborhood of $[2/3, 3]$). The previous characterization makes it possible to describe the norm-minimizing equivalent vector-field to a given vector-field $M \in [L^p(\Omega)]^n$. Here, two vector-fields are equivalent if their magnetic potentials outside Ω differ by a constant, or equivalently if the vector-fields differ by a silent source. To this effect, we exploited the fact that the norm-minimizing equivalent source can be written as $\langle M, \nabla \phi \rangle |\nabla \phi|^{q-2} \nabla \phi$ with ϕ a harmonic function in $W^{1,q}(\Omega)$ with $\nabla \phi$ orthogonal to all silent magnetizations in $[L^p(\Omega)]^n$. This yields in passing a nonlinear Hodge-like decomposition, valid for all $1 < p < \infty$ on arbitrary bounded open sets, containing an extra p-divergent-free term. Moreover, $\nabla \phi$ maximizes $\langle M, \nabla \phi \rangle$ among all harmonic gradients orthogonal to silent sources in $[L^p(\Omega)]^n$ with L^q-norm 1. In practice, when Ω is Lipschitz, one can further use that ϕ can be expressed as a single layer potential, say $\mathscr{S} f^*$ with f^* maximizing the duality pairing $\langle P_M(y), f(y) \rangle$ on $\partial\Omega$ (here P_M is the Newton potential of the (distributional) divergence of M) under appropriate constraints. Then, a Galerkin-type method can be used to approach the problem numerically, using a frame of the space of distributions $W^{\frac{-1}{q},q}(\partial\Omega)$ that can be constructed as in [78]. Alternatively, in the case where $n = 3$ and Ω is a $C^{1,1}$ simply connected domain, a minimization procedure that alternates between approximating $M \in [W^{1,p}(\Omega)]^3$ by a silent gradient and a silent divergence-free vector-field can be used. This alternating minimization procedure makes connection with a non-linear stationary Maxwell system and a q-Laplace boundary value problem both of which can be solved via Galerkin methods with the use of finite elements methods. Of course, it would be very interesting to derive counterparts of these results to the case $p = 1$, on smooth domains at least.

Besides, an analysis of harmonic equivalent sources, and a uniqueness result (injectivity of the forward operator) for piecewise constant magnetizations on parallelepipeds was obtained in [16].

In another connection, a version of the Hardy-Hodge decomposition is being developed for the Helmholtz equation, in collaboration with H. Haddar from the team DEFI (Inria Saclay); here, there is an extra 1-dimensional summand whose nonzero elements are non-silent from inside and outside.

7.7   Identification of resonating frequencies of compact metallic objects in electromagnetic inverse scattering

Participants: Laurent Baratchart, Juliette Leblond, Martine Olivi, Fabien Seyfert.

We started an academic collaboration with Leat (Univ. Côte d'Azur, France, J.-Y. Dauvignac, N. Fortino, Y. Zaki) on the topic of inverse scattering using frequency dependent measurements. As opposed to classical electromagnetic imaging where several spatially located sensors are used to identify the shape of an object by means of scattering data at a single frequency, a discrimination process between different metallic objects is here being sought for by means of a single, or a reduced number of sensors that operate on a whole frequency band. For short the spatial multiplicity and complexity of antenna sensors is here traded against a simpler architecture performing a frequency sweep.

The setting is shown on Figure 12. The total field $E_t(r, \theta, \phi)$ is the sum of the incident field E_i (here a plane wave) and scattered field E_s, that is at every point in space we have $E_t = E_i + E_s$. A harmonic time dependency ($e^{j\omega t}$, where j is the imaginary unit: $j^2 = -1$) is supposed for the incident wave, so that by
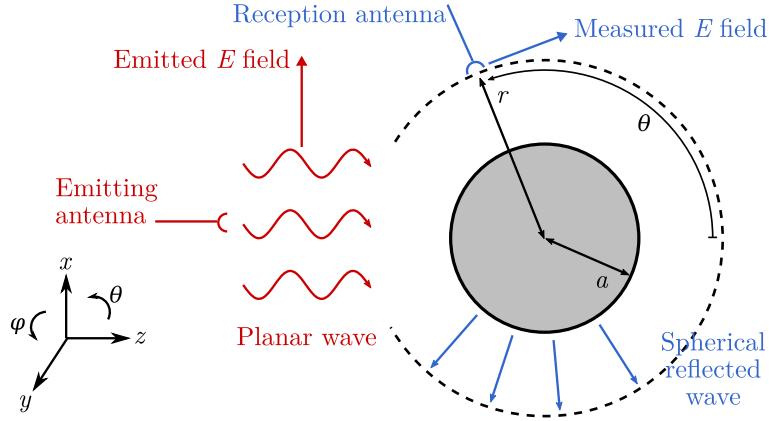
Figure 12: Sphere illuminated by an electromagnetic plane wave - measurement of the scattered wave

linearity of Maxwell equations and after a transient state, following holds,

$$E_s(r_o,\theta_o,\phi_o) = H(s = j\omega,\theta_o,\phi_o)E_i(r_e,\theta_e,\phi_e).$$

The subscripts o and e stand here for «observation point» and «emission point»: the scattered field at the observation point is therefore related to the emitted planar wave field at the emission point via the transfer function $H(s = j\omega,\theta_o,\phi_o)$. The emission point is here supposed fixed, so the dependency in e is omitted in H. Under regularity conditions on the scatterer's boundary the function H can be shown to admit an analytic continuation into the complex left half plane for the s variable, away from a discrete set (with a possible accumulation point a infinity) where it admits poles. Thus, H is a meromorphic function in the variable s. Its poles are called the resonating frequencies of the scattering object. Recovering these resonating frequencies from frequency scattering measurement, that is measurements of H at particular $s = j\omega_i's$ is the primary objective of this project.

In order to gain some insight we started a full study of the particular case when the scatterer is a spherical PEC (Perfectly Electric Conductor). In this case Maxwell equations can be solved «explicitly» by means of expansions in series of vectorial spherical harmonics. We showed in particular that in this case H admits following simple structure:

$$H(\omega,\theta_o,\phi_o) = R(s,\theta_o,\phi_o)e^{-\tau_1(\theta_o,\phi_o)s} + C(\theta_o,\phi_o)e^{-\tau_2(\theta_o,\phi_o)s},$$

where R is a meromorphic functions with poles at zeros of the spherical Hankel functions and their derivatives and C is independent of the frequency. Identification procedures, surprisingly close to the ones we developed in connection with amplifier stability analysis, are currently being studied to gain information about the resonating frequencies by means of a rational approximation of the function R once it has been de-embedded. A preliminary study resulted in a common publication with the Leat team [91].

In order to perform the rational approximation of the function H, its behavior outside the range of measured frequencies, specifically at high frequencies, is being studied for the particular case when the scatterer is a spherical PEC (Perfectly Electric Conductor). In this case, H can be written as:

$$H(\omega,\theta_0,\phi_0) = H_O(\omega,\theta_0,\phi_0) + H_C(\omega,\theta_0,\phi_0)$$

where H_O and H_C are respectively the optic and creeping wave parts. Their high-frequency behaviors are given by:

$$H_O(\omega,\theta_0,\phi_0) \underset{\omega\to\infty}{\sim} \sum_{n=0}^{\infty} \frac{A_n(\theta_0,\phi_0)}{(i\omega)^n} e^{ik\omega(\theta_0,\phi_0)},$$

$$H_C(\omega,\theta_0,\phi_0) \underset{\omega\to\infty}{\sim} \omega^{\frac{4}{3}} B_0 e^{i\frac{\pi}{3}} \exp\left(iB_1\omega - e^{-i\frac{\pi}{6}} B_2\omega^{1/3}\right).$$

where the coefficients $B_0 \in \mathbb{R}$, $B_1 \in \mathbb{R}$, $B_2 \in \mathbb{R}^+$ and $A_n \in \mathbb{R}$ are identified when the observation point (r_o, θ_o, ϕ_o) and the emission point (r_e, θ_e, ϕ_e) coincide. The above asymptotics of H_O is called the Luneberg-Kline expansion; its first terms were analytically computed (solving eikonal and transport equations).

Numerical simulations showed that even though the creeping wave part is negligible in front of the optic part at high frequencies, it needs to be taken into account around the band of measured frequencies for the rational approximation.

Furthermore, the physical interpretation of these two terms leads to consider that the creeping wave part should carry more information about the scatterer and we want to investigate the conjecture that the poles of H are those of H_C hence that H_O is analytic.

We plan in the future to investigate a generalization of this form for other PEC scatterers.

8 Bilateral contracts and grants with industry

8.1 Bilateral Contracts with Industry

Contract Inria-Inoveos We had a contract with the SMB company Inoveos in order to build a prototypical robot dedicated to the automatic tuning of microwave devices. In addition to Inria, this project included the university of Limoges Xlim and the engineering center Cisteme.

9 Partnerships and cooperations

9.1 European initiatives

9.1.1 Other european programs/initiatives

Factas is part of the European Research Network on System Identification (ERNSI) since 1992. System identification deals with the derivation, estimation and validation of mathematical models of dynamical phenomena from experimental data.

9.2 National initiatives

ANR REPKA ANR-18-CE40-0035, "REProducing Kernels in Analysis and beyond" (2019–2023).

Led by Aix-Marseille Univ. (IMM), involving Factas team, together with Bordeaux (IMB), Paris-Est, Toulouse Universities.

The project consists of several interrelated tasks dealing with topical problems in modern complex analysis, operator theory and their important applications to other fields of mathematics including approximation theory, probability, and control theory. The project is centered around the notion of the so-called reproducing kernel of a Hilbert space of holomorphic functions. Reproducing kernels are very powerful objects playing an important role in numerous domains such as determinantal point processes, signal theory, Sturm-Liouville and Schrödinger equations.

This project supports the PhD of M. Nemaire within Factas, co-advised by IMB partners.

GDR AFHP GDR "Analyse Fonctionnelle, Harmonique et Probabilités".

Led by Gustave Eiffel Univ. (LAMA), involving Factas team, together with several universities.

The GDR is concerned with five main axes: linear dynamics, Banach spaces and their operators, holomorphic dynamics, harmonic analysis, analysis and probability, and with the interactions between them.

9.3 Regional initiatives

Arch-AI-Story The collaborative project Arch-AI-Story funded by the Idex UCA$^{\text{JEDI}}$ started last year brings together researchers in archaeology and history, physical, and mathematical sciences, with, in particular, the purpose of modeling and detecting low level signals in 3-D images of ancient potteries, see Section 4.5. It involves scientists from CEPAM-CNRS-UCA (project coordinator: Isabelle Théry), Nice, the

team Morpheme, UCA-CNRS-I3S-Inria, together with other institutions, and supports the post-doctoral research of Vanna Lisa Coli.

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: selection

Member of the conference program committees

- L. Baratchart was on the program committee of 19th IFAC Symposium on System Identification (SYSID 2021), Padova, Italy, July 14-16, 2021

Reviewer

- J. Leblond was a reviewer for the 19th IFAC Symposium on System Identification (SYSID 2021), Padova, Italy, July 14-16, 2021.

- F. Seyfert was a reviewer for IMFW 2021, Perugia, Italy, November 17-19, 2021.

10.1.2 Journal

Member of the editorial boards

- L. Baratchart is a member of the editorial board of "Computational methods and function theory" (CMFT) and "Complex analysis and operator theory" (CAOT).

Reviewer - reviewing activities

- F. Seyfert is a frequent reviewer of IEEE Journal of Microwave Theory and Techniques.

10.1.3 Invited talks

- L. Baratchart was an invited speaker at the *Workshop on Analysis and Control Theory*, on-line 2021.

10.1.4 Research administration

- J. Leblond is a member of the "Conseil Scientifique" and of the "Commission Administrative Paritaire" of Inria, and of the "équipe transformation" of the Research Center.

- M. Olivi was a member of the CLDD (Commission Locale de Développement Durable) and in charge of coordination.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching

- **Colles**: S. Chevillard gives "Colles" (oral examination preparing undergraduate students for the competitive examination to enter French Engineering Schools) at Centre International de Valbonne (CIV) (1 hour per week).

10.2.2   Supervision

- Internship: A. Yousfi, *Study of electromagnetic waves scattering from a sphere*, April-September 2021, advisor: F. Seyfert.

- PhD: G. Bose, *Méthodologies et outils de synthèse pour des fonctions de filtrage chargées par des impédances complexes*, since December 2016, defended January 8, 2021, advisors: F. Ferrero (Leat), F. Seyfert and M. Olivi.

- PhD in progress: P. Asensio, *Inverse source estimation problems in EEG and MEG*, since November 2019, advisors: L. Baratchart, J. Leblond.

- PhD in progress: M. Nemaire, *Inverse potential problems with application to quasi-static electro-magnetics*, since October 2019, advisors: L. Baratchart, J. Leblond, S. Kupin (IMB, Univ. Bordeaux).

- Pre-PhD in progress: A. Yousfi, *Methods to estimate the net magnetic moment of rocks*, advisor: S. Chevillard, since October 2021.

- Post-doc.: V. L. Coli, *Multiscale Tomography: imaging and modeling ancient materials*, since March 2018, advisors: J. Leblond, L. Blanc-Féraud (project-team Morpheme, I3S-CNRS/Inria Sophia/iBV), D. Binder (CEPAM-CNRS, Nice).

- Post-doc.: C. Villalobos-Guillén, *Inverse source problems for magnetization , primary current and inverse scattering*, since December 2021; advisor: L. Baratchart.

10.2.3   Juries

- M. Olivi was a member of the PhD committee of Joanna Blisch (Univ. Lille, Oct).

- J. Leblond was a member of the HDR committee of Maya De Buhan (Univ. Paris, Jun), and of the PhD committees of Boris Wembe (Univ. Toulouse Paul Sabatier, Nov), Mohamed Aziz Boukraa (Univ. Caen Normandie, Dec), Karine Fouchet-Isambard (Aix-Marseille Univ., Dec).

10.3   Popularization

10.3.1   Internal or external Inria responsibilities

- M. Olivi was a member of the Committee MASTIC (Commission d'Animation et de Médiation Scientifique) and of Terra Numerica.

10.3.2   Articles and contents

- M. Olivi participated in the creation of the MOOC "Impacts environnementaux du numérique" distributed by FUN(France Université Numérique).

10.3.3   Interventions

- M. Olivi participated to the event "Fête de la science" in Mouans-Sartoux, she animated the activity "bricoleurs scientifiques" with Terra Numerica.

- S. Chevillard gave a talk at the "Journées scientifiques Inria" presenting our work on the evaluation of the carbon footprint of an Inria team (see 5.1).

11   Scientific production

11.1   Major publications

[1]   S. Amari, F. Seyfert and M. Bekheit. 'Theory of Coupled Resonator Microwave Bandpass Filters of Arbitrary Bandwidth'. In: *Microwave Theory and Techniques, IEEE Transactions on* 58.8 (Aug. 2010), pp. 2188–2203.

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[12] Y. Zhang, F. Seyfert, S. Amari, M. Olivi and K.-L. Wu. 'General Synthesis Method for Dispersively Coupled Resonator Filters With Cascaded Topologies'. In: *IEEE Transactions on Microwave Theory and Techniques* (Dec. 2020), p. 15. DOI: 10.1109/TMTT.2020.3041223. URL: https://hal.inria.fr/hal-03070731.

11.2 Publications of the year

International journals

[13] L. Baratchart, S. Chevillard, A. Cooman, M. Olivi and F. Seyfert. 'Linearized Active Circuits: Transfer Functions and Stability'. In: *Mathematics in Engineering* 4.5 (12th Oct. 2021), pp. 1–18. DOI: 10.3934/mine.2022039. URL: https://hal.inria.fr/hal-01667606.

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International peer-reviewed conferences

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Reports & preprints

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11.3   Cited publications

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