RESEARCH CENTRE

Nancy - Grand Est

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Université de Lorraine

2021

ACTIVITY REPORT

Project-Team GAMBLE

Geometric Algorithms & Models Beyond the Linear & Euclidean realm

IN COLLABORATION WITH: Laboratoire lorrain de recherche en informatique et ses applications (LORIA)

DOMAIN

Algorithmics, Programming, Software and Architecture

THEME

Algorithmics, Computer Algebra and Cryptology

Contents

Project-Team GAMBLE

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Keywords

Computer sciences and digital sciences

A5.5.1. – Geometrical modeling

A5.10.1. – Design

A7.1. – Algorithms

A8.1. – Discrete mathematics, combinatorics

A8.3. – Geometry, Topology

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Other research topics and application domains

B1.1.1. – Structural biology

B1.2.3. – Computational neurosciences

B2.6. – Biological and medical imaging

B3.3. – Geosciences

B5.5. – Materials

B5.6. – Robotic systems

B5.7. – 3D printing

B6.2.2. – Radio technology

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2 Overall objectives

Starting in the eighties, the emerging computational geometry community has put a lot of effort into designing and analyzing algorithms for geometric problems. The most commonly used framework was to study the worst-case theoretical complexity of geometric problems involving linear objects (points, lines, polyhedra...) in Euclidean spaces. This so-called *classical computational geometry* has some known limitations:

- Objects: dealing with objects only defined by linear equations.
- Ambient space: considering only Euclidean spaces.
- Complexity: worst-case complexities often do not capture realistic behaviour.
- Dimension: complexities are often exponential in the dimension.
- Robustness: ignoring degeneracies and rounding errors.

Even if these limitations have already got some attention from the community [39], a quick look at the proceedings of the flagship conference SoCG[1] shows that these topics still need a big effort.

It should be stressed that, in this document, the notion of certified algorithms is to be understood with respect to robustness issues. In other words, certification does not refer to programs that are proven correct with the help of mechanical proof assistants such as Coq, but to algorithms that are proven correct on paper even in the presence of degeneracies and computer-induced numerical rounding errors.

We address several of the above limitations:

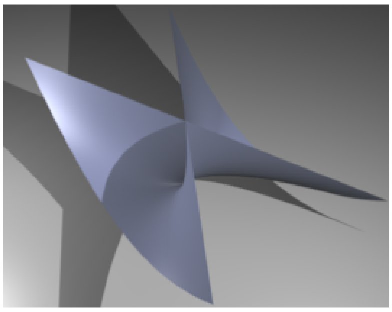
• **Non-linear computational geometry.** Curved objects are ubiquitous in the world we live in. However, despite this ubiquity and decades of research in several communities, curved objects are far from being robustly and efficiently manipulated by geometric algorithms. Our work on, for instance, quadric intersections and certified drawing of plane curves has proven that dramatic improvements can be accomplished when the right mathematics and computer science concepts are put into motion. In this direction, many problems are fundamental and solutions have potential industrial impact in Computer Aided Design and Robotics for instance. Intersecting NURBS (Non-uniform rational basis splines) and meshing singular surfaces in a certified manner are important examples of such problems.

• **Non-Euclidean computational geometry.** Triangulations are central geometric data structures in many areas of science and engineering. Traditionally, their study has been limited to the Euclidean setting. Needs for triangulations in non-Euclidean settings have emerged in many areas dealing with objects whose sizes range from the nuclear to the astrophysical scale, and both in academia and in industry. It has become timely to extend the traditional focus on $\mathbb{R}^d$ of computational geometry and encompass non-Euclidean spaces.

• **Probability in computational geometry.** The design of efficient algorithms is driven by the analysis of their complexity. Traditionally, worst-case input and sometimes uniform distributions are considered and many results in these settings have had a great influence on the domain. Nowadays, it is necessary to be more subtle and to prove new results in between these two extreme settings. For instance, smoothed analysis, which was introduced for the simplex algorithm and which we applied successfully to convex hulls, proves that such promising alternatives exist.

• **Discrete geometric structures.** Many geometric algorithms work, explicitly or implicitly, over discrete structures such as graphs, hypergraphs, lattices that are induced by the geometric input data. For example, convex hulls or straight-line graph drawing are essentially based on orientation predicates, and therefore operate on the so-called *order type* of the input point set. Order types are a subclass of oriented matroids that remains poorly understood: for instance, we do not even know how to sample this space with reasonable bias. One of our goals is to contribute to the development of these foundations by better understanding these discrete geometric structures.

[1] Symposium on Computational Geometry. www.computational-geometry.org/.



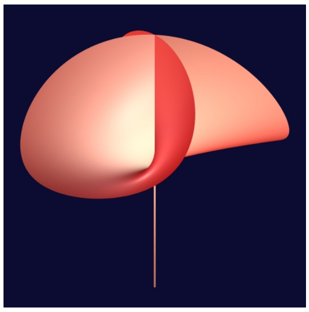


Figure 1: Two views of the Whitney umbrella (on the left, the "stick" of the umbrella, i.e., the negative z-axis, is missing). Right picture from [Wikipedia], left picture from [Lachaud et al.].

3 Research program

3.1 Non-linear computational geometry

As mentioned above, curved objects are ubiquitous in real world problems and in computer science and, despite this fact, there are very few problems on curved objects that admit robust and efficient algorithmic solutions without first discretizing the curved objects into meshes. Meshing curved objects induces a loss of accuracy which is sometimes not an issue but which can also be most problematic depending on the application. In addition, discretization induces a combinatorial explosion which could cause a loss in efficiency compared to a direct solution on the curved objects (as our work on quadrics has demonstrated with flying colors [45, 46, 47, 49, 53]). But it is also crucial to know that even the process of computing meshes that approximate curved objects is far from being resolved. As a matter of fact there is no algorithm capable of computing in practice meshes with certified topology of even rather simple singular 3D surfaces, due to the high constants in the theoretical complexity and the difficulty of handling degenerate cases. Part of the difficulty comes from the unintuitive fact that the structure of an algebraic object can be quite complicated, as depicted in the Whitney umbrella (see Figure 1), the surface with equation $x^2 = y^2 z$ whose origin (the "special" point of the surface) is a vertex of the arrangement induced by the surface while the singular locus is simply the whole z-axis. Even in 2D, meshing an algebraic curve with the correct topology, that is in other words producing a correct drawing of the curve (without knowing where the domain of interest is), is a very difficult problem on which we have recently made important contributions [32, 33, 54].

Thus producing practical, robust, and efficient algorithmic solutions to geometric problems on curved objects is a challenge on all and even the most basic problems. The basicness and fundamentality of the two problems we mentioned above on the intersection of 3D quadrics and on the drawing in a topologically certified way of plane algebraic curves show rather well that the domain is still in its infancy. And it should be stressed that these two sets of results were not anecdotal but flagship results produced during the lifetime of the VEGAS team (the team preceding GAMBLE).

There are many problems in this theme that are expected to have high long-term impacts. Intersecting NURBS (Non-uniform rational basis splines) in a certified way is an important problem in computer-aided design and manufacturing. As hinted above, meshing objects in a certified way is important when topology matters. The 2D case, that is essentially drawing plane curves with the correct topology, is a fundamental problem with far-reaching applications in research or R&D. Notice that on such elementary problems it is often difficult to predict the reach of the applications; as an example, we were astonished by the scope of the applications of our software on 3D quadric intersection[2] which was used by researchers in, for instance, photochemistry, computer vision, statistics and mathematics.

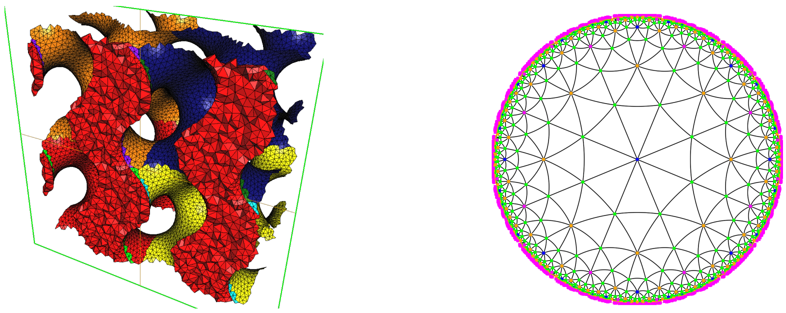


Figure 2: Left: 3D mesh of a gyroid (triply periodic surface) [56]. Right: Simulation of a periodic Delaunay triangulation of the hyperbolic plane [28].

3.2 Non-Euclidean computational geometry

Triangulations, in particular Delaunay triangulations, in the *Euclidean space* $\mathbb{R}^d$ have been extensively studied throughout the 20th century and they are still a very active research topic. Their mathematical properties are now well understood, many algorithms to construct them have been proposed and analyzed (see the book of Aurenhammer *et al.* [27]). Some members of GAMBLE have been contributing to these algorithmic advances (see, e.g. [31, 63, 42, 30]); they have also contributed robust and efficient triangulation packages through the state-of-the-art Computational Geometry Algorithms Library CGAL whose impact extends far beyond computational geometry. Application fields include particle physics, fluid dynamics, shape matching, image processing, geometry processing, computer graphics, computer vision, shape reconstruction, mesh generation, virtual worlds, geophysics, and medical imaging.[3]

It is fair to say that little has been done on non-Euclidean spaces, in spite of the large number of questions raised by application domains. Needs for simulations or modeling in a variety of domains[4] ranging from the infinitely small (nuclear matter, nano-structures, biological data) to the infinitely large (astrophysics) have led us to consider 3D periodic Delaunay triangulations, which can be seen as Delaunay triangulations of the 3D *flat torus*, i.e., the quotient of $\mathbb{R}^3$ under the action of some group of translations [37]. This work has already yielded a fruitful collaboration with astrophysicists [50, 64] and new collaborations with physicists are emerging. To the best of our knowledge, our CGAL package [36] is the only publicly available software that computes Delaunay triangulations of a 3D flat torus, in the special case where the domain is cubic. This case, although restrictive, is already useful.[5] We have also generalized this algorithm to the case of general d-dimensional compact flat manifolds [38]. As far as non-compact manifolds are concerned, past approaches, limited to the two-dimensional case, have stayed theoretical [55].

Interestingly, even for the simple case of triangulations on the *sphere*, the software packages that are currently available are far from offering satisfactory solutions in terms of robustness and efficiency [35].

Moreover, while our solution for computing triangulations in hyperbolic spaces can be considered as ultimate [28], the case of *hyperbolic manifolds* has hardly been explored. Hyperbolic manifolds are quotients of a hyperbolic space by some group of hyperbolic isometries. Their triangulations can be seen as hyperbolic periodic triangulations. Periodic hyperbolic triangulations and meshes appear for instance in geometric modeling [59], neuromathematics [40], or physics [60]. Even the case of the Bolza surface (a surface of genus 2, whose fundamental domain is the regular octagon in the hyperbolic plane) shows mathematical difficulties [29, 52].

[2] QI: web.
[3] See Projects using CGAL for details.
[4] See CGAL Prospective Workshop on Geometric Computing in Periodic Spaces, Subdivide and Tile: Triangulating spaces for understanding the world, Computational geometry in non-Euclidean spaces, Shape Up 2015 : Exercises in Materials Geometry and Topology
[5] See examples at Projects using CGAL

3.3 Probability in computational geometry

In most computational geometry papers, algorithms are analyzed in the worst-case setting. This often yields too pessimistic complexities that arise only in pathological situations that are unlikely to occur in practice. On the other hand, probabilistic geometry provides analyses with great precision [57, 58, 34], but using hypotheses with much more randomness than in most realistic situations. We are developing new algorithmic designs improving state-of-the-art performance in random settings that are not overly simplified and that can thus reflect many realistic situations.

Sixteen years ago, smooth analysis was introduced by Spielman and Teng analyzing the simplex algorithm by averaging on some noise on the data [62] (and they won the Gödel prize). In essence, this analysis smoothes the complexity around worst-case situations, thus avoiding pathological scenarios but without considering unrealistic randomness. In that sense, this method makes a bridge between full randomness and worst case situations by tuning the noise intensity. The analysis of computational geometry algorithms within this framework is still embryonic. To illustrate the difficulty of the problem, we started working in 2009 on the smooth analysis of the size of the convex hull of a point set, arguably the simplest computational geometry data structure; then, only one very rough result from 2004 existed [41] and we only obtained in 2015 breakthrough results, but still not definitive [44, 43, 48].

Another example of a problem of different flavor concerns Delaunay triangulations, which are rather ubiquitous in computational geometry. When Delaunay triangulations are computed for reconstructing meshes from point clouds coming from 3D scanners, the worst-case scenario is, again, too pessimistic and the full randomness hypothesis is clearly not adapted. Some results exist for “good samplings of generic surfaces” [26] but the big result that everybody wishes for is an analysis for random samples (without the extra assumptions hidden in the “good” sampling) of possibly non-generic surfaces.

Trade-offs between full randomness and worst case may also appear in other forms such as dependent distributions, or random distributions conditioned to be in some special configurations. Simulating these kinds of geometric distributions is currently out of reach for more than a few hundred points [51] although it has practical applications in physics or networks.

3.4 Discrete geometric structures

Our work on discrete geometric structures develops in several directions, each one probing a different type of structure. Although these objects appear unrelated at first sight, they can be tackled by the same set of probabilistic and topological tools.

A first research topic is the study of *Order types.* Order types are combinatorial encodings of finite (planar) point sets, recording for each triple of points the orientation (clockwise or counterclockwise) of the triangle they form. This already determines properties such as convex hulls or half-space depths, and the behaviour of algorithms based on orientation predicates. These properties for all (infinitely many) n-point sets can be studied through the finitely many order types of size n. Yet, this finite space is poorly understood: its estimated size leaves an exponential margin of error, no method is known to sample it without concentrating on a vanishingly small corner, the effect of pattern exclusion or VC dimension-type restrictions are unknown. These are all directions we actively investigate.

A second research topic is the study of *Embedded graphs and simplicial complexes.* Many topological structures can be effectively discretized, for instance combinatorial maps record homotopy classes of embedded graphs and simplicial complexes represent a large class of topological spaces. This raises many structural and algorithmic questions on these discrete structures; for example, given a closed walk in an embedded graph, can we find a cycle of the graph homotopic to that walk? (The complexity status of that problem is unknown.) Going in the other direction, some purely discrete structures can be given an associated topological space that reveals some of their properties (*e.g.* the Nerve theorem for intersection patterns). An open problem is for instance to obtain fractional Helly theorems for set systems of bounded topological complexity.

Another research topic is that of *Sparse inclusion-exclusion formulas.* For any family of sets $A_1, A_2, \ldots, A_n$, by the principle of inclusion-exclusion we have

$$\mathbb{1}_{\bigcup_{i=1}^n A_i} = \sum_{I \subseteq \{1,2,\ldots,n\}} (-1)^{|I|+1} \mathbb{1}_{\bigcap_{i\in I} A_i} \qquad (1)$$

where $\mathbb{1}_X$ is the indicator function of X. This formula is universal (it applies to any family of sets) but its number of summands grows exponentially with the number n of sets. When the sets are balls, the formula remains true if the summation is restricted to the regular triangulation; we proved that similar simplifications are possible whenever the Venn diagram of the A_i is sparse. There is much room for improvements, both for general set systems and for specific geometric settings. Another interesting problem is to combine these simplifications with the inclusion-exclusion algorithms developed, for instance, for graph coloring.

4 Application domains

Many domains of science can benefit from the results developed by GAMBLE. Curves and surfaces are ubiquitous in all sciences to understand and interpret raw data as well as experimental results. Still, the non-linear problems we address are rather basic and fundamental, and it is often difficult to predict the impact of solutions in that area. The short-term industrial impact is likely to be small because, on basic problems, industries have used ad hoc solutions for decades and have thus got used to it.

The example of our work on quadric intersection is typical: even though we were fully convinced that intersecting 3D quadrics is such an elementary/fundamental problem that it ought to be useful, we were the first to be astonished by the scope of the applications of our software [6] (which was the first and still is the only one —to our knowledge— to compute robustly and efficiently the intersection of 3D quadrics) which has been used by researchers in, for instance, photochemistry, computer vision, statistics, and mathematics. Our work on certified drawing of plane (algebraic) curves falls in the same category. It seems obvious that it is widely useful to be able to draw curves correctly (recall also that part of the problem is to determine where to look in the plane) but it is quite hard to come up with specific examples of fields where this is relevant. A contrario, we know that certified meshing is critical in mechanical-design applications in robotics, which is a non-obvious application field. There, the singularities of a manipulator often have degrees higher than 10 and meshing the singular locus in a certified way is currently out of reach. As a result, researchers in robotics can only build physical prototypes for validating, or not, the approximate solutions given by non-certified numerical algorithms.

The fact that several of our pieces of software for computing non-Euclidean triangulations had already been requested by users long before they become public in CGAL is a good sign for their wide future impact. This will not come as a surprise, since most of the questions that we have been studying followed from discussions with researchers outside computer science and pure mathematics. Such researchers are either users of our algorithms and software, or we meet them in workshops. Let us only mention a few names here. Rien van de Weijgaert [50, 64] (astrophysicist, Groningen, NL) and Michael Schindler [61] (theoretical physicist, ENSPCI, CNRS, France) used our software for 3D periodic weighted triangulations. Stephen Hyde and Vanessa Robins (applied mathematics and physics at Australian National University) used our package for 3D periodic meshing. Olivier Faugeras (neuromathematics, INRIA Sophia Antipolis) had come to us and mentioned his needs for good meshes of the Bolza surface [40] before we started to study them. Such contacts are very important both to get feedback about our research and to help us choose problems that are relevant for applications. These problems are at the same time challenging from the mathematical and algorithmic points of view. Note that our research and our software are generic, i.e., we are studying fundamental geometric questions, which do not depend on any specific application. This recipe has made the sucess of the CGAL library.

Probabilistic models for geometric data are widely used to model various situations ranging from cell phone distribution to quantum mechanics. The impact of our work on probabilistic distributions is twofold. On the one hand, our studies of properties of geometric objects built on such distributions will yield a better understanding of the above phenomena and has potential impact in many scientific domains. On the other hand, our work on simulations of probabilistic distributions will be used by other teams, more maths oriented, to study these distributions.

[6] QI: web.

5 Highlights of the year

We developed a breakthrough method for the two fundamental problems of evaluating a polynomial on multiple points and finding its complex roots, with a bit complexity quasi-linear in the degree. In particular, our approach solves a problem that has been open for 50 years. It was selected to the prestigious conference FOCS [18], and the practical efficiency of this method led to a transfer contract with the private company WATERLOO MAPLE INC.

6 New software and platforms

6.1 New software

6.1.1 CGAL Package: 2D triangulations on the sphere

Keywords: Computational geometry, Delaunay triangulation

Functional Description: This package enables the construction and manipulation of Delaunay triangulations on the 2-sphere. Triangulations are built incrementally and can be modified by insertion or removal of vertices. Point location querying and primitives to build the dual Voronoi diagram are provided.

News of the Year: Integration into CGAL 5.3

URL: https://doc.cgal.org/latest/Manual/packages.html#PkgTriangulationOnSphere2

Publication: hal-03469649

Authors: Mael Rouxel-Labbé, Monique Teillaud, Claudia Werner, Sébastien Loriot, Olivier Rouiller

Contact: Monique Teillaud

Partner: GeometryFactory

6.1.2 hefroots

Name: Hyperbolic, Elliptic and Flat ROOT Solver

Keywords: Polynomial equations, Complex number

Scientific Description: This software for solving polynomial equations is based on a recent result which consists in approximating a polynomial of large degree by a piecewise polynomial function on the complex plane.

Functional Description: This software takes as input a file containing the coefficients of a univariate polynomial and returns the list of its complex roots.

URL: https://gitlab.inria.fr/gmoro/hefpoly

Publication: hal-03249123

Author: Guillaume Moroz

Contact: Guillaume Moroz

7 New results

7.1 Non-Linear Computational Geometry

Participants: Laurent Dupont, Nuwan Herath Mudiyanselage, George Krait, Sylvain Lazard, Guillaume Moroz, Marc Pouget.

7.1.1 An improved complexity bound for computing the topology of a real algebraic space curve

We propose a new algorithm to compute the topology of a real algebraic space curve. The novelties of this algorithm are a new technique to achieve the lifting step which recovers points of the space curve in each plane fiber from several projections and a weakened notion of generic position. As opposed to previous work, our sweep generic position does not require that x-critical points have different x-coordinates. The complexity of achieving this sweep generic position is thus no longer a bottleneck in terms of complexity. The bit complexity of our algorithm is $O(d^{18}+d^{17}t)$ where d and t bound the degree and the bitsize of the integer coefficients of the defining polynomials of the curve and polylogarithmic factors are ignored. To the best of our knowledge, this improves upon the best currently known results at least by a factor of d^2 [20].

7.1.2 Certified numerical algorithm for isolating the singularities of the plane projection of generic smooth space curves

Isolating the singularities of a plane curve is the first step towards computing its topology. For this, numerical methods are efficient but not certified in general. We are interested in developing certified numerical algorithms for isolating the singularities. In order to do so, we restrict our attention to the special case of plane curves that are projections of smooth curves in higher dimensions. This type of curve appears naturally in robotics applications and scientific visualization. In this setting, we show that the singularities can be encoded by a regular square system whose solutions can be isolated with certified numerical methods. Our analysis is conditioned by assumptions that we prove to be generic using transversality theory. We also provide a semi-algorithm to check their validity. Finally, we present experiments, some of which are not reachable by other methods, and discuss the efficiency of our method [15, 19].

7.1.3 Fast guaranteed drawing of high degree algebraic curves

We address the problem of computing a guaranteed drawing of high resolution of a plane curve defined by a bivariate polynomial equation $P(x, y) = 0$. The drawing is a subset of pixels defined on a grid of a given resolution with the guarantee that the pixels enclose all the parts of the curve that intersect the grid. Our main contribution is to use a non-uniform grid based on the Chebyshev nodes to take advantage of multipoint evaluation techniques via the Discrete Cosine Transform. We propose two new algorithms that compute guaranteed drawings and compare them experimentally on several classes of high degree polynomials. Notably, one of those approaches is faster than state-of-the-art drawing software, even when these softwares are not guaranteed [25].

7.1.4 Fast real and complex root-finding methods for well-conditioned polynomials

Given a polynomial p of degree d and a bound κ on a condition number of p, we present the first root-finding algorithms that return all its real and complex roots with a number of bit operations quasi-linear in $d\log^2(\kappa)$. More precisely, several condition numbers can be defined depending on the norm chosen on the coefficients of the polynomial. Let

$$p(x) = \sum_{k=0}^{d} a_k x^k = \sum_{k=0}^{d} \sqrt{\binom{d}{k}}\, b_k x^k.$$

We call the condition number associated with a perturbation of the a_k the hyperbolic condition number κ_h, and the one associated with a perturbation of the b_k the elliptic condition number κ_e. For each of these condition numbers, we present algorithms that find the real and the complex roots of p in $O\big(d\log^2(d\kappa)\ \mathrm{polylog}(\log(d\kappa))\big)$ bit operations.

Our algorithms are well suited for random polynomials since κ_h (resp. κ_e) is bounded by a polynomial in d with high probability if the a_k (resp. the b_k) are independent, centered Gaussian variables of variance 1 [24].

7.1.5 New data structure for univariate polynomial approximation and applications to root isolation, numerical multipoint evaluation, and other problems

We present a new data structure to approximate accurately and efficiently a polynomial f of degree d given as a list of coefficients f_i. Its properties allow us to improve the state-of-the-art bounds on the bit complexity for the problems of root isolation and approximate multipoint evaluation. This data structure also leads to a new geometric criterion to detect ill-conditioned polynomials, implying notably that the standard condition number of the zeros of a polynomial is at least exponential in the number of roots of modulus less than 1/2 or greater than 2.

Given a polynomial f of degree d with $\|f\|_1 = \sum |f_i| \le 2^\tau$ for $\tau \ge 1$, isolating all its complex roots or evaluating it at d points can be done with a quasi-linear number of arithmetic operations. However, considering the bit complexity, the state-of-the-art algorithms require at least $d^{3/2}$ bit operations even for well-conditioned polynomials and when the accuracy required is low. Given a positive integer m, we can compute our new data structure and evaluate f at d points in the unit disk with an absolute error less than 2^{-m} in $\widetilde{O}(d(\tau + m))$ bit operations, where $\widetilde{O}(\cdot)$ means that we omit logarithmic factors. We also show that if κ is the absolute condition number of the zeros of f, then we can isolate all the roots of f in $\widetilde{O}(d(\tau + \log \kappa))$ bit operations. Moreover, our algorithms are simple to implement. For approximating the complex roots of a polynomial, we implemented a small prototype in `Python/NumPy` that is an order of magnitude faster than the state-of-the-art solver `MPSolve` for high degree polynomials with random coefficients [18].

7.2 Non-Euclidean Computational Geometry

Participants: Vincent Despré, Loïc Dubois, Benedikt Kolbe, Alba Marina Málaga Sabogal, Monique Teillaud.

7.2.1 Experimental analysis of Delaunay flip algorithms on genus two hyperbolic surfaces

Guided by insights on the mapping class group of a surface, we give experimental evidence that the upper bound recently proven on the diameter of the flip graph of a surface by Despré, Schlenker, and Teillaud [3] is largely overestimated. To obtain this result, we propose a set of techniques allowing us to actually perform experiments. We solve arithmetic issues by proving a density result on rationally described genus two hyperbolic surfaces, and we rely on a description of surfaces allowing us to propose a data structure on which flips can be efficiently implemented [21].

7.2.2 Representing infinite hyperbolic periodic Delaunay triangulations using finitely many Dirichlet domains

The Delaunay triangulation of a set of points P on a hyperbolic surface is the projection of the Delaunay triangulation of the set $\widetilde{P}$ of lifted points in the hyperbolic plane. Since $\widetilde{P}$ is infinite, the algorithms to compute Delaunay triangulations in the plane do not generalize naturally. Assuming that the surface comes with a Dirichlet domain, we exhibit a finite set of points that captures the full triangulation. Indeed, we prove that an edge of a Delaunay triangulation has a combinatorial length (a notion we define in the paper) smaller than $12g - 6$ with respect to a Dirichlet domain. On the way, we prove that both the edges of a Delaunay triangulation and of a Dirichlet domain have some kind of distance minimizing properties that are of intrinsic interest. The bounds produced in this paper depend only on the topology of the surface. They provide mathematical foundations for hyperbolic analogs of the algorithms to compute periodic Delaunay triangulations in Euclidean space [22].

7.2.3 Delaunay triangulations of generalized Bolza surfaces

The Bolza surface can be seen as the quotient of the hyperbolic plane, represented by the Poincaré disk model, under the action of the group generated by the hyperbolic isometries identifying opposite sides of a regular octagon centered at the origin. We consider generalized Bolza surfaces M_g, where the octagon is replaced by a regular $4g$-gon, leading to a genus g surface. We propose an extension of Bowyer's algorithm to these surfaces. In particular, we compute the value of the systole of M_g. We also propose algorithms computing small sets of points on M_g that are used to initialize Bowyer's algorithm [23]. *In collaboration with Matthijs Ebbens and Gert Vegter (Bernoulli Institute for Mathematics and Computer Science and Artificial Intelligence, University of Groningen).*

7.3 Probabilistic Analysis of Geometric Data Structures and Algorithms

Participants: Olivier Devillers, Charles Duménil, Xavier Goaoc, Guillaume Moroz, Ji Won Park.

7.3.1 Stochastic analysis of empty-region graphs

Given a set of points X, an empty-region graph is a graph in which $p, q \in X$ are neighbors if some region defined by (p, q) does not contain any point of X. We provide expected analyses of the degree of a point and the possibility of having *far* neighbors in such a graph when X is a planar Poisson point process. Namely the expected degree of a point in the empty axis-aligned-ellipse graph for a Poisson point process of intensity λ in the unit square is $\Theta(\ln\lambda)$. It is $\Theta(\ln\beta)$ if the ellipses are constrained to have an aspect ratio between 1 and $\beta > 1$, and $\Theta(\beta)$ when the aspect ratio is constrained but ellipses are not axis-aligned [16].

7.4 Discrete Geometric structures

Participants: Xavier Goaoc.

7.4.1 A stepping-up lemma for topological set systems

Intersection patterns of convex sets in $\mathbb{R}^d$ have the remarkable property that for $d+1 \le k \le \ell$, in any sufficiently large family of convex sets in $\mathbb{R}^d$, if a constant fraction of the k-element subfamilies have nonempty intersection, then a constant fraction of the ℓ-element subfamilies must also have nonempty intersection. Here, we prove that a similar phenomenon holds for any topological set system $\mathscr{F}$ in $\mathbb{R}^d$. Quantitatively, our bounds depend on how complicated the intersection of ℓ elements of $\mathscr{F}$ can be, as measured by the sum of the $\lceil \frac{d}{2} \rceil$ first Betti numbers. As an application, we improve the fractional Helly 2 number of set systems with bounded topological complexity due to the third author, from a Ramsey number down to $d+1$. We also shed some light on a conjecture of Kalai and Meshulam on intersection patterns of sets with bounded homological VC dimension. A key ingredient in our proof is the use of the stair convexity of Bukh, Matoussěk and Nivasch to recast a simplicial complex as a homological minor of a cubical complex [17].

In collaboration with Andreas Holmsen (KAIST) and Zuzana Patáková (Charles University of Praha)

7.4.2 An experimental study of forbidden patterns in geometric permutations by combinatorial lifting

We study the problem of deciding if a given triple of permutations can be realized as geometric permutations of disjoint convex sets in $\mathbb{R}^3$. We show that this question, which is equivalent to deciding the emptiness of certain semi-algebraic sets bounded by cubic polynomials, can be "lifted" to a purely combinatorial problem. We propose an effective algorithm for that problem, and use it to gain new insights into the structure of geometric permutations. In particular, we prove that all triples of permutations of size 5 are realizable, and give a complete list of triples of size 6 that are not [13].

In collaboration with Andreas Holmsen (KAIST) and Cyril Nicaud (LIGM)

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

8.1.1 WATERLOO MAPLE INC.

Participants: Laurent Dupont, Sylvain Lazard, Guillaume Moroz, Marc Pouget.

Company: WATERLOO MAPLE INC.
Duration: 2 years
Participants: GAMBLE and OURAGAN Inria teams

Abstract: A renewable two-years licence and cooperation agreement was signed on April 1st, 2018 between WATERLOO MAPLE INC., Ontario, Canada (represented by Laurent Bernardin, its Executive Vice President Products and Solutions) and Inria. On the Inria side, this contract involves the teams GAMBLE and OURAGAN (Paris), and it is coordinated by Fabrice Rouillier (OURAGAN).

F. Rouillier and GAMBLE are the developers of the ISOTOP software for the computation of topology of curves. The transfer of a version of ISOTOP to WATERLOO MAPLE INC. should be done on the long run.

This contract is amended this year to include the new software HEFROOTS for the isolation of the complex roots of a univariate polynomial. The transfer of HEFROOTS to WATERLOO MAPLE INC. started this year with the help of the independent contractor Rémi Imbach.

8.1.2 GEOMETRYFACTORY

Participants: Monique Teillaud.

Company: GEOMETRYFACTORY
Duration: permanent
Participants: INRIA and GEOMETRYFACTORY
Abstract: CGAL packages developed in GAMBLE are commercialized by GEOMETRYFACTORY.

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 Inria associate team not involved in an IIL or an international program

FIP

Participants: Xavier Goaoc, Ji Won Park.

Title: Finite point sets and Intersection Patterns

Duration: 4 years

Starting date: 2021

Coordinator: Andreas Holmsen (andreash@kaist.edu)

Partners:

- Korea Advanced Institute of Science and Technology

Inria contact: Xavier Goaoc

Summary: This project involves, besides Andreas Holmsen and Xavier Goaoc who are discrete and computational geometers, three specialists in combinatorics and probability theory: Mathilde Bouvel (LORIA), Philippe Chassaing (IECL) and Valentin Féray (IECL). This collaboration tackles questions from two classical research directions:

- Intersection patterns of geometric set systems. The theory of combinatorial convexity revealed many striking properties of families of convex sets that found surprising applications going far beyond their geometric origins. An established line of research explores generalizations of these properties beyond convexity and some far-reaching conjectures were made by Kalai and Meshulam regarding a homological analogue of the Vapnik-Chervonenkis dimension from learning theory.
- Combinatorial abstractions of finite point sets. Geometric algorithms are often designed over the reals, taking advantage of properties of continuity, closure under arithmetic operations, and geometric figures of $\mathbb{R}^d$. These algorithms do, however, effectively operate on a combinatorial abstraction of the geometric input, as their courses are determined not by the numerical values given in input, but by the output of certain predicate functions. A better understanding of these combinatorial abstractions would allow better design and finer analysis of geometric algorithms, as well as better testing of algorithms or geometric conjectures dealing with finite point sets. This is a classical line of research; an example of a long standing open problem is the tightening of the bounds on the number of halving lines in n-point sets, that is the number of ways in which a line can partition it in two equal-size subsets, known to be $ne^{\Omega(\sqrt{\log n})}$ and $O(n^{4/3})$.

9.1.2 Participation in other International Programs

SoS

Participants: Vincent Despré, Loïc Dubois, Benedikt Kolbe, Alba Marina Málaga Sabogal, Monique Teillaud.

Title: Structures on Surfaces

Duration: 4 years + 1 year Covid'19 extension

Starting date: April 1st, 2018

Coordinator: Monique Teillaud

Partners:

- Gamble project-team, Inria.
- LIGM (Laboratoire d'Informatique Gaspard Monge), Université Gustave Eiffel. Local Coordinator: Éric Colin de Verdière.
- RMATH (Mathematics Research Unit), University of Luxembourg. National Coordinator: Hugo Parlier.

Inria contact: Monique Teillaud

Summary: SoS is co-funded by ANR (ANR-17-CE40-0033) and FNR (INTER/ANR/16/11554412/SoS) as a PRCI (Projet de Recherche Collaborative Internationale).

The central theme of this project is the study of geometric and combinatorial structures related to surfaces and their moduli. Even though they work on common themes, there is a real gap between communities working in geometric topology and computational geometry and SoS aims to create a long-lasting bridge between them. Beyond a common interest, techniques from both ends are relevant and the potential gain in perspective from long-term collaborations is truly thrilling.

In particular, SoS aims to extend the scope of computational geometry, a field at the interface between mathematics and computer science that develops algorithms for geometric problems, to a variety of unexplored contexts. During the last two decades, research in computational geometry has gained wide impact through CGAL, the Computational Geometry Algorithms Library. In parallel, the needs for non-Euclidean geometries are arising, e.g., in geometric modeling, neuromathematics, or physics. Our goal is to develop computational geometry for some of these non-Euclidean spaces and make these developments readily available for users in academia and industry.

To reach this aim, SoS follows an interdisciplinary approach, gathering researchers whose expertise cover a large range of mathematics, algorithms and software. A mathematical study of the objects considered is performed, together with the design of algorithms when applicable. Algorithms are analyzed both in theory and in practice after prototype implementations, which are improved whenever it makes sense to target longer-term integration into CGAL.

Our main objects of study are Delaunay triangulations and circle patterns on surfaces, polyhedral geometry, and systems of disjoint curves and graphs on surfaces.

Project website: sos.loria.fr/.

9.2 National initiatives

9.2.1 ANR PRC

ANR Aspag

Participants: Olivier Devillers, Charles Duménil, Xavier Goaoc, Sylvain Lazard, Guillaume Moroz, Ji Won Park, Marc Pouget.

Title: Analyse et Simulation Probabilistes d'Algorithmes Géométriques

Duration: 4 years + 1 year Covid'19 extension

Starting date: January 1st, 2018

Coordinator: Olivier Devillers

Partners:

- Gamble project-team, Inria.
- Labri (Laboratoire Bordelais de Recherche en Informatique), Université de Bordeaux. Local Coordinator: Philippe Duchon.
- Laboratoire de Mathématiques Raphaël Salem, Université de Rouen. Local Coordinator: Pierre Calka.
- LAMA (Laboratoire d'Analyse et de Mathématiques Appliquées), Université Paris-Est Marne-la-Vallée. Local Coordinator: Matthieu Fradelizi

Inria contact: Olivier Devillers

Summary: The ASPAG projet is funded by ANR under number ANR-17-CE40-0017 .

The analysis and processing of geometric data has become routine in a variety of human activities ranging from computer-aided design in manufacturing to the tracking of animal trajectories in ecology or geographic information systems in GPS navigation devices. Geometric algorithms and probabilistic geometric models are crucial to the treatment of all this geometric data, yet the current available knowledge is in various ways much too limited: many models are far from matching real data, and the analyses are not always relevant in practical contexts. One of the reasons for this state of affairs is that the breadth of expertise required is spread among different scientific communities (computational geometry, analysis of algorithms and stochastic geometry) that historically had very little interaction. The Aspag project brings together experts of these communities to address the problem of geometric data. We will more specifically work on the following three interdependent directions.

(1) Dependent point sets: One of the main issues of most models is the core assumption that the data points are independent and follow the same underlying distribution. Although this may be relevant in some contexts, the independence assumption is too strong for many applications.

(2) Simulation of geometric structures: The phenomena studied in (1) involve intricate random geometric structures subject to new models or constraints. A natural first step would be to build up our understanding and identify plausible conjectures through simulation. Perhaps surprisingly, the tools for an effective simulation of such complex geometric systems still need to be developed.

(3) Understanding geometric algorithms: the analysis of algorithms is an essential step in assessing the strengths and weaknesses of algorithmic principles, and is crucial to guide the choices made when designing a complex data processing pipeline. Any analysis must strike a balance between realism and tractability; the current analyses of many geometric algorithms are notoriously unrealistic. Aside from the purely scientific objectives, one of the main goals of Aspag is to bring the communities closer in the long term. As a consequence, the funding of the project is crucial to ensure that the members of the consortium will be able to interact on a very regular basis, a necessary condition for significant progress on the above challenges.

Project website: members.loria.fr/Olivier.Devillers/aspag/.

ANR MinMax

Participants: Xavier Goaoc.

Title: MIN-MAX

Duration: 4 years

Starting date: January 1st, 2019

Coordinator: Stéphane Sabourau (Université Paris-Est Créteil)

Partners:

- Université Paris Est Créteil, Laboratoire d'Analyse et de Mathématiques Appliquées (LAMA). Local coordinator: Stéphane Sabourau
- Université de Tours, Institut Denis Poisson. Local coordinator: Laurent Mazet. This node includes two participants from Nancy, Benoît Daniel (IECL) and Xavier Goaoc (Loria, GAMBLE).

Inria contact: Xavier Goaoc

Summary: The MinMax projet is funded by ANR under number ANR-19-CE40-0014

This collaborative research project aims to bring together researchers from various areas – namely, geometry and topology, minimal surface theory and geometric analysis, and computational geometry and algorithms – to work on a precise theme around min-max constructions and waist estimates.

Project website: perso.math.u-pem.fr/sabourau.stephane/min-max/min-max.html

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Member of the organizing committees A. Málaga co-organized a Fabrikathon in Marseille, October 11-13, 2021; a workshop on design and production of mathematical objects.

Seminars Five virtual seminars were organized in the framework of the SoS project (web)

10.1.2 Journal

Member of the editorial boards Monique Teillaud is a managing editor of JoCG, Journal of Computational Geometry.

Reviewer - reviewing activities All members of the team are regular reviewers for the journals of our field, namely Discrete and Computational Geometry (DCG), Journal of Computational Geometry (JoCG), International Journal on Computational Geometry and Applications (IJCGA), Journal on Symbolic Computations (JSC), SIAM Journal on Computing (SICOMP), Mathematics in Computer Science (MCS), SIAM Journal on Discrete Mathematics (SIDMA), etc.

10.1.3 Invited talks

Members of the team were invited to give a talk at the following venues: Stochastic Geometry Days (O. Devillers, (web)), Séminaire francilien de géométrie algorithmique et combinatoire (M. Teillaud, (web)), Journées Nationales de Calcul Formel 2021 (G. Moroz, (web)), Copenhagen-Jerusalem combinatorics seminar (X. Goaoc, (web)), DGeCo seminar (X. Goaoc, (web)), Séminaire de systèmes dynamiques de l'IMT (Alba Málaga, (web)), Séminaire commun de géometrie de l'IECL (Alba Málaga), Séminaire virtuel francophone Groupes et Géométrie (Alba Málaga, (web), as well as the seminars of the INRIA project teams LFANT (X. Goaoc) and OURAGAN (M. Teillaud).

10.1.4 Leadership within the scientific community

Member of the editorial boards of software projects Marc Pouget and Monique Teillaud are members of the CGAL editorial board.

10.1.5 Scientific expertise

Members of Gamble are occasionally reviewing proposals or applications for foreign research agencies (e.g., FWF, NWO, NSERC, ISF, etc.) or foreign universities. We are not giving more details here to preserve anonymity.

10.1.6 Research administration

Team members are involved in various committees managing the scientific life of the lab or at a national level.

Hiring committees:

- M. Teillaud was a member of the admission committee for INRIA senior researchers (DR)
- M. Teillaud was a member of the hiring committee for a permanent engineer (IR) position at INRIA Nancy - Grand Est
- X. Goaoc chaired the hiring committee for a position of Maitre de conférences at Université de Lorraine.

Local:

- S. Lazard is the chair of the LORIA department "Algorithms, computation, image and geometry" and a member of the scientific council of LORIA,
- G. Moroz is the chair of the *Comité des utilisateurs des moyens informatiques* of INRIA Nancy - Grand Est,
- S. Lazard is the chair of the PhD and postdoc hiring committee of INRIA Nancy - Grand Est,
- M. Teillaud is the Scientific Advisor for Technologic Development of INRIA Nancy - Grand Est,
- Team members participate in several INRIA local commissions: *Développement technologique* (G. Moroz), *Commission de gestion AGOS* (M. Pouget), CLHSCT (G. Moroz), *Information et Édition Scientifique* (L. Dupont),
- Team members participate in various university councils: Pôle AM2I de l'*Université de Lorraine* (O. Devillers), *Bureau CMI École doctorale IAEM* (S. Lazard, X. Goaoc), *Fédération Charles Hermite* (X. Goaoc),
- Team members participate in local councils: *Comité de centre INRIA NAncy Grand - Est* (X. Goaoc), *Conseil de laboratoire du LORIA* (M. Teillaud).

National:

- S. Lazard is the chair of the INRIA *Mission Jeunes Chercheurs*.

10.2 Teaching - Supervision - Juries

10.2.1 Teaching Committees

- V. Despré: Head of the Engineer diploma speciality SIR, Systèmes d'Information et Réseaux, Polytech Nancy, Université de Lorraine.
- L. Dupont is the secretary of Commission Pédagogique Nationale Carrières Sociales / Information-Communication / Métiers du Multimédia et de l'Internet (2017-2022).
- L. Dupont represents the Commission Pédagogique Nationale Carrières Sociales / Information-Communication / Métiers du Multimédia et de l'Internet at the national working group on D.U.T/B.U.T reform
- L. Dupont: Head of the Bachelor diploma Licence Professionnelle Animateur, Facilitateur de Tiers-lieux Eco-Responsables, Université de Lorraine,
- L. Dupont: Responsible of fablab "Charlylab" of I.U.T. Nancy-Charlemagne,
- X. Goaoc is a member of the Conseil d'administration de l'École des Mines.

10.2.2 Teaching

- Licence: V. Despré, *Programmation orientée objet*, 62h, L2, Polytech Nancy, France.
- Licence: V. Despré, *Algorithmique*, 66h, L3, Polytech Nancy, France.
- Master: V. Despré, *Programmation réseau*, 68h, M1, Polytech Nancy, France.
- Master: V. Despré, *Algorithmique distribuée*, 48h, M1, Polytech Nancy, France.
- Master: O. Devillers, *Modèles d'environnements, planification de trajectoires*, 18h, M2 AVR, Université de Lorraine, France (web).
- Licence: C. Duménil, *Découverte de l'informatique*, 103h, L1, Polytech Nancy, France.

- Licence: C. Duménil, *Algorithmique*, 81h, L2, Polytech Nancy, France.
- Master: C. Duménil, Jury Projets J2E, 8h, M1, Polytech Nancy, France.
- Licence: L. Dupont, *Web development*, 35h, L1, Université de Lorraine, France.
- Licence: L. Dupont, *Web development*, 150h, L2, Université de Lorraine, France.
- Licence: L. Dupont *Web development and Social networks* 100h L3, Université de Lorraine, France.
- Licence: L. Dupont, *3D printing and CAO* L3, Université de Lorraine, France.
- Licence : X. Goaoc, *Operation research*, 18 HETD, L3, École des Mines de Nancy, France.
- Licence : X. Goaoc, *Algorithms*, 28 HETD, L3, École des Mines de Nancy, France.
- Master: X. Goaoc, *Algorithms*, 12 HETD, M1, École des Mines de Nancy, France.
- Master: X. Goaoc, *Computer architecture*, 32 HETD, M1, École des Mines de Nancy Nancy, France.
- Master: X. Goaoc, *Introduction to blockchains*, 60 HETD, M1, École des Mines de Nancy + Polytech Nancy, France.
- Licence: B. Kolbe, *Algorithmique*, 42 HETD, L3, Polytech Nancy.
- Master, A. Málaga, *Mathematical software*, course in the Master in Mathematics at University of Granada, invited lecturer, 10h, Spain (taught online in 2021).
- Licence: A. Málaga, *Information systems and databases*, 86h, L1, Université de Lorraine, France.
- Master: M. Pouget, *Introduction to computational geometry*, 10.5h, M2, École Nationale Supérieure de Géologie, France.
- Licence : G. Moroz, *Programmation et structures de données*, 20 HETD, L3, École des Mines de Nancy, France.

10.2.3 Supervision

- PhD: George Krait, Isolating the singularities of the plane projection of generic space curves and applications in robotics, defended on May 4th, supervised by Sylvain Lazard, Guillaume Moroz and Marc Pouget.
- PhD in progress: Charles Duménil, Probabilistic analysis of geometric structures, started in Oct. 2016, supervised by Olivier Devillers.
- PhD in progress: Nuwan Herath, Fast algorithm for the visualization of surfaces, started in Nov. 2019, supervised by Sylvain Lazard, Guillaume Moroz and Marc Pouget.
- PhD in progress: Léo Valque, Rounding 3D meshes, started in Sept. 2020, supervised by Sylvain Lazard.
- PhD in progress: Matthias Fresacher, A toolbox for hyperbolic surfaces, started in Nov. 2021, supervised by Monique Teillaud and Vincent Despré.

10.2.4 Juries

- O. Devillers was a member of the PhD defense committee of Loïc Crombez, Université Clermont Auvergne.
- M. Teillaud chaired the Habilitation defense committee of Jeanne Pellerin, Université de Lorraine
- M. Teillaud chaired the PhD defense committee of Thomas Magnard, Université Gustave Eiffel, Marne-la-Vallée
- X. Goaoc was a member of the PhD defense committee of Matthijs Ebbens, Groningen University.

10.3 Popularization

10.3.1 Articles and contents

A. Málaga co-authored the winning image in the competition marking the 40th anniversary of the mathematical meeting center CIRM, related to the non-euclidean geometry research questions treated in the Gamble team.

10.3.2 Education

- O. Devillers and M. Teillaud went to highschools to introduce research in computer science, in the framework of the program Chiche (web).
- G. Moroz is a member of the Mathematics Olympiads committee of the Nancy-Metz academy.
- A. Málaga is a member of the scientific council of the "Maths en scène" association.

11 Scientific production

11.1 Major publications

[1] N. Bonichon, P. Bose, J.-L. De Carufel, V. Despré, D. Hill and M. Smid. 'Improved Routing on the Delaunay Triangulation'. In: *ESA 2018 - 26th Annual European Symposium on Algorithms*. Helsinki, Finland, 2018. DOI: 10.4230/LIPIcs.ESA.2018.22. URL: https://hal.archives-ouvertes.fr/hal-01881280.

[2] N. Chenavier and O. Devillers. 'Stretch Factor in a Planar Poisson-Delaunay Triangulation with a Large Intensity'. In: *Advances in Applied Probability* 50.1 (2018), pp. 35–56. DOI: 10.1017/apr.2018.3. URL: https://hal.inria.fr/hal-01700778.

[3] V. Despré, J.-M. Schlenker and M. Teillaud. 'Flipping Geometric Triangulations on Hyperbolic Surfaces'. In: *SoCG 2020 - 36th International Symposium on Computational Geometry*. Zurich, Switzerland, 2020. DOI: 10.4230/LIPIcs.SoCG.2020.35. URL: https://hal.inria.fr/hal-02886493.

[4] O. Devillers, S. Lazard and W. Lenhart. 'Rounding meshes in 3D'. In: *Discrete and Computational Geometry* (Apr. 2020). DOI: 10.1007/s00454-020-00202-2. URL: https://hal.inria.fr/hal-02549290.

[5] X. Goaoc, P. Paták, Z. Patáková, M. Tancer and U. Wagner. 'Shellability is NP-complete'. In: *Journal of the ACM (JACM)* 66.3 (2019). DOI: 10.1145/3314024. URL: https://hal.inria.fr/hal-02050505.

[6] X. Goaoc and E. Welzl. 'Convex Hulls of Random Order Types'. In: *SoCG 2020 - 36th International Symposium on Computational Geometry*. Ed. by S. Cabello and D. Z. Chen. Vol. 164. 36th International Symposium on Computational Geometry (SoCG 2020). Best paper award. Zürich / Virtual, Switzerland, 2020, 49:1–49:15. DOI: 10.4230/LIPIcs.SoCG.2020.49. URL: https://hal.inria.fr/hal-02879289.

[7] R. Imbach, G. Moroz and M. Pouget. 'Reliable Location with Respect to the Projection of a Smooth Space Curve'. In: *Reliable Computing* 26 (2018), pp. 13–55. URL: https://hal.archives-ouvertes.fr/hal-01920444.

[8] I. Iordanov and M. Teillaud. 'Implementing Delaunay Triangulations of the Bolza Surface'. In: *33rd International Symposium on Computational Geometry (SoCG 2017)*. Brisbane, Australia, July 2017, 44:1–44:15. DOI: 10.4230/LIPIcs.SoCG.2017.44. URL: https://hal.inria.fr/hal-01568002.

[9] R. Jha, D. Chablat, L. Baron, F. Rouillier and G. Moroz. 'Workspace, Joint space and Singularities of a family of Delta-Like Robot'. In: *Mechanism and Machine Theory* 127 (Sept. 2018), pp. 73–95. DOI: 10.1016/j.mechmachtheory.2018.05.004. URL: https://hal.archives-ouvertes.fr/hal-01796066.

[10] S. Lazard, M. Pouget and F. Rouillier. 'Bivariate triangular decompositions in the presence of asymptotes'. In: *Journal of Symbolic Computation* 82 (2017), pp. 123–133. DOI: 10.1016/j.jsc.2017.01.004. URL: https://hal.inria.fr/hal-01468796.

11.2 Publications of the year

International journals

[11] F. Balacheff, V. Despré and H. Parlier. 'Systoles and diameters of hyperbolic surfaces'. In: *Kyoto Journal of Mathematics* (2022). URL: https://hal.archives-ouvertes.fr/hal-03480483.

[12] L. Decreusefond and G. Moroz. 'Optimal transport between determinantal point processes and application to fast simulation'. In: *Modern Stochastics: Theory and Applications* 8.2 (2021), pp. 209–237. DOI: 10.15559/21-VMSTA180. URL: https://hal.telecom-paris.fr/hal-02984323.

[13] X. Goaoc, A. Holmsen and C. Nicaud. 'An experimental study of forbidden patterns in geometric permutations by combinatorial lifting'. In: *Journal of Computational Geometry*. Special Issue of Selected Papers from SoCG 2019 11.2 (7th Jan. 2021), pp. 131–161. DOI: 10.20382/jocg.v11i2a6. URL: https://hal.inria.fr/hal-03152958.

[14] B. Kolbe and M. E. Evans. 'Enumerating Isotopy Classes of Tilings guided by the symmetry of Triply-Periodic Minimal Surfaces'. In: *SIAM Journal on Applied Algebra and Geometry* (Nov. 2021). URL: https://hal.inria.fr/hal-03482422.

[15] G. Krait, S. Lazard, G. Moroz and M. Pouget. 'Certified numerical algorithm for isolating the singularities of the plane projection of generic smooth space curves'. In: *Journal of Computational and Applied Mathematics* (2021). DOI: 10.1016/j.cam.2021.113553. URL: https://hal.inria.fr/hal-03161393.

International peer-reviewed conferences

[16] O. Devillers and C. Duménil. 'Stochastic Analysis of Empty-Region Graphs'. In: CCCG 2021 - 33rd Canadian Conference on Computational Geometry. Halifax / Virtual, Canada, 2021. URL: https://hal.inria.fr/hal-03296186.

[17] X. Goaoc, A. Holmsen and Z. Patáková. 'A Stepping-Up Lemma for Topological Set Systems'. In: 37th International Symposium on Computational Geometry (SoCG 2021). Vol. 189. Buffalo/virtual, United States, 7th June 2021, 40:1–40:17. DOI: 10.4230/LIPIcs.SoCG.2021.40. URL: https://hal.inria.fr/hal-03341599.

[18] G. Moroz. 'New data structure for univariate polynomial approximation and applications to root isolation, numerical multipoint evaluation, and other problems'. In: *2021 IEEE 62nd Annual Symposimum on Foundations of Computer Science (FOCS)*. FOCS 2021 - 62nd Annual IEEE Symposium on Foundations of Computer Science. Denver, United States, Dec. 2021. URL: https://hal.archives-ouvertes.fr/hal-03249123.

Doctoral dissertations and habilitation theses

[19] G. Krait. 'Isolating the Singularities of the Plane Projection of Generic Space Curves and Applications in Robotics'. Université de Lorraine, 4th May 2021. URL: https://tel.archives-ouvertes.fr/tel-03241794.

Reports & preprints

[20] J.-S. Cheng, K. Jin, M. Pouget, J. Wen and B. Zhang. *An Improved Complexity Bound for Computing the Topology of a Real Algebraic Space Curve*. 8th Dec. 2021. URL: https://hal.inria.fr/hal-03469996.

[21] V. Despré, L. Dubois, B. Kolbe and M. Teillaud. *Experimental analysis of Delaunay flip algorithms on genus two hyperbolic surfaces*. 2nd Dec. 2021. URL: https://hal.inria.fr/hal-03462834.

[22] V. Despré, B. Kolbe and M. Teillaud. *Representing infinite hyperbolic periodic Delaunay triangulations using finitely many Dirichlet domains*. July 2021. URL: https://hal.archives-ouvertes.fr/hal-03045921.

[23] M. Ebbens, I. Iordanov, M. Teillaud and G. Vegter. *Delaunay triangulations of generalized Bolza surfaces*. Mar. 2021. URL: https://hal.inria.fr/hal-03080125.

[24] G. Moroz. *Fast real and complex root-finding methods for well-conditioned polynomials*. 8th Feb. 2021. URL: https://hal.archives-ouvertes.fr/hal-03134259.

[25] N. H. Mudiyanselage, G. Moroz and M. Pouget. *Fast Guaranteed Drawing of High Degree Algebraic Curves*. 3rd Dec. 2021. URL: https://hal.inria.fr/hal-03465123.

11.3 Cited publications

[26] D. Attali, J.-D. Boissonnat and A. Lieutier. 'Complexity of the Delaunay triangulation of points on surfaces: the smooth case'. In: *Proceedings of the 19th Annual Symposium on Computational Geometry*. 2003, pp. 201–210. DOI: 10.1145/777792.777823. URL: http://dl.acm.org/citation.cfm?id=777823.

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[28] M. Bogdanov, O. Devillers and M. Teillaud. 'Hyperbolic Delaunay complexes and Voronoi diagrams made practical'. In: *Journal of Computational Geometry* 5 (2014), pp. 56–85.

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