

RESEARCH CENTRE

Bordeaux - Sud-Ouest

2021

ACTIVITY REPORT

Team

REALOPT

## **Reformulations based algorithms for Combinatorial Optimization**

Inria teams are typically groups of researchers working on the definition of a common project, and objectives, with the goal to arrive at the creation of a project-team. Such project-teams may include other partners (universities or research institutions)

### **DOMAIN**

**Applied Mathematics, Computation and  
Simulation**

### **THEME**

**Optimization, machine learning and  
statistical methods**

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## Team REALOPT

*Creation of the Team: 2020 December 15*

### Keywords

#### Computer sciences and digital sciences

- A6.2.6. – Optimization
- A7.1.3. – Graph algorithms
- A8.1. – Discrete mathematics, combinatorics
- A8.2. – Optimization
- A8.2.1. – Operations research
- A8.7. – Graph theory
- A9.7. – AI algorithmics

#### Other research topics and application domains

- B3.1. – Sustainable development
- B3.1.1. – Resource management
- B4.2. – Nuclear Energy Production
- B4.4. – Energy delivery
- B6.5. – Information systems
- B7. – Transport and logistics
- B9.5.2. – Mathematics

## **1 Team members, visitors, external collaborators**

### **Research Scientists**

- Gael Guillot [Univ de Bordeaux, Researcher, until Jan 2021]
- Ruslan Sadykov [Inria, Researcher, HDR]

### **Faculty Members**

- François Clautiaux [Team leader, Univ de Bordeaux, Professor, HDR]
- Boris Detienne [Univ de Bordeaux, Associate Professor]
- Aurelien Froger [Univ de Bordeaux, Associate Professor]
- Arnaud Pecher [Univ de Bordeaux, Professor, HDR]
- Pierre Pesneau [Univ de Bordeaux, Associate Professor]

### **Post-Doctoral Fellow**

- Eduardo Queiroga [Inria, from Nov 2021]

### **PhD Students**

- Komlanvi Parfait Ametana [Univ de Bordeaux, from Oct 2021]
- Isaac Balster [Inria]
- Xavier Blanchot [Réseau de transport d'électricité, CIFRE]
- Mickael Gaury [Ecole de Commerce KEDGE Business School]
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- Daniil Khachai [Ecole de Commerce KEDGE Business School]
- Johan Leveque [La Poste, CIFRE]
- Orlando Rivera Letelier [Universidad Adolfo Ibanez - Santiago Chili, until Jun 2021]

### **Interns and Apprentices**

- Komlanvi Parfait Ametana [Inria, from Mar 2021 until Aug 2021]
- Nicolas Guillemain [Inria, from Mar 2021 until Aug 2021]

### **Administrative Assistant**

- Joelle Rodrigues [Inria]

## External Collaborators

- Artur Alves Pessoa [Universidade Federal Fluminense - Niteroi Brazil]
- Ayse Nur Arslan [INSA Rennes]
- Imen Ben Mohamed [Ecole de Commerce KEDGE Business School, until Sep 2021]
- Philippe Depouilly [CNRS]
- Laurent Facq [CNRS]
- Cédric Joncour [Univ du Havre]
- Walid Klibi [Ecole de Commerce KEDGE Business School]
- Philippe Meurdesoif [Univ de Bordeaux]
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## 2 Overall objectives

Reformulation techniques in Mixed Integer Programming (MIP), Polyhedral approaches (cut generation), Robust Optimization, Approximation Algorithms, Extended formulations, Lagrangian Relaxation (Column Generation) based algorithms, Dantzig and Benders Decomposition, Primal Heuristics, Graph Theory, Constraint Programming.

Quantitative modeling is routinely used in both industry and administration to design and operate transportation, distribution, or production systems. Optimization concerns every stage of the decision-making process: long term investment budgeting and activity planning, tactical management of scarce resources, or the control of day-to-day operations. In many optimization problems that arise in decision support applications the most important decisions (control variables) are discrete in nature: such as on/off decision to buy, to invest, to hire, to send a vehicle, to allocate resources, to decide on precedence in operation planning, or to install a connection in network design. Such *combinatorial optimization* problems can be modeled as linear or nonlinear programs with integer decision variables and extra variables to deal with continuous adjustments. The most widely used modeling tool consists in defining the feasible decision set using linear inequalities with a mix of integer and continuous variables, so-called Mixed Integer Programs (MIP), which already allow a fair description of reality and are also well-suited for global optimization. The solution of such models is essentially based on enumeration techniques and is notoriously difficult given the huge size of the solution space.

Commercial solvers have made significant progress but remain quickly overwhelmed beyond a certain problem size. A key to further progress is the development of better problem formulations that provide strong continuous approximations and hence help to prune the enumerative solution scheme. Effective solution schemes are a complex blend of techniques: cutting planes to better approximate the convex hull of feasible (integer) solutions, extended reformulations (combinatorial relations can be formulated better with extra variables), constraint programming to actively reduce the solution domain through logical implications along variable fixing based on reduced cost, Lagrangian decomposition methods to produce powerful relaxations, and Bender's decomposition to project the formulation, reducing the problem to the important decision variables, and to implement multi-level programming that models a hierarchy of decision levels or recourse decision in the case of data adjustment, primal heuristics and meta-heuristics (greedy, local improvement, or randomized partial search procedures) to produce good candidates at all stage of the solution process, and branch-and-bound or dynamic programming enumeration schemes to find a global optimum, with specific strong strategies for the selection on the sequence of fixings. The real challenge is to integrate the most efficient methods in one global system so as to prune what is essentially an enumeration based solution technique. The progress are measured in terms of the large scale of input data that can now be solved, the integration of many decision levels into planning models, and not least, the account taken for random (or dynamically adjusted)

data by way of modeling expectation (stochastic approaches) or worst-case behavior (robust approaches).

Building on complementary expertise, our team's overall goals are threefold:

- (i) **Methodologies:** To design tight formulations for specific combinatorial optimization problems and generic models, relying on delayed cut and column generation, decomposition, extended formulations and projection tools for linear and nonlinear mixed integer programming models. To develop generic methods based on such strong formulations by handling their large scale dynamically. To generalize algorithmic features that have proven efficient in enhancing performance of exact optimization approaches. To develop approximation schemes with proven optimality gap and low computational complexity. More broadly, to contribute to theoretical and methodological developments of exact and approximate approaches in combinatorial optimization, while extending the scope of applications and their scale.
- (ii) **Problem solving:** To demonstrate the strength of cooperation between complementary exact mathematical optimization techniques, dynamic programming, robust and stochastic optimization, constraint programming, combinatorial algorithms and graph theory, by developing "efficient" algorithms for specific mathematical models. To tackle large-scale real-life applications, providing provably good approximate solutions by combining exact, approximate, and heuristic methods.
- (iii) **Software platform & Transfer:** To provide prototypes of modelers and solvers based on generic software tools that build on our research developments, writing code that serves as the proof-of-concept of the genericity and efficiency of our approaches, while transferring our research findings to internal and external users.

## 3 Research program

### 3.1 Introduction

**Keywords:** integer programming, graph theory, decomposition approaches, polyhedral approaches, quadratic programming approaches, constraint programming..

*Combinatorial optimization* is the field of discrete optimization problems. In many applications, the most important decisions (control variables) are binary (on/off decisions) or integer (indivisible quantities). Extra variables can represent continuous adjustments or amounts. This results in models known as *mixed integer programs* (MIP), where the relationships between variables and input parameters are expressed as linear constraints and the goal is defined as a linear objective function. MIPs are notoriously difficult to solve: good quality estimations of the optimal value (bounds) are required to prune enumeration-based global-optimization algorithms whose complexity is exponential. In the standard approach to solving an MIP is so-called *branch-and-bound algorithm*: (i) one solves the linear programming (LP) relaxation using the simplex method; (ii) if the LP solution is not integer, one adds a disjunctive constraint on a fractional component (rounding it up or down) that defines two sub-problems; (iii) one applies this procedure recursively, thus defining a binary enumeration tree that can be pruned by comparing the local LP bound to the best known integer solution. Commercial MIP solvers are essentially based on branch-and-bound (such IBM-CPLEX, FICO-Xpress-mp, or GUROBI). They have made tremendous progress over the last decade (with a speedup by a factor of 60). But extending their capabilities remains a continuous challenge; given the combinatorial explosion inherent to enumerative solution techniques, they remain quickly overwhelmed beyond a certain problem size or complexity.

Progress can be expected from the development of tighter formulations. Central to our field is the characterization of polyhedra defining or approximating the solution set and combinatorial algorithms to identify "efficiently" a minimum cost solution or separate an unfeasible point. With properly chosen formulations, exact optimization tools can be competitive with other methods (such as meta-heuristics) in constructing good approximate solutions within limited computational time, and of course has the important advantage of being able to provide a performance guarantee through the relaxation

bounds. Decomposition techniques are implicitly leading to better problem formulation as well, while constraint propagation are tools from artificial intelligence to further improve formulation through intensive preprocessing. A new trend is robust optimization where recent progress have been made: the aim is to produce optimized solutions that remain of good quality even if the problem data has stochastic variations. In all cases, the study of specific models and challenging industrial applications is quite relevant because developments made into a specific context can become generic tools over time and see their way into commercial software.

Our project brings together researchers with expertise in mathematical programming (polyhedral approaches, decomposition and reformulation techniques in mixed integer programming, robust and stochastic programming, and dynamic programming), graph theory (characterization of graph properties, combinatorial algorithms) and constraint programming in the aim of producing better quality formulations and developing new methods to exploit these formulations. These new results are then applied to find high quality solutions for practical combinatorial problems such as routing, network design, planning, scheduling, cutting and packing problems, High Performance and Cloud Computing.

### 3.2 Polyhedral approaches for MIP

Adding valid inequalities to the polyhedral description of an MIP allows one to improve the resulting LP bound and hence to better prune the enumeration tree. In a cutting plane procedure, one attempt to identify valid inequalities that are violated by the LP solution of the current formulation and adds them to the formulation. This can be done at each node of the branch-and-bound tree giving rise to a so-called *branch-and-cut algorithm* [47]. The goal is to reduce the resolution of an integer program to that of a linear program by deriving a linear description of the convex hull of the feasible solutions. Polyhedral theory tells us that if  $X$  is a mixed integer program:  $X = P \cap \mathbb{Z}^n \times \mathbb{R}^p$  where  $P = \{x \in \mathbb{R}^{n+p} : Ax \leq b\}$  with matrix  $(A, b) \in \mathbb{Q}^{m \times (n+p+1)}$ , then  $\text{conv}(X)$  is a polyhedron that can be described in terms of linear constraints, i.e. it writes as  $\text{conv}(X) = \{x \in \mathbb{R}^{n+p} : Cx \leq d\}$  for some matrix  $(C, d) \in \mathbb{Q}^{m' \times (n+p+1)}$  although the dimension  $m'$  is typically quite large. A fundamental result in this field is the equivalence of complexity between solving the combinatorial optimization problem  $\min\{cx : x \in X\}$  and solving the *separation problem* over the associated polyhedron  $\text{conv}(X)$ : if  $\tilde{x} \notin \text{conv}(X)$ , find a linear inequality  $\pi x \geq \pi_0$  satisfied by all points in  $\text{conv}(X)$  but violated by  $\tilde{x}$ . Hence, for NP-hard problems, one can not hope to get a compact description of  $\text{conv}(X)$  nor a polynomial time exact separation routine. Polyhedral studies focus on identifying some of the inequalities that are involved in the polyhedral description of  $\text{conv}(X)$  and derive efficient *separation procedures* (cutting plane generation). Only a subset of the inequalities  $Cx \leq d$  can offer a good approximation, that combined with a branch-and-bound enumeration techniques permits to solve the problem. Using *cutting plane algorithm* at each node of the branch-and-bound tree, gives rise to the algorithm called *branch-and-cut*.

### 3.3 Decomposition-and-reformulation-approaches

An hierarchical approach to tackle complex combinatorial problems consists in considering separately different substructures (subproblems). If one is able to implement relatively efficient optimization on the substructures, this can be exploited to reformulate the global problem as a selection of specific subproblem solutions that together form a global solution. If the subproblems correspond to subset of constraints in the MIP formulation, this leads to Dantzig-Wolfe decomposition. If it corresponds to isolating a subset of decision variables, this leads to Bender's decomposition. Both lead to extended formulations of the problem with either a huge number of variables or constraints. Dantzig-Wolfe approach requires specific algorithmic approaches to generate subproblem solutions and associated global decision variables dynamically in the course of the optimization. This procedure is known as *column generation*, while its combination with branch-and-bound enumeration is called *branch-and-price*. Alternatively, in Bender's approach, when dealing with exponentially many constraints in the reformulation, the *cutting plane procedures* that we defined in the previous section are well-suited tools. When optimization on a substructure is (relatively) easy, there often exists a tight reformulation of this substructure typically in an extended variable space. This gives rise powerful reformulation of the global problem, although it might be impractical given its size (typically pseudo-polynomial). It can be possible to project (part of) the extended formulation in a smaller dimensional space if not the original variable



space to bring polyhedral insight (cuts derived through polyhedral studies can often be recovered through such projections).

### 3.4 Integration of Artificial Intelligence Techniques in Integer Programming

When one deals with combinatorial problems with a large number of integer variables, or tightly constrained problems, mixed integer programming (MIP) alone may not be able to find solutions in a reasonable amount of time. In this case, techniques from artificial intelligence can be used to improve these methods. In particular, we use variable fixing techniques, primal heuristics and constraint programming.

Primal heuristics are useful to find feasible solutions in a small amount of time. We focus on heuristics that are either based on integer programming (rounding, diving, relaxation induced neighborhood search, feasibility pump), or that are used inside our exact methods (heuristics for separation or pricing subproblem, heuristic constraint propagation, ...). Such methods are likely to produce good quality solutions only if the integer programming formulation is of top quality, i.e., if its LP relaxation provides a good approximation of the IP solution.

In the same line, variable fixing techniques, that are essential in reducing the size of large scale problems, rely on good quality approximations: either tight formulations or tight relaxation solvers (as a dynamic program combined with state space relaxation). Then if the dual bound derives when the variable is fixed to one exceeds the incumbent solution value, the variable can be fixed to zero and hence removed from the problem. The process can be applied sequentially by refining the degree of relaxation.

Constraint Programming (CP) focuses on iteratively reducing the variable domains (sets of feasible values) by applying logical and problem-specific operators. The latter propagates on selected variables the restrictions that are implied by the other variable domains through the relations between variables that are defined by the constraints of the problem. Combined with enumeration, it gives rise to exact optimization algorithms. A CP approach is particularly effective for tightly constrained problems, feasibility problems and min-max problems. Mixed Integer Programming (MIP), on the other hand, is known to be effective for loosely constrained problems and for problems with an objective function defined as the weighted sum of variables. Many problems belong to the intersection of these two classes. For such problems, it is reasonable to use algorithms that exploit complementary strengths of Constraint Programming and Mixed Integer Programming.

### 3.5 Robust Optimization

Decision makers are usually facing several sources of uncertainty, such as the variability in time or estimation errors. A simplistic way to handle these uncertainties is to overestimate the unknown parameters. However, this results in over-conservatism and a significant waste in resource consumption. A better approach is to account for the uncertainty directly into the decision aid model by considering mixed integer programs that involve uncertain parameters. Stochastic optimization accounts for the expected realization of random data and optimizes an expected value representing the average situation. Robust optimization on the other hand entails protecting against the worst-case behavior of unknown data. There is an analogy to game theory where one considers an oblivious adversary choosing the realization that harms the solution the most. A full worst case protection against uncertainty is too conservative and induces very high over-cost. Instead, the realization of random data are bound to belong to a restricted feasibility set, the so-called uncertainty set. Stochastic and robust optimization rely on very large scale programs where probabilistic scenarios are enumerated. There is hope of a tractable solution for realistic size problems, provided one develops very efficient ad-hoc algorithms. The techniques for dynamically handling variables and constraints (column-and-row generation and Bender's projection tools) that are at the core of our team methodological work are specially well-suited to this context.

### 3.6 Polyhedral Combinatorics and Graph Theory

Many fundamental combinatorial optimization problems can be modeled as the search for a specific structure in a graph. For example, ensuring connectivity in a network amounts to building a *tree* that spans all the nodes. Inquiring about its resistance to failure amounts to searching for a minimum

cardinality *cut* that partitions the graph. Selecting disjoint pairs of objects is represented by a so-called *matching*. Disjunctive choices can be modeled by edges in a so-called *conflict graph* where one searches for *stable sets* – a set of nodes that are not incident to one another. Polyhedral combinatorics is the study of combinatorial algorithms involving polyhedral considerations. Not only it leads to efficient algorithms, but also, conversely, efficient algorithms often imply polyhedral characterizations and related min-max relations. Developments of polyhedral properties of a fundamental problem will typically provide us with more interesting inequalities well suited for a branch-and-cut algorithm to more general problems. Furthermore, one can use the fundamental problems as new building bricks to decompose the more general problem at hand. For problem that let themselves easily be formulated in a graph setting, the graph theory and in particular graph decomposition theorem might help.

## 4 Application domains

### 4.1 Network Design and Routing Problems

We are actively working on problems arising in network topology design, implementing a survivability condition of the form “at least two paths link each pair of terminals”. We have extended polyhedral approaches to problem variants with bounded length requirements and re-routing restrictions [45]. Associated to network design is the question of traffic routing in the network: one needs to check that the network capacity suffices to carry the demand for traffic. The assignment of traffic also implies the installation of specific hardware at transient or terminal nodes.

To accommodate the increase of traffic in telecommunication networks, today’s optical networks use grooming and wavelength division multiplexing technologies. Packing multiple requests together in the same optical stream requires to convert the signal in the electrical domain at each aggregation of disaggregation of traffic at an origin, a destination or a bifurcation node. Traffic grooming and routing decisions along with wavelength assignments must be optimized to reduce opto-electronics system installation cost. We developed and compared several decomposition approaches [60] to deal with backbone optical network with relatively few nodes (around 20) but thousands of requests for which traditional multi-commodity network flow approaches are completely overwhelmed. We also studied the impact of imposing a restriction on the number of optical hops in any request route. We also developed a branch-and-cut approach to a problem that consists in placing sensors on the links of a network for a minimum cost [48].

The Dial-a-Ride Problem is a variant of the pickup and delivery problem with time windows, where the user inconvenience must be taken into account. In [54], ride time and customer waiting time are modeled through both constraints and an associated penalty in the objective function. We develop a column generation approach, dynamically generating feasible vehicle routes. Handling ride time constraints explicitly in the pricing problem solver requires specific developments. Our dynamic programming approach for pricing problem makes use of a heuristic dominance rule and a heuristic enumeration procedure, which in turns implies that our overall branch-and-price procedure is a heuristic. However, in practice our heuristic solutions are experimentally very close to exact solutions and our approach is numerically competitive in terms of computation times.

In [51, 52], we consider the problem of covering an urban area with sectors under additional constraints. We adapt the aggregation method to our column generation algorithm and focus on the problem of disaggregating the dual solution returned by the aggregated master problem.

We studied several time dependent formulations for the unit demand vehicle routing problem [37, 36]. We gave new bounding flow inequalities for a single commodity flow formulation of the problem. We described their impact by projecting them on some other sets of variables, such as variables issued of the Picard and Queyranne formulation or the natural set of design variables. Some inequalities obtained by projection are facet defining for the polytope associated with the problem. We are now running more numerical experiments in order to validate in practice the efficiency of our theoretical results.

We considered the multi-commodity transportation problem. Applications of this problem arise in, for example, rail freight service design, “less than truckload” trucking, where goods should be delivered between different locations in a transportation network using various kinds of vehicles of large capacity. A particularity here is that, to be profitable, transportation of goods should be consolidated. This means

that goods are not delivered directly from the origin to the destination, but transferred from one vehicle to another in intermediate locations. We proposed an original Mixed Integer Programming formulation for this problem which is suitable for resolution by a Branch-and-Price algorithm and intelligent primal heuristics based on it.

For the problem of routing freight railcars, we proposed two algorithms based on the column generation approach. These algorithms have been tested on a set of real-life instances coming from a real Russian freight transportation company. Our algorithms have been faster on these instances than the current solution approach being used by the company.

## 4.2 Packing and Covering Problems

Reopt team has a strong experience on exact methods for cutting and packing problems. These problems occur in logistics (loading trucks), industry (wood or steel cutting), computer science (parallel processor scheduling).

We developed a branch-and-price algorithm for the Bin Packing Problem with Conflicts which improves on other approaches available in the literature [59]. The algorithm uses our methodological advances like the generic branching rule for the branch-and-price and the column based heuristic. One of the ingredients which contributes to the success of our method are fast algorithms we developed for solving the subproblem which is the Knapsack Problem with Conflicts. Two variants of the subproblem have been considered: with interval and arbitrary conflict graphs.

We also developed a branch-and-price algorithm for a variant of the bin-packing problem where the items are fragile. In [28] we studied empirically different branching schemes and different algorithms for solving the subproblems.

We studied a variant of the knapsack problem encountered in inventory routing problem: we faced a multiple-class integer knapsack problem with setups [49] (items are partitioned into classes whose use implies a setup cost and associated capacity consumption). We showed the extent to which classical results for the knapsack problem can be generalized to this variant with setups and we developed a specialized branch-and-bound algorithm.

We studied the orthogonal knapsack problem, with the help of graph theory. Fekete and Schepers proposed to model multi-dimensional orthogonal placement problems by using an efficient representation of all geometrically symmetric solutions by a so called *packing class* involving one *interval graph* for each dimension. Though Fekete & Schepers' framework is very efficient, we have however identified several weaknesses in their algorithms: the most obvious one is that they do not take advantage of the different possibilities to represent interval graphs. We propose to represent these graphs by matrices with consecutive ones on each row. We proposed a branch-and-bound algorithm for the 2D knapsack problem that uses our 2D packing feasibility check. We are currently developing exact optimization tools for glass-cutting problems in a collaboration with Saint-Gobain [31]. This 2D-3stage-Guillotine cut problems are very hard to solve given the scale of the instance we have to deal with. Moreover one has to issue cutting patterns that avoid the defaults that are present in the glass sheet that are used as raw material. There are extra sequencing constraints regarding the production that make the problem even more complex.

We have also organized a European challenge on packing with society Renault. This challenge was about loading trucks under practical constraints.

## 4.3 Planning, Scheduling, and Logistic Problems

Inventory routing problems combine the optimization of product deliveries (or pickups) with inventory control at customer sites. We considered an industrial application where one must construct the planning of single product pickups over time; each site accumulates stock at a deterministic rate; the stock is emptied on each visit. We have developed a branch-and-price algorithm where periodic plans are generated for vehicles by solving a multiple choice knapsack subproblem, and the global planning of customer visits is coordinated by the master program [50]. We previously developed approximate solutions to a related problem combining vehicle routing and planning over a fixed time horizon (solving

instances involving up to 6000 pick-ups and deliveries to plan over a twenty day time horizon with specific requirements on the frequency of visits to customers [46].

Together with our partner company GAPSO from the associate team SAMBA, we worked on the equipment routing task scheduling problem [53] arising during port operations. In this problem, a set of tasks needs to be performed using equipments of different types with the objective to maximize the weighted sum of performed tasks.

We participated to the project on an airborne radar scheduling. For this problem, we developed fast heuristics [44] and exact algorithms [30]. A substantial research has been done on machine scheduling problems. A new compact MIP formulation was proposed for a large class of these problems [29]. An exact decomposition algorithm was developed for the NP-hard maximizing the weighted number of late jobs problem on a single machine [55]. A dominant class of schedules for malleable parallel jobs was discovered in the NP-hard problem to minimize the total weighted completion time [57]. We proved that a special case of the scheduling problem at cross docking terminals to minimize the storage cost is polynomially solvable [58], [56].

Another application area in which we have successfully developed MIP approaches is in the area of tactical production and supply chain planning. In [27], we proposed a simple heuristic for challenging multi-echelon problems that makes effective use of a standard MIP solver. [26] contains a detailed investigation of what makes solving the MIP formulations of such problems challenging; it provides a survey of the known methods for strengthening formulations for these applications, and it also pinpoints the specific substructure that seems to cause the bottleneck in solving these models. Finally, the results of [32] provide demonstrably stronger formulations for some problem classes than any previously proposed. We are now working on planning phytosanitary treatments in vineries.

We have been developing robust optimization models and methods to deal with a number of applications like the above in which uncertainty is involved. In [42, 41], we analyzed fundamental MIP models that incorporate uncertainty and we have exploited the structure of the stochastic formulation of the problems in order to derive algorithms and strong formulations for these and related problems. These results appear to be the first of their kind for structured stochastic MIP models. In addition, we have engaged in successful research to apply concepts such as these to health care logistics [33]. We considered train timetabling problems and their re-optimization after a perturbation in the network [25, 35]. The question of formulation is central. Models of the literature are not satisfactory: continuous time formulations have poor quality due to the presence of discrete decision (re-sequencing or re-routing); arc flow in time-space graph blow-up in size (they can only handle a single line timetabling problem). We have developed a discrete time formulation that strikes a compromise between these two previous models. Based on various time and network aggregation strategies, we develop a 2-stage approach, solving the contiguous time model having fixed the precedence based on a solution to the discrete time model.

Currently, we are conducting investigations on a real-world planning problem in the domain of energy production, in the context of a collaboration with EDF [40, 39, 38]. The problem consists in scheduling maintenance periods of nuclear power plants as well as production levels of both nuclear and conventional power plants in order to meet a power demand, so as to minimize the total production cost. For this application, we used a Dantzig-Wolfe reformulation which allows us to solve realistic instances of the deterministic version of the problem [43]. In practice, the input data comprises a number of uncertain parameters. We deal with a scenario-based stochastic demand with help of a Benders decomposition method. We are working on Multistage Robust Optimization approaches to take into account other uncertain parameters like the duration of each maintenance period, in a dynamic optimization framework. The main challenge addressed in this work is the joint management of different reformulations and solving techniques coming from the deterministic (Dantzig-Wolfe decomposition, due to the large scale nature of the problem), stochastic (Benders decomposition, due to the number of demand scenarios) and robust (reformulations based on duality and/or column and/or row generation due to maintenance extension scenarios) components of the problem [34].

## 5 Social and environmental responsibility

### 5.1 Footprint of research activities

Our research involves a large amount of computational experiments.

### 5.2 Impact of research results

The objective of our research is to reduce the quantity of energy/material used to realize some large projects, including energy production and distribution, chemical treatments, and distribution of goods.

## 6 Highlights of the year

Imen BEN MOHAMED has received the second prize for the best PhD thesis in transportation and logistics in France (edition 2021). (<https://perso.isima.fr/lacomme/GT2L/>)

Boris Detienne has defended his Habilitation thesis [18].

This was the last year of RealOpt project. Our new Inria team EDGE will start in 2022.

## 7 New software and platforms

### 7.1 New software

#### 7.1.1 BaPCod

**Name:** A generic Branch-And-Price-And-Cut Code

**Keywords:** Column Generation, Branch-and-Price, Branch-and-Cut, Mixed Integer Programming, Mathematical Optimization, Benders Decomposition, Dantzig-Wolfe Decomposition, Extended Formulation

**Functional Description:** BaPCod is a prototype code that solves Mixed Integer Programs (MIP) by application of reformulation and decomposition techniques. The reformulated problem is solved using a branch-and-price-and-cut (column generation) algorithms, Benders approaches, network flow and dynamic programming algorithms. These methods can be combined in several hybrid algorithms to produce exact or approximate solutions (primal solutions with a bound on the deviation to the optimum).

**Release Contributions:** First public version of the software. The source code has been cleaned up.

**News of the Year:** First public release.

**URL:** <https://bapcod.math.u-bordeaux.fr/>

**Publication:** [hal-03340548](https://hal.archives-ouvertes.fr/hal-03340548)

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**Partners:** Université de Bordeaux, CNRS, IPB, Universidade Federal Fluminense

### 7.1.2 VRPSolver

**Name:** VRPSolver

**Keywords:** Column Generation, Vehicle routing, Numerical solver

**Scientific Description:** Major advances were recently obtained in the exact solution of Vehicle Routing Problems (VRPs). Sophisticated Branch-Cut-and-Price (BCP) algorithms for some of the most classical VRP variants now solve many instances with up to a few hundreds of customers. However, adapting and reimplementing those successful algorithms for other variants can be a very demanding task. This work proposes a BCP solver for a generic model that encompasses a wide class of VRPs. It incorporates the key elements found in the best recent VRP algorithms: ng-path relaxation, rank-1 cuts with limited memory, and route enumeration, all generalized through the new concept of "packing set". This concept is also used to derive a new branch rule based on accumulated resource consumption and to generalize the Ryan and Foster branch rule. Extensive experiments on several variants show that the generic solver has an excellent overall performance, in many problems being better than the best existing specific algorithms. Even some non-VRPs, like bin packing, vector packing and generalized assignment, can be modeled and effectively solved.

**Functional Description:** This solver allows one to model and solve to optimality many combinatorial optimization problems, belonging to the class of vehicle routing, scheduling, packing and network design problems. The problem is formulated using variables, linear objective function, linear and integrality constraints, definition of graphs, resources, and mapping between graph arcs and variables. A complex Branch-Cut-and-Price algorithm is used to solve the model. A new concept of elementarity and packing sets is used to pass an additional information to the solver, so that several state-of-the-art Branch-Cut-and-Price components can be used to improve radically the efficiency of the solver. The interface of the solver is implemented in Julia using JuMP package. To simplify the installation and usage, the solver is distributed as a docker image. The solver can be used only for academic purposes.

**Release Contributions:** Version 0.4.1 brings new features as well as correction of some bugs

**News of the Year:** New version is released

**URL:** <https://vrpsolver.math.u-bordeaux.fr/>

**Publication:** [hal-02178171v2](https://hal.archives-ouvertes.fr/hal-02178171v2)

**Contact:** Ruslan Sadykov

**Participants:** Ruslan Sadykov, Eduardo Uchoa Barboza, Artur Alves Pessoa, Eduardo Queiroga, Teobaldo Bulhões, Laurent Facq

**Partners:** Universidade Federal Fluminense, Universidade Federal da Paraíba

## 8 New results

### 8.1 Algorithms for optimization under uncertainty

In [21], we introduce a new exact algorithm to solve two-stage stochastic linear programs. Based on the multicut Benders reformulation of such problems, with one subproblem for each scenario, this method relies on a partition of the subproblems into batches. By detecting as soon as possible the non-optimality of a first-stage candidate, it solves only a small proportion of the subproblems at most iterations. We also propose a general framework to stabilize our algorithm, and show its finite convergence and exact behavior. We report an extensive computational study on large-scale instances of stochastic optimization literature that shows the efficiency of the proposed algorithm compared to six alternative algorithms from the literature (monocut and multicut implementation of the Benders decomposition algorithm with and without an in-out stabilization approach, a monocut implementation of a level bundle method, IBM

ILOG CPLEX 12.10 built-in Benders decomposition). We also obtain significant additional computational time savings using the primal stabilization schemes.

We have studied a class of two-stage robust binary optimization problems with objective uncertainty where recourse decisions are restricted to be mixed-binary [1]. For these problems, we present a deterministic equivalent formulation through the convexification of the recourse feasible region. We then explore this formulation under the lens of a relaxation, showing that the specific relaxation we propose can be solved using the branch-and-price algorithm. We present conditions under which this relaxation is exact, and describe alternative exact solution methods when this is not the case. Despite the two-stage nature of the problem, we provide NP-completeness results based on our reformulations. Finally, we present various applications in which the methodology we propose can be applied. We compare our exact methodology to those approximate methods recently proposed in the literature under the name K-adaptability. Our computational results show that our methodology is able to produce better solutions in less computational time compared to the K-adaptability approach, as well as to solve bigger instances than those previously managed in the literature.

We further extend this work in [22], where we address general problems in which all constraints (including those linking the first and the second stages) are defined by convex functions and involve mixed-integer variables, thus extending the existing literature to a much wider class of problems. We show how these problems can be reformulated using Fenchel duality, allowing to derive an enumerative exact algorithm, for which we prove  $\epsilon$ -convergence in a finite number of operations. An implementation of the resulting algorithm, embedding a column generation scheme, is then computationally evaluated on two different problems, using instances that are derived starting from the existing literature. To the best of our knowledge, this is the first approach providing results on the practical solution of this class of problems.

## 8.2 Arc-flow models

Network flow formulations are among the most successful tools to solve optimization problems. Such formulations correspond to determining an optimal flow in a network. One particular class of network flow formulations is the arc flow, where variables represent flows on individual arcs of the network. For hard problems, polynomial-sized arc-flow models typically provide weak linear relaxations and may have too much symmetry to be efficient in practice. Instead, arc flow models with a pseudo-polynomial size usually provide strong relaxations and are efficient in practice. The interest in pseudo-polynomial arc-flow formulations has grown considerably in the last twenty years, in which they have been used to solve many open instances of hard problems. A remarkable advantage of pseudo-polynomial arc-flow models is the possibility to solve practical-sized instances directly by a Mixed Integer Linear Programming solver, avoiding the implementation of complex methods based on column generation.

In [8], we present theoretical foundations of pseudo-polynomial arc-flow formulations, by showing a relation between their network and Dynamic Programming (DP). This relation allows a better understanding of the strength of these formulations, through a link with models obtained by Dantzig-Wolfe decomposition. The relation with DP also allows a new perspective to relate state-space relaxation methods for DP with arc-flow models. We also present a dual point of view to contrast the linear relaxation of arc-flow models with that of models based on paths and cycles. To conclude, we review the main solution methods and applications of arc-flow models based on DP in several domains such as cutting, packing, scheduling, and routing.

## 8.3 Machine scheduling problems

In [23], we study the prize-collecting job sequencing problem with one common and multiple secondary resources. In this problem, a set of jobs is given, each with a profit, multiple time windows for its execution, and a duration during which it requires the main resource. Each job also requires one of the secondary resources before, during, and after its use of the main resource. The goal is to select and schedule the subset of jobs that maximize the total profit. The problem has application in particle therapy scheduling and in pre-runtime scheduling of avionic systems among others. We present a new mixed integer linear programming formulation of the problem and a branch-cut-and-price algorithm as exact solution methods. We also introduce a heuristic algorithm to tackle larger instances. Extensive numerical

experiments show that our exact algorithms can solve to optimality literature instances with up to 500 jobs for a particular dataset and up to 250 jobs for another dataset with different characteristics. Our heuristic builds high-quality solutions in a small computational time. It computes new best-known solutions for most of the larger instances.

## 8.4 Vehicle routing problems

The Shortest Path Problem with Resource Constraints (SPPRC) arises as a subproblem in state-of-the-art Branch-Cut-and-Price algorithms for vehicle routing problems, including the BCP solver described just above. In [5], we propose a variant of the bi-directional label correcting algorithm in which the labels are stored and extended according to the so-called bucket graph. Such organization of labels helps to decrease significantly the number of dominance checks and the running time of the algorithm. We also show how the forward/backward route symmetry can be exploited and how to eliminate arcs from the bucket graph using reduced costs. The proposed algorithm can be especially beneficial for vehicle routing instances with large vehicle capacity and/or with time window constraints. Computational experiments were performed on instances from the distance constrained vehicle routing problem, including multi-depot and site-dependent variants, on the vehicle routing problem with time windows, and on the "nightmare" instances of the heterogeneous fleet vehicle routing problem. Significant improvements over the best algorithms in the literature were achieved and many instances could be solved for the first time. The proposed algorithm is the central part of the generic solver for vehicle routing and related problems proposed in [4]. This solver is based on BaPCod library [24] which is being developed in the team.

In [3], we examine the robust counterpart of the classical Capacitated Vehicle Routing Problem (CVRP). We consider two types of uncertainty sets for the customer demands: the classical budget polytope introduced by Bertsimas and Sim (2003), and a partitioned budget polytope proposed by Gounaris et al. (2013). We show that using the set-partitioning formulation it is possible to reformulate our problem as a deterministic heterogeneous vehicle routing problem. Thus, many state-of-the-art techniques for exactly solving deterministic VRPs can be applied for the robust counterpart, and a modern branch-and-cut-and-price algorithm can be adapted to our setting by keeping the number of pricing subproblems strictly polynomial. More importantly, we introduce new techniques to significantly improve the efficiency of the algorithm. We present analytical conditions under which a pricing subproblem is infeasible. This result is general and can be applied to other combinatorial optimization problems with knapsack uncertainty. We also introduce robust capacity cuts which are provably stronger than the ones known in the literature. Finally, a fast iterated local search algorithm is proposed to obtain heuristic solutions for the problem. Using our branch-and-cut-and-price algorithm incorporating existing and new techniques, we are able to solve to optimality all but one open instances from the literature.

In [14], we propose a partial optimization metaheuristic under special intensification conditions (POPMUSIC) for the classical capacitated vehicle routing problem (CVRP). The proposed approach uses a branch-cut-and-price algorithm as a powerful heuristic to solve subproblems whose dimensions are typically between 25 and 200 customers. The whole algorithm can be seen as the application of local search over very large neighborhoods, starting from a single initial solution. The main computational experiments were carried out on instances having between 302 and 1000 customers. Using initial solutions generated by some of the best available metaheuristics for the problem, POPMUSIC was able to obtain consistently better solutions for long runs of up to 32 hours. In a final experiment, starting from the best known solutions available in CVRP library (CVRPLIB), POPMUSIC was able to find new best solutions for several instances, including some very large ones.

## 8.5 Cutting and packing problems

In [15], we introduce and motivate a variant of the bin packing problem where bins are assigned to time slots, and minimum and maximum lags are required between some pairs of items. We suggest two integer programming formulations for the problem: a compact one, and a stronger formulation with an exponential number of variables and constraints. We propose a branch-cut-and-price approach which exploits the latter formulation. For this purpose, we devise separation algorithms based on a mathematical characterization of feasible assignments for two important special cases of the problem.



Computational experiments are reported for instances inspired from a real-case application of chemical treatment planning in vineyards, as well as for literature instances for special cases of the problem. The experimental results show the efficiency of our branch-cut-and-price approach, as it outperforms the compact formulation of newly proposed instances, and is able to obtain improved lower and upper bounds for literature instances.

In [13], we propose branch-cut-and-price algorithms for the classic bin packing problem and also for the following related problems: vector packing, variable sized bin packing and variable sized bin packing with optional items. The algorithms are defined as models for VRPSolver, a generic solver for vehicle routing problems. In that way, a simple parameterization enables the use of several branch-cut-and-price advanced elements: automatic stabilization by smoothing, limited-memory rank-1 cuts, enumeration, hierarchical strong branching and limited discrepancy search diving heuristics. As an original theoretical contribution, we prove that the branching over accumulated resource consumption, that does not increase the difficulty of the pricing subproblem, is sufficient for those bin packing models. Extensive computational results on instances from the literature show that the VRPSolver models have a performance that is very robust over all those problems, being often superior to the existing exact algorithms on the hardest instances. Several instances could be solved to optimality for the first time.

We have developed an approach to solve the temporal knapsack problem (TKP) based on a very large size dynamic programming formulation [7]. In this generalization of the classical knapsack problem, selected items enter and leave the knapsack at fixed dates. We solve the TKP with a dynamic program of exponential size, which is solved using a method called Successive Sublimation Dynamic Programming (SSDP). This method starts by relaxing a set of constraints from the initial problem, and iteratively reintroduces them when needed. We show that a direct application of SSDP to the temporal knapsack problem does not lead to an effective method, and that several improvements are needed to compete with the best results from the literature.

## 8.6 Network Design Problems

More than ever, data networks have demonstrated their central role in the world economy, but also in the well-being of humanity that needs fast and reliable networks. In parallel, with the emergence of Network Function Virtualization (NFV) and Software Defined Networking (SDN), efficient network algorithms considered too hard to be put in practice in the past now have a second chance to be considered again. In this context, as new networks will be deployed and current ones get significant upgrades, it is thus time to rethink the network dimensioning problem with protection against failures. In [17], we consider a path-based protection scheme with the global rerouting strategy in which, for each failure situation, there may be a new routing of all the demands. Our optimization task is to minimize the needed amount of bandwidth. After discussing the hardness of the problem, we develop two scalable mathematical models that we handle using both Column Generation and Benders Decomposition techniques. Through extensive simulations on real-world IP network topologies and on randomly generated instances, we show the effectiveness of our methods: they lead to savings of 40 to 48% of the bandwidth to be installed in a network to protect against failures compared to traditional schemes. Finally, our implementation in OpenDaylight demonstrates the feasibility of the approach. Its evaluation with Mininet shows that our solution provides sub-second recovery times, but the way it is implemented may greatly impact the amount of signaling traffic exchanged. In our evaluations, the recovery phase requires only a few tens of milliseconds for the fastest implementation, compared to a few hundreds of milliseconds for the slowest one.

## 8.7 Energy

Optimizing nuclear unit outages is of significant economic importance for the French electricity company EDF, as these outages induce a substitute production by other more expensive means to fulfill electricity demand. This problem is quite challenging given the specific operating constraints of nuclear units, the stochasticity of both the demand and non-nuclear units availability, and the scale of the instances. To tackle these difficulties we use a combined decomposition approach in [10]. The operating constraints of the nuclear units are built into a Dantzig-Wolfe pricing subproblem whose solutions define the columns of a demand covering formulation. The scenarios of demand and non-nuclear units availability are

handled in a Benders decomposition. Our approach is shown to scale up to the real-life instances of the French nuclear fleet.

## 8.8 Sustainable agriculture

In [20], we investigate the robust planning and scheduling of activities in agriculture and in particular the application of phytosanitary treatments. The crops are subject to many diseases that may arise during different time windows of the planning horizon. In response, a phytosanitary treatment can be applied to protect against a subset of these diseases. However, the effective duration of some treatments is uncertain, it depends on the type of treatment applied as well as on the weather conditions. In this study we introduce a penalty function based approach to handle this uncertainty without being overly conservative akin to light robustness approach proposed in the literature. We discuss different forms for this penalty function and elaborate on solution methodologies for the resulting models. We test the effectiveness of our approach with realistically-sized instances, which correspond to a typical vineyard in Bordeaux area, and present a numerical analysis of different optimization models and solution methods.

Our work in [15] cited above also finds its practical application in the context of application of phytosanitary treatments.

## 9 Bilateral contracts and grants with industry

### 9.1 Bilateral contracts with industry

We have a contract with RTE to develop strategies inspired from stochastic gradient methods to speed-up Benders' decomposition. The PhD thesis of Xavier Blanchot is part of this contract.

### 9.2 Bilateral grants with industry

Our joint project with Atoptima start-up "Solution methods for the inventory routing problem: application to waste collection in the urban environment" has been supported in 2020 by Nouvelle Aquitaine region (appel à projet "Recherche et Enseignement Supérieur"). The project is financing one half of a PhD thesis.

We received the funding for a one year post-doctoral position from the PGMFO foundation, for a joint project with EDF. This project aims at improving robustness and stability of power plants planning given the uncertainty associated with production fleet availability (nuclear outage durations, nuclear availability, growth of renewable sources, imponderable events such as the Covid-19 crisis...).

## 10 Partnerships and cooperations

### 10.1 National initiatives

**ANR DE-SIDE**

**Title:** DE-SIDE

**Duration:** January 2021 - December 2023 (4 years)

**Coordinator:** F. Clautiaux (Université Bordeaux, Inria BSO)

**Partners:**

- KEDGE (France)
- Sobolev Institute (Russia)

**Summary:** The main objective of this proposal is to provide new mathematical models and optimization approaches for design of spatio-temporal networks in stochastic and dynamic environment. Optimization approaches and mathematical modelling will concern strategic, tactical and operational

levels. More specifically, such combinatorial NP-hard problems as Facility Location and Vehicle Routing problems will be considered jointly through optimizing the following decisions: (1) optimum number of facilities, (2) optimal facility location, (3) optimal relocation of facilities according to dynamic evolving parameters, (4) allocation of clusters to each facility, over the time period where the information about spatio-temporal parameters is incomplete or inexact, (5) various formulations of routing problems among the facilities of the network will be considered under different optimality criteria.

## 10.2 Regional initiatives

### ESR 2020

**Title:** Solution approaches for the Inventory Routing Problem

**Duration:** November 2020 - October 2023 (3 years)

**Coordinator:** R. Sadykov (Inria BSO)

**Partners:** Atoptima start-up

**Summary:** The recent progress made in solving vehicle routing problems allows us to tackle more complex variants such as the planning of routes over a multi-period horizon combined with the management of inventory levels at the customer sites. This problem, known in the literature as the Inventory Routing Problem (IRP), is not yet within the reach of exact mathematical optimization methods. It combines three levels of decisions to be made for each period: (i) which customer to serve, (ii) how much to deliver or pick up, (iii) which routes to use. It becomes even more complex with the arrival of multi-level and multi-modal logistics solutions: intermediate depots are delivered via large trucks, while the last mile is delivered via light and non-polluting vehicles from these intermediate depots. Finally, it is necessary to be able not only to optimize a tactical schedule, but also to be able to re-optimize this schedule in real time in the light of the hazards of the solution's deployment. The real applications for this optimization model are multiple throughout the urban logistics sector: whether it is in the collection of recycled waste, the delivery of gas stations, the collection of milk in farms, or in maintenance problems with a prescribed time between two services, as well as sales representatives found with this same type of characteristics. It is important to focus on producing new optimization approaches capable of handling this level of complexity by advancing the state-of-the-art.

## 11 Dissemination

### 11.1 Promoting scientific activities

#### 11.1.1 Scientific events: organisation

**Member of the organizing committees** François Clautiaux has been a member of the organizing committee of Dataquitaine, a local scientific event gathering researchers, students, and companies in relation with data science, AI, and operations research.

#### 11.1.2 Scientific events: selection

**Member of the conference program committees** François Clautiaux has been a member of the scientific committee of ROADEF 2021 (Mulhouse).

#### 11.1.3 Journal

**Member of the editorial boards** François Clautiaux is a member of the editorial board of OJMO (Open Journal on Mathematical Optimization)

Ruslan Sadykov is an associate editor of the EURO Journal on Computational Optimization.

**Reviewer - reviewing activities** François Clautiaux has been reviewer for Math. Prog. Computing, Informatics Journal on Computing, European Journal of Operational Research, and Computers and Operations Research.

Aurélien Froger has been reviewer for European Journal of Operational Research, EURO Journal on Transportation and Logistics, INFORMS Journal on Computing, Transportation Research Part C: Emerging technologies, Transportation Research Part E: Logistics and Transportation Review, and Transportation Science.

Ruslan Sadykov has been reviewer for the following international journals: Computers and Operations Research, IIE Transactions, Ad Hoc Networks, International Journal of Production Research, INFORMS Journal on Computing, Expert Systems With Applications, Transportation Research Part B: Methodological, European Journal on Operational Research, Omega, Journal of Scheduling, and Transportation Science.

Pierre Pesneau has been reviewer for European Journal of Operational Research, and Discrete Optimization Journal.

#### 11.1.4 Invited talks

François Clautiaux. Irkutsk - Extended network flow formulations, Plenary speaker for MOTOR, Baikal (Russia).

François Clautiaux. Paris - Synergies between Dynamic Programming and Mixed Integer Programming, Invited talk at the joint ROADEF / ORBEL seminar.

Boris Detienne. Paris - A finite epsilon-convergence algorithm for 0-1 mixed-integer convex two-stage robust optimization with objective uncertainty, Invited talk at the Workshop on robust and stochastic optimization methods (ENPC).

Ruslan Sadykov. A generic exact solver for vehicle routing problems and its applications, Invited talk at the seminar of LIPN (Laboratoire Informatique du Paris Nord).

Ruslan Sadykov. Developing Branch-Cut-and-Price solver for Vehicle Routing and Related Problems, Invited talk at the seminar of Huawei Research Center, Minsk, Belarus (online).

#### 11.1.5 Leadership within the scientific community

François Clautiaux is president of the French operations research society ROADEF.

François Clautiaux is a member of the scientific committee of GDR Recherche Opérationnelle.

## 11.2 Teaching - Supervision - Juries

### 11.2.1 Teaching

Boris Detienne is **head of the Master Program in Operations Research** of the University of Bordeaux.

Pierre Pesneau is **head of the Master of Engineering in Mathematical Optimization** (CMI OPTIM) of the University of Bordeaux.

François Clautiaux is **head of the Master in Applied Mathematics** (180 students) of the University of Bordeaux.

Aurélien Froger has organized a one-week workshop on Optimisation and decision, in the Graduate Research Program NUMERICS, Université de Bordeaux, France

- Licence : François Clautiaux, Projet d'optimisation, L3, Université de Bordeaux, France
- Licence : François Clautiaux, Grands domaines de l'optimisation, L1, Université de Bordeaux, France
- Master : François Clautiaux, Introduction à la programmation en variables entières, M1, Université de Bordeaux, France
- Master : François Clautiaux, Integer Programming, M2, Université de Bordeaux, France
- Master : François Clautiaux, Algorithmes pour l'optimisation en nombres entiers, M1, Université de Bordeaux, France

- Master : François Clautiaux, Programmation linéaire, M1, Université de Bordeaux, France
- Master: Boris Detienne, Combinatoire et routage, ENSEIRB INPB
- Licence : Boris Detienne, Optimisation, L2, Université de Bordeaux
- Licence : Boris Detienne, Groupe de travail applicatif, L3, Université de Bordeaux
- Master : Boris Detienne, Optimisation continue, M1, Université de Bordeaux
- Master : Boris Detienne, Integer Programming, M2, Université de Bordeaux
- Master : Boris Detienne, Optimisation dans l'incertain, M2, Université de Bordeaux
- Licence : Aurélien Froger, Optimisation, L2, Université de Bordeaux, France
- Licence : Aurélien Froger, Groupe de travail applicatif, L3, Université de Bordeaux, France
- Master : Aurélien Froger, Programmation linéaire, M1, Université de Bordeaux, France
- Master : Aurélien Froger, Optimisation dans les graphes, M1, Université de Bordeaux, France
- Master : Ruslan Sadykov, Introduction to Constraint Programming, M2, Université de Bordeaux, France
- Licence : Pierre Pesneau, Grands domaines de l'optimisation, L1, Université de Bordeaux, France
- Licence : Pierre Pesneau, Programmation pour le calcul scientifique, L2, Université de Bordeaux, France
- Licence : Pierre Pesneau, Recherche Opérationnelle, ENSEIRB INPB, France
- Master : Pierre Pesneau, Algorithmique et Programmation 1, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Introduction à la programmation en variables entières, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Programmation linéaire, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Projet Algorithmes de flot, M1, Université de Bordeaux, France
- Master : Pierre Pesneau, Integer Programming, M2, Université de Bordeaux, France

### 11.2.2 Supervision

- Orlando Rivera Letelier defended PhD thesis "Applications of Integer Programming and Decomposition to Scheduling Problems: the Strategic Mine Planning Problem and the Bin Packing Problem with Time Lag" [19] on February 26, 2021, under the supervision of Ruslan Sadykov and François Clautiaux.
- François Clautiaux supervises three PhD students: Mellila Kechir, Xavier Blanchot and Parfait Ametana.
- Boris Detienne supervises two PhD students: Parfait Ametana and Mickaël Gaury.
- Aurelien Froger supervises one PhD student: Xavier Blanchot.
- Ruslan Sadykov supervises two PhD students: Isaac Balster and Daniil Khachay.
- Nicolas Guillemin defended master thesis "Branch-cut-and-price algorithms for vehicle routing and scheduling problems with complex objective functions" in September 2021, under the supervision of Ruslan Sadykov and Aurelien Froger.

- Paul Fleurance defended master thesis "Optimisation des chaînes de fonctions de service dans les réseaux télécom" in October 2021, under the supervision of Ruslan Sadykov. The work was done in collaboration with Orange Télécom.
- Laura Codazzi defended master thesis "Charge scheduling for a fleet of electric buses considering Vehicle-to-Grid technologies" in July 2021, under the supervision of Aurélien Froger. The student was from Politecnico di Milano and the work was also supervised by a researcher from this institution.

### 11.2.3 Juries

François Clautiaux has been in the following PhD committees: Ilia Tarasov (Toulouse), Alexandre Le Jean (Grenoble), Franco Quesada (Paris CNAM), Sébastien Deschamps (Ponts ParisTech).

François Clautiaux is jury member for EDSA: EURO Distinguished Service Award 2021 (awarded by the European Society of Operations Research).

Boris Detienne has been in the PhD committee of Boukhalfa Zahout (Tours).

Ruslan Sadykov has been in the PhD committee of Gabriel Volte (University of Montpellier).

## 11.3 Popularization

### 11.3.1 Articles and contents

François Clautiaux has been associated editor for the special issue of *Tangente* on Operations Research.

François Clautiaux and Pierre Pesneau have published a popularization paper on Integer Linear Programming in *Tangente*.

Boris Detienne and Ayşe Nur Arslan have published a popularization paper on optimization under uncertainty in *Tangente*.

### 11.3.2 Local clusters

François Clautiaux is a member of the board of DOMEX AI/Data Science in Région Nouvelle Aquitaine. This entity aims at developing company activities in AI, data science and operations research in Nouvelle Aquitaine.

## 12 Scientific production

### 12.1 Major publications

- [1] A. N. Arslan and B. Detienne. 'Decomposition-based approaches for a class of two-stage robust binary optimization problems'. In: *INFORMS Journal on Computing* (2021). DOI: [10.1287/ijoc.2021.1061](https://doi.org/10.1287/ijoc.2021.1061). URL: <https://hal.inria.fr/hal-02190059>.
- [2] F. Clautiaux, S. Hanafi, R. Macedo, M.-E. Vogé and C. Alves. 'Iterative aggregation and disaggregation algorithm for pseudo-polynomial network flow models with side constraints'. In: *European Journal of Operational Research* 258 (2017), pp. 467–477. DOI: [10.1016/j.ejor.2016.09.051](https://doi.org/10.1016/j.ejor.2016.09.051). URL: <https://hal.inria.fr/hal-01410170>.
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- [4] A. A. Pessoa, R. Sadykov, E. Uchoa and F. Vanderbeck. 'A Generic Exact Solver for Vehicle Routing and Related Problems'. In: *Mathematical Programming* 183 (2020), pp. 483–523. DOI: [10.1007/s10107-020-01523-z](https://doi.org/10.1007/s10107-020-01523-z). URL: <https://hal.inria.fr/hal-02178171>.
- [5] R. Sadykov, A. A. Pessoa and E. Uchoa. 'A Bucket Graph Based Labelling Algorithm for Vehicle Routing'. In: *Transportation Science* 55.1 (2021), pp. 4–28. DOI: [10.1287/trsc.2020.0985](https://doi.org/10.1287/trsc.2020.0985). URL: <https://hal.inria.fr/hal-02378624>.

## 12.2 Publications of the year

### International journals

- [6] A. N. Arslan and B. Detienne. ‘Decomposition-based approaches for a class of two-stage robust binary optimization problems’. In: *INFORMS Journal on Computing* (2021). DOI: [10.1287/ijoc.2021.1061](https://doi.org/10.1287/ijoc.2021.1061). URL: <https://hal.inria.fr/hal-02190059>.
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