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Project-Team CAGE

Control and Geometry

IN COLLABORATION WITH: Laboratoire Jacques-Louis Lions (LJLL)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Optimization and control of dynamic systems

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Contents

Project-Team CAGE

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Keywords

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2 Overall objectives

CAGE's activities take place in the field of mathematical control theory, with applications in several directions: control of quantum mechanical systems, stability and stabilization, in particular in presence of uncertain dynamics, optimal control, and geometric models for vision. Although control theory is nowadays a mature discipline, it is still the subject of intensive research because of its crucial role in a vast array of applications. Our focus is on the analytical and geometrical aspects of control applications.

At the core of the scientific activity of the team is the **geometric control** approach, that is, a distinctive viewpoint issued in particular from (elementary) differential geometry, to tackle questions of controllability, motion planning, stability, and optimal control. The emphasis of such a geometric approach is in intrinsic properties, and it is particularly well adapted to study nonlinear and nonholonomic phenomena [89, 65]. The geometric control approach has historically been associated with the development of finite-dimensional control theory. However, its impact in the study of distributed parameter control systems and, in particular, systems of **controlled partial differential equations** has been growing in the last decades, complementing analytical and numerical approaches by providing dynamical, qualitative, and intrinsic insight [81]. CAGE has the ambition to be at the core of this development.

One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures (e.g., Lagrangian or Hamiltonian structures) can be used to characterize minimizing trajectories, prove regularity properties, and describe invariants. The geometric theory of **quantum control**, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to design adapted control schemes and to characterize their qualitative properties.

3 Research program

3.1 Research domain

Our contributions are in the area of **mathematical control theory**, which is to say that we are interested in the analytical and geometrical aspects of control applications. In this approach, a control system is modeled by a system of equations (of many possible types: ordinary differential equations, partial differential equations, stochastic differential equations, difference equations,...), possibly not explicitly known in all its components, which are studied in order to establish qualitative and quantitative properties concerning the actuation of the system through the control.

Motion planning is, in this respect, a cornerstone property: it denotes the design and validation of algorithms for identifying a control law steering the system from a given initial state to (or close to) a target one. Initial and target positions can be replaced by sets of admissible initial and final states as, for instance, in the motion planning task towards a desired periodic solution. Many specifications can be added to the pure motion planning task, such as robustness to external or endogenous disturbances, obstacle avoidance or penalization criteria. A more abstract notion is that of **controllability**, which denotes the property of a system for which any two states can be connected by a trajectory corresponding to an admissible control law. In mathematical terms, this translates into the surjectivity of the so-called **end-point map**, which associates with a control and an initial state the final point of the corresponding trajectory. The analytical and topological properties of endpoint maps are therefore crucial in analyzing the properties of control systems.

One of the most important additional objective which can be associated with a motion planning task is **optimal control**, which corresponds to the minimization of a cost (or, equivalently, the maximization of a gain) [117]. Optimal control theory is clearly deeply interconnected with calculus of variations, even if the non-interchangeable nature of the time-variable results in some important specific features, such as the occurrence of **abnormal extremals** [93]. Research in optimal control encompasses different aspects, from numerical methods to dynamic programming and non-smooth analysis, from regularity of minimizers to high order optimality conditions and curvature-like invariants.

Another domain of control theory with countless applications is **stabilization**. The goal in this case is to make the system converge towards an equilibrium or some more general safety region. The main difference with respect to motion planning is that here the control law is constructed in feedback form. One of the most important properties in this context is that of **robustness**, i.e., the performance of the stabilization protocol in presence of disturbances or modeling uncertainties. A powerful framework which has been developed to take into account uncertainties and exogenous non-autonomous disturbances is that of hybrid and switched systems [104, 94, 110]. The central tool in the stability analysis of control systems is that of **control Lyapunov function**. Other relevant techniques are based on algebraic criteria or dynamical systems. One of the most important stability property which is studied in the context of control system is **input-to-state stability** [108], which measures how sensitive the system is to an external excitation.

One of the areas where control applications have nowadays the most impressive developments is in the field of **biomedicine and neurosciences**. Improvements both in modeling and in the capability of finely actuating biological systems have concurred in increasing the popularity of these subjects. Notable advances concern, in particular, identification and control for biochemical networks [102] and models for neural activity [86]. Therapy analysis from the point of view of optimal control has also attracted a great attention [106].

Biological models are not the only one in which stochastic processes play an important role. Stock-markets and energy grids are two major examples where optimal control techniques are applied in the non-deterministic setting. Sophisticated mathematical tools have been developed since several decades to allow for such extensions. Many theoretical advances have also been required for dealing with complex systems whose description is based on **distributed parameters** representation and **partial differential equations**. Functional analysis, in particular, is a crucial tool to tackle the control of such systems [114].

Let us conclude this section by mentioning another challenging application domain for control theory: the decision by the European Union to fund a flagship devoted to the development of quantum technologies is a symptom of the role that quantum applications are going to play in tomorrow's society. **Quantum control** is one of the bricks of quantum engineering, and presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way [87].

3.2 Scientific foundations

At the core of the scientific activity of the team is the **geometric control** approach. One of the features of the geometric control approach is its capability of exploiting **symmetries and intrinsic structures** of control systems. Symmetries and intrinsic structures can be used to characterize minimizing trajectories, prove regularity properties and describe invariants. An egregious example is given by mechanical systems, which inherently exhibit Lagrangian/Hamiltonian structures which are naturally expressed using the language of symplectic geometry [77]. The geometric theory of quantum control, in particular, exploits the rich geometric structure encoded in the Schrödinger equation to engineer adapted control schemes and to characterize their qualitative properties. The Lie–Galerkin technique that we proposed starting in [78] builds on this premises in order to provide powerful tests for the controllability of quantum systems defined on infinite-dimensional Hilbert spaces.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in optimal control. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [113, 105]. Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Numerical optimal control methods with geometric insight are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Optimal control is not the only area where the geometric approach has a great impact. Let us mention, for instance, motion planning, where different geometric approaches have been developed: those based on the **Lie algebra** associated with the control system [98, 95], those based on the differentiation of nonlinear flows such as the **return method** [82, 83], and those exploiting the **differential flatness** of the system [85].

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. Two occurrences of this property play an important role in the activities of CAGE:

- geometric control theory as a tool to investigate properties of mathematical structures;
- geometric control theory as a modeling tool for neurophysical phenomena and for synthesizing biomimetic algorithms based on such models.

Examples of the first type, concern, for instance, hypoelliptic heat kernels [64] or shape optimization [68]. Examples of the second type are inactivation principles in human motricity [70] or neurogeometrical models for image representation of the primary visual cortex in mammals [75].

A particularly relevant class of control systems, both from the point of view of theory and applications, is characterized by the linearity of the controlled vector field with respect to the control parameters. When the controls are unconstrained in norm, this means that the admissible velocities form a distribution in the tangent bundle to the state manifold. If the distribution is equipped with a point-dependent quadratic form (encoding the cost of the control), the resulting geometrical structure is said to be **sub-Riemannian**. Sub-Riemannian geometry appears as the underlying geometry of nonlinear control systems: in a similar way as the linearization of a control system provides local informations which are readable using the Euclidean metric scale, sub-Riemannian geometry provides an adapted non-isotropic class of lenses which are often much more informative. As such, its study is fundamental for control design. The importance of sub-Riemannian geometry goes beyond control theory and it is an active field of research both in differential geometry [97], geometric measure theory [66] and hypoelliptic operator theory [71].

4 Application domains

4.1 First axis: Quantum control

Quantum control is one of the bricks of quantum engineering, since manipulation of quantum mechanical systems is ubiquitous in applications such as quantum computation, quantum cryptography, and quantum sensing (in particular, imaging by nuclear magnetic resonance).

Quantum control presents many peculiarities with respect to standard control theory, as a consequence of the specific properties of the systems described by the laws of quantum physics. Particularly important for technological applications is the capability of inducing and reproducing coherent state superpositions and entanglement in a fast, reliable, and efficient way. The efficiency of the control action has a dramatic impact on the quality of the coherence and the robustness of the required manipulation. Minimal time constraints and interaction of time scales are important factors for characterizing the efficiency of a quantum control strategy. CAGE works for the improvement of quantum control paradigms, especially for what concerns quantum systems evolving in infinite-dimensional Hilbert spaces. The controllability of quantum system is a well-established topic when the state space is finite-dimensional [84], thanks to general controllability methods for left-invariant control systems on compact Lie groups [76, 90]. When the state space is infinite-dimensional, it is known that in general the bilinear Schrödinger equation is not exactly controllable [115]. The Lie–Galerkin technique [78] combines finite-dimensional geometric control techniques and the distributed parameter framework in order to provide the most powerful available tests for the approximate controllability of quantum systems defined on infinite-dimensional Hilbert spaces. Another important technique to the development of which we contribute is **adiabatic quantum control**. Adiabatic approximation theory and, in particular, adiabatic evolution [99, 111, 118] is well-known to improve the robustness of the control strategy and is strongly related to time scales analysis. The advantage of the adiabatic control is that it is constructive and produces control laws which are both smooth and robust to parameter uncertainty [119, 92, 74].

4.2 Second axis: Stability and stabilization

A control application with a long history and still very challenging open problems is **stabilization**. For infinite-dimensional systems, in particular nonlinear ones, the richness of the possible functional analytical frameworks makes feedback stabilization a challenging and active domain of research. Of particular interest are the different types of stabilization that may be obtained: exponential, polynomial, finite-time, ... Another important aspect of stabilization concerns control of systems with uncertain dynamics, i.e., with dynamics including possibly non-autonomous parameters whose value and dependence on time cannot be anticipated. **Robustification**, i.e., offsetting uncertainties by suitably designing the control strategy, is a widespread task in automatic control theory, showing up in many applicative domains such as electric circuits or aerospace motion planning. If dynamics are not only subject to static uncertainty, but may also change as time goes, the problem of controlling the system can be recast within the theory of switched and hybrid systems, both in a deterministic and in a probabilistic setting. **Switched and hybrid systems** constitute a broad framework for the description of the heterogeneous systems in which continuous dynamics (typically pertaining to physical quantities) interact with discrete/logical components. The development of the switched and hybrid paradigm has been motivated by a broad range of applications, including automotive and transportation industry [107], energy management [100] and congestion control [96]. Even if both controllability [109] and observability [91] of switched and hybrid systems raise several important research issues, the central role in their study is played by uniform stability and stabilizabilization [94, 110]. Uniformity is considered with respect to all signals in a given class, and it is well-known that stability of switched systems depends not only on the dynamics of each subsystem but also on the properties of the considered class of switching signals. In many situations it is interesting for modeling purposes to specify the features of the switched system by introducing **constrained switching rules**. A typical constraint is that each mode is activated for at least a fixed minimal amount of time, called the dwell-time. Our approach to constrained switching is based on the idea of relating the analytical properties of the classes of constrained switching laws (shift-invariance, compactness, closure under concatenation, ...) to the stability behavior of the corresponding switched systems. One can introduce **probabilistic uncertainties** by endowing the classes of admissible signals with suitable probability measures. The interest of this approach is that probabilistic stability analysis filters out highly 'exceptional' worst-case trajectories. Although less explicitly characterized from a dynamical viewpoint than its deterministic counterpart, the probabilistic notion of uniform exponential stability can be studied using several reformulations of Lyapunov exponents proposed in the literature [69, 80, 116].

4.3 Third axis: Motion planning and optimal control

Geometric optimal control allows the development of general techniques, applying to wide classes of nonlinear optimal control problems, that can be used to characterize the behavior of optimal trajectories and in particular to establish regularity properties for them and for the cost function. Hence, geometric optimal control can be used to obtain powerful optimal syntheses results and to provide deep geometric insights into many applied problems. Geometric optimal control methods are in particular important to handle subtle situations such as rigid optimal paths and, more generally, optimal syntheses exhibiting abnormal minimizers.

Although the focus of geometric control theory is on qualitative properties, its impact can also be disruptive when it is used in combination with quantitative analytical tools, in which case it can dramatically improve the computational efficiency. This is the case in particular in **optimal control**. Classical optimal control techniques (in particular, Pontryagin Maximum Principle, conjugate point theory, associated numerical methods) can be significantly improved by combining them with powerful modern techniques of geometric optimal control, of the theory of numerical continuation, or of dynamical system theory [113, 105]. Applications of optimal control theory considered by CAGE concern, in particular, motion planning problems for aerospace (atmospheric re-entry, orbit transfer, low cost interplanetary space missions, ...) [72, 112].

4.4 Fourth axis: Geometric models for vision and sub-Riemannian geometry

Geometric control theory is not only a powerful framework to investigate control systems, but also a useful tool to model and study phenomena that are not *a priori* control-related. In particular, we use control theory to investigate the properties of sub-Riemannian structures, both for the sake of mathematical understanding and as a modeling tool for image and sound perception and processing . We recall that **sub-Riemannian geometry** is a geometric framework which is used to measure distances in nonholonomic contexts and which has a natural and powerful optimal control interpretation in terms control-linear systems with quadratic cost. Sub-Riemannian geometry, and in particular the theory of their associated (hypoelliptic) diffusive processes, plays a crucial role in the neurogeometrical model of the primary visual cortex due to Petitot, Citti and Sarti, based on the functional architecture first described by Hubel and Wiesel [88, 101, 79, 103]. Such a model can be used as a powerful paradigm for bio-inspired image processing, as already illustrated in the literature [75, 73]. Our contributions to **geometry of vision** are based not only on this approach, but also on another geometric and sub-Riemannian framework for vision, based on pattern matching in the group of diffeomorphisms. In this case admissible diffeomorphisms correspond to deformations which are generated by vector fields satisfying a set of nonholonomic constraints. A sub-Riemannian metric on the infinite-dimensional group of diffeomorphisms is induced by a length on the tangent distribution of admissible velocities [67]. Nonholonomic constraints can be especially useful to describe distortions of sets of interconnected objects (e.g., motions of organs in medical imaging).

5 New software and platforms

5.1 New software

5.1.1 Stellacode

Keywords: Plasma physics, Stellarator, Inverse problem, Magnetic fusion, Electromagnetics, Shape optimization

Functional Description: The goal of Stellacode is to optimize the Coil Winding Surface of a stellarator. What does this means and what for? A stellarator is a nuclear fusion reactor which produces energy from the fusion of nuclei in a very hot plasma. The confinement of this plasma is a very complicated task and a stellarator is able to achieve it thanks to a complex set of non-planar supraconductor coils lying on the so-called "Coil Winding Surface". Stellacode is able to perform efficiently the shape optimization of this surface, taking into account both plasma confinement figures of merit and engineering costs.

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Contact: Rémi Robin

5.1.2 Astus2

Name: Astus2

Keyword: Optimal control

Functional Description: Confidentiel

Contact: Emmanuel Trélat

5.1.3 IMODAL

Name: Implicit Modular Deformations Analysis Library

Keywords: Registration, Optimal control

Functional Description: IMODAL is a python librairy allowing to register shapes (curves, meshes, images) with structured large deformations. The structures are incorporated via deformation modules which generate vector fields of particular, chosen types. They can be defined explicitly (generating local scalings or rotations for instance) or implicitly from constraints. In addition, it is possible to combine them so that a complex structure can be easily defined as the superimposition of simple ones. Trajectories of such modular vector fields can then be integrated to build modular large deformations. Their parameters can be optimized to register observed shapes and analyzed.

Contact: Barbara Gris

6 New results

6.1 Quantum control: new results

Let us list here our new results in quantum control theory.

- In [35], we present a quantum optimal control problem which exhibits a chattering phenomenon. This is the first instance of such a process in quantum control. Using the Pontryagin Maximum Principle and a general procedure due to V. F. Borisov and M. I. Zelikin, we characterize the local optimal synthesis, which is then globalized by a suitable numerical algorithm. We illustrate the importance of detecting chattering phenomena because of their impact on the efficiency of numerical optimization procedures. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

- In [12], we study up to which extent we can apply adiabatic control strategies to a quantum control model obtained by rotating wave approximation. In particular, we show that, under suitable assumptions on the asymptotic regime between the parameters characterizing the rotating wave and the adiabatic approximations, the induced flow converges to the one obtained by considering the two approximations separately and by combining them formally in cascade. As a consequence, we propose explicit control laws which can be used to induce desired populations transfers, robustly with respect to parameter dispersions in the controlled Hamiltonian.

- Quantum optimal control, a toolbox for devising and implementing the shapes of external fields that accomplish given tasks in the operation of a quantum device in the best way possible, has evolved into one of the cornerstones for enabling quantum technologies. The last few years have seen a rapid evolution and expansion of the field. We review in [24] recent progress in our understanding of the controllability of open quantum systems and in the development and application of quantum control techniques to quantum technologies. We also address key challenges and sketch a roadmap for future developments.

- In [25], we prove complete controllability for rotational states of an asymmetric top molecule belonging to degenerate values of the orientational quantum number M. Based on this insight, we construct a pulse sequence that energetically separates population initially distributed over degenerate M-states, as a precursor for orientational purification. Introducing the concept of enantio-selective controllability, we determine the conditions for complete enantiomer-specific population transfer in chiral molecules and construct pulse sequences realizing this transfer for population initially distributed over degenerate M-states. This degeneracy presently limits enantiomer-selectivity for any initial state except the rotational ground state. Our work thus shows how to overcome an important obstacle towards separating, with electric fields only, left-handed from right-handed molecules in a racemic mixture.

- In the physics literature it is common to see the rotating wave approximation and the adiabatic approximation used "in cascade" to justify the use of chirped pulses for two-level quantum systems driven by one external field, in particular when the resonance frequency of the system is not known precisely. Both approximations need relatively long time and are essentially based on averaging theory of dynamical systems. Unfortunately, the two approximations cannot be done independently since, in a sense, the two time scales interact. The purpose of [34] is to study how the cascade of the two approximations can be justified and how large becomes the final time as the fidelity goes to one, while preserving the robustness of the adiabatic strategy. Our first result, based on high-order averaging techniques, gives a precise quantification of the uncertainty interval of the resonance frequency for which the population inversion works. As a byproduct of this result, we prove that it is possible to control an ensemble of spin systems by a single real-valued control, providing a non-trivial extension of a celebrated result of ensemble controllability with two controls by Khaneja and Li. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

- In [19], we study, in the semiclassical sense, the global approximate controllability in small time of the quantum density and quantum momentum of the 1-D semiclassical cubic Schrödinger equation with two controls between two states with positive quantum densities. We first control the asymptotic expansions of the zeroth and first order of the physical observables via the Agrachev–Sarychev method. Then we conclude the proof through techniques of semiclassical approximation of the nonlinear Schrödinger equation.
- In [55], we establish some properties of quantum limits on a product manifold, proving for instance that, under appropriate assumptions, the quantum limits on the product of manifolds are absolutely continuous if the quantum limits on each manifolds are absolutely continuous. On a product of Riemannian manifolds satisfying the minimal multiplicity property, we prove that a periodic geodesic can never be charged by a quantum limit.
- In [32] we present an analytical approach to construct the Lie algebra of finite-dimensional subsystems of the driven asymmetric top rotor. Each rotational level is degenerate due to the isotropy of space, and the degeneracy increases with rotational excitation. For a given rotational excitation, we determine the nested commutators between drift and drive Hamiltonians using a graph representation. We then generate the Lie algebra for subsystems with arbitrary rotational excitation using an inductive argument.

6.2 Stability and stabilization: new results

Let us list here our new results about stability and stabilization of control and hybrid systems.

- The paper [53] concerns some spectral properties of the scalar dynamical system defined by a linear delay-differential equation with two positive delays. More precisely, the existing links between the delays and the maximal multiplicity of the characteristic roots are explored, as well as the dominance of such roots compared with the spectrum localization. As a by-product of the analysis, the pole placement issue is revisited with more emphasis on the role of the delays as control parameters in defining a partial pole placement guaranteeing the closed-loop stability with an appropriate decay rate of the corresponding dynamical system.
- In [15], we consider finite and infinite-dimensional first-order consensus systems with timeconstant interaction coefficients. For symmetric coefficients, convergence to consensus is classically established by proving, for instance, that the usual variance is an exponentially decreasing Lyapunov function. We investigate here the convergence to consensus in the non-symmetric case: we identify a positive weight which allows to define a weighted mean corresponding to the consensus, and obtain exponential convergence towards consensus. Moreover, we compute the sharp exponential decay rate.
- In [17], we recall some general properties of extremal and Barabanov norms and we give a necessary and sufficient condition for a finite-dimensional continuoustime linear switched system to admit a Barabanov norm.
- In the paper [48], we consider the problem of determining the stability properties, and in particular assessing the exponential stability, of a singularly perturbed linear switching system. One of the challenges of this problem arises from the intricate interplay between the small parameter of singular perturbation and the rate of switching, as both tend to zero. Our approach consists in characterizing suitable auxiliary linear systems that provide lower and upper bounds for the

asymptotics of the maximal Lyapunov exponent of the linear switching system as the parameter of the singular perturbation tends to zero.

- The proceeding [38] presents some results on the uniform exponential stability of singularly perturbed linear systems undergoing switching. In absence of dwell-time constraints, the switching parameter can evolve on the same time scale as the fast variables, or even faster. We investigate the effect of switching laws evolving at a time scale comparable with the fast variables, describing the corresponding asymptotic effect on the slow variable. Based on this analysis, we propose stability criteria for the overall system, uniform for small values of the parameter of singular perturbation.

- In the article [18], we study the so-called water tank system. In this system, the behavior of water contained in a 1-D tank is modelled by Saint-Venant equations, with a scalar distributed control. It is well-known that the linearized systems around uniform steady-states are not controllable, the uncontrollable part being of infinite dimension. Here we will focus on the linearized systems around non-uniform steady states, corresponding to a constant acceleration of the tank. We prove that these systems are controllable in Sobolev spaces, using the moments method and perturbative spectral estimates. Then, for steady states corresponding to small enough accelerations, we design an explicit Proportional Integral feedback law (obtained thanks to a well-chosen dynamic extension of the system) that stabilizes these systems exponentially with arbitrarily large decay rate. Our design relies on feedback equivalence/backstepping.

- The paper [43] deals with stability of linear periodic time-varying difference delay systems, i.e. dynamical systems where a finite dimensional signal at a certain time is given as a linear time-varying function of its values at a finite number of delayed times. We give a necessary and sufficient condition for exponential stability, that is a generalisation of the one by Henry and Hale in the 1970s. It has a control theoretic interpretation in terms of the harmonic transfer function (HTF) of a corresponding linear control system.

- Adaptive control using the σ-modification provides an easily implementable way to stabilize systems with uncertain or fluctuating parameters. Motivated by a specific application from neuroscience, we extend in [31] this methodology to nonlinear time-delay systems ruled by globally Lipschitz dynamics. In order to make the result more handy in practice, we provide an explicit construction of a Lyapunov–Krasovskii functional (LKF) with linear bounds and strict dissipation rate based on the knowledge of an LKF with quadratic bounds and point-wise dissipation rate. When applied to a model of neuronal populations involved in Parkinson's disease, the benefits with respect to a pure proportional stabilization scheme are discussed through numerical simulations.

- The paper [26] is concerned with the Proportional Integral (PI) regulation control of the left Neumann trace of a one-dimensional semilinear wave equation. The control input is selected as the right Neumann trace. The control design goes as follows. First, a preliminary (classical) velocity feedback is applied in order to shift all but a finite number of the eivenvalues of the underlying unbounded operator into the open left half-plane. We then leverage on the projection of the system trajectories into an adequate Riesz basis to obtain a truncated model of the system capturing the remaining unstable modes. Local stability of the resulting closed-loop infinite-dimensional system composed of the semilinear wave equation, the preliminary velocity feedback, and the PI controller, is obtained through the study of an adequate Lyapunov function. Finally, an estimate assessing the set point tracking performance of the left Neumann trace is derived.

- In [14], we investigate the asymptotic formation of consensus for several classes of time-dependent cooperative graphon dynamics. After motivating the use of this type of macroscopic models to describe multi-agent systems, we adapt the classical notion of scrambling coefficient to this setting, leverage it to establish sufficient conditions ensuring the exponential convergence to consensus with respect to the L^∞-norm topology. We then shift our attention to consensus formation expressed in terms of the L^2-norm, and prove three different consensus result for symmetric, balanced and strongly connected topologies, which involve a suitable generalisation of the notion of algebraic connectivity to this infinite-dimensional framework. We then show that, just as in the finite-dimensional setting, the notion of algebraic connectivity that we propose encodes information about the connectivity properties of the underlying interaction topology. We finally use the corresponding results to shed some light on the relation between L^2 and L^∞ -consensus formation, and illustrate our contributions by a series of numerical simulations.

- In [44], we consider the problem of boundary feedback control of single-input-single-output (SISO) one-dimensional linear hyperbolic systems when sensing and actuation are anti-located. The main issue of the output feedback stabilization is that it requires dynamic control laws that include delayed values of the output (directly or through state observers) which may not be robust to infinitesimal uncertainties on the characteristic velocities. The purpose of this paper is to highlight some features of this problem by addressing the feedback stabilization of an unstable open-loop system which is made up of two interconnected transport equations and provided with anti-located boundary sensing and actuation. The main contribution is to show that the robustness of the control against delay uncertainties is recovered as soon as an arbitrary small diffusion is present in the system. Our analysis also reveals that the effect of diffusion on stability is far from being an obvious issue by exhibiting an alternative simple example where the presence of diffusion has a destabilizing effect instead.

- In [22], we deal with infinite-dimensional nonlinear forward complete dynamical systems which are subject to external disturbances. We first extend the well-known Datko lemma to the framework of the considered class of systems. Thanks to this generalization, we provide characterizations of the uniform (with respect to disturbances) local, semi-global, and global exponential stability, through the existence of coercive and non-coercive Lyapunov functionals. The importance of the obtained results is underlined through some applications concerning 1) exponential stability of nonlinear retarded systems with piecewise constant delays, 2) exponential stability preservation under sampling for semilinear control switching systems, and 3) the link between input-to-state stability and exponential stability of semilinear switching systems.

- Hyperbolic systems in one dimensional space are frequently used in modeling of many physical systems. In our recent works, we introduced time independent feedbacks leading to the finite stabilization for the optimal time of homogeneous linear and quasilinear hyperbolic systems. In [63], we present Lyapunov's functions for these feedbacks and use estimates for Lyapunov's functions to rediscover the finite stabilization results.

- In [57] we discuss the notion of universality for classes of candidate common Lyapunov functions of linear switched systems. On the one hand, we prove that a family of absolutely homogeneous functions is universal as soon as it approximates arbitrarily well every convex absolutely homogeneous function for the C^0 topology of the unit sphere. On the other hand, we prove several obstructions for a class to be universal, showing, in particular, that families of piecewise-polynomial continuous functions whose construction involves at most l polynomials of degree at most m (for given positive integers l, m) cannot be universal.

- Consider a non-autonomous continuous-time linear system in which the time-dependent matrix determining the dynamics is piecewise constant and takes finitely many values $A_1, \dots, A_N$. Our article [49] studies the equality cases between the maximal Lyapunov exponent associated with the set of matrices $\{A_1, \dots, A_N\}$, on the one hand, and the corresponding ones for piecewise deterministic Markov processes with modes $A_1, \dots, A_N$, on the other hand. A fundamental step in this study consists in establishing a result of independent interest, namely, that any sequence of Markov processes associated with the matrices $A_1, \dots, A_N$ converges, up to extracting a subsequence, to a Markov process associated with a suitable convex combination of those matrices.

6.3 Motion planning and optimal control: new results

Let us list here our new results on controllability and motion planning algorithms, including optimal control, beyond the quantum control framework.

- In [56], we consider a nonlinear system of two parabolic equations, with a distributed control in the first equation and an odd coupling term in the second one. We prove that the nonlinear system is smalltime locally null-controllable. The main difficulty is that the linearized system is not null-controllable. To overcome this obstacle, we extend in a nonlinear setting the strategy introduced by one of the authors that consists in constructing odd controls for the linear heat equation. The proof relies on three main steps. First, we obtain from the classical L 2 parabolic Carleman estimate, conjugated with maximal regularity results, a weighted L p observability inequality for the nonhomogeneous heat equation. Secondly, we perform a duality argument, close to the well-known Hilbert Uniqueness Method in a reflexive Banach setting, to prove that the heat equation perturbed by a source term is null-controllable thanks to odd controls. Finally, the nonlinearity is handled with a Schauder fixed-point argument.

- In [16], self-organization and control around flocks and mills is studied for second-order swarming systems involving self-propulsion and potential terms. It is shown that through the action of constrained control, is it possible to control any initial configuration to a flock or a mill. The proof builds on an appropriate combination of several arguments: LaSalle invariance principle and Lyapunov-like decreasing functionals, control linearization techniques, and quasi-static deformations. A stability analysis of the second-order system guides the design of feedback laws for the stabilization to flock and mills, which are also assessed computationally.

- In [62], we focus on turnpike phenomenom, which stipulates that the solution of an optimal control problem in large time, remains essentially close to a steady-state of the dynamics, itself being the optimal solution of an associated static optimal control problem. Under general assumptions, it is known that not only the optimal state and the optimal control, but also the adjoint state coming from the application of the Pontryagin maximum principle, are exponentially close to a steady-state, except at the beginning and at the end of the time frame. In such results, the turnpike set is a singleton, which is a steady-state. In this paper, we establish a turnpike result for finite-dimensional optimal control problems in which some of the coordinates evolve in a monotone way, and some others are partial steady-states of the dynamics. We prove that the discrepancy between the optimal trajectory and the turnpike set is then linear, but not exponential: we thus speak of a linear turnpike theorem.

- In [23], given any measurable subset ω of a closed Riemannian manifold and given any $T > 0$, defining $l^T(\omega) \in [0,1]$ as the smallest average time over $[0,T]$ spent by all geodesic rays in ω, our first main result, which is of geometric nature, states that, under regularity assumptions, $1/2$ is the maximal possible discrepancy of l^T when taking the closure. Our second main result is of probabilistic nature: considering a regular checkerboard on the flat two-dimensional torus made of n^2 square white cells, constructing random subsets ω_ε^n by darkening cells randomly with a probability ε, we prove that the random law $l^T(\omega_\varepsilon^n)$ converges in probability to ε as $n \to +\infty$. We discuss the consequences in terms of observability of the wave equation, that is related to the controllability of the wave equation by the well-known H.U.M. method.

- The work [20] studies the reachable space of infinite dimensional control systems which are null controllable in any positive time, the typical example being the heat equation controlled from the boundary or from an arbitrary open set. The focus is on the robustness of the reachable space with respect to linear or nonlinear perturbations of the generator. More precisely, our first main results asserts that this space is invariant under perturbations which are small (in an appropriate sense). A second main result asserts the invariance of the reachable space with respect to perturbations which are compact (again in an appropriate sense), provided that a Hautus type condition is satisfied. Moreover, our methods give precise information on the behavior of the reachable space when the generator is perturbed by a class of nonlinear operators. When applied to the classical heat equation, our results provide detailed information on the reachable space when the generator is perturbed by a small potential or by a class of non local operators, and in particular in one space dimension, we deduce from our analysis that the reachable space for perturbations of the 1-d heat equation is a space of holomorphic functions. We also show how our approach leads to reachability results for a class of semi-linear parabolic equations.

- In [46], we survey on numerics for finite-dimensional nonlinear optimal control. The chapter is written as a guide to practitioners who wish to get rapidly acquainted with the main numerical methods used to efficiently solve an optimal control problem. We consider throughout two classical examples, quite simple but representative enough to be complexified and generalized to other problems: the Zermelo and the Goddard problems. We provide their solving codes that are available on the web and make the point on the most up-to-date and efficient methods existing nowadays. We range on direct and indirect methods, on Hamilton-Jacobi approaches and we end with optimistic planning. Our examples illustrate the pros and cons of the methods and we also show how those various approaches can be combined in view of augmenting the efficiency of the numerical solving.

- The article [40] follows and complements where we have established the turnpike property for some optimal shape design problems. Considering linear parabolic partial differential equations where the shapes to be optimized acts as a source term, we want to minimize a quadratic criterion. Existence of optimal shapes is proved under some appropriate assumptions. We prove and provide numerical evidence of the turnpike phenomenon for those optimal shapes, meaning that the extremal time-varying optimal solution remains essentially stationary; actually, it remains essentially close to the optimal solution of an associated static problem.

- In [59], we consider the internal control of linear parabolic equations through on-off shape controls, i.e., controls of the form $M(t)1_{\omega(t)}$ with $M(t) \geq 0$ and $\omega(t)$ with a prescribed maximal measure. We establish small-time approximate controllability towards all possible final states allowed by the comparison principle with nonnegative controls. We manage to build controls with constant amplitude $M(t) = M$. In contrast, if the moving control set $\omega(t)$ is confined to evolve in some region of the whole domain, we prove that approximate controllability fails to hold for small times. The method of proof is constructive. Using Fenchel-Rockafellar duality and the bathtub principle, the on-off shape control is obtained as the bang-bang solution of an optimal control problem, which we design by relaxing the constraints. Our optimal control approach is outlined in a rather general form for linear constrained control problems, paving the way for generalisations and applications to other PDEs and constraints.

- The article [29] revisits the optimal control problem with maximum cost with the objective to provide different equivalent reformulations suitable to numerical methods. We propose two reformulations in terms of extended Mayer problems with constraint, and another one in terms of a differential inclusion with upper-semi continuous right member but without constraint. For this last one we also propose an approximation scheme of the optimal value from below. These approaches are illustrated and discussed on several examples.

- In [28], we give the explicit solution of the optimal control problem which consists in minimizing the epidemic peak in the SIR model when the control is an attenuation factor of the infectious rate, subject to a L 1 constraint on the control which represents a budget constraint. The optimal strategy is given as a feedback control which consists in a singular arc maintaining the infected population at a constant level until the immunity threshold is reached, and no intervention outside the singular arc. We discuss and compare this strategy with the one that minimizes the peak when fixing the duration of a single intervention, as already proposed in the literature. Numerical simulations illustrate the benefits of the proposed control.

- In [33], we are interested in the design of stellarators, devices for the production of controlled nuclear fusion reactions alternative to tokamaks. The confinement of the plasma is entirely achieved by a helical magnetic field created by the complex arrangement of coils fed by high currents around a toroidal domain. Such coils describe a surface called "coil winding surface" (CWS). In this paper, we model the design of the CWS as a shape optimization problem, so that the cost functional reflects both optimal plasma confinement properties, through a least square discrepancy, and also manufacturability, thanks to geometrical terms involving the lateral surface or the curvature of the CWS. We completely analyze the resulting problem: on the one hand, we establish the existence of an optimal shape, prove the shape differentiability of the criterion, and provide the expression of the differential in a workable form. On the other hand, we propose a numerical method and perform simulations of optimal stellarator shapes. We discuss the efficiency of our approach with respect to the literature in this area. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

- During the Covid-19 pandemic a key role is played by vaccination to combat the virus. There are many possible policies for prioritizing vaccines, and different criteria for optimization: minimize death, time to herd immunity, functioning of the health system. Using an age-structured population compartmental finite-dimensional optimal control model, our results in [27] suggest that the eldest to youngest vaccination policy is optimal to minimize deaths. Our model includes the possible infection of vaccinated populations. We apply our model to real-life data from the US Census for New Jersey and Florida, which have a significantly different population structure. We also provide various estimates of the number of lives saved by optimizing the vaccine schedule and compared to no vaccination.

- In the proceeding [37], we deal with combining direct and indirect methods to have the best of both worlds is an efficient method to solve numerically optimal control problems. A direct solver will typically provide information on the structure of the optimal control, allowing an educated guess for indirect shooting. The control toolbox ct offers such possibilities and is presented on two examples. The first example has a bang-singular solution and is solved by chaining direct and indirect solvers. The second one consists in computing conjugate and cut loci on an ellipsoid of revolution, which is performed using a more advanced combination of indirect methods with differential continuation.

- The work [58] tackles the Open Pit planning problem in an optimal control framework. We study the optimality conditions for the so-called continuous formulation using Pontryagin's Maximum Principle, and introduce a new, semicontinuous formulation that can handle the optimization of a 2D mine profile. Numerical simulations are provided for several test cases, including global optimization for the 1D Final Open Pit, and first results for the 2D Sequential Open Pit. Results indicate a good consistency between the different approaches, and with the theoretical optimality conditions.

- In [39], we survey the main numerical techniques for finite-dimensional nonlinear optimal control. The chapter is written as a guide to practitioners who wish to get rapidly acquainted with the main numerical methods used to efficiently solve an optimal control problem. We consider two classical examples, simple but significant enough to be enriched and generalized to other settings: Zermelo and Goddard problems. We provide sample of the codes used to solve them and make these codes available online. We discuss direct and indirect methods, Hamilton-Jacobi approach, ending with optimistic planning. The examples illustrate the pros and cons of each method, and we show how these approaches can be combined into powerful tools for the numerical solution of optimal control problems for ordinary differential equations.

- In [51], we consider the small-time local controllability property of a water tank modeled by 1D Saint-Venant equations, where the control is the acceleration of the tank. It is known from the work of Dubois et al. that the linearized system is not controllable. Moreover, concerning the linearized system, they showed that a traveling time * is necessary to bring the tank from one position to another for which the water is still at the beginning and at the end. Concerning the nonlinear system, Coron showed that local controllability around equilibrium states holds for a time large enough. In this paper, we show that for the local controllability of the nonlinear system around the equilibrium states, the necessary time is at least 2 * even for the tank being still at the beginning and at the end. The key point of the proof is a coercivity property for the quadratic approximation of the water-tank system.

- In the paper [54], we prove the small-time global null-controllability of forward (resp. backward) semilinear stochastic parabolic equations with globally Lipschitz nonlinearities in the drift and diffusion terms (resp. in the drift term). In particular, we solve the open question posed by S. Tang and X. Zhang, in 2009. We propose a new twist on a classical strategy for controlling linear stochastic systems. By employing a new refined Carleman estimate, we obtain a controllability result in a weighted space for a linear system with source terms. The main novelty here is that the Carleman parameters are made explicit and are then used in a Banach fixed point method. This allows to circumvent the well-known problem of the lack of compactness embeddings for the solutions spaces arising in the study of controllability problems for stochastic PDEs.

- It has been proved by Zuazua in the nineties that the internally controlled semilinear 1D wave equation $\partial_{tt} y - \partial_{xx} y + g(y) = f 1_\omega$, with Dirichlet boundary conditions, is exactly controllable in $H^1_0(0,1) \times L^2(0,1)$ with controls $f \in L^2((0,1)\times(0,1))$, for any nonempty open subset ω of $(0,1)$ and T large enough, assuming that $g \in C^1(\mathbb{R})$ does not grow faster than $\beta \log^2(|x|)$ at infinity for some $\beta > 0$ small enough. The proof, based on the Leray-Schauder fixed point theorem, is however not constructive. In [30], we design a constructive proof and algorithm for the exact controllability of semilinear 1D wave equations. Assuming that g does not grow faster than $\beta \log^2(|x|)$ at infinity for some $\beta > 0$ small enough and that g is uniformly Hölder continuous on $\mathbb{R}$ with exponent $s \in [0,1]$, we design a least-squares algorithm yielding an explicit sequence converging to a controlled solution for the semilinear equation, at least with order $1+s$ after a finite number of iterations.

- The goal of the article [52] is to obtain observability estimates for non-homogeneous elliptic equations in the presence of a potential, posed on a smooth bounded domain Ω in $\mathbb{R}^2$ and observed from a non-empty open subset $\omega \subset \Omega$. More precisely, for $V \in L^\infty(\Omega;\mathbb{R})$, our main result shows that, when $\Omega \subset \mathbb{R}^2$ has a finite number of holes, the observability constant of the elliptic operator $-\Delta + V$, with domain $H^2(\Omega) \cap H^1_0(\Omega)$, is of the form $C \exp\left(C \|V\|^{1/2}_{L^\infty(\Omega)} \log^{1/2}\left(\|V\|_{L^\infty(\Omega)}\right)\right)$ where C is a positive constant depending only on Ω and ω. Our methodology of proof is crucially based on the one recently developed by Logunov, Malinnikova, Nadirashvili, and Nazarov, in the context of the Landis conjecture on exponential decay of solutions to homogeneous elliptic equations in the plane $\mathbb{R}^2$. The main difference and additional difficulty compared to Logunov, Malinnikova, Nadirashvili, and Nazarov is that the zero set of the solutions to elliptic equations with source term can be very intricate and should be dealt with carefully. As a consequence of these new observability estimates, we obtain new results concerning control of semi-linear elliptic equations in the spirit of Fernández-Cara, Zuazua's open problem concerning small-time global null-controllability of slightly super-linear heat equations.

- Magnetic confinement devices for nuclear fusion can be large and expensive. Compact stellarators are promising candidates for costreduction, but introduce new difficulties: confinement in smaller volumes requires higher magnetic field, which calls for higher coil-currents and ultimately causes higher Laplace forces on the coils-if everything else remains the same. This motivates the inclusion of force reduction in stellarator coil optimization. In the present paper [36] we consider a coil winding surface, we prove that there is a natural and rigorous way to define the Laplace force (despite the magnetic field discontinuity across the current-sheet), we provide examples of cost associated (peak force, surface-integral of the force squared) and discuss easy generalizations to parallel and normal force-components, as these will be subject to different engineering constraints. Such costs can then be easily added to the figure of merit in any multi-objective stellarator coil optimization code. We demonstrate this for a generalization of the REGCOIL code [1], which we rewrote in python, and provide numerical examples for the NCSX (now QUASAR) design. We present results for various definitions of the cost function, including peak force reductions by up to 40%, and outline future work for further reduction. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

- In [61], we study the small-time global null controllability of the generalized Burgers' equations $y_t + \gamma |y|^{\gamma-1} y_x - y_{xx} = u(t)$ on the segment $[0,1]$. The scalar control $u(t)$ is uniform in space and plays a role similar to the pressure in higher dimension. We set a right Dirichlet boundary condition $y(t,1) = 0$, and allow a left boundary control $y(t,0) = v(t)$. Under the assumption $\gamma > 3/2$ we prove that the system is small-time global null controllable. Our proof relies on the return method and a careful analysis of the shape and dissipation of a boundary layer. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

- The paper [47] deals with the controllability of finite-dimensional linear difference delay equations, i.e., dynamics for which the state at a given time t is obtained as a linear combination of the control evaluated at time t and of the state evaluated at finitely many previous instants of time $t-\Lambda_1,\dots,t-\Lambda_N$. Based on the realization theory developed by Y. Yamamoto for general infinite-dimensional dynamical systems, we obtain necessary and sufficient conditions, expressed in the frequency domain, for the approximate controllability in finite time in L^q spaces, $q \in [1,+\infty)$. We also provide a necessary condition for L^1 exact controllability, which can be seen as the closure of the L^1 approximate controllability criterion. Furthermore, we provide an explicit upper bound on the minimal times of approximate and exact controllability, given by $d\max\{\Lambda_1,\dots,\Lambda_N\}$, where d is the dimension of the state space.

- The article [60] deals with the existence of hypersurfaces minimizing general shape functionals under certain geometric constraints. We consider as admissible shapes orientable hypersurfaces satis- fying a so-called reach condition, also known as the uniform ball property, which ensures $C^{1,1}$ regularity of the hypersurface. In this paper, we revisit and generalise the results of Guo et al and, J. Dalphin. We provide a simpler framework and more concise proofs of some of the results contained in these references and extend them to a new class of problems involving PDEs. Indeed, by using the signed distance introduced by Delfour and Zolesio, we avoid the intensive and technical use of local maps, as was the case in the above references. Our approach, originally developed to solve an existence problem in a recent work by the same authors dedicated to optimal shape issues for Plasma Physics, can be easily extended to costs involving different mathematical objects associated with the domain, such as solutions of elliptic equations on the hypersurface. The article is also part of the Ph.D. thesis [42] of Rémi Robin.

6.4 Geometric models for vision and sub-Riemannian geometry: new results

Let us list here our new results in the geometry of vision axis and, more generally, on hypoelliptic diffusion and sub-Riemannian geometry.

- In [50], we study spectral properties of sub-Riemannian Laplacians, which are hypoelliptic operators. The main objective is to obtain quantum ergodicity results, what we have achieved in the 3D contact case. In the general case we study the small-time asymptotics of sub-Riemannian heat kernels. We prove that they are given by the nilpotentized heat kernel. In the equiregular case, we infer the local and microlocal Weyl law, putting in light the Weyl measure in sR geometry. This measure coincides with the Popp measure in low dimension but differs from it in general. We prove that spectral concentration occurs on the shief generated by Lie brackets of length r-1, where r is the degree of nonholonomy. In the singular case, like Martinet or Grushin, the situation is more involved but we obtain small-time asymptotic expansions of the heat kernel and the Weyl law in some cases. Finally, we give the Weyl law in the general singular case, under the assumption that the singular set is stratifiable.
- Given a surface S in a 3D contact sub-Riemannian manifold M, we investigate in [13] the metric structure induced on S by M, in the sense of length spaces. First, we define a coefficient at characteristic points that determines locally the characteristic foliation of S. Next, we identify some global conditions for the induced distance to be finite. In particular, we prove that the induced distance is finite for surfaces with the topology of a sphere embedded in a tight coorientable distribution, with isolated characteristic points.
- In [21], we study the isoperimetric problem for anisotropic left-invariant perimeter measures on $\mathbb{R}^3$, endowed with the Heisenberg group structure. The perimeter is associated with a left-invariant norm ϕ on the horizontal distribution. We first prove a representation formula for the ϕ-perimeter of regular sets and, assuming some regularity on ϕ and on its dual norm ϕ^*, we deduce a foliation property by sub-Finsler geodesics of C^2-smooth surfaces with constant ϕ-curvature. We then prove that the characteristic set of C^2-smooth surfaces that are locally extremal for the isoperimetric problem is made of isolated points and horizontal curves satisfying a suitable differential equation. Based on such a characterization, we characterize C^2-smooth ϕ-isoperimetric sets as the sub-Finsler analogue of Pansu's bubbles. We also show, under suitable regularity properties on ϕ, that such sub-Finsler candidate isoperimetric sets are indeed C^2-smooth. By an approximation

procedure, we finally prove a conditional minimality property for the candidate solutions in the general case (including the case where ϕ is crystalline).

7 Bilateral contracts and grants with industry

Participants: Emmanuel Trélat, Aymeric Nayet, Veljko Askovic, Georgy Scholten.

7.1 Bilateral contracts with industry

Contract CIFRE with ArianeGroup (les Mureaux), 2019–2022, funding the thesis of A. Nayet. Participants : M. Cerf (ArianeGroup), E. Trélat (coordinator). A new contract will start in 2023

Contract with MBDA (Palaiseau), 2021–2023. Subject: "Contrôle optimal pour la planification de trajectoires et l'estimation des ensembles accessibles". Pariticpants: V. Askovic (MBDA & CAGE), E. Trélat (coordinator).

7.2 Bilateral grants with industry

Grant by AFOSR (Air Force Office of Scientific Research), 2020–2023. Participants : Mohab Safey El Din (LIP6), E. Trélat.

8 Partnerships and cooperations

8.1 International research visitors

8.1.1 Visits of international scientists

Inria International Chair Andrei Agrachev visited CAGE, in the framework of his Inria International Chair, in November and December 2022.

Other international visits to the team Riccardo Adami visited CAGE and the LJLL in November and December 2022.

8.1.2 Visits to international teams

Research stays abroad

Jean-Michel Coron visited the École Polytechnique Fédérale de Lausanne in June 2022

8.2 National initiatives

The Inria Exploratory Action "StellaCage" is supporting since Spring 2020 a collaboration between CAGE, Yannick Privat (Inria team TONUS), and the startup Renaissance Fusion, based in Grenoble.

StellaCage approaches the problem of designing better stellarators (yielding better confinement, with simpler coils, capable of higher fields) by combining geometrical properties of magnetic field lines from the control perspective with shape optimization techniques.

8.2.1 ANR

- ANR TRECOS, for *New Trends in Control and Stabilization: Constraints and non-local terms*, coordinated by Sylvain Ervedoza, University of Bordeaux. The ANR started in 2021 and runs up to 2024. TRECOS' focus is on control theory for partial differential equations, and in particular models from ecology and biology.

- ANR QUACO, for *QUAntum COntrol: PDE systems and MRI applications*, coordinated by Thomas Chambrion, started in 2017 and will run until mid 2023 (after extension granted by ANR). Other partners: Burgundy University. QUACO aims at contributing to quantum control theory in two directions: improving the comprehension of the dynamical properties of controlled quantum systems in infinite-dimensional state spaces, and improve the efficiency of control algorithms for MRI.

8.3 Regional initiatives

Barbara Gris is the PI of a Bourse Emergence(s) by the Ville de Paris.

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

- Ugo Boscain, Jean-Michel Coron, Kévin Le Balc'h, Mario Sigalotti, and Emmanuel Trélat are members of the scientific committee of the *Groupe de Travail Contrôle*.
- Jean-Michel Coron and Emmanuel Trélat were co-organizers and members of the scientific committee of the "Congrès pour honorer la mémoire de Roland Glowinski", Sorbonne Université, July 2022.
- Kévin Le Balc'h was co-organizer of a thematic session on control of parabolic equations at the conference "IX Partial differential equations, optimal design and numerics", Benasque, Spain, August 2022.
- Emmanuel Trélat was member of the scientific and organization committee of the Workshop "Round mean field crowd opinion cells", Rome, Italy, September 2022
- Emmanuel Trélat was organizer of the conference Horizon Maths 2022 "Maths et gravitation", Sorbonne Université, December 2022.

9.1.2 Journal

Member of the editorial boards

- Ugo Boscain is Associate editor of SIAM Journal of Control and Optimization
- Ugo Boscain is Managing editor of Journal of Dynamical and Control Systems
- Jean-Michel Coron is Editor-in-chief of Comptes Rendus Mathématique
- Jean-Michel Coron is Associate editor of Journal of Evolution Equations
- Jean-Michel Coron is Associate editor of Asymptotic Analysis
- Jean-Michel Coron is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Jean-Michel Coron is Associate editor of Applied Mathematics Research Express
- Jean-Michel Coron is Associate editor of Advances in Differential Equations
- Jean-Michel Coron is Associate editor of Mathematics of Control, Signals, and Systems
- Jean-Michel Coron is Associate editor of Annales de l'IHP, Analyse non linéaire
- Mario Sigalotti is Associate editor of ESAIM: Control, Optimisation and Calculus of Variations
- Mario Sigalotti is Associate editor of Journal on Dynamical and Control Systems

- Emmanuel Trélat is Editor-in-chief of ESAIM: Control, Optimisation and Calculus of Variations
- Emmanuel Trélat is Associate editor of SIAM Review
- Emmanuel Trélat is Associate editor of Systems & Control Letters
- Emmanuel Trélat is Associate editor of Journal on Dynamical and Control Systems
- Emmanuel Trélat is Associate editor of Bollettino dell'Unione Matematica Italiana
- Emmanuel Trélat is Associate editor of ESAIM: Mathematical Modelling and Numerical Analysis
- Emmanuel Trélat is Editor of BCAM Springer Briefs
- Emmanuel Trélat is Associate editor of IEEE Transactions on Automatic Control
- Emmanuel Trélat is Associate editor of Journal of Optimization Theory and Applications
- Emmanuel Trélat is Associate editor of Mathematical Control & Related Fields
- Emmanuel Trélat is Associate editor of Mathematics of Control, Signals, and Systems
- Emmanuel Trélat is Associate editor of Optimal Control Applications and Methods
- Emmanuel Trélat is Associate editor of Advances in Continuous and Discrete Models: Theory and Modern Applications

9.1.3 Invited talks

- Ugo Boscain was invited speaker at the Mini-Workshop "Zero-Range and Point-Like Singular Perturbations: For a Spillover to Analysis, PDE and Differential Geometry", Oberfolfach, Germany.
- Ugo Boscain was invited speaker at the conference "Analysis and Control of (bi)linear PDEs", Rome, Italy.
- Ugo Boscain was invited speaker at the AMS-SMF-EMS Joint international meeting, Grenoble.
- Ugo Boscain was invited speaker at the Workshop on "Optimal Control Theory", Rouen.
- Jean-Michel Coron was invited speaker at the Isaac Newton Institute.
- Jean-Michel Coron was invited speaker at the Conference on Analysis of Partial Differential Equations.
- Jean-Michel Coron was invited speaker at 2022 MS101 Nonlocal Conservation Laws (visio).
- Jean-Michel Coron was invited speaker at EPFL, Switzerland.
- Jean-Michel Coron was invited speaker at the conference Analysis and Control of (bi)linear PDEs, Rome, Italy.
- Jean-Michel Coron was invited speaker at the Séminaire AMAC: EDP-AIRSEA-CVGI.
- Jean-Michel Coron was invited speaker at ICoCTA 2022, Chengdu (visio).
- Jean-Michel Coron was invited speaker at the conference *Contrôle des EDPs : approches en mathématique et en automatique*, GDR MACs and H-Code conference d'inauguration de la Fédération de Mathématiques de CentraleSupélec, Gif-sur-Yvette.
- Jean-Michel Coron was plenary speaker at the conference ICSC 2022, Marseille.
- Barbara Gris was invited speaker at SIAM conference on Imaging Science (visio).
- Barbara Gris was invited speaker at the conference "Geometry, Topology and Statistics in Data Sciences", IHP, Paris.

- Kévin Le Balc'h was invited speaker at *Séminaire d'analyse*, Nantes.
- Kévin Le Balc'h was invited speaker at *Séminaire d'EDP*, Rennes.
- Kévin Le Balc'h was invited speaker at *Séminaire d'analyse*, Bordeaux.
- Kévin Le Balc'h was invited speaker at *Seminario de EDP e Matematica Aplicada*, online.
- Kévin Le Balc'h was invited speaker at *Séminaire d'EDP et d'analyse numérique*, Sevilla, Spain.
- Kévin Le Balc'h was invited speaker at *Séminaire du Laboratoire de Mathématiques Appliquées de Compiègne*, Compiègne.
- Kévin Le Balc'h was invited speaker at *Workshop of the ANR Trecos*, Marseille.
- Kévin Le Balc'h was invited speaker at the confernece *IX Partial differential equations, optimal design and numerics*, Benasque, Spain.
- Kévin Le Balc'h was invited speaker at the Workshop *Inverse Problems and Related Fields*, Marseille.
- Mario Sigalotti was invited speaker at the Workshop on optimal control theory, Rouen, June 2022.
- Mario Sigalotti was invited speaker at the FAU DCN-AvH Seminar, Erlangen, Germany, November 2022.
- Mario Sigalotti was invited speaker at the LJLL-Day, November 2022.
- Mario Sigalotti was invited speaker at the Workshop "Mathematics for Quantum Technologies", Nice, March 2022.
- Emmanuel Trélat was plenary speaker at SMAI Mode, Limoges, June 2022.
- Emmanuel Trélat was invited speaker at *Séminaire de Mathématiques Appliquées du Collège de Franc*, March 2022.
- Emmanuel Trélat was invited speaker at *Colloquium du Centre Inria Paris*, January 2022.
- Emmanuel Trélat was invited speaker at the Workshop on control problems, Dortmund (online), October 2022.
- Emmanuel Trélat was invited speaker at the workshop "Round meanfield crowd-opinion-cells", Rome, Italy, September 2022.
- Emmanuel Trélat was invited speaker at the workshop "Analysis and Control of (bi)linear PDEs", Rome, Italy, September 2022.
- Emmanuel Trélat was invited speaker at the Workshop on optimal control theory, Rouen, June 2022.
- Emmanuel Trélat was invited speaker at the online geometric analysis seminar, IIT Bombay, December 2022.
- Emmanuel Trélat was invited speaker at the RTE seminar, Les Cle‘s de la re‘ussite, November 2022.
- Emmanuel Trélat was invited speaker at the online analysis and applied mathematics seminar, WMU (Michigan), November 2022.
- Emmanuel Trélat was invited speaker at the local seminar at Institut Fourier, Grenoble, November 2022.
- Emmanuel Trélat was invited speaker at the local seminar at Orle‘ans, October 2022.
- Emmanuel Trélat was invited speaker at the local seminar at Konstanz, May 2022.
- Emmanuel Trélat was invited speaker at the Virtual Informal Systems Seminar, McGill, January 2022.

9.1.4 Leadership within the scientific community

Emmanuel Trélat is Head of the Laboratoire Jacques-Louis Lions (LJLL).

9.1.5 Scientific expertise

- Emmanuel Trélat is member of the *conseil scientifique de la Fédération de Mathématiques de CentraleSupelec*.
- Emmanuel Trélat is member of the Advisory Board of the Department of Data Science, FAU (Erlangen), Germany.

9.1.6 Research administration

- Emmanuel Trélat is member of the *Bureau de comité des équipes-projets*, Inria Paris center.

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Ugo Boscain thought “Quantum Geometric Control” to PhD students at Politecnico di Torino, Italy.
- Ugo Boscain and Mario Sigalotti thought “Geometric control theory” to M2 students at Sorbonne Université.
- Ugo Boscain thought “Automatic control with applications in robotics and in quantum engineering” at Ecole Polytechnique.
- Ugo Boscain thought “Diffusion in almost-Riemannian Geometry” to PhD students at *42nd Winter School on Geometry and Physics*, Srni, Czech Republic.
- Kévin Le Balc’h thought *Encadrement de leçons d’agrégation externe de mathématiques* to M2 students at Sorbonne Université.
- Kévin Le Balc’h thought “calcul différentiel et d’optimisation” to L3 students at Sorbonne Université.
- Mario Sigalotti thought “Équations d’évolution, stabilité et contrôle” to M1 students at Sorbonne Université
- Emmanuel Trélat thought “Contrôle en dimension finie et infinie” to M2 students at Sorbonne Université
- Emmanuel Trélat thought “Optimisation numérique et sciences des données” to M1 students at Sorbonne Université

9.2.2 Supervision

- PhD: Emilio Molina, “Application of optimal control techniques to natural systems management”, September 2022. Supervisors: Héctor Ramirez (Santiago, Chile), Pierre Martinon, and Mario Sigalotti.
- PhD: Aymeric Nayet, “Improvement of a trajectory optimization software for future Ariana missions”, June 2022. Supervisor: Emmanuel Trélat.
- PhD: Rémi Robin, “Control and Optimization of Physical Systems: Quantum Dynamics and Magnetic Confinement in Stellarators”, September 2022. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Kala Agbo Bidi, “Robust pest control strategies”. Supervisors: Luis Almeida and Jean-Michel Coron.

- PhD in progress: Veljko Askovic, “Planification de trajectoires par HJB & PMP”, started in 2020. Supervisors: Emmanuel Trélat and Hasnaa Zidani (INSA, Rouen).
- PhD in progress: Liangying Chen, “Sensitivity, Verification and Conjugate Times in Stochastic Optimal Control”, started in 2021. Supervisors: Emmanuel Trélat and Xu Zhang (Chengdu, China).
- PhD in progress: Ruikang Liang, “The quantum speed limit in Quantum Control”, started in 2022. Supervisors: Ugo Boscain and Mario Sigalotti.
- PhD in progress: Robin Roussel, “Magnetic field lines and confinement in stellarators: a Hamiltonian perspective”, started in 2021. Supervisors: Ugo Boscain and Mario Sigalotti.

9.2.3 Juries

- Ugo Boscain was member of the PhD jury of T. Schmoderer, Insa de Rouen.
- Ugo Boscain was referee and member of the HDR jury of N. Amini, LSS, CentraleSupélec, Saclay.
- Jean-Michel Coron was member of the PhD jury of L. Guan, Gipsa-LAB, Grenoble.
- Jean-Michel Coron was member of the PhD jury of I. Darghoum, Metz.
- Barbara Gris was member of the PhD jury of P-L. Antonsanti, Université de Paris.
- Mario Sigalotti was referee and member of the PhD jury of A. Scagliotti, SISSA, Trieste, Italy.
- Mario Sigalotti was referee and member of the PhD jury of M. Bournissou, École Normale Supérieure de Rennes.
- Mario Sigalotti was referee and member of the PhD jury of R. Manriquez, Université Paris-Saclay.
- Mario Sigalotti was member of the PhD jury of S. Farinelli, SISSA, Trieste, Italy.
- Mario Sigalotti was member of the PhD jury of N. Vanspranghe, Université Grenoble Alpes.
- Mario Sigalotti was member of the PhD jury of A. Nayet, Sorbonne Université.
- Emmanuel Trélat was member of the HDR jury of M. Laleg, Univiversité Paris-Saclay.
- Emmanuel Trélat was member of the HDR jury of C. Bertucci, École polytechnique.
- Emmanuel Trélat was president of the HDR jury of F. Di Meglio, École des Mines de Paris.
- Emmanuel Trélat was referee and member of the PhD jury of B. Danhane, Université de Lorraine.
- Emmanuel Trélat was president of the PhD jury of E. Berthier, ENS Ulm.
- Emmanuel Trélat was member of the PhD jury of E. Molina, University of Chili and Sorbone Université.
- Emmanuel Trélat was referee and member of the PhD jury of R. Manriquez, Université Paris-Saclay.

9.3 Popularization

9.3.1 Articles and contents

E. Trélat, Les courants de gravité : un ticket gratuit pour l’exploration spatiale, La Recherche 569 (2022).

9.3.2 Education

Mario Sigalotti participated to *Fête de la science 2022* at the *École polyvalente publique d’application Enfants d’Izieu*, Paris.

9.3.3 Interventions

Mario Sigalotti spoke about “Fusion nucléaire par confinement magnétique : quelques questions mathématiques autour des stellarators” at the *Demi-heure de science* of the Paris Inria Research Center, February 2022.

10 Scientific production

10.1 Major publications

[1] D. Barilari, Y. Chitour, F. Jean, D. Prandi and M. Sigalotti. ‘On the regularity of abnormal minimizers for rank 2 sub-Riemannian structures’. In: *Journal de Mathématiques Pures et Appliquées* 133 (2020), pp. 118–138. DOI: 10.1016/j.matpur.2019.04.008. URL: https://hal.archives-ouvertes.fr/hal-01757343.

[2] R. Bonalli, B. Hérissé and E. Trélat. ‘Optimal Control of Endo-Atmospheric Launch Vehicle Systems: Geometric and Computational Issues’. In: *IEEE Transactions on Automatic Control* 65.6 (2020), pp. 2418–2433. DOI: 10.1109/tac.2019.2929099. URL: https://hal.archives-ouvertes.fr/hal-01626869.

[3] Y. Colin de Verdière, L. Hillairet and E. Trélat. *Spectral asymptotics for sub-Riemannian Laplacians*. 5th Dec. 2022. URL: https://hal.archives-ouvertes.fr/hal-03885610.

[4] J.-M. Coron, A. Hayat, S. Xiang and C. Zhang. ‘Stabilization of the linearized water tank system’. In: *Archive for Rational Mechanics and Analysis* 244.3 (2022), pp. 1019–1097. URL: https://hal.archives-ouvertes.fr/hal-03161523.

[5] J.-M. Coron, F. Marbach and F. Sueur. ‘Small-time global exact controllability of the Navier-Stokes equation with Navier slip-with-friction boundary conditions’. In: *Journal of the European Mathematical Society* 22.5 (May 2020), pp. 1625–1673. DOI: 10.4171/JEMS/952. URL: https://hal.archives-ouvertes.fr/hal-01422161.

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[8] M. Leibscher, E. Pozzoli, C. Pérez, M. Schnell, M. Sigalotti, U. Boscain and C. P. Koch. ‘Full quantum control of enantiomer-selective state transfer in chiral molecules despite degeneracy’. In: *Communications Physics* (6th May 2022). DOI: 10.1038/s42005-022-00883-6. URL: https://hal.inria.fr/hal-02972059.

[9] O. Öktem, B. Gris and C. Chen. ‘Image reconstruction through metamorphosis’. In: *Inverse Problems* 36 (2020). DOI: 10.1088/1361-6420/ab5832. URL: https://hal.archives-ouvertes.fr/hal-01773633.

[10] Y. Privat, R. Robin and M. Sigalotti. ‘Optimal shape of stellarators for magnetic confinement fusion’. In: *Journal de Mathématiques Pures et Appliquées* (2022). DOI: 10.1016/j.matpur.2022.05.005. URL: https://hal.inria.fr/hal-03472623.

[11] R. Robin, N. Augier, U. Boscain and M. Sigalotti. ‘Ensemble qubit controllability with a single control via adiabatic and rotating wave approximations’. In: *Journal of Differential Equations* 318 (5th May 2022). DOI: 10.1016/j.jde.2022.02.042. URL: https://hal.archives-ouvertes.fr/hal-02504532.

10.2 Publications of the year

International journals

[12] N. Augier, U. Boscain and M. Sigalotti. 'Effective adiabatic control of a decoupled Hamiltonian obtained by rotating wave approximation'. In: *Automatica* 136 (2022), p. 110034. DOI: 10.1016/j.automatica.2021.110034. URL: https://hal.inria.fr/hal-02562363.

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[14] B. Bonnet, N. Pouradier Duteil and M. Sigalotti. 'Consensus Formation in First-Order Graphon Models with Time-Varying Topologies'. In: *Mathematical Models and Methods in Applied Sciences* (2022). URL: https://hal.science/hal-03418399.

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[34] R. Robin, N. Augier, U. Boscain and M. Sigalotti. 'Ensemble qubit controllability with a single control via adiabatic and rotating wave approximations'. In: *Journal of Differential Equations* 318 (5th May 2022). DOI: 10.1016/j.jde.2022.02.042. URL: https://hal.science/hal-02504532.

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[42] R. Robin. ‘Control and Optimization of Physical Systems: Quantum Dynamics and Magnetic Confinement in Stellarators’. Sorbone Université, 16th Sept. 2022. URL: https://hal.inria.fr/tel-03779871.

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