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ACTIVITY REPORT

Project-Team

MATHRISK

Mathematical Risk handling

IN COLLABORATION WITH: Centre d'Enseignement et de Recherche en Mathématiques et Calcul Scientifique (CERMICS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Stochastic approaches

Inria

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Project-Team MATHRISK

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2 Overall objectives

The Inria project team **MathRisk** team was created in 2013. It is the follow-up of the MathFi project team founded in 2000. MathFi was focused on financial mathematics, in particular on computational methods for pricing and hedging increasingly complex financial products. The 2007 global financial crisis and its "aftermath crisis" has abruptly highlighted the critical importance of a better understanding and management of risk.

The project team MathRisk addresses broad research topics embracing risk management in quantitative finance and insurance and in other related domains as economy and sustainable development. In these contexts, the management of risk appears at different time scales, from high frequency data to long term life insurance management, raising challenging renewed modeling and numerical issues. We aim at both producing advanced mathematical tools, models, algorithms, and software in these domains, and developing collaborations with various institutions involved in risk control. The scientific issues we consider include:

Option pricing and hedging, and risk-management of portfolios in finance and insurance. These remain crucial issues in finance and insurance, with the development of increasingly complex products and various regulatory legislations. Models must take into account the multidimensional features, incompleteness issues, model uncertainties and various market imperfections and defaults. It is also important to understand and capture the joint dynamics of the underlying assets and their volatilities. The insurance activity faces a large class of risk, including financial risk, and is submitted to strict regulatory requirements. We aim at proposing modelling frameworks which catch the main specificity of life insurance contracts.

Systemic risk and contagion modeling. These last years have been shaped by ever more interconnectedness among all aspects of human life. Globalization and economics growth as well as technological progress have led to more complex dependencies worldwide. While these complex networks facilitate physical, capital and informational transmission, they have an inherent potential to create and propagate distress and risk. The financial crisis 2007-2009 has illustrated the significance of network structure on the amplification of initial shocks in the banking system to the level of the global financial system, leading to an economic recession. We are contributing on the issues of systemic risk and financial networks, aiming at developing adequate tools for monitoring financial stability which capture accurately the risks due to a variety of interconnections in the financial system.

(Martingale) Optimal transport. Optimal transport problems arise in a wide range of topics, from economics to physics. In mathematical finance, an additional martingale constraint is considered to take the absence of arbitrage opportunities into account. The minimal and maximal costs provide price bounds robust to model risk, i.e. the risk of using an inadequate model. On the other hand, optimal transport is also useful to analyse mean-field interactions. We are in particular interested in particle approximations of McKean-Vlasov stochastic differential equations (SDEs) and the study of mean-field backward SDEs with applications to systemic risk quantization.

Advanced numerical probability methods and Computational finance. Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. Financial institutions and insurance companies, submitted to more and more stringent regulatory legislations, such as FRTB or XVA computation, are facing numerical implementation challenges and research focused on numerical efficiency is strongly needed. Overcoming the curse of dimensionality in computational finance is a crucial issue that we address by developing advanced stochastic algorithms and deep learning techniques.

The **MathRisk** project is strongly devoted to the development of new mathematical methods and numerical algorithms. Mathematical tools include stochastic modeling, stochastic analysis, in particular various aspects of stochastic control and optimal stopping with nonlinear expectations, Malliavin calculus, stochastic optimization, random graphs, (martingale) optimal transport, mean-field systems, numerical probability and generally advanced numerical methods for effective solutions. The numerical platform Premia that MathRisk is developing in collaboration with a consortium of financial institutions, focuses on the computational challenges the recent developments in financial mathematics encompass, in particular risk control in large dimensions.

3 Research program

3.1 Systemic risk in financial networks

After the recent financial crisis, systemic risk has emerged as one of the major research topics in mathematical finance. Interconnected systems are subject to contagion in time of distress. The scope is to understand and model how the bankruptcy of a bank (or a large company) may or not induce other bankruptcies. By contrast with the traditional approach in risk management, the focus is no longer on modeling the risks faced by a single financial institution, but on modeling the complex interrelations between financial institutions and the mechanisms of distress propagation among these.

The mathematical modeling of default contagion, by which an economic shock causing initial losses and default of a few institutions is amplified due to complex linkages, leading to large scale defaults, can be addressed by various techniques, such as network approaches (see in particular R. Cont et al. [52] and A. Minca [93]) or mean field interaction models (Garnier-Papanicolaou-Yang [75]).

The goal of our project is to develop a model that captures the dynamics of a complex financial network and to provide methods for the control of default contagion, both by a regulator and by the institutions themselves.

We have contributed in the last years to the research on the control of contagion in financial systems in the framework of random graph models (see PhD thesis of R. Chen [67]).

In [55, 94], [8], we consider a financial network described as a weighted directed graph, in which nodes represent financial institutions and edges the exposures between them. The distress propagation is modeled as an epidemics on this graph. We study the optimal intervention of a lender of last resort who seeks to make equity infusions in a banking system prone to insolvency and to bank runs, under complete and incomplete information of the failure cluster, in order to minimize the contagion effects. The paper [8] provides in particular important insight on the relation between the value of a financial system, connectivity and optimal intervention.

The results show that up to a certain connectivity, the value of the financial system increases with connectivity. However, this is no longer the case if connectivity becomes too large. The natural question remains how to create incentives for the banks to attain an optimal level of connectivity. This is studied in [68], where network formation for a large set of financial institutions represented as nodes is investigated. Linkages are source of income, and at the same time they bear the risk of contagion, which is endogeneous and depends on the strategies of all nodes in the system. The optimal connectivity of the nodes results from a game. Existence of an equilibrium in the system and stability properties is studied. The results suggest that financial stability is best described in terms of the mechanism of network formation than in terms of simple statistics of the network topology like the average connectivity.

In [7], H. Amini (University of Florida), A. Minca (Cornell University) and A. Sulem study Dynamic Contagion Risk Model With Recovery Features. We introduce threshold growth in the classical threshold contagion model, in which nodes have downward jumps when there is a failure of a neighboring node. We are motivated by the application to financial and insurance-reinsurance networks, in which thresholds represent either capital or liquidity. An initial set of nodes fail exogenously and affect the nodes connected to them as they default on financial obligations. If those nodes' capital or liquidity is insufficient to absorb the losses, they will fail in turn. In other terms, if the number of failed neighbors reaches a node's threshold, then this node will fail as well, and so on. Since contagion takes time, there is the potential for the capital to recover before the next failure. It is therefore important to introduce a notion of growth. Choosing the configuration model as underlying graph, we prove fluid limits for the baseline model, as well as extensions to the directed case, state-dependent inter-arrival times and the case of growth driven by upward jumps. We then allow nodes to choose their connectivity by trading off link benefits and contagion risk. Existence of an asymptotic equilibrium is shown as well as convergence of the sequence of equilibria on the finite networks. In particular, these results show that systems with higher overall growth may have higher failure probability in equilibrium.

3.2 Stochastic Control, optimal stopping and non-linear backward stochastic differential equations (BSDEs) with jumps

Option pricing in incomplete and nonlinear financial market models with default. A. Sulem with M.C. Quenez and M. Grigorova have studied option pricing and hedging in nonlinear incomplete financial markets model with default. The underlying market model consists of a risk-free asset and a risky asset driven by a Brownian motion and a compensated default martingale. The portfolio processes follow nonlinear dynamics with a nonlinear driver f, which encodes the imperfections or constraints of the market. A large class of imperfect market models can fit in this framework, including imperfections coming from different borrowing and lending interest rates, taxes on profits from risky investments, or from the trading impact of a large investor seller on the market prices and the default probability. Our market is *incomplete*, in the sense that not every contingent claim can be replicated by a portfolio. In this framework, we address in [13] the problem of pricing and (super)hedging of European options. By using a dynamic programming approach, we provide a dual formulation of the seller's superhedging price as the supremum over a suitable set of equivalent probability measures $Q \in \mathcal{Q}$ of the non-linear $\mathcal{E}^f_Q$-expectation under Q of the payoff. We also provide a characterization of this price as the minimal supersolution of a *constrained* BSDE with default. In [78], we study the superhedging problem for American options with irregular payoffs. We establish a dual formulation of the seller's price in terms of the value of a non-linear mixed optimal control/stopping problem. We also characterize the seller's price process as the minimal supersolution of a *reflected* BSDE with *constraints*. We then prove a duality result for the *buyer's* price in terms of the value of a non-linear optimal control/stopping *game* problem. A crucial step in the proofs is to establish a non-linear optional and a non-linear predictable decomposition for processes which are $\mathcal{E}^f_Q$-strong supermartingales under Q, for all $Q \in \mathcal{Q}$. American option pricing in a non-linear *complete* market model with default is previously studied in [70]. A complete analysis of BSDEs driven by a Brownian motion and a compensated default jump process with intensity process (λ_t) is achieved in [69]. Note that these equations do not correspond to a particular case of BSDEs with Poisson random measure, and are particularly useful in default risk modeling in finance.

Optimal stopping. The theory of optimal stopping in connection with American option pricing has been extensively studied in recent years. Our contributions in this area concern:

(i) *The analysis of the binomial approximation of the American put price in the Black-Scholes model.* We proved that the rate of convergence is, up to a logarithmic factor, of the order $1/n$, where n is the number of discretization time points [87]; (ii) *The American put in the Heston stochastic volatility model.* We have results about existence and uniqueness for the associated variational inequality, in suitable weighted Sobolev spaces, following up on the work of P. Feehan et al. (2011, 2015, 2016) (cf [89]). We also established some qualitative properties of the value function (monotonicity, strict convexity, smoothness) [88]. (iii) *A probabilistic approach to the smoothness of the free boundary in the optimal stopping of a one-dimensional diffusion* (work in progress with T. De Angelis)(University of Torino),

Stochastic control with jumps. The 3rd edition of the book *Applied Stochastic Control of Jump diffusions* (Springer, 2019) by B. Øksendal and A. Sulem [15] contains recent developments within stochastic control and its applications. In particular, there is a new chapter devoted to a comprehensive presentation of financial markets modelled by jump diffusions, one on backward stochastic differential equations and risk measures, and an advanced stochastic control chapter including optimal control of mean-field systems, stochastic differential games and stochastic Hamilton-Jacobi-Bellman equations.

3.3 Insurance modeling

Asset Liability Management. Life insurance contracts are popular and involve very large portfolios, for a total amount of trillions of euros in Europe. To manage them in a long run, insurance companies perform Asset and Liability Management (ALM) : it consists in investing the deposit of policyholders in different asset classes such as equity, sovereign bonds, corporate bonds, real estate, while respecting a performance warranty with a profit sharing mechanism for the policyholders. A typical question is how to determine an allocation strategy which maximizes the rewards and satisfies the regulatory constraints. The management of these portfolios is quite involved: the different cash reserves imposed by the regulator, the profit sharing mechanisms, and the way the insurance company determines the crediting rate to its policyholders make the whole dynamics path-dependent and rather intricate. A. Alfonsi et al. have developed in [46] a synthetic model that takes into account the main features of the life insurance business. This model is then used to determine the allocation that minimizes the Solvency Capital Requirement (SCR). In [47], numerical methods based on Multilevel Monte-Carlo algorithms are proposed to calculate the SCR at future dates, which is of practical importance for insurance companies. The standard formula prescribed by the regulator is basically obtained from conditional expected losses given standard shocks that occur in the future.

3.4 (Martingale) Optimal Transport and Mean-field systems

3.4.1 Numerical methods for Optimal transport

Optimal transport problems arise in a wide range of topics, from economics to physics. There exists different methods to solve numerically optimal transport problems. A popular one is the Sinkhorn algorithm which uses an entropy regularization of the cost function and then iterative Bregman projections. Alfonsi et al. [49] have proposed an alternative relaxation that consists in replacing the constraint of matching exactly the marginal laws by constraints of matching some moments. Using Tchakaloff's theorem, it is shown that the optimum is reached by a discrete measure, and the optimal transport is found by using a (stochastic) gradient descent that determines the weights and the points of the discrete measure. The number of points only depends of the number of moments considered, and therefore does not depend on the dimension of the problem. The method has then been developed in [48] in the case of symmetric multimarginal optimal transport problems. These problems arise in quantum chemistry with the Coulomb interaction cost. The problem is in dimension $(\mathbb{R}^3)^M$ where M is the number of electrons, and the method is particularly relevant since the optimal discrete measure weights only $N+2$ points, where N is the number of moments constraint on the distribution of each electron. Numerical examples up to $M = 100$ can be thus investigated while existing methods could not go beyond $M \approx 10$.

3.4.2 Mean-field systems

Mean-field systems and optimal transport. In [66], O.Bencheikh and B. Jourdain prove that the weak error between a stochastic differential equation with nonlinearity in the sense of McKean given by moments and its approximation by the Euler discretization with time-step h of a system of N interacting particles is $\mathcal{O}(N^{-1}+h)$. The challenge was to improve the $\mathcal{O}(N^{-1/2})$ strong rate of convergence in the number of particles. In [23], they prove the same estimation for the Euler discretization of a system interacting particles with mean-field rank based interaction in the drift coefficient. To deal with the initialization error, they investigate in [21] the approximation rate in Wasserstein distance with index $\rho \geq 1$ of a probability measure μ on the real line with finite moment of order ρ by the empirical measure of N deterministic points.
In [85], B. Jourdain and A. Tse propose a generalized version of the central limit theorem for nonlinear functionals of the empirical measure of i.i.d. random variables, provided that the functional satisfies some regularity assumptions for the associated linear functional derivatives of various orders. Using this result to deal with the contribution of the initialization, they check the convergence of fluctuations between the empirical measure of particles in an interacting particle system and its mean-field limiting measure. In [39], R. Flenghi and B. Jourdain pursue their study of the central limit theorem for nonlinear functionals of the empirical measure of random variables by relaxing the i.i.d. assumption to deal with the successive values of an ergodic Markov chain. In [50], A. Alfonsi and B. Jourdain show that any optimal coupling for the quadratic Wasserstein distance $\mathcal{W}_2^2(\mu,\nu)$ between two probability measures μ and ν on $\mathbf{R}^d$ is the composition of a martingale coupling with an optimal transport map. They prove that $\sigma \mapsto \mathcal{W}_2^2(\sigma,\nu)$ is differentiable at μ in both Lions and the geometric senses iff there is a unique optimal coupling between μ and ν and this coupling is given by a map.

3.4.3 Martingale Optimal Transport

In mathematical finance, optimal transport problems with an additional martingale constraint are considered to handle the model risk, i.e. the risk of using an inadequate model. The Martingale Optimal Transport (MOT) problem introduced in [65] provides model-free hedges and bounds on the prices of exotic options. The market prices of liquid call and put options give the marginal distributions of the underlying asset at each traded maturity. Under the simplifying assumption that the risk-free rate is zero, these probability measures are in increasing convex order, since by Strassen's theorem this property is equivalent to the existence of a martingale measure with the right marginal distributions. For an exotic payoff function of the values of the underlying on the time-grid given by these maturities, the model-free upper-bound (resp. lower-bound) for the price consistent with these marginal distributions is given by the following martingale optimal transport problem : maximize (resp. minimize) the integral of the payoff with respect to the martingale measure over all martingale measures with the right marginal distributions. Super-hedging (resp. sub-hedging) strategies are obtained by solving the dual problem. With J. Corbetta, A. Alfonsi and B. Jourdain [5] have studied sampling methods preserving the convex order for two probability measures μ and ν on $\mathbf{R}^d$, with ν dominating μ. Their method is the first generic approach to tackle the martingale optimal transport problem numerically and it can also be applied to several marginals.

Martingale Optimal Transport provides thus bounds for the prices of exotic options that take into account the risk neutral marginal distributions of the underlying assets deduced from the market prices of vanilla options. For these bounds to be robust, the stability of the optimal value with respect to these marginal distributions is needed. Because of the global martingale constraint, stability is far less obvious than in optimal transport (it even fails in multiple dimensions). B. Jourdain has advised the PhD of W. Margheriti devoted to this issue and related problems. He also initiated a collaboration on this topic with M. Beiglböck, one of the founders of MOT theory. In [82], B. Jourdain and W. Margheriti exhibit a new family of martingale couplings between two one-dimensional probability measures μ and ν in the convex order. The integral of $|x-y|$ with respect to each of these couplings is smaller than twice the $\mathcal{W}^1$ distance between μ and ν. Moreover, for $\rho > 1$, replacing $|x-y|$ and $\mathcal{W}_1$ respectively with $|x-y|^\rho$ and $\mathcal{W}_\rho^\rho$ does not lead to a finite multiplicative constant. In [27], they show that a finite constant is recovered when replacing $\mathcal{W}_\rho^\rho$ with the product of $\mathcal{W}_\rho$ times the centred ρ-th moment of the second marginal to the power $\rho-1$ and they study the generalisation of this stability inequality to higher dimension. In [28], they give a direct construction of the projection in adapted Wasserstein distance onto the set of martingale couplings of a coupling between two probability measures on the real line in the convex order which satisfies the barycentre dispersion assumption. Under this assumption, Wiesel had given a clear algorithmic construction of the projection for finitely supported marginals before getting rid of the finite support condition by a rather messy limiting procedure. In [20], with M. Beiglböck and G. Pammer they establish stability of martingale couplings in dimension one : when approximating in Wasserstein distance the two marginals of a martingale coupling by probability measures in the convex order, it is possible to construct a sequence of martingale couplings between these probability measures converging in adapted Wasserstein distance to the original coupling. In [64], they deduce the stability of the Weak Martingale Optimal Transport Problem with respect to the marginal distributions in dimension one which is important since financial data can give only imprecise information on these marginals. As application, this yields the stability of the superreplication bound for VIX futures and of the stretched Brownian motion. In [40], B. Jourdain et al. prove that, in dimension one, contrary to the minimum and maximum in the convex order, the Wasserstein projections of μ (resp. ν) on the set of probability measures dominated by ν (resp. dominating μ) in the convex order are Lipschitz continuous in (μ,ν) for the Wasserstein distance. The thesis of K. Shao (advisers: B. Jourdain, A. Sulem) focuses so far on optimal couplings for costs $|y-x|^\rho$ in dimension one.

Quantization. In order to exploit the natural links between quantization and convex order in view of numerical methods for (Weak) Martingale Optimal Transport, B. Jourdain has initiated a fruitful collaboration with G. Pagès, one of the leading experts of quantization. For two compactly supported probability measures in the convex order, any stationary quadratic primal quantization of the smaller remains dominated by any dual quantization of the larger. B. Jourdain and G. Pagès prove in [30] that any martingale coupling between the original probability measures can be approximated by a martingale coupling between their quantizations in Wassertein distance with a rate given by the quantization errors but also in the much finer adapted Wassertein distance. In [29], in order to approximate a sequence of more than two probability measures in the convex order by finitely supported probability measures still in the convex order, they propose to alternate transitions according to a martingale Markov kernel mapping a probability measure in the sequence to the next and dual quantization steps. In the case of ARCH models, the noise has to be truncated to enable the dual quantization steps. They exhibit conditions under which the ARCH model with truncated noise is dominated by the original ARCH model in the convex order and also analyse the error of the scheme combining truncation of the noise according to primal quantization with the dual quantization steps. In [84], they prove that for compactly supported one dimensional probability distributions having a log-concave density, L^r-optimal dual quantizers are unique at each level N. In the quadratic $r = 2$ case, they propose an algorithm which computes this unique optimal dual quantizer with geometric rate of convergence.

3.5 Advanced numerical probability methods and Computational finance

Our project team is very much involved in numerical probability, aiming at pushing numerical methods towards the effective implementation. This numerical orientation is supported by a mathematical expertise which permits a rigorous analysis of the algorithms and provides theoretical support for the study of rates of convergence and the introduction of new tools for the improvement of numerical methods. This activity in the MathRisk team is strongly related to the development of the Premia software.

3.5.1 Approximation of stochastic differential equations

High order schemes. The approximation of SDEs and more general Markovian processes is a very active field. One important axis of research is the analysis of the weak error, that is the error between the law of the process and the law of its approximation. A standard way to analyse this is to focus on marginal laws, which boils down to the approximation of semigroups. The weak error of standard approximation schemes such as the Euler scheme has been widely studied, as well as higher order approximations such as those obtained with the Richardson-Romberg extrapolation method.

Stochastic Volterra Equations. Stochastic Volterra Equations (SVE) provide a wide family of non-Markovian stochastic processes. They have been introduced in the early 80's by Berger and Mizel and have received a recent attention in mathematical finance to model the volatility : it has been noticed that SVEs with a fractional convolution kernel $G(t) = c_H t^{H-1/2}$ reproduce some important empirical features. The problem of approximating these equations has been tackled by Zhang [100] and Richard et al. [99] who show under suitable conditions a strong convergence rate of $O(n^{-H-})$ for the Euler scheme, where n is the number of time steps. We almost recover the rate for classical SDEs when $H \to 1/2$. However, an important drawback is that the required computation time is proportional to n^2.

Abstract Malliavin calculus and convergence in total variation. In collaboration with L. Caramellino and G. Poly, V. Bally has settled a Malliavin type calculus for a general class of random variables, which are not supposed to be Gaussian (as it is the case in the standard Malliavin calculus). This is an alternative to the Γ-calculus settled by Bakry, Gentile and Ledoux. The main application is the estimate in total variation distance of the error in general convergence theorems. This is done in [62].

Invariance principles. As an application of the above methodology, V. Bally et al. have studied several limit theorems of Central Limit type (see [58] and [61]). In particular they estimate the total variation distance between random polynomials, and prove a universality principle for the variance of the number of roots of trigonometric polynomials with random coefficients [63]).

Analysis of jump type SDEs. V. Bally, L. Caramellino and A. Kohatsu Higa, study the regularity properties of the law of the solutions of jump type SDE's [18]. They use an interpolation criterion (proved in [57]) combined with Malliavin calculus for jump processes. They also use a Gaussian approximation of the solution combined with Malliavin calculus for Gaussian random variables. Another approach to the same regularity property, based on a semigroup method has been developed by Bally and Caramellino in [59]. An application for the Bolzmann equation is given by V. Bally in [57]. In the same line but with different application, the total variation distance between a jump equation and its Gaussian approximation is studied by V. Bally and his PhD student Y. Qin [19] and by V. Bally, V. Rabiet, D. Goreac [63]. A general discussion on the link between total variation distance and integration by parts is done in [62]. Finally V. Bally et al. estimate in [60] the probability that a diffusion process remains in a tube around a smooth function.

3.5.2 Monte-Carlo and Multi-level Monte-Carlo methods

Error bounds of MLMC. In [81], B. Jourdain and A. Kebaier are interested in deriving non-asymptotic error bounds for the multilevel Monte Carlo method. As a first step, they deal with the explicit Euler discretization of stochastic differential equations with a constant diffusion coefficient. As long as the deviation is below an explicit threshold, they check that the multilevel estimator satisfies a Gaussian-type concentration inequality optimal in terms of the variance.

Approximation of conditional expectations. The approximation of conditional expectations and the computation of expectations involving nested conditional expectations are important topics with a broad range of applications. In risk management, such quantities typically occur in the computation of the regulatory capital such as future Value-at-Risk or CVA. A. Alfonsi et al. [47] have developed a Multilevel Monte-Carlo (MLMC) method to calculate the Solvency Capital Ratio of insurance companies at future dates. The main advantage of the method is that it avoids regression issues and has the same computational complexity as a plain Monte-Carlo method (i.e. a computational time in $O(\varepsilon^{-2})$ to reach a precision of order ε). In other contexts, one may be interested in approximating conditional expectations. To do so, the classical method consists in considering a parametrized family $\varphi(\alpha,\cdot)$ of functions, and to minimize the empirical L^2-distance $\frac{1}{M}\sum_{k=1}^{M}(Y_i-\varphi(\alpha,X_i))^2$ between the observations and their prediction. In general, it is assumed to have as many observations as explanatory variables. However, when these variables are sampled, it may be possible to sample K values of Y's for a given X_i and to minimize $\frac{1}{M}\sum_{k=1}^{M}(\frac{1}{K}\sum_{k=1}^{K}Y_i^k-\varphi(\alpha,X_i))^2$. A. Alfonsi, J. Lelong and B. Lapeyre [32] have determined the optimal value of K which minimizes the computation time for a given precision. They show that K is large when the family approximates well the conditional expectation. The computational gain can be important, especially if the computational cost of sampling Y given X is small with respect to the cost of sampling X.

3.6 Remarks

We have focused above on the research program of the last four years. We refer to the previous MathRisk activity report for a description of the research done earlier, in particular on Liquidity and Market Microstructure [51, 45], [4], dependence modelling [86], interest rate modeling [42], Robust option pricing in financial markets with imperfections [69, 98], [12, 11], Mean field control and Stochastic Differential Games [96, 80, 97], Stochastic control and optimal stopping (games) under nonlinear expectation [70, 73, 71, 72], robust utility maximization [95, 97, 74], Generalized Malliavin calculus and numerical probability.

4 Application domains

4.1 Financial Mathematics, Insurance

The domains of application are quantitative finance and insurance with emphasis on risk modeling and control. In particular, the project-team Mathrisk focuses on financial modeling and calibration, systemic risk, option pricing and hedging, portfolio optimization, risk measures.

5 Social and environmental responsibility

Our work aims to contribute to a better management of risk in the banking and insurance systems, in particular by the study of systemic risk, asset price modeling, stability of financial markets.

6 New software and platforms

6.1 New software

6.1.1 PREMIA

Keywords: Financial products, Computational finance, Option pricing, Machine learning

Scientific Description: Premia is a numerical platform for computational finance. It is designed for option pricing, hedging and financial model calibration. Premia is developed by the MathRisk project team in collaboration with a consortium of financial institutions. The Premia project keeps track of the most recent advances in the field of computational finance in a well-documented way. It focuses on the implementation of numerical analysis techniques for both probabilistic and deterministic numerical methods. An important feature of the platform Premia is the detailed documentation which provides extended references in option pricing. Premia contains various numerical algorithms: deterministic methods (Finite difference and finite element algorithms for partial differential equations, wavelets, Galerkin, sparse grids ...), stochastic algorithms (Monte-Carlo simulations, quantization methods, Malliavin calculus based methods), tree methods, approximation methods (Laplace transforms, Fast Fourier transforms...) These algorithms are implemented for the evaluation of vanilla and exotic options on equities, interest rate, credit, energy and insurance products. Moreover Premia provides a calibration toolbox for Libor Market model and a toolbox for pricing Credit derivatives. The latest developments of the software address evaluation of financial derivative products, risk management and computations of risk measures required by new financial regulation. They include the implementation of advanced numerical algorithms taking into account model dependence, counterparty credit risk, hybrid features, rough volatility and various nonlinear effects. A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging,

Functional Description: Premia is a software designed for quantitative finance, developed by the MathRisk project team in collaboration with a consortium of financial institutions presently composed of Crédit Agricole CIB and NATIXIS. The Premia project keeps track of the most recent advances in computational finance and focuses on the implementation of numerical techniques to solve financial problems. An important feature of the platform Premia is its detailed documentation which provides extended references in computational finance. Premia is a powerful tool to assist Research and Development professional teams in their day-to-day duty. It is also a useful support for academics who wish to perform tests on new algorithms or pricing methods. Besides being a single entry point for accessible overviews and basic implementations of various numerical methods, the aim of the Premia project is: - to elaborate a powerful testing platform for comparing different numerical methods between each other, - to build a link between professional financial teams and academic researchers, - to provide a useful teaching support for Master and PhD students in mathematical finance. The project Premia has started in 1999 and is now considered as a standard reference platform for quantitative finance among the academic mathematical finance community.

Release Contributions: A big effort has been put these last years on the development and implementation of deep learning techniques using neural network approximations, and Machine Learning algorithms in finance, in particular for high-dimensional American option pricing, high-dimensional PDEs, deep hedging.The latest developments of the software address also the evaluation of financial derivative products, risk management and computations of risk measures by advanced numerical algorithms taking into account model dependence, counterparty credit risk (computations of XVA), hybrid features, rough stochastic volatility models and various new regulations. Nested Monte Carlo strategies with GPU optimizations, and Chebyshev Interpolation method for Parametric Option Pricing have been implemented. We have also developed our activity on insurance contracts, in particular on the computation of risk measures (Value at Risk, Condition Tail Expectation) of variable annuities contracts like GMWB (guaranteed minimum withdrawal benefit) including taxation and customers mortality modeling.

News of the Year: Release 24 of the Premia software has been delivered to the Consortium in March 2022. It contains the following new implemented algorithms dedicated to Machine Learning in finance: • Deep Hedging. H.Buhler, L. Gonon, J. Teichmann, B. Wood Quantitative Finance, Volume 19-8, 2019. • Deep neural network framework based on backward stochastic differential equations for pricing and hedging American options in high dimensions. Y.Chen J.W.L. Wan Quantitative Finance, 21-1, 2021 • Deep learning for ranking response surfaces with applications to optimal stopping problems. R.Hu Quantitative Finance, 21-1, 2021 • DGM: A deep learning algorithm for solving partial differential equations. J.Sirignano K.Spiliopoulos Journal of Computational Physics 375, 2018 • Financial option valuation by unsupervised learning with artificial neural networks. B. Salvador, C. W. Oosterlee, R. van der Meer • Equal Risk Pricing of Derivatives with Deep Hedging. A.Carbonneau F.Godin Quantitative Finance, 2020 • Quant GANs: Deep Generation of Financial Time Series. M.Wiese, R. Knobloch, R.Korn, P.Kretschmer Quantitative Finance, 20-9, 2020 • Artificial neural network for option pricing with and without asymptotic correction. H.Funahashi Quantitative Finance, 2020 • Pricing Bermudan options using regression trees/random forests. Z. El Filali Ech-Chafiq, P. Henry-Labordere, J.Lelong • Moving average options: Machine Learning and Gauss-Hermite quadrature for a double non-Markovian problem. L.Goudenge A.Molent A.Zanette European Journal of Operational Research 2022

Pricing of Equity Derivatives: • American options in the Volterra Heston model. S.Pulido, E.Chevalier, E.Zuniga. • Sinh-Acceleration: Efficient Evaluation of Probability Distributions, Option Pricing, and Monte-Carlo Simulations. S. Boyarchenko, S.Levendorskii International Journal of Theoretical and Applied Finance 22-03,2019 • A Simple Wiener-Hopf Factorization Approach for Pricing Double Barrier Options. O. Kudryavtsev In: Karapetyants A.N., Pavlov I.V., Shiryaev A.N. (eds) Operator Theory and Har- monic Analysis. OTHA 2020. Springer Proceedings in Mathematics Statistics, vol 358.

URL: `http://www.premia.fr`

Publications: hal-03436046, hal-01940715, hal-01873346, hal-03810106, hal-03526905, hal-03013606, hal-02183587

Contact: Agnes Sulem

Participants: Agnes Sulem, Antonino Zanette, Aurélien Alfonsi, Benjamin Jourdain, Jerome Lelong, Bernard Lapeyre, Ahmed Kebaier, Ludovic Goudenège

Partners: Inria, Ecole des Ponts ParisTech

7 New results

7.1 Control of systemic risk in a dynamic framework

Participants: A. Sulem, H. Amini, A. Minca, , Z. Cao.

Default cascades in sparse heterogeneous financial networks. A. Sulem and H. Amini are supervising a PhD student (Z. Cao) on the control of interbank contagion, dynamics and stability of complex financial networks, by using techniques from random graphs and stochastic control. We have obtained limit results for default cascades in sparse heterogeneous financial networks subject to an exogenous macroeconomic shock in [53]. These limit theorems for different system-wide wealth aggregation functions allow us to provide systemic risk measures in relation with the structure and heterogeneity of the financial network. These results are applied to determine the optimal policy for a social planner to target interventions during a financial crisis, with a budget constraint and under partial information of the financial network. Banks can impact each other due to large-scale liquidations of similar assets or non-payment of liabilities. In [54], we present a general tractable framework for understanding the joint impact of fire sales and default cascades on systemic risk in complex financial networks. The effect of heterogeneity in network structure and price impact function on the final size of default cascade and fire sales loss is investigated.

7.2 Mean-field (Graphon) Backward Stochastic Differential Equations and systemic risk measures

Participants: A. Sulem, R. Chen, A. Minca, R. Dumitrescu, Z. Cao, H. Amini.

Agnès Sulem, Rui Chen, Andreea Minca, Roxana Dumitrescu have studied mean-field BSDEs with a generalized mean-field operator which can capture system influence with higher order interactions such as those occurring in an inhomogeneous random graph.

We interpret the BSDE solution as a dynamic *global* risk measure for a representative bank whose risk attitude is influenced by the system. This influence can come in a wide class of choices, including the average system state or average intensity of system interactions [24].

This opens the path towards using dynamic risk measures induced by mean-field BSDE as a complementary approach to systemic risk measurement.

Extensions to *Graphon BSDEs* with jumps are studied by H. Amini, A. Sulem, and their PhD student Z. Cao in [35]. The use of graphons has emerged recently in order to analyze heterogeneous interaction in mean-field systems and game theory. Existence, uniqueness, measurability and stability of solutions under some regularity assumptions are established. We also prove convergence results for interacting mean-field particle systems with inhomogeneous interactions to graphon mean-field BSDE systems.

7.3 Optimal stopping

Participants: D. Lamberton.

D. Lamberton and Tiziano De Angelis (University of Torino) are working on the optimal stopping problem of a one dimensional diffusion in finite horizon. They develop a probabilistic approach to the regularity of the associated free boundary problem.

7.4 (Martingale) Optimal transport and mean-field systems

Participants: A. Alfonsi, B. Jourdain, V. Bally, G. Pagès, A. Tse, R. Flenghi, A. Sulem.

7.4.1 Optimal transport

B. Jourdain and A. Tse have extended to non-linear functionals of the empirical measure of independent and identically distributed random vectors the central limit theorem which is well known for linear functionals. The main tool permitting this extension is the linear functional derivative, one of the notions of derivation on the Wasserstein space of probability measures that have recently been developed. In [39], R. Flenghi and B. Jourdain relax first the equal distribution assumption and then the independence property to be able to deal with the successive values of an ergodic Markov chain.

Wasserstein projections in the convex order were first considered in the framework of weak optimal transport, and found application in various problems such as concentration inequalities and martingale optimal transport. In dimension one, it is well-known that the set of probability measures with a given mean is a lattice w.r.t. the convex order. In [40], B. Jourdain, W. Margheriti and G. Pammer prove that, contrary to the minimum and maximum in the convex order, the Wasserstein projections are Lipschitz continuity w.r.t. the Wasserstein distance in dimension one. Moreover, they provide examples that show sharpness of the obtained bounds for the 1-Wasserstein distance.

7.4.2 Convex order

In [41], B. Jourdain and G. Pagès are interested in comparing solutions to stochastic Volterra equations for the convex order on the space of continuous $\mathbb{R}^d$-valued paths and for the monotonic convex order when $d = 1$. Even if in general these solutions are neither semi-martingales nor Markov processes, they are able to exhibit conditions on their coefficients enabling the comparison. The approach consists in first comparing their Euler schemes and then taking the limit as the time step vanishes. They consider two types of Euler schemes depending on the way the Volterra kernels are discretized. The conditions ensuring the comparison are slightly weaker for the first scheme than for the second one and this is the other way round for convergence. Moreover, they extend the integrability needed on the starting values in the existence and convergence results in the literature to be able to only assume finite first order moments, which is the natural framework for convex ordering.

7.4.3 Approximation of Boltzmann and Mc-Kean Vlasov equations

V. Bally and A. Alfonsi have studied existence, uniqueness and Euler approximation for jump type stochastic equations of Bolzmann and Mc Kean-Vlasov type [44]. The specificity of their approach is to use a methodology based on stochastic flows and the sewing lemma which has been introduced in the rough path theory. Recently, V. Bally and his Phd student Y. Qin use Malliavin calculus for jump processes to prove convergence of the Euler scheme (for the above mentioned equations) in total variation distance [19](while in [44] only the Wasserstein distance is considered). Moreover, they study the approximation of the invariant measure by an adaptative Euler scheme, as introduced by Lamberton and Pagès and recently studied by Panloup and Pagès.

7.5 Financial markets modeling

Participants: J. Guyon, S. Mustapha.

Joint SPX/VIX calibration. J. Guyon and S. Mustapha calibrate neural stochastic differential equations jointly to S&P 500 smiles, VIX futures, and VIX smiles. Drifts and volatilities are modeled as neural networks. Minimizing a suitable loss allows them to fit market data for multiple S&P 500 and VIX maturities. A one-factor Markovian stochastic local volatility model is shown to fit both smiles and VIX futures within bid-ask spreads. The joint calibration actually makes it a pure path-dependent volatility model, confirming the findings in [79].

7.6 Insurance modeling

Participants: A. Alfonsi, B. Jourdain, N. Vadillo Fernandez.

Validation of economic scenarios. Real-world economic scenarios provide stochastic forecasts of economic variables like interest rates, equity stocks or indices, or inflation and are widely used in the insurance sector. Unlike risk-neutral scenarios, they aim to be realistic in view of the historical data and/or expert expectations of future incomes. In [37], H. Andrës, A. Boumezoued and B. Jourdain propose a new approach for the validation of real-world economic scenario motivated by insurance applications. This approach is based on the statistical test developed by Chevyrev and Oberhauser and relies on the notions of signature and maximum mean distance. This test allows to check whether two samples of stochastic processes paths come from the same distribution. Their contribution is to apply this test to two stochastic processes, namely the fractional Brownian motion and the Black-Scholes dynamics. They analyze its statistical power based on numerical experiments under two constraints: 1. they work in an asymetric setting in which they compare a large sample that represents simulated real-world scenarios and a small sample that mimics information from historical data, both with a monthly time step as often considered in practice and 2. they make the two samples identical from the perspective of validation methods used in practice, i.e. they impose that the marginal distributions of the two samples are the same at a given one-year horizon. By performing specific transformations of the signature, they obtain high statistical powers and demonstrate the potential of this validation approach for real-world economic scenarios. they also discuss several challenges related to the numerical implementation of this approach, and highlight its domain of validity in terms of distance between models and the volume of data at hand.

Interest rates for insurance: efficient calibration and derivatives valuation. In view of the long lifetime of insurance contracts, major part of insurers/reinsurers assets portfolios is composed of bonds. Consequently, the main financial risk the insurers are exposed to is the interest rates risk. The thesis of S. Mehalla [92] (superviser B. Lapeyre) is dedicated to the study of efficient calibration procedures for interest rates models, such as as LIBOR Market Model, used by insurance/reinsurance undertakings. Financial models with stochastic volatility factor are studied. A new method to efficiently price swap rates derivatives under the LIBOR Market Model with Stochastic Volatility and Displaced Diffusion is proposed (see [17]). Fast calibration based on optimization algorithms are studied (see [16], [56]).

Stochastic modeling of the Temperature for pricing weather derivatives Insurance companies sell insurance contracts that give a protection against weather fluctuations, typically on rain or temperature. These contracts are managed then using weather derivatives. In [34], A. Alfonsi and his PhD student N. Vadillo Fernandez propose a new stochastic volatility model for the temperature in order to price derivatives on the climate (Heating Degree Day index). They develop a simple and efficient estimation of the model parameters on historical data and provide efficient numerical pricing methods.

7.7 Numerical probability

Participants: A. Alfonsi, B. Jourdain, A. Kebaier, V. Bally, O. Bencheikh, B. Jourdain, B. Lapeyre, J. Lelong, A. Zanette, L. Goudenège, A. Molent, H. Madmoun.

7.7.1 Approximations of Stochastic Differential Equations (SDEs)

High order schemes. A. Alfonsi and V. Bally [43] have proposed a method to construct high order approximations from an elementary approximation scheme by using it on suitable random grids. This method can be applied to any general semigroup, provided that the "elementary scheme" satisfies some properties, which hold e.g. for the Euler scheme for SDEs with regular coefficients. In [33], A. Alfonsi and E. Lombardo develop high order schemes for the weak error for the Cox-Ingersoll-Ross process, that has singular diffusion coefficient. This process is widely used in mathematical finance to model the interest rate or the volatility as in the Heston model. The method is based on the construction proposed in [43].

Approximation of Stochastic Volterra Equations (SVE). Alfonsi and Kebaier [31] have analyzed the L^2-error when approximating a SVE by another SVE with an approximating kernels. Then, for completely monotone kernels, they develop approximations obtained as weighted sums of exponentials. The approximating SVE can be written as an SDE (in higher dimension) for which the Euler scheme has a time complexity in $O(n)$. A special effort is done to get accurate approximation of the fractional kernel; the efficiency of the method is illustrated on the rough Heston model. A. Alfonsi and E. Lombardo are working on Multilevel Monte-Carlo methods for these processes.

Euler-Maruyama approximation. O. Bencheikh and B. Jourdain have studied the Euler-Maruyama approximation of SDEs with constant diffusion coefficient, when the drift coefficient is bounded and merely measurable. In [22], they prove weak convergence with order 1/2 in total variation distance. In [83], this result is generalized by B. Jourdain and S. Menozzi to the case when the drift coefficient has $L_t^q - L_x^\rho$ integrability : under the condition $\frac{d}{\rho} + \frac{2}{q} < 1$ supposed by Krylov and Röckner to obtain the existence of a unique strong solution in dimension d, they prove that both the diffusion and the Euler scheme admit transition densities and that the difference between these densities is bounded from above by the time-step to $\frac{1}{2}(1-(\frac{d}{\rho}+\frac{2}{q}))$ multiplied by some centered Gaussian density.

7.7.2 Numerical approximation of conditional expectations

A.Alfonsi, B. Lapeyre and J. Lelong have revisited the classical Monte-Carlo regression problem to approximate the conditional expectation $\mathbb{E}[Y|X]$ when it is possible to sample Y given X and then to generate $K \in \mathbb{N}^*$ values of $Y_{i,k}$ for a given value X_i (See [32]). They determine the optimal value of K that minimizes the computation cost for a given precision.

7.7.3 Optimal transport and mean-field SDEs

In [23], O. Bencheikh and B. jourdain analyse the rate of convergence of a system of interacting particles with mean-field rank based interaction in the drift coefficient and constant diffusion coefficient.

7.7.4 Abstract Malliavin calculus and convergence in total variation

In [19], V. Bally and his PhD student Yifen Qin obtain total variation distance result between a jump-equation and its Gaussian approximation by Malliavin calculus techniques.

7.8 Deep learning for large dimensional financial problems

Participants: B. Lapeyre, J. Lelong, A. Zanette, L. Goudenège, A. Molent, H. Madmoun.

Neural networks and Machine Learning techniques for high dimensional American options. The pricing of American option or its Bermudan approximation amounts to solving a backward dynamic programming equation, in which the main difficulty comes from the conditional expectation involved in the computation of the continuation value.

In [90], B. Lapeyre and J. Lelong study neural networks approximations of conditional expectations. They prove the convergence of the well-known Longstaff and Schwartz algorithm when the standard least-square regression on a finite-dimensional vector space is replaced by a neural network approximation, and illustrate the numerical efficiency of the method on several numerical examples. Its stability with respect to a change of parameters as interest rate and volatility is shown. The numerical study proves that training neural network with only a few chosen points in the grid of parameters permits to price efficiently for a whole range of parameters.

In [76], two efficient techniques, called GPR Tree (GRP-Tree) and GPR Exact Integration (GPR-EI), are proposed to compute the price of American basket options. Both techniques are based on Machine Learning, exploited together with binomial trees or with a closed formula for integration. On the exercise dates, the value of the option is first computed as the maximum between the exercise value and the continuation value and then approximated by means of Gaussian Process Regression. In [77], an efficient method is provided to compute the price of multi-asset American options, based on Machine Learning, Monte Carlo simulations and variance reduction techniques. Numerical tests show that the proposed algorithm is fast and reliable, and can handle American options on very large baskets of assets, overcoming the curse of dimensionality issue.

• *Machine Learning in the Energy and Commodity Market.* Evaluating moving average options is a computational challenge for the energy and commodity market, as the payoff of the option depends on the prices of underlying assets observed on a moving window. An efficient method for pricing Bermudan style moving average options is presented in [25], based on Gaussian Process Regression and Gauss-Hermite quadrature. This method is tested in the Clewlow-Strickland model, the reference framework for modeling prices of energy commodities, the Heston (non-Gaussian) model and the rough-Bergomi model, which involves a double non-Markovian feature, since the whole history of the volatility process impacts the future distribution of the process.

Investment strategies based on Machine learning algorithms. The goal of the thesis of H. Madmoun [91] (Supervisor: B. Lapeyre) is to show how portfolio allocation can benefit from the development of Machine Learning algorithms. Two investment strategies based on such algorithms are proposed. The first strategy (Low Turbulence Model) is an asset allocation method based on an interpretable low-dimensional representation framework of financial time series combining signal processing, deep neural networks, and bayesian statistics. The second one (Attention Based Ranking Model) is a stock picking strategy for large- cap US stocks.

8 Bilateral contracts and grants with industry

8.1 Bilateral contracts with industry

- Consortium PREMIA, Natixis - INRIA
- Consortium PREMIA, Crédit Agricole Corporate Investment Bank (CA - CIB) - INRIA
- AXA Joint Research Initiative on Numerical methods for the ALM, from September 2017 to August 2020. PhD grant of Adel Cherchali, Supervisor: A. Alfonsi.
- CIFRE agreement Braham gardens/Ecole des Ponts

  PhD thesis of Hachem Madmoun: "Creating Investment Strategies based on Machine Learning Algorithms"
- CIFRE agreement AXA Climate/ENPC PhD thesis of Nerea Vadillo Fernandez. Supervisor: A. Alfonsi

8.2 Bilateral grants with industry

- Chair Ecole Polytechnique-Ecole des Ponts ParisTech-Sorbonne Université-Société Générale "Financial Risks" of the Risk fondation.

  Participants: Aurélien Alfonsi, Benjamin Jourdain.

  Postdoctoral grants : G.Szulda
- Chair Ecole des Ponts ParisTech - Université Paris-Cité - BNP Paribas "Futures of Quantitative Finance"

  Participants: Julien Guyon.

9 Partnerships and cooperations

Participants: Mathrisk .

9.1 International initiatives

9.1.1 Participation in International Programs

Edoardo Lombardo, international PhD grant between ENPC and Univ. Tor Vergata, Roma

9.2 International research visitors

9.2.1 Visits of international scientists

International visits to the team Hamed Amini, Associate Professor (University of Florida): Collaboration with Agnès Sulem on Dynamic contagion in financial networks; co-advising of the PhD thesis of Z. Cao, Research stay, Spring and Summmer 2022.

9.3 National initiatives

- FMSP (Fondation Sciences Mathématiques de Paris) PhD grants :
  - Cofund MathInParis program: K. Shao (2021 - Present)(INRIA)
  - DIM Math Innov: Z. Cao (2020- Present)(INRIA)
  - DIM Math Innov: Y. Qin (2020- Present)(UGE)
- Labex Bezout

10 Dissemination

Mathrisk Members

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

- A. Alfonsi:

  Co-organizer of the Mathrisk seminar “Méthodes stochastiques et finance”

  Co-organizer of the Bachelier (Mathematical Finance) seminar (IHP, Paris).

- V. Bally

  - Organizer of the seminar of the LAMA laboratory, Université Gustave Eiffel.

  - Organizer of the mini symposium " Stochastic Equations", XVth French-Romanian Colloquium on Applied Mathematics, Toulouse, 29 August - 2 September 2022

- A. Sulem

  - Co-organizer of the seminar INRIA-MathRisk /Université Paris Diderot LPSM “Numerical probability and mathematical finance”

  - Member of the scientific committee of the London-Paris Bachelier workshop (September 2022), IHP, Paris.

10.1.2 Journal

Member of the editorial boards

- J. Guyon

  Associate editor of

  - Finance and Stochastics
  - Quantitative Finance
  - SIAM Journal on Financial Mathematics
  - Journal of Dynamics and Games

- B. Jourdain

  Associate editor of

  - ESAIM : Proceedings and Surveys
  - Stochastic Processes and their Applications (SPA)
  - Stochastic and Partial Differential Equations : Analysis and Computations

- D. Lamberton

  Associate editor of

  - Mathematical Finance,
  - ESAIM Probability & Statistics

- A. Sulem

  Associate editor of

  - *Mathematics*, (Financial Mathematics Section)
  - *Journal of Mathematical Analysis and Applications* (JMAA)
  - *SIAM Journal on Financial Mathematics* (SIFIN)

Reviewer - reviewing activities

- J. Guyon Reviewer for Canadian Journal of Statistics
- B. Jourdain : Reviewer for *Mathematical Reviews*
- A. Sulem: Reviewer for *Mathematical Reviews*

10.1.3 Invited talks

- A. Alfonsi
  - 06 01 2022: "A synthetic model for ALM in life insurance and numerical methods for SCR computation", Berlin Seminar on Mathematical Finance.
  - 22 03 2022: "Approximation of Optimal Transport problems with marginal moments constraints", Stochastic Mass Transports (22w5166), Banff.
  - 07 04 2022: "Approximation of Optimal Transport problems with marginal moments constraints", Séminaire commun MathRisk-LPSM
  - 21 04 2022: "A synthetic model for ALM in life insurance and numerical methods for SCR computation", Labex Bezout.
- V. Bally
  - Conference *Kolmogorov Operators and their Applications*; Cortona June 13-17, 2022. Talk : Construction of Boltzmann and Mc-Kean Vlasov Type Flows.
  - Conference Stochastic Analysis and Partial differential Equations, Barcelona 30 May - 03 June 2022. talk: Construction of Boltzmnn and Mc-Kean Vlasov Type Flows
- J. Guyon
  - Institut Henri Poincaré, Bachelier seminar, September 2022.
  - Ecole des Ponts ParisTech, Stochastic Methods in Finance seminar, October 2022.
  - ETH Zurich, Talks in Financial and Insurance Mathematics, October 2022.
  - WBS, 18th Quantitative Finance Conference, Dubrovnik, October 2022.
  - QuantMinds 2022 Conference, Barcelona, November 2022.
  - University of Oxford, Statistics and Machine Learning in Finance Seminar Series, December 2022.

- B. Jourdain
  - Worshop on *Advances in Stochastic Control and Optimal Stopping*, Marseille, 12 September 2022 : Approximation of martingale couplings on the real line in the weak adapted topology
  - 9th colloquium on BSDEs and Mean Field Systems, Annecy, 29 June 2022 : Central limit theorem over nonlinear functionals of empirical measures and fluctuations of interacting particle systems
  - Séminaire de probabilitÈs, Sorbonne université, 31 may 2022 : Stabilité du problème de transport optimal martingale faible
  - BIRS workshop Stochastic Mass Transports, 25 march 2022 : Approximation of martingale couplings on the real line in the weak adapted topology
  - Ritsumeikan University Seminar, 13 January 2022 : Convergence rate of the Euler-Maruyama scheme applied to diffusion processes with irregular drift coefficient and additive noise
- D. Lamberton
  - A probabilistic approach to the regularity of a free boundary. XVth French-Romanian Colloquium on Applied Mathematics, Toulouse, August 2022.
  - Regularity of the free boundary: a probabilistic approach. Advances in Stochastic Control and Optimal Stopping with Applications in Economics and Finance. Luminy, September 2022.
  - Régularité de la frontière libre d'un problème d'arrêt optimal : une approche probabiliste. Groupe de travail Mathématiques financières et actuarielles, probabilités numériques. Sorbonne Université, October 2022.
- A. Sulem

  Keynote speaker, 19th e-Summer School in Risk Finance and Stochastics 28/9 - 30/9/2022, Athens

10.1.4 Research administration

- A. Alfonsi
  - Deputy director of the CERMICS.
  - In charge of the Master "Finance and Data" at the Ecole des Ponts.
- V. Bally
  - Responsible of the Master 2, option finance, Université Gustave Eiffel
  - Member of the LAMA committee, UGE.
- A. Sulem
  - Member of the Scientific Committee of AMIES(Agence pour les Mathématiques en Interaction avec l'Entreprise et la Société)
  - Corresponding member of the Operational Committee for the assesment of Legal and Ethical risks (COERLE) at INRIA Paris research center
  - Member of the Committee for INRIA international Chairs
  - Member of the Nominating Committee of the Bachelier Finance Society

10.2 Teaching - Supervision - Juries

- A. Alfonsi
  - “Probabilités”, first year course at the Ecole des Ponts.
  - “Données Haute Fréquence en finance”, lecture for the Master at UPEMLV.
  - “Mesures de risque”, Master course of UPEMLV and Sorbonne Université.
  - Professeur chargé de cours at Ecole Polytechnique.
- V. Bally
  - "Taux d’Intêret", M2 Finance, Université Gustave Eiffel
  - "Calcul de Malliavin et applications en finance", M2 Finance, UGE
  - 2018-Present: "Analyse du risque" M2 Actuariat, UGE
  - 2018-Present: "Processus Stochastiques" M2 Recherche, UGE
  - ProbabilitÈs approfondies M1, UGE
- J. Guyon

  course "Probability Theory", 1st year ENPC
- B. Jourdain
  - course "Mathematical finance", 2nd year ENPC
  - course "Monte-Carlo methods", 3rd year ENPC and Research Master MathÈmatiques et Application, university Gustave Eiffel
  - J.-F. Delmas, B.Jourdain course "Jump processes with applications to energy markets", 3rd year ENPC and Research Master MathÈmatiques et Application, university Gustave Eiffel
  - course "Monte-Carlo Markov chain methods and particle algorithms", Research Master ProbabilitÈs et ModËles AlÈatoires, Sorbonne UniversitÈ
- D. Lamberton
  - "Calcul stochastique pour la finance", master 1 course, Université Gustave Eiffel.
  - "Arbitrage, volatilité et gestion de portefeuille", master 2 course, Université Gustave Eiffel.
  - "Intégration et probabilités", L3 course, Université Gustave Eiffel.
- A. Sulem

  Master of Mathematics, Université du Luxembourg, Responsible of the course on "Numerical Methods in Finance", and lectures (22 hours)

10.2.1 Supervision

- PhD defended :
  - Hachem Madmoun, defended at Ecole des Ponts Paris Tech, 30 November 2022, Adviser: B. Lapeyre.
- PhD in progress :
  - Roberta Flenghi (started in January 2021) "Central limit theorems for nonlinear functionals of the empirical measure of correlated random variables", supervised by B. Jourdain
  - HervÈ AndrËs (started in June 2021) "Dependence modelling in economic scenario generation for insurance", supervised by B. Jourdain

  - Kexin Shao (started in october 2021) "Martingale optimal transport and financial applications", supervised by B. Jourdain and A. Sulem
  - Zhonguyan Cao, "Dynamics and Stability of Complex Financial networks", Université Paris-Dauphine, Supervisor: Agnès Sulem, Dim Mathinnov doctoral allocation, started October 2020
  - Edoardo Lombardo, "High order numerical approximation for some singular stochastic processes and related PDEs", started in November 2020, International PhD, advisors: Aurélie Alfonsi and Lucia Caramellino (Tor Vegata Roma University),
  - Nerea Vadillo Fernandez (CIFRE AXA Climate), "Risk valuation for weather derivatives in index-based insurance", started in November 2020, supervised by A. Alfonsi
  - Yfen Qin, "Regularity properties for non linear problems", Dim-Mathinnov doctoral allocation, supervisor: V. Bally

10.2.2 Juries

- B. Jourdain
  - PhD of Antoine Bienvenu, defended on July 4, Sorbonne université,
  - PhD of Laetitia Della Maestra, defended on November 30, Université Paris-Dauphine
  - PhD of Mohan Yang, defended on December 8, Université Paris Cité, B. Jourdain reviewer
- J. Guyon

  PhD defense of William Lefebvre, LPSM, Université Paris-Cité, 09/12/2022,
- A. Sulem
  - President of the jury for the Prix Marc Yor 2022
  - Member of the Committee for the recruitment of an associate Professor in "Financial Mathematics, statistics, numerical probability", Université Paris-Cité,
  - President of the jury for the PhD defense of William Lefebvre, *Méthodes de Contrôle stochastique appliquées à la construction de portefeuille, au contrôle avec retard et à la résolution d'EDPs*, LPSM, Université Paris-Cité, 09/12/2022,
  - President of the jury for the PhD defense of Hachem Madmoun,*Creating Investment Strategies based on Machine Learning Algorithms*, ENPC, 30/11/2022
  - Reviewer for the PhD thesis of Zineb El Filali Ech-Chafiq, *Applications de l'apprentissage automatique en finance*, Univ. Grenoble Alpes, 27/09/2022
  - PhD of Maximilien Germain, *Méthodes d'apprentissage automatique pour la résolution de problèmes de contrôle stochastique et d'équations aux dérivées partielles en grande dimension*, LPSM, Université Paris-Cité, 20/06/2022

10.3 Popularization

- Ligue des champions : le Bayern Munich, adversaire le plus probable du PSG en huitièmes de finale, Le Monde, November 7, 2022, J. Guyon
- Interview in the French sports daily L'Equipe on the calculation of draw probabilities for the Round of 16 of the Champions League, November 3, 2022, J. Guyon

11 Scientific production

11.1 Major publications

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