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ACTIVITY REPORT

Project-Team

MOKAPLAN

Advances in Numerical Calculus of Variations

IN COLLABORATION WITH: CEREMADE

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

Contents

Project-Team MOKAPLAN

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2 Overall objectives

2.1 Introduction

The last decade has witnessed a remarkable convergence between several sub-domains of the calculus of variations, namely optimal transport (and its many generalizations), infinite dimensional geometry of diffeomorphisms groups and inverse problems in imaging (in particular sparsity-based regularization). This convergence is due to (i) the mathematical objects manipulated in these problems, namely sparse measures (e.g. coupling in transport, edge location in imaging, displacement fields for diffeomorphisms) and (ii) the use of similar numerical tools from non-smooth optimization and geometric discretization schemes. Optimal Transportation, diffeomorphisms and sparsity-based methods are powerful modeling tools, that impact a rapidly expanding list of scientific applications and call for efficient numerical strategies. Our research program shows the important part played by the team members in the development of these numerical methods and their application to challenging problems.

2.2 Static Optimal Transport and Generalizations

Optimal Transport, Old and New. *Optimal Mass Transportation* is a mathematical research topic which started two centuries ago with Monge's work on the "Théorie des déblais et des remblais" (see [101]). This engineering problem consists in minimizing the transport cost between two given mass densities. In the 40's, Kantorovich [110] introduced a powerful linear relaxation and introduced its dual formulation. The *Monge-Kantorovich* problem became a specialized research topic in optimization and Kantorovich obtained the 1975 Nobel prize in economics for his contributions to resource allocations problems. Since the seminal discoveries of Brenier in the 90's [64], Optimal Transportation has received renewed attention from mathematical analysts and the Fields Medal awarded in 2010 to C. Villani, who gave important contributions to Optimal Transportation and wrote the modern reference monographs [134, 135], arrived at a culminating moment for this theory. Optimal Mass Transportation is today a mature area of mathematical analysis with a constantly growing range of applications. Optimal Transportation has also received a lot of attention from probabilists (see for instance the recent survey [114] for an overview of the Schrödinger problem which is a stochastic variant of the Benamou-Brenier dynamical formulation of optimal transport). The development of numerical methods for Optimal Transportation and Optimal Transportation related problems is a difficult topic and comparatively underdeveloped. This research field has experienced a surge of activity in the last five years, with important contributions of the MOKAPLAN group (see the list of important publications of the team). We describe below a few of recent and less recent Optimal Transportation concepts and methods which are connected to the future activities of MOKAPLAN :

Brenier's theorem [66] characterizes the unique optimal map as the gradient of a convex potential. As such Optimal Transportation may be interpreted as an infinite dimensional optimisation problem under "convexity constraint": i.e. the solution of this infinite dimensional optimisation problem is a convex potential. This connects Optimal Transportation to "convexity constrained" non-linear variational problems such as, for instance, Newton's problem of the body of minimal resistance. The value function of the optimal transport problem is also known to define a distance between source and target densities called the *Wasserstein distance* which plays a key role in many applications such as image processing.

Monge-Ampère Methods. A formal substitution of the optimal transport map as the gradient of a convex potential in the mass conservation constraint (a Jacobian equation) gives a non-linear Monge-Ampère equation. Caffarelli [70] used this result to extend the regularity theory for the Monge-Ampère equation. In the last ten years, it also motivated new research on numerical solvers for non-linear degenerate Elliptic equations [93] [118] [56] [57] and the references therein. Geometric approaches based on Laguerre diagrams and discrete data [121] have also been developed. Monge-Ampère based Optimal Transportation solvers have recently given the first linear cost computations of Optimal Transportation (smooth) maps.

Generalizations of OT. In recent years, the classical Optimal Transportation problem has been extended in several directions. First, different ground costs measuring the "physical" displacement have been considered. In particular, well posedness for a large class of convex and concave costs has been established by McCann and Gangbo [100]. Optimal Transportation techniques have been applied for example to a Coulomb ground cost in Quantum chemistry in relation with Density Functional theory [87]. Given the densities of electrons

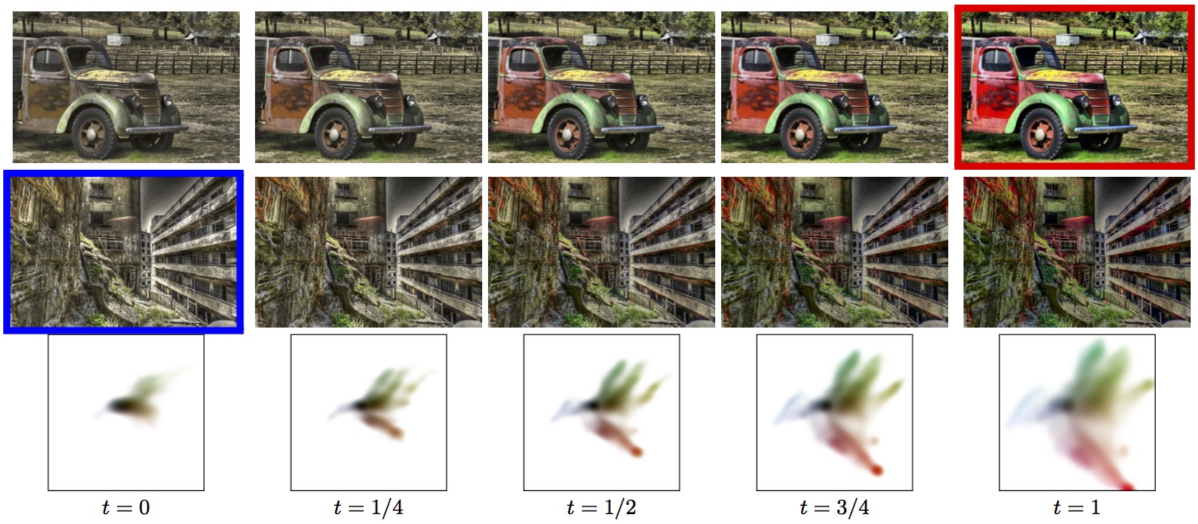

Figure 1: Example of color transfer between two images, computed using the method developed in [55], see also [130]. The image framed in red and blue are the input images. *Top and middle row:* adjusted image where the color of the transported histogram has been imposed. *Bottom row:* geodesic (displacement) interpolation between the histogram of the chrominance of the image.

Optimal Transportation models the potential energy and their relative positions. For more than more than 2 electrons (and therefore more than 2 densities) the natural extension of Optimal Transportation is the so called Multi-marginal Optimal Transport (see [125] and the references therein). Another instance of multi-marginal Optimal Transportation arises in the so-called Wasserstein barycenter problem between an arbitrary number of densities [42]. An interesting overview of this emerging new field of optimal transport and its applications can be found in the recent survey of Ghoussoub and Pass [126].

Numerical Applications of Optimal Transportation. Optimal transport has found many applications, starting from its relation with several physical models such as the semi-geostrophic equations in meteorology [105, 90, 89, 53, 117], mesh adaptation [116], the reconstruction of the early mass distribution of the Universe [98, 67] in Astrophysics, and the numerical optimisation of reflectors following the Optimal Transportation interpretation of Oliker [71] and Wang [136]. Extensions of OT such as multi-marginal transport has potential applications in Density Functional Theory , Generalized solution of Euler equations [65] (DFT) and in statistics and finance [51, 99] .... Recently, there has been a spread of interest in applications of OT methods in imaging sciences [60], statistics [58] and machine learning [91]. This is largely due to the emergence of fast numerical schemes to approximate the transportation distance and its generalizations, see for instance [55]. Figure 1 shows an example of application of OT to color transfer. Figure 2 shows an example of application in computer graphics to interpolate between input shapes.

2.3 Diffeomorphisms and Dynamical Transport

Dynamical transport. While the optimal transport problem, in its original formulation, is a static problem (no time evolution is considered), it makes sense in many applications to rather consider time evolution. This is relevant for instance in applications to fluid dynamics or in medical images to perform registration of organs and model tumor growth.

In this perspective, the optimal transport in Euclidean space corresponds to an evolution where each particule of mass evolves in straight line. This interpretation corresponds to the *Computational Fluid Dynamic* (CFD) formulation proposed by Brenier and Benamou in [52]. These solutions are time curves in the space of densities and geodesics for the Wasserstein distance. The CFD formulation relaxes the non-linear mass conservation constraint into a time dependent continuity equation, the cost function remains convex but is highly non smooth. A remarkable feature of this dynamical formulation is that it can be re-cast as a convex but non smooth optimization problem. This convex dynamical formulation finds many non-trivial extensions and applications, see for instance [54]. The CFD formulation also appears to be a limit case of *Mean Fields games*

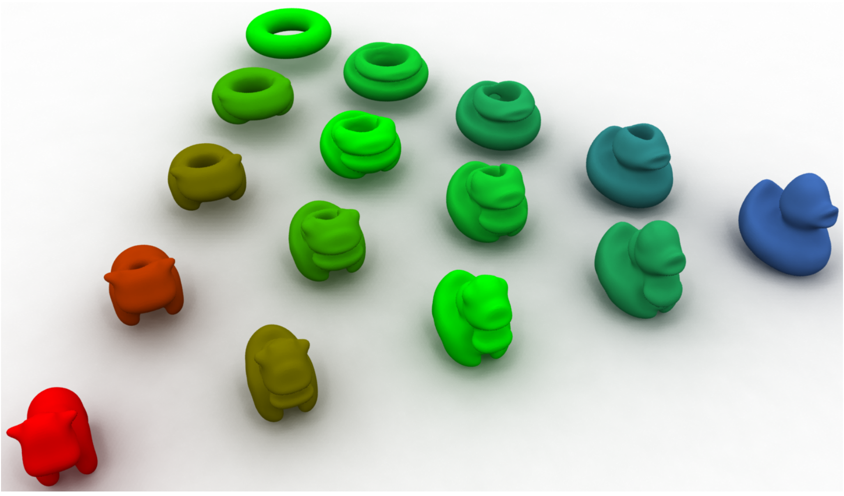

Figure 2: Example of barycenter between shapes computed using optimal transport barycenters of the uniform densities inside the 3 extremal shapes, computed as detailed in [130]. Note that the barycenters are not in general uniform distributions, and we display them as the surface defined by a suitable level-set of the density.

(MFGs), a large class of economic models introduced by Lasry and Lions [112] leading to a system coupling an Hamilton-Jacobi with a Fokker-Planck equation. In contrast, the Monge case where the ground cost is the euclidan distance leads to a static system of PDEs [61].

Gradient Flows for the Wasserstein Distance. Another extension is, instead of considering geodesic for transportation metric (i.e. minimizing the Wasserstein distance to a target measure), to make the density evolve in order to minimize some functional. Computing the steepest descent direction with respect to the Wasserstein distance defines a so-called Wasserstein gradient flow, also known as *JKO gradient flows* after its authors [109]. This is a popular tool to study a large class of non-linear diffusion equations. Two interesting examples are the Keller-Segel system for chemotaxis [108, 79] and a model of congested crowd motion proposed by Maury, Santambrogio and Roudneff-Chupin [120]. From the numerical point of view, these schemes are understood to be the natural analogue of implicit scheme for linear parabolic equations. The resolution is however costly as it involves taking the derivative in the Wasserstein sense of the relevant energy, which in turn requires the resolution of a large scale convex but non-smooth minimization.

Geodesic on infinite dimensional Riemannian spaces. To tackle more complicated warping problems, such as those encountered in medical image analysis, one unfortunately has to drop the convexity of the functional involved in defining the gradient flow. This gradient flow can either be understood as defining a geodesic on the (infinite dimensional) group of diffeomorphisms [50], or on a (infinite dimensional) space of curves or surfaces [137]. The de-facto standard to define, analyze and compute these geodesics is the "Large Deformation Diffeomorphic Metric Mapping" (LDDMM) framework of Trouvé, Younes, Holm and co-authors [50, 104]. While in the CFD formulation of optimal transport, the metric on infinitesimal deformations is just the L^2 norm (measure according to the density being transported), in LDDMM, one needs to use a stronger regularizing metric, such as Sobolev-like norms or reproducing kernel Hilbert spaces (RKHS). This enables a control over the smoothness of the deformation which is crucial for many applications. The price to pay is the need to solve a non-convex optimization problem through geodesic shooting method [122], which requires to integrate backward and forward the geodesic ODE. The resulting strong Riemannian geodesic structure on spaces of diffeomorphisms or shapes is also pivotal to allow us to perform statistical analysis on the tangent space, to define mean shapes and perform dimensionality reduction when analyzing large collection of input shapes (e.g. to study evolution of a diseases in time or the variation across patients) [72].

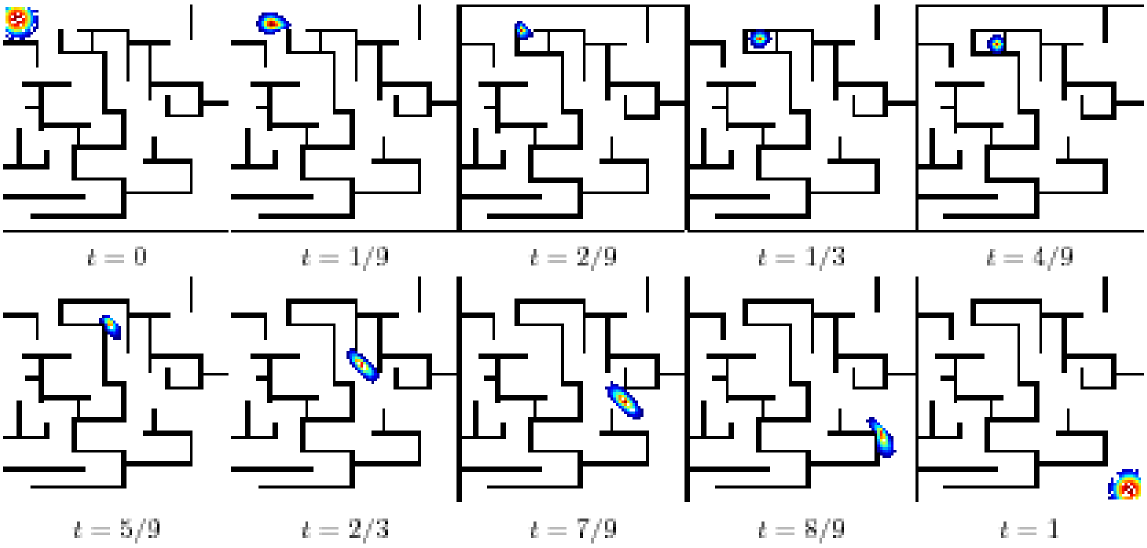

Figure 3: Examples of displacement interpolation (geodesic for optimal transport) according to a non-Euclidean Riemannian metric (the mass is constrained to move inside a maze) between to input Gaussian distributions. Note that the maze is dynamic: its topology change over time, the mass being "trapped" at time $t = 1/3$.

2.4 Sparsity in Imaging

Sparse ℓ^1 regularization. Beside image warping and registration in medical image analysis, a key problem in nearly all imaging applications is the reconstruction of high quality data from low resolution observations. This field, commonly referred to as "inverse problems", is very often concerned with the precise location of features such as point sources (modeled as Dirac masses) or sharp contours of objects (modeled as gradients being Dirac masses along curves). The underlying intuition behind these ideas is the so-called sparsity model (either of the data itself, its gradient, or other more complicated representations such as wavelets, curvelets, bandlets [119] and learned representation [138]).

The huge interest in these ideas started mostly from the introduction of convex methods to serve as proxy for these sparse regularizations. The most well known is the ℓ^1 norm introduced independently in imaging by Donoho and co-workers under the name "Basis Pursuit" [84] and in statistics by Tibshirani [131] under the name "Lasso". A more recent resurgence of this interest dates back to 10 years ago with the introduction of the so-called "compressed sensing" acquisition techniques [73], which make use of randomized forward operators and ℓ^1-type reconstruction.

Regularization over measure spaces. However, the theoretical analysis of sparse reconstructions involving real-life acquisition operators (such as those found in seismic imaging, neuro-imaging, astro-physical imaging, etc.) is still mostly an open problem. A recent research direction, triggered by a paper of Candès and Fernandez-Granda [75], is to study directly the infinite dimensional problem of reconstruction of sparse measures (i.e. sum of Dirac masses) using the total variation of measures (not to be mistaken for the total variation of 2-D functions). Several works [74, 95, 94] have used this framework to provide theoretical performance guarantees by basically studying how the distance between neighboring spikes impacts noise stability.

Low complexity regularization and partial smoothness. In image processing, one of the most popular methods is the total variation regularization [129, 68]. It favors low-complexity images that are piecewise constant, see Figure 4 for some examples on how to solve some image processing problems. Beside applications in image processing, sparsity-related ideas also had a deep impact in statistics [131] and machine learning [45]. As a typical example, for applications to recommendation systems, it makes sense to consider sparsity of the singular values of matrices, which can be relaxed using the so-called nuclear norm (a.k.a. trace norm) [46]. The underlying methodology is to make use of low-complexity regularization models, which turns out to be

Segmentation input output Zooming input output

Figure 4: Two example of application of the total variation regularization of functions. *Left:* image segmentation into homogeneous color regions. *Right:* image zooming (increasing the number of pixels while keeping the edges sharp).

equivalent to the use of partly-smooth regularization functionals [115, 133] enforcing the solution to belong to a low-dimensional manifold.

2.5 MOKAPLAN unified point of view

The dynamical formulation of optimal transport creates a link between optimal transport and geodesics on diffeomorphisms groups. This formal link has at least two strong implications that MOKAPLAN will elaborate on: (i) the development of novel models that bridge the gap between these two fields ; (ii) the introduction of novel fast numerical solvers based on ideas from both non-smooth optimization techniques and Bregman metrics.

In a similar line of ideas, we believe a unified approach is needed to tackle both sparse regularization in imaging and various generalized OT problems. Both require to solve related non-smooth and large scale optimization problems. Ideas from proximal optimization has proved crucial to address problems in both fields (see for instance [52, 128]). Transportation metrics are also the correct way to compare and regularize variational problems that arise in image processing (see for instance the Radon inversion method proposed in [55]) and machine learning (see [91]).

3 Research program

Since its creation, the Mokaplan team has made important contributions in Optimal Transport both on the theoretical and the numerical side, together with applications such as fluid mechanics, the simulation biological systems, machine learning. We have also contributed to to the field of inverse problems in signal and image processing (super-resolution, nonconvex low rank matrix recovery). In 2022, the team was renewed with the following research program which broadens our spectrum and addresses exciting new problems.

3.1 OT and related variational problems solvers *encore et toujours*

Participants: Flavien Léger , Jean-David Benamou , Guillaume Carlier , Thomas Gallouët , François-Xavier Vialard , Guillaume Chazareix , Adrien Vacher , Paul Pegon.

Asymptotic analysis of entropic OT for a small entropic parameter is well understood for regular data on compact manifolds and standard quadratic ground cost [85], the team will extend this study to more general settings and also establish rigorous asymptotic estimates for the transports maps. This is important to provide a sound theoretical background to efficient and useful debiasing approaches like Sinkhorn Divergences [96]. Guillaume Carlier, Paul Pegon and Luca Tamanini are investigating speed of convergence and quantitative stability results under general conditions on the cost (so that optimal maps may not be continuous or even fail to exist). Some sharp bounds have already been obtained, the next challenging goal is to extend the Laplace method to a nonsmooth setting and understand what entropic OT really selects when there are several optimal OT plans.

High dimensional - Curse of dimensionality We will continue to investigate the computation or approximation of high-dimensional OT losses and the associated transports [132] in particular in relation with their use in ML. In particular for Wasserstein 2 metric but also the repulsive Density Functional theory cost [97].

Back-and-forth The back-and-forth method [107, 106] is a state-of-the-art solver to compute optimal transport with convex costs and 2-Wasserstein gradient flows on grids. Based on simple but new ideas it has great potential to be useful for related problems. We plan to investigate: OT on point clouds in low dimension, the principal-agent problem in economics and more generally optimization under convex constraints [111, 123].

Transport and diffusion The diffusion induced by the entropic regularization is fixed and now well understood. For recent variations of the OT problem (Martingale OT, Weak OT see [47]) the diffusion becomes an explicit constraint or the control itself [102]. The entropic regularisation of these problems can then be understood as metric/ground cost learning [77] (see also [127]) and offers a tractable numerical method.

Wasserstein Hamiltonian systems We started to investigate the use of modern OT solvers for the SG equation [88, 53] Semi-Discrete and entropic regularization. This is a special instance Hamiltonian Systems in the sense of [43]. with an OT component in the Energy.

Nonlinear fourth-order diffusion equations such as thin-films or (the more involved) DLSS quantum drift equations are WGF. Such WGF are challenging both in terms of mathematical analysis (lack of maximum principle...) and of numerics. They are currently investigated by Jean-David Benamou, Guillaume Carlier in collaboration with Daniel Matthes. Note also that Mokaplan already contributed to a related topic through the TV-JKO scheme [78].

Lagrangian approaches for fluid mechanics More generally we want to extend the design and implementation of Lagrangian numerical scheme for a large class of problem coming from fluids mechanics (WHS or WGF) using semi-discrete OT or entropic regularization. We will also take a special attention to link this approaches with problems in machine/statistical learning. To achieve this part of the project we will join forces with colleagues in Orsay University: Y. Brenier, H. Leclerc, Q. Mérigot, L. Nenna.

L^∞ **optimal transport** is a variant of OT where we want to minimize the maximal displacement of the transport plan, instead of the average distance. Following the seminal work of [83], and more recent developments [92], Guillaume Carlier, Paul Pegon and Luigi De Pascale are working on the description of *restrictable* solutions (which are cyclically ∞-monotone) through some potential maps, in the spirit of Mange-Kantorovich potentials provided by a duality theory. Some progress has been made to partially describe cyclically quasi-motonone maps (related in some sense to cyclically ∞-monotone maps), through quasi-convex potentials.

3.2 Application of OT numerics to non-variational and non convex problems

Participants: Flavien Léger , Guillaume Carlier , Jean-David Benamou , François-Xavier Vialard.

Market design Z-mappings form a theory of non-variational problems initiated in the '70s but that has been for the most part overlooked by mathematicians. We are developing a new theory of the algorithms associated with convergent regular splitting of Z-mappings. Various well-established algorithms for matching models can be grouped under this point of view (Sinkhorn, Gale–Shapley, Bertsekas' auction) and this new perspective has the potential to unlock new convergence results, rates and accelerated methods.

Non Convex inverse problems The PhD [139] provided a first exploration of Unbalanced Sinkhorn Divergence in this context. Given enough resources, a branch of PySit, a public domain software to test misfit functions in the context of Seismic imaging will be created and will allow to test other signal processing strategies in Full Waveform Inversion. Likewise the numerical method tested for 1D reflectors in [11] coule be develloped further (in particular in 2D).

Equilibrium and transport Equilibrium in labor markets can often be expressed in terms of the Kantorovich duality. In the context of urban modelling or spatial pricing, this observation can be fruitfully used to compute equilibrium prices or densities as fixed points of operators involving OT, this was used in [49] and [48]. Quentin Petit, Guillaume Carlier and Yves Achdou are currently developing a (non-variational) new semi-discrete model for the structure of cities with applications to tele-working.

Non-convex Principal-Agent problems Guillaume Carlier, Xavier Dupuis, Jean-Charles Rochet and John Thanassoulis are developing a new saddle-point approach to non-convex multidimensional screening problems arising in regulation (Barron-Myerson) and taxation (Mirrlees).

3.3 Inverse problems with structured priors

Participants: Irène Waldspurger , Antonin Chambolle , Vincent Duval , Robert Tovey , Romain Petit .

Off-the-grid reconstruction of complex objects Whereas, very recently, some methods were proposed for the reconstruction of curves and piecewise constant images on a continuous domain ([63] and [14]), those are mostly proofs of concept, and there is still some work to make them competitive in real applications. As they are much more complex than point source reconstruction methods, there is room for improvements (parametrization, introduction of several atoms…). In particular, we are currently working on an improvement of the algorithm [63] for inverse problems in imaging which involve Optimal Transport as a regularizer (see [40] for preliminary results). Moreover, we need to better understand their convergence and the robustness of such methods, using sensitivity analysis.

Correctness guarantees for Burer-Monteiro methods Burer-Monteiro methods work well in practice and are therefore widely used, but existing correctness guarantees [62] hold under unrealistic assumptions only. In the long term, we aim at proposing new guarantees, which would be slightly weaker but would hold in settings more relevant to practice. A first step is to understand the “average” behavior of Burer-Monteiro methods, when applied to random problems, and could be the subject of a PhD thesis.

3.4 Geometric variational problems, and their interactions with transport

Participants: Vincent Duval , Paul Pegon , Antonin Chambolle , Joao-Miguel Machado .

Approximation of measures with geometric constraints Optimal Transport is a powerful tool to compare and approximate densities, but its interaction with geometric constraints is still not well understood. In applications such as optimal design of structures, one aims at approximating an optimal pattern while taking into account fabrication constraints [59]. In Magnetic Resonance Imaging (MRI), one tries to sample the Fourier transform of the unknown image according to an optimal density but the acquisition device can only proceed along curves with bounded speed and bounded curvature [113]. Our goal is to understand how OT interacts with energy terms which involve, e.g. the length, the perimeter or the curvature of the support… We want to understand the regularity of the solutions and to quantify the approximation error. Moreover, we want to design numerical methods for the resolution of such problems, with guaranteed performance.

Discretization of singular measures Beyond the (B)Lasso and the total variation (possibly off-the-grid), numerically solving branched transportation problems requires the ability to faithfully discretize and represent 1-dimensional structures in the space. The research program of A. Chambolle consists in part in developing the numerical analysis of variational problems involving singular measures, such as lower-dimensional currents or free surfaces. We will explore both phase-field methods (with P. Pegon, V. Duval) [80, 124] which easily represent non-convex problems, but lack precision, and (with V. Duval) precise discretizations of convex problems, based either on finite elements (and relying to the FEM discrete exterior calculus [44], cf [81] for the case of the total variation), or on finite differences and possibly a clever design of dual constraints as studied in [86, 82] again for the total variation.

Transport problems with metric optimization In urban planning models, one looks at building a network (of roads, metro or train lines, etc.) so as to minimize a transport cost between two distributions, penalized by the cost for building the network, usually its length. A typical transport cost is Monge cost MK_ω with a metric $\omega = \omega_\Sigma$ which is modified as a fraction of the euclidean metric on the network Σ. We would like to consider general problems involving a construction cost to generate a conductance field σ (having in mind 1-dimensional integral of some function of σ), and a transport cost depending on this conductance field. The afore-mentioned case studied in [69] falls into this category, as well as classical branched transport. The biologically-inspired network evolution model of [103] seems to provide such an energy in the vanishing diffusivity limit, with a cost for building a 1-dimensional permeability tensor and an L^2 congested transport cost with associated resistivity metric ; such a cost seems particularly relevant to model urban planning. Finally, we would like to design numerical methods to solve such problems, taking advantage of the separable structure of the whole cost.

4 Application domains

4.1 Natural Sciences

FreeForm Optics, Fluid Mechanics (Incompressible Euler, Semi-Geostrophic equations), Quantum Chemistry (Density Functional Theory), Statistical Physics (Schroedinger problem), Porous Media.

4.2 Signal Processing and inverse problems

Full Waveform Inversion (Geophysics), Super-resolution microscopy (Biology), Satellite imaging (Meteorology)

4.3 Social Sciences

Mean-field games, spatial economics, principal-agent models, taxation, nonlinear pricing.

5 Highlights of the year

Guillaume Carlier has published a new reference book in optimization [76].

6 New results

6.1 Stability of optimal traffic plans in the irrigation problem

Participants: Maria Colombo, Antonio De Rosa, Andrea Marchese, , Paul Pegon, Antoine Prouff.

We prove in [17] the stability of optimal traffic plans in branched transport. In particular, we show that any limit of optimal traffic plans is optimal as well. This result goes beyond the Eulerian stability proved in [Colombo, De Rosa, Marchese ; 2021], extending it to the Lagrangian framework.

6.2 Convergence of a Lagrangian discretization for barotropic fluids and porous media flow

Participants: Thomas Gallouët, Quentin Merigot, Andrea Natale.

When expressed in Lagrangian variables, the equations of motion for compressible (barotropic) fluids have the structure of a classical Hamiltonian system in which the potential energy is given by the internal energy of the fluid. The dissipative counterpart of such a system coincides with the porous medium equation, which can be cast in the form of a gradient flow for the same internal energy. Motivated by these related variational structures, we propose in [19] a particle method for both problems in which the internal energy is replaced by its Moreau-Yosida regularization in the L2 sense, which can be efficiently computed as a semi-discrete optimal transport problem. Using a modulated energy argument which exploits the convexity of the problem in Eulerian variables, we prove quantitative convergence estimates towards smooth solutions. We verify such estimates by means of several numerical tests.

6.3 Point Source Regularization of the Finite Source Reflector Problem

Participants: Jean-David Benamou, Guillaume Chazareix, Wilbert L Ijzerman, Giorgi Rukhaia.

We address in [11] the “freeform optics” inverse problem of designing a reflector surface mapping a prescribed source distribution of light to a prescribed far field distribution, for a finite light source. When the finite source reduces to a point source, the light source distribution has support only on the optics ray directions. In this setting the inverse problem is well posed for arbitrary source and target probability distributions. It can be recast as an Optimal Transportation problem and has been studied both mathematically and nu-merically. We are not aware of any similar mathematical formulation in the finite source case: i.e. the source has an “ ´etendue” with support both in space and directions. We propose to leverage the well-posed variational formulation of the point source problem to build a smooth parameterization of the reflec-tor and the reflection map. Under this parameterization we can construct a smooth loss/misfit function to optimize for the best solution in this class of reflectors. Both steps, the parameterization and the loss, are related to Optimal Transportation distances. We also take advantage of recent progress in the numerical approximation and resolution of these mathematical objects to perform a numerical study.

6.4 On the linear convergence of the multi-marginal Sinkhorn algorithm

Participants: Guillaume Carlier.

The aim of [12] is to give an elementary proof of linear convergence of the Sinkhorn algorithm for the entropic regularization of multi-marginal optimal transport. The proof simply relies on: i) the fact that Sinkhorn iterates are bounded, ii) strong convexity of the exponential on bounded intervals and iii) the convergence analysis of the coordinate descent (Gauss-Seidel) method of Beck and Tetruashvili.

6.5 "FISTA" in Banach spaces with adaptive discretisations

Participants: Antonin Chambolle, Robert Tovey.

FISTA is a popular convex optimisation algorithm which is known to converge at an optimal rate whenever the optimisation domain is contained in a suitable Hilbert space. We propose in [16] a modified algorithm where each iteration is performed in a subspace, and that subspace is allowed to change at every iteration.

Analytically, this allows us to guarantee convergence in a Banach space setting, although at a reduced rate depending on the conditioning of the specific problem. Numerically we show that a greedy adaptive choice of discretisation can greatly increase the time and memory efficiency in infinite dimensional Lasso optimisation problems.

6.6 Accelerated Bregman primal-dual methods applied to optimal transport and Wasserstein Barycenter problems

Participants: Antonin Chambolle, Juan Pablo Contreras.

We discuss in [15] the efficiency of Hybrid Primal-Dual (HPD) type algorithms to approximate solve discrete Optimal Transport (OT) and Wasserstein Barycenter (WB) problems, with and without entropic regularization. Our first contribution is an analysis showing that these methods yield state-of-the-art convergence rates, both theoretically and practically. Next, we extend the HPD algorithm with linesearch proposed by Malitsky and Pock in 2018 to the setting where the dual space has a Bregman divergence, and the dual function is relatively strongly convex to the Bregman's kernel. This extension yields a new method for OT and WB problems based on smoothing of the objective that also achieves state-of-the-art convergence rates. Finally, we introduce a new Bregman divergence based on a scaled entropy function that makes the algorithm numerically stable and reduces the smoothing, leading to sparse solutions of OT and WB problems. We complement our findings with numerical experiments and comparisons.

6.7 Towards Off-the-grid Algorithms for Total Variation Regularized Inverse Problems

Participants: Yohann De Castro, Vincent Duval, Romain Petit.

We introduce in [14] an algorithm to solve linear inverse problems regularized with the total (gradient) variation in a gridless manner. Contrary to most existing methods, that produce an approximate solution which is piecewise constant on a fixed mesh, our approach exploits the structure of the solutions and consists in iteratively constructing a linear combination of indicator functions of simple polygons.

6.8 Mass concentration in rescaled first order integral functionals

Participants: Antonin Monteil, Paul Pegon.

We consider in [39] first order local minimization problems $\min \int_{\mathbb{R}^N} f(u, \nabla u)$ under a mass constraint $\int_{\mathbb{R}^N} u = m \in \mathbb{R}$. We prove that the minimal energy function $H(m)$ is always concave on $(-\infty, 0)$ and $(0, +\infty)$, and that relevant rescalings of the energy, depending on a small parameter ε, Γ-converge in the weak topology of measures towards the H-mass, defined for atomic measures $\sum_i m_i \delta_{x_i}$ as $\sum_i H(m_i)$. We also consider space dependent Lagrangians $f(x, u, \nabla u)$, which cover the case of space dependent H-masses $\sum_i H(x_i, m_i)$, and also the case of a family of Lagrangians $(f_\varepsilon)_\varepsilon$ converging as $\varepsilon \to 0$. The Γ-convergence result holds under mild assumptions on f, and covers several situations including homogeneous H-masses in any dimension $N \geq 2$ for exponents above a critical threshold, and all concave H-masses in dimension $N = 1$. Our result yields in particular the concentration of Cahn-Hilliard fluids into droplets, and is related to the approximation of branched transport by elliptic energies.

6.9 Dynamical Programming for off-the-grid dynamic Inverse Problems

Participants: Robert Tovey, Vincent Duval.

In [40], we consider algorithms for reconstructing time-varying data into a finite sum of discrete trajectories, alternatively, an off-the-grid sparse-spikes decomposition which is continuous in time. Recent work showed that this decomposition was possible by minimising a convex variational model which combined a quadratic data fidelity with dynamical Optimal Transport. We generalise this framework and propose new numerical methods which leverage efficient classical algorithms for computing shortest paths on directed acyclic graphs. Our theoretical analysis confirms that these methods converge to globally optimal reconstructions which represent a finite number of discrete trajectories. Numerically, we show new examples for unbalanced Optimal Transport penalties, and for balanced examples we are 100 times faster in comparison to the previously known method.

6.10 A geometric Laplace method

Participants: Flavien Léger , François-Xavier Vialard.

A classical tool for approximating integrals is the Laplace method. The first-order, as well as the higher-order Laplace formula is most often written in coordinates without any geometrical interpretation. In [8], motivated by a situation arising, among others, in optimal transport, we give a geometric formulation of the first-order term of the Laplace method. The central tool is the Kim–McCann Riemannian metric which was introduced in the field of optimal transportation. Our main result expresses the first-order term with standard geometric objects such as volume forms, Laplacians, covariant derivatives and scalar curvatures of two different metrics arising naturally in the Kim–McCann framework. Passing by, we give an explicitly quantified version of the Laplace formula, as well as examples of applications.

6.11 Convergence rate of general entropic optimal transport costs

Participants: Guillaume Carlier , Paul Pegon , Luca Tamanini.

We investigate in [32] the convergence rate of the optimal entropic cost vε to the optimal transport cost as the noise parameter $\varepsilon \to 0$. We show that for a large class of cost functions c on $\mathbb{R}^d \times \mathbb{R}^d$ (for which optimal plans are not necessarily unique or induced by a transport map) and compactly supported and L^∞ marginals, one has $v_\varepsilon - v_0 = d/2\varepsilon \log(1/\varepsilon) + O(\varepsilon)$. Upper bounds are obtained by a block approximation strategy and an integral variant of Alexandrov's theorem. Under an infinitesimal twist condition on c, i.e. invertibility of $\nabla^2_{xy} c$, we get the lower bound by establishing a quadratic detachment of the duality gap in d dimensions thanks to Minty's trick.

7 Partnerships and cooperations

7.1 International research visitors

7.1.1 Visits of international scientists

Other international visits to the team

Luigi De Pascale

Status Associate Professor

Institution of origin: Università di Firenze

Country: Italy

Dates: October 5th-october 20th

Context of the visit: research with Guillaume Carlier, Luca Nenna and Paul Pegon, mentoring of the PhD of Camilla Brizzi

Mobility program/type of mobility: research stay and lecture

Giuseppe Buttazzo

Status Full Professor

Institution of origin: Università di Pisa

Country: Italy

Dates: march 13th-march 17th and december 11th-14th

Context of the visit: research with Guillaume Carlier and Katharina Eichinger, PhD defense of Katharina Eichinger

Mobility program/type of mobility: research stay, lecture and Ph.D defense.

7.1.2 Visits to international teams

Research stays abroad

Flavien Léger

Visited institution: Fields Institute, Toronto, ON

Country: Canada

Dates: November 7, 2022 to November 24, 2022

Context of the visit: long-term visit to the Thematic Program on Nonsmooth Riemannian and Lorentzian Geometry

Mobility program/type of mobility: research stay

Jean-David Benamou

Visited institution: Imperial College

Country: Royaume-Uni

Dates: April 23 to April 28 and September 26 to December 19.

Context of the visit: Invitation and Nelder Fellowship

Mobility program/type of mobility: research stay and lecture.

Guillaume Carlier

Visited institution: TUM (Munich)

Country: Allemagne

Dates: may 22nd-may 26th and november 6th-november 11th

Context of the visit: research with Daniel Matthes and with Gero Friesecke, workshop calculus of variations

Mobility program/type of mobility: research stay, lecture

7.2 European initiatives

7.2.1 H2020 projects

ROMSOC

Participants: Jean-David Benamou, Giorgi Rukhaia.

ROMSOC project on cordis.europa.eu

Title: Reduced Order Modelling, Simulation and Optimization of Coupled systems

Duration: From September 1, 2017 to August 31, 2022

Partners:

- INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET AUTOMATIQUE (INRIA), France
- DANIELI OFFICINE MECCANICHE SpA, Italy
- ABB SCHWEIZ AG (ABB SUISSE SA ABB SWITZERLAND LTD), Switzerland
- FRIEDRICH-ALEXANDER-UNIVERSITAET ERLANGEN-NUERNBERG (FAU), Germany
- MATHCONSULT GMBH (MATHCONSULT GMBH), Austria
- Math.Tec GmbH (Math.Tec), Austria
- MICROGATE SRL (MICROGATE S.R.L.), Italy
- FORSCHUNGSVERBUND BERLIN EV, Germany
- STICHTING EUROPEAN SERVICE NETWORK OF MATHEMATICS FOR INDUSTRY AND INNOVATION (EUROPEAN SERVICE NETWORK OF MATHEMATICS FOR INDUSTRY AND INNOVATION), Netherlands
- SCUOLA INTERNAZIONALE SUPERIORE DI STUDI AVANZATI DI TRIESTE (SISSA), Italy
- SAGIV TECH LTD (SAGIVTECH), Israel
- UNIVERSITAT LINZ (JOHANNES KEPLER UNIVERSITAT LINZ UNIVERSITY OF LINZ JOHANNES KEPLER UNIVERSITY OF LINZ JKU), Austria
- TECHNISCHE UNIVERSITAT BERLIN (TUB), Germany
- CONSORCIO CENTRO DE INVESTIGACIÓN E TECNOLOXÍA MATEMÁTICA DE GALICIA (CITMAGA), Spain
- UNIVERSITAET BREMEN (UBREMEN), Germany
- BERGISCHE UNIVERSITAET WUPPERTAL (BUW), Germany
- POLITECNICO DI MILANO (POLIMI), Italy
- ARCELORMITTAL INNOVACION INVESTIGACION E INVERSION SL (AMIII), Spain
- STMICROELECTRONICS SRL, Italy
- MICROFLOWN TECHNOLOGIES BV (MICROFLOWN), Netherlands
- DB Schenker Rail Polska S.A. (DB Schenker Rail Polska), Poland
- SIGNIFY NETHERLANDS BV (Signify Netherlands BV), Netherlands
- CorWave (CorWave), France

Inria contact: Jean-David Benamou

Coordinator:

Summary: The development of high quality products and processes is essential for the future competitiveness of the European economy. In most key technology areas product development is increasingly based on simulation and optimization via mathematical models that allow to optimize design and functionality using free design parameters. Best performance of modelling, simulation and optimization (MSO) techniques is obtained by using a model hierarchy ranging from very fine to very coarse models obtained by model order reduction (MOR) techniques and to adapt the model and the methods to the user-defined requirements in accuracy and computational speed.

ROMSOC will work towards this goal for high dimensional and coupled systems that describe different physical phenomena on different scales; it will derive a common framework for different industrial applications and train the next generation of researchers in this highly interdisciplinary field. It will focus on the three major methodologies: coupling methods, model reduction methods, and optimization methods, for industrial applications in well selected areas, such as optical and electronic systems, economic processes, and materials. ROMSOC will develop novel MSO techniques and associated software with adaptability to user-defined accuracy and efficiency needs in different scientific disciplines. It will transfer synergies between different industrial sectors, in particular for SMEs.

To lift this common framework to a new qualitative level, a joint training programme will be developed which builds on the strengths of the academic and industrial partners and their strong history of academic/industrial cooperation. By delivering early-career training embedded in a cutting-edge research programme, ROMSOC will educate highly skilled interdisciplinary researchers in mathematical MSO that will become facilitators in the transfer of innovative concepts to industry. It will thus enhance the capacity of European research and development.

7.3 National initiatives

ANR MAGA (2016-2022) To study and to implement discretizations of optimal transport and Monge-Ampère equations which rely on tools from computational geometry (Laguerre diagrams). to apply these solvers to concrete problems from various fields involving optimal transport. .

PRAIRIE chair : Irène Waldspurger.

ANR CIPRESSI (2019-) is a JCJC grant (149k€) carried by Vincent Duval. Its aim is to develop off-the-grid methods for inverse problems involving the reconstruction of complex objects.

8 Dissemination

8.1 Promoting scientific activities

8.1.1 Scientific events: organisation

Member of the organizing committees

- Antonin Chambolle was a co-organizer of the Oberwolfach 2234 seminar: Mathematical Imaging and Surface Processing (August, 21-27 2022)
- Antonin Chambolle is a co-organizer of the Séminaire Parisien d'Optimisation (SPO)
- Vincent Duval is a co-organizer of the Imaging in Paris seminar.
- Paul Pegon is a co-organizer of the workgoup on Calculus of Variation GT CalVa
- Thomas Gallouët is co-organizer of the journées MAGA at Autrans (February, 2-4 2022)

8.1.2 Journal

Member of the editorial boards

- Irène Waldspurger is associate editor for the journal IEEE Transactions on Signal Processing.

Reviewer - reviewing activities

- Vincent Duval has reviewed submissions for the journals Math in Action and SIAM Journal on Imaging Sciences (SIIMS).
- Irène Waldspurger has reviewed submissions for the journals Applied and Computational Harmonic Analysis, IEEE Transactions on Information Theory, IEEE Transactions on Signal Processing and the SIAM journal on Optimization, as well as for the NeurIPS conference.
- Flavien Léger has reviewed submission for the journals IMA Journal of Applied Mathematics, ESAIM: Control, Optimisation and Calculus of Variations and Communications in Partial Differential Equations.
- Paul Pegon has reviewed submissions for the SIAM Journal on Mathematical Analysis (SIMA) and the Journal of Mathematical Analysis and Applications (JMAA).
- Thomas Gallouët has reviewed submissions for Foundations of Computational Mathematics, Journal of Convex Analysis.

8.1.3 Invited talks

- Antonin Chambolle was invited to give talks at the *Inverse Problem on Large Scales* workshop at RICAM (Linz, Austria), at the *IEEE ICIP 2022 Int. Conference on Image Processing*, and at the *Summerschool on Analysis and Applied Mathematics* at Münster, Germany.
- Vincent Duval was invited to give a talk at the Inverse Problems: Modelling and Simulation (IPMS) conference, and the "Probabilités et Statistiques" seminar at IECL (Nancy).
- Irène Waldspurger was invited to give a talk at the *Learning and Optimization in Luminy* workshop, and at the CIS/MINDS seminar at Johns Hopkins University.
- Flavien Léger was invited to give a talk at the *Institut de mathématique d'Orsay*, the *Laboratoire Jean Kuntzmann Université Grenoble Alpes*, the *Laboratoire Paul Painlevé, Université de Lille* and the *Fields Institute Applied Mathematics Colloquium*.
- Paul Pegon was invited to give a talk at the *Séminaire parisien d'optimisation* (IHP) and at the *Gdt Transport Optimal* (Laboratoire de Mathématiques d'Orsay, Université Paris-Saclay).
- Thomas Gallouët was invited to give a talk at the Séminaire ANEDP at Lille University, at the the workgoup on *Calculus of Variation* GT CalVa at Dauphine University and at the conference on Optimal transport, geometric and stochastic Hydrodynamics at Lisbon, June 21-24, 2022.

8.1.4 Research administration

- Irène Waldspurger was a member of a selection committee at Université Aix Marseille.
- Vincent Duval was a member of a selection committee at Sorbonne Université (LJLL)
- Thomas Gallouët is a member of the Commission d'Évaluation Scientifique (CES) of Inria Paris.

8.2 Teaching - Supervision - Juries

8.2.1 Teaching

- Master: Antonin Chambolle Optimisation Continue, ??h, niveau M2, Université Paris Dauphine-PSL, FR
- Master : Vincent Duval, Problèmes Inverses, 22,5 h équivalent TD, niveau M1, Université PSL/Mines ParisTech, FR
- Master : Vincent Duval, Optimization for Machine Learning, 6h, niveau M2, Université PSL/ENS, FR
- Licence : Irène Waldspurger, Pré-rentrée raisonnement, 31,2 h équivalent TD, niveau L1, Université Paris-Dauphine, FR

- Licence : Irène Waldspurger, Analyse 4, 58,5 h équivalent TD, niveau L1, Université Paris-Dauphine, FR
- Master : Irène Waldspurger, Optimization for Machine Learning, 6h, niveau M2, Université PSL/ENS, FR
- Licence : Guillaume Carlier, algebre 1, L1 78h, Dauphine, FR
- Master : Guillaume Carlier Variational and transport methods in economics, M2 Masef, 27h, Dauphine, FR
- Licence : Paul Pegon, Analyse 2, 51 H. équivalent TD, TD niveau L1, Université Paris-Dauphine, FR
- Licence : Paul Pegon, Intégrale de Lebesgue et probabilités, 44 H. équivalent TD, TD niveau L3, Université Paris-Dauphine, FR
- Licence : Paul Pegon, Analyse fonctionnelle et hilbertienne, 44 H. équivalent TD, TD niveau L3, Université Paris-Dauphine, FR
- Licence : Paul Pegon, Méthodes numériques pour l'optimisation, 33 H. équivalent TD, TD/TP niveau L3, Université Paris-Dauphine, FR
- Agregation : Thomas Gallouët, Optimisation, Analyse numérique, 48h équivalent TD, niveau M2, Université d'Orsay), FR
- Guillaume Carlier: Licence Algèbre 1, Dauphine 70h, M2 Masef: Variatioanl and transport problems in economics, 18h
- Flavien Léger: Graduate course, two lectures in 'math+econ+code' masterclass on equilibrium transport and matching models in economics, NYU Paris. 5h.

8.2.2 Supervision

- PhD completed: Quentin Petit, *Mean field games and optimal transport in urban modelling*, defended on 18/02/2022, Supervised by Guillaume Carlier, Y. Achdou, D. Tonon.
- PhD completed: Romain Petit, *Reconstruction of piecewise constant images via total variation regularization*, defended on 12/12/2022, Supervised by Vincent Duval and Yohann De Castro.
- PhD in completed: Katharina Eichinger, *Problèmes variationnels pour l'interpolation dans l'espace de Wasserstein* 1/09/2019. Supervised by Guillaume Carlier.
- PhD in progress : Joao-Miguel Machado, Transport optimal et structures géométriques, 01/10/2021, Co-supervised by Vincent Duval and Antonin Chambolle.
- PhD in progress : Adrien Vacher 1/10/2020. Co-supervised by François-Xavier Vialard and Jean-David Benamou
- Summer internship: Mitchell Gaudet, an undergraduate student at the University of Toronto, Canada. *The Kim–McCann geometry in applied optimal transport.* Supervised by Flavien Léger and Robert McCann.
- PhD in progress : Erwan Stämplfi 1/10/2021, *Around multiphasic flows.* Co-supervised by Yann Brenier and Thomas Gallouët.
- PhD in progress : Siwan Boufadene 1/10/2022, *Gradient flows for energy distance.* Co-supervised by François-Xavier Vialard and Thomas Gallouët.
- PhD in progress : Chazareix Guillaume 1/08/2021, *Non Linear Parabolic equations and Volatility Calibration.* Co-supervised by Jean-David Benamou and G. Loeper.
- PhD in progress : Malamut Hugo 1/09/2022, *Régularisation Entropique et Transport Optimal Généralisé.* Supervised by Jean-David Benamou.

8.2.3 Juries

- Antonin Chambolle was a member of the jury for the PhD theses of Ulysse Marteau-Ferey (ENS), Romain Petit (Dauphine Paris-PSL), and for the habilitation thesis of Andrea Simonetto. He was the reviewer of the PhD theses of Jordan Michelet (Univ. La Rochelle) and Garry Terii (Univ. Lyon 1).
- Vincent Duval was a member of the jury for the PhD Theses of Thomas Debarre (EPFL) and Zoé Lambert (INSA Rouen).
- Irène Waldspurger was a member of the jury of the PhD thesis of Pierre-Hugo Vial (ENSEEIHT, Toulouse).
- Jean-David Benamou was a reviewer of the PhD thesis Jean-François Quilbert (U. Joseph Fourier, Grenoble) and Charlie Egan (Heriot Watt, Edimbourg).

8.3 Popularization

8.3.1 Internal or external Inria responsibilities

- Vincent Duval is a member of the CSD (*comité de suivi doctoral*) of the Inria Paris Centre.
- Thomas Gallouët is a member of the Commission d'Évaluation Scientifique (CES) of Inria Paris.

8.3.2 Education

- Vincent Duval has given a 10h lecture on Inverse Problems at the CIMPA school "Mathématiques en analyse et traitement du signal, des images et des données" at Thiès (Sénégal).

8.3.3 Interventions

- Irène Waldspurger has given a talk at the Journée des doctorant·e·s en mathématiques des Hauts-de-France.

9 Scientific production

9.1 Major publications

[1] P.-C. Aubin-Frankowski, A. Korba and F. Léger. 'Mirror Descent with Relative Smoothness in Measure Spaces, with application to Sinkhorn and EM'. In: NeurIPS 2022 - Thirty-sixth Conference on Neural Information Processing Systems. New Orleans, United States, 2022. URL: https://hal.science/hal-03811583.

[2] J.-D. Benamou, G. Carlier, M. Cuturi, L. Nenna and G. Peyré. 'Iterative Bregman Projections for Regularized Transportation Problems'. In: *SIAM Journal on Scientific Computing* 2.37 (2015), A1111–A1138. DOI: 10.1137/141000439. URL: https://hal.science/hal-01096124.

[3] J.-D. Benamou, T. Gallouët and F.-X. Vialard. 'Second order models for optimal transport and cubic splines on the Wasserstein space'. In: *Foundations of Computational Mathematics* (Oct. 2019). DOI: 10.1007/s10208-019-09425-z. URL: https://hal.science/hal-01682107.

[4] C. Boyer, A. Chambolle, Y. de Castro, V. Duval, F. de Gournay and P. Weiss. 'On Representer Theorems and Convex Regularization'. In: *SIAM Journal on Optimization* 29.2 (9th May 2019), pp. 1260–1281. DOI: 10.1137/18M1200750. URL: https://hal.archives-ouvertes.fr/hal-01823135.

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[7] G. Carlier, P. Pegon and L. Tamanini. *Convergence rate of general entropic optimal transport costs*. 7th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03689945.

[8] F. Léger and F.-X. Vialard. *A geometric Laplace method*. 22nd Dec. 2022. URL: https://hal.science/hal-03911149.

[9] I. Waldspurger. 'Phase retrieval with random Gaussian sensing vectors by alternating projections'. In: *IEEE Transactions on Information Theory* 64.5 (2018), pp. 3301–3312. URL: https://hal.science/hal-01645081.

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9.2 Publications of the year

International journals

[11] J.-D. Benamou, G. Chazareix, G. Rukhaia and W. L. Ijzerman. 'Point Source Regularization of the Finite Source Reflector Problem'. In: *Journal of Computational Physics* (1st May 2022). URL: https://hal.inria.fr/hal-03344571.

[12] G. Carlier. 'On the linear convergence of the multi-marginal Sinkhorn algorithm'. In: *SIAM Journal on Optimization* 32.2 (2022), pp. 786–794. URL: https://hal.archives-ouvertes.fr/hal-03176512.

[13] G. Carlier, V. Chernozhukov, G. de Bie and A. Galichon. 'Correction to: Vector Quantile Regression and Optimal Transport, from Theory to Numerics'. In: *Empirical Economics* 62.1 (Jan. 2022), pp. 63–63. DOI: 10.1007/s00181-020-01933-0. URL: https://hal-sciencespo.archives-ouvertes.fr/hal-03896159.

[14] Y. de Castro, V. Duval and R. Petit. 'Towards Off-the-grid Algorithms for Total Variation Regularized Inverse Problems'. In: *Journal of Mathematical Imaging and Vision* (25th July 2022). DOI: 10.1007/s10851-022-01115-w. URL: https://hal.inria.fr/hal-03406710.

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[16] A. Chambolle and R. Tovey. '"FISTA" in Banach spaces with adaptive discretisations'. In: *Computational Optimization and Applications* (2022). URL: https://hal.inria.fr/hal-03119773.

[17] M. Colombo, A. de Rosa, A. Marchese, P. Pegon and A. Prouff. 'Stability of optimal traffic plans in the irrigation problem'. In: *Discrete and Continuous Dynamical Systems - Series A* (2022). DOI: 10.3934/dcds.2021167. URL: https://hal.archives-ouvertes.fr/hal-02519237.

[18] P. Descloux, C. Boyer, J. Josse, A. Sportisse and S. Sardy. 'Robust Lasso-Zero for sparse corruption and model selection with missing covariates'. In: *Scandinavian Journal of Statistics* (2022). URL: https://hal.archives-ouvertes.fr/hal-02569696.

[19] T. Gallouët, Q. Merigot and A. Natale. 'Convergence of a Lagrangian discretization for barotropic fluids and porous media flow'. In: *SIAM Journal on Mathematical Analysis* 54.3 (2022). DOI: 10.1137/21M1422756. URL: https://hal.archives-ouvertes.fr/hal-03234144.

[20] A. Natale and G. Todeschi. 'A mixed finite element discretization of dynamical optimal transport'. In: *Journal of Scientific Computing* (2022). DOI: 10.1007/s10915-022-01821-y. URL: https://hal.archives-ouvertes.fr/hal-02501634.

International peer-reviewed conferences

[21] P.-C. Aubin-Frankowski, A. Korba and F. Léger. 'Mirror Descent with Relative Smoothness in Measure Spaces, with application to Sinkhorn and EM'. In: NeurIPS 2022 - Thirty-sixth Conference on Neural Information Processing Systems. New Orleans, United States, 2022. URL: https://hal.science/hal-03811583.

Edition (books, proceedings, special issue of a journal)

[22] A. Ayme, C. Boyer, A. Dieuleveut and E. Scornet, eds. *Near-optimal rate of consistency for linear models with missing values*. 2022. URL: https://hal.archives-ouvertes.fr/hal-03552109.

Doctoral dissertations and habilitation theses

[23] V. Duval. 'Faces and extreme points of convex sets for the resolution of inverse problems'. Ecole doctorale SDOSE, 23rd June 2022. URL: https://theses.hal.science/tel-03718371.

Reports & preprints

[24] Y. Achdou, G. Carlier, Q. Petit and D. Tonon. *A mean field model for the interactions between firms on the markets of their inputs*. 11th July 2022. URL: https://hal.archives-ouvertes.fr/hal-03720158.

[25] Y. Achdou, G. Carlier, Q. Petit and D. Tonon. *A simple city equilibrium model with an application to teleworking*. 25th July 2022. URL: https://hal.archives-ouvertes.fr/hal-03737457.

[26] L. Arnould, C. Boyer and E. Scornet. *Is interpolation benign for random forests?* 29th Apr. 2022. URL: https://hal.archives-ouvertes.fr/hal-03560047.

[27] D. Bucur, A. Chambolle, A. Giacomini and M. Nahon. *A FREE DISCONTINUITY APPROACH TO OPTIMAL PROFILES IN STOKES FLOWS*. 3rd Jan. 2023. URL: https://hal-cnrs.archives-ouvertes.fr/hal-03920819.

[28] G. Buttazzo, G. Carlier and K. Eichinger. *Wasserstein interpolation with constraints and application to a parking problem*. 14th Nov. 2022. URL: https://hal.archives-ouvertes.fr/hal-03851536.

[29] G. Carlier. *Fenchel-Young inequality with a remainder and applications to convex duality and optimal transport*. 16th Apr. 2022. URL: https://hal.archives-ouvertes.fr/hal-03614052.

[30] G. Carlier, L. Chizat and M. Laborde. *Lipschitz Continuity of the Schrödinger Map in Entropic Optimal Transport*. 1st Oct. 2022. URL: https://hal.archives-ouvertes.fr/hal-03793562.

[31] G. Carlier, A. Delalande and Q. Merigot. *Quantitative Stability of Barycenters in the Wasserstein Space*. 20th Sept. 2022. URL: https://hal.archives-ouvertes.fr/hal-03781835.

[32] G. Carlier, P. Pegon and L. Tamanini. *Convergence rate of general entropic optimal transport costs*. 7th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03689945.

[33] A. Chambolle and V. Crismale. *A general compactness theorem in <i>G(S)BD</i>*. 10th Oct. 2022. URL: https://hal.archives-ouvertes.fr/hal-03807668.

[34] A. Chambolle, C. Delplancke, M. J. Ehrhardt, C.-B. Schönlieb and J. Tang. *Stochastic Primal Dual Hybrid Gradient Algorithm with Adaptive Step-Sizes*. 6th Jan. 2023. URL: https://hal.science/hal-03927644.

[35] A. Chambolle and M. Novaga. *L1-Gradient Flow of Convex Functionals*. 7th Oct. 2022. URL: https://hal.archives-ouvertes.fr/hal-03805962.

[36] E. Facca, G. Todeschi, A. Natale and M. Benzi. *Efficient preconditioners for solving dynamical optimal transport via interior point methods*. 2022. URL: https://hal.inria.fr/hal-03766668.

[37] T. Gallouët, A. Natale and G. Todeschi. *From geodesic extrapolation to a variational BDF2 scheme for Wasserstein gradient flows*. 28th Sept. 2022. URL: https://hal.archives-ouvertes.fr/hal-03790981.

[38] F. Léger and F.-X. Vialard. *A geometric Laplace method*. 22nd Dec. 2022. URL: https://hal.science/hal-03911149.

[39] A. Monteil and P. Pegon. *Mass concentration in rescaled first order integral functionals*. 2nd Mar. 2022. URL: https://hal.archives-ouvertes.fr/hal-03517074.

[40] R. Tovey and V. Duval. *Dynamical Programming for off-the-grid dynamic Inverse Problems*. 23rd Dec. 2022. URL: https://hal.inria.fr/hal-03500048.

[41] A. Vacher and F.-X. Vialard. *Stability of Semi-Dual Unbalanced Optimal Transport: fast statistical rates and convergent algorithm*. 15th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03609629.

9.3 Cited publications

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