RESEARCH CENTRE

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2022
ACTIVITY REPORT

Project-Team

POEMS

Wave propagation: mathematical analysis and simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

Inria

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2 Overall objectives

The propagation of waves is one of the most common physical phenomena in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) to the scales of the universe (electromagnetic waves, gravity waves) and of the atoms (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us. The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in various domains of physics and engineering sciences. The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas altogether justify a research project in applied mathematics and scientific computing devoted to this topic.

3 Research program

3.1 Expertises

The activity of the team is oriented towards the design, the analysis and the numerical approximation of mathematical models for all types of problems involving wave propagation phenomena, in mechanics, physics and engineering sciences. Let us briefly describe our core business and current expertise, in order to clarify the new challenges that we want to address in the short and long terms.

Typically, our works are based on *boundary value problems* established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation of the hyperbolic type, whose prototype is the scalar wave equation, or the Helmholtz equation if time-periodic solutions are considered. More generally, we systematically consider both the transient problem, in the time domain, and the time-harmonic problem, in the frequency domain. Let us mention that, even if different waves share a lot of common properties, the transition from the scalar acoustic equation to the vectorial electromagnetism and elastodynamics systems raises a lot of mathematical and numerical difficulties, and requires a specific expertise.

A notable particularity of the problems that we consider is that they are generally set in *unbounded domains*: for instance, for radar applications, it is necessary to simulate the interaction of the electromagnetic waves with the airplane only, without any complex environment perturbing the wave phenomena. This raises an intense research activity, both from a theoretical and a numerical point of view. There exist several approaches which all consist in rewriting the problem (or an approximation of it) in a bounded domain, the new formulation being well-suited for classical mathematical and numerical techniques.

One class of methods consists in applying an appropriate condition on some boundary enclosing the zone of interest. In the frequency domain, one can use a non-local transparent condition, which can be expressed by a convolution with a Green function like in integral equation techniques, or by a modal decomposition when a separation of variables is applicable. But for explicit schemes in the time domain, local radiation conditions at a finite distance are generally preferred (constructed as local approximations at various orders of the exact non-local condition). A second class of methods consists in surrounding the computational domain by so called *Perfectly Matched absorbing Layers* (PML), which are very popular because they are easy to implement. POEMS members have provided several contributions to these two classes of methods for more than twenty-five years. Among them, on can mention the understanding of the instability of PMLs in anisotropic media and in dispersive media, the derivation of transparent boundary conditions in periodic media or the improvement of Fast Multipole techniques for elastodynamic integral equations.

In addition to more classical domains of applied mathematics that we are led to use (variational analysis and functional analysis, interpolation and approximation theory, linear algebra of large systems, etc...), we have acquired a deep expertise in *spectral theory*. Indeed, the analysis of wave phenomena is intimately linked to the study of some associated spectral problems. Acoustic resonance frequencies of a cavity correspond to the eigenvalues of a selfadjoint Laplacian operator, modal solutions in a waveguide correspond to a spectral problem set in the cross section. In these two examples, if the cavity or the cross-section is unbounded, a part of the spectrum is a continuum. Again, POEMS has produced several contributions in this field. In particular, a large number of significant results have been obtained for the existence or non-existence of guided modes in open waveguides and of trapped modes in infinite domains.

To end this far from exhaustive presentation of our main expertise domains, let us mention the *asymptotic techniques* with respect to some small scale appearing in the model: it can be the wavelength compared to the size of the scatterer, or on the contrary, the scale of the scatterer compared to the wavelength, it can be the scale of some microstructure in a composite material or the width of a thin layer or a thin tube. In each case, the objective, in order to avoid the use of costly meshes, is to derive effective simplified models. Our specificity here is that we can combine skills in physics, mathematics and numerics: in particular, we take care of the mathematical properties of the effective model, which are used to ensure the robustness of the numerical method, and also to derive error estimates with respect to the small parameter. There has been a lot of contributions of POEMS to this topic, going from the modeling of electromagnetic coatings to the justification of models for piezoelectric sensors. Let us mention that effective models for small scatterers and thin coatings have been used to improve imaging techniques that we are developing (topological gradient, time reversal or sampling techniques).

3.2 Recent evolutions

In order to consider more and more challenging problems (involving non-deterministic, large-scale and more realistic models), we decided recently to enlarge our domain of expertise in three directions.

Firstly, we want to reinforce our activity on *efficient solvers for large-scale wave propagation problems*. Since its inception, POEMS has frequently contributed to the development and the analysis of numerical methods that permit the fast solution of large-scale problems, such as high-order finite element methods, boundary elements methods and domain decomposition methods. Nevertheless, implementing these methods in parallel programming environments and dealing with large-scale benchmarks have generally not been done by the team. We want to continue our activities on these methods and, in a more comprehensive approach, we will incorporate modern algebraic strategies and high-performance computing (HPC) aspects in our methodology. In collaboration with academic or industrial partners, we would like to address industrial-scale benchmarks to assess the performance of our approaches. We believe that taking all these aspects into consideration will allow us to design more efficient wave-specific computational tools for large-scale simulations.

Secondly, up to now, *probabilistic methods* were outside the expertise of POEMS team, restricting us to deterministic approaches for wave propagation problems. We however firmly believe in the importance and usefulness of addressing uncertainty and randomness inherent to many propagation phenomena. Randomness may occur in the description of complex propagation media (for example in the modeling of ultrasound waves in concrete for the simulation of non-destructive testing experiments) or of data uncertainties. To quantify the effect of such uncertainties on the design, behavior, performance or reliability of many systems is then a natural goal in diverse fields of application.

Thirdly and lastly, we wish to develop and strengthen collaborations allowing a *closer interaction between our mathematical, modeling and computing activities and physical experiments*, where the latter may either provide reality checks on existing models or strongly affect the choice of modeling assumptions. Within our typical domain of activities, we can mention four areas for which such considerations are highly relevant. One is musical acoustics, where POEMS has made several well-recognized contributions dealing with the simulation of musical instruments. Another area is inverse problems, whose very purpose is to extract useful information from actual measurements with the help of (propagation) models. This is a core of our partnership with CEA on ultrasonic Non Destructive Testing. A third area is the modelling of effective (acoustic or electromagnetic) metamaterials, where predictions based on homogenized models have to be confirmed by experiments. Finally, a fourth area of expertise is the modeling and simulations of waves in reactive media, where the development of simple mathematical models is of great importance in order to better understand the complex dynamics of reactive flows.

4 Application domains

Our research finds applications in many fields where acoustic, elastic, electromagnetic and aquatic waves are involved. Topics that have given rise to industrial partnerships include aircraft noise reduction (aeroacoustics), ultrasonic non-destructive testing of industrial structures, and seismic wave simulations in the subsoil, for the oil exploration.

Nowadays, the numerical techniques for solving the basic academic problems are well mastered, and significant progress has been made during the last twenty years for handling problems closer to real applications. But several bottlenecks remain, among which one can mention the high-frequency problems for radar applications, the multiscale problems that arise for instance in nanotechnologies or the multi-physics couplings, like in *aeroacoustics*. Moreover, in the recent period, new challenges have emerged, related to new discoveries in physics (like negative index metamaterials) or to the fantastic development of information and communication techniques. For example, the growing development of increasingly connected objects (internet of things) and the forthcoming availability of autonomous vehicles depend crucially on electromagnetic waves, raising important issues about radar performance, sensor reliability, component miniaturization and electromagnetic compatibility. Generally, there are a lot of application domains which could benefit from advanced research on waves phenomena. Enhancing ultrasound-based methods for *detection* and *imaging*, which are already intensively used in e.g. medicine, could permit real-time health monitoring of aircrafts or nuclear plants. Guarding against seismic risks still requires considerable advances in the simulation of elastic waves in large and complex media. And many other applications motivating our research and our prospects could be added to this far-from-comprehensive list.

5 Highlights of the year

- The conference WAVES 2022 (15th International Conference on Mathematical and Numerical Aspects of Wave Propagation), chaired by Sonia Fliss and Christophe Hazard, was held at ENSTA Paris in July. It brought together 230 participants, including 75 doctoral students, from 17 countries, including 198 from Europe (110 from France), 29 from America (26 from the North, 3 from the South) and 3 from Asia. In addition to the 8 plenary sessions, 169 oral communications of 30 minutes spread over 5 parallel sessions were presented. Most of the participants were accommodated on site, in the ENSTA Paris student residence, and the main hall of ENSTA Paris was transformed for the occasion into a vast convivial area hosting catering and spaces for relaxation and exchange. This event was a first which showed that ENSTA Paris has the capacity to host a conference of this size in excellent conditions. In the opinion of many participants, this edition of the WAVES series was a success, both scientifically and humanely.

- POEMS participated in the photographic exhibition organized by Inria to promote its partnerships with universities and major academic partners. This exhibition was presented during the Inria Scientific Days (November 23-25, 2022 in Rocquencourt) and will circulate in all 9 Inria centers and partner campuses in 2023. POEMS chose the theme of musical composition for the photo of its members (by Frédéric Stucin), which resonates with its research activities.

6 New software and platforms

6.1 New software

6.1.1 COFFEE

Keywords: Numerical simulations, Wave propagation, Boundary element method

Functional Description: COFFEE is an adapted fast BEM solver to model acoustic and elastic wave propagation (full implementation in Fortran 90). The 3-D acoustic or elastodynamic equations are solved with the boundary element method accelerated by the multi-level fast multipole method or a hierarchical-matrices based representation of the system matrix. The fundamental solutions for the infinite space are used in this implementation. A boundary element-boundary element coupling strategy is also implemented so multi-region problems (strata inside a valley for example) can be solved. In order to accelerate the convergence of the iterative solver, various analytic or algebraic preconditioners are available. Finally, an anisotropic mesh adaptation strategy is used to further reduce the computational times.

URL: https://uma.ensta-paris.fr/soft/COFFEE/

Author: Stéphanie Chaillat

Contact: Stéphanie Chaillat

6.1.2 XLiFE++

Name: eXtended Library of Finite Elements in C++

Keywords: Numerical simulations, Finite element modelling, Boundary element method

Functional Description: XLiFE++ is an FEM-BEM C++ library developed by POEMS laboratory, that can solve 1D/2D/3D, scalar/vector, transient/stationary/harmonic problems.

URL: https://uma.ensta-paris.fr/soft/XLiFE++/

Contact: Eric Lunéville

6.1.3 HTool

Keyword: Hierarchical matrices

Functional Description: HTool is a C++ header-only library implementing compression techniques (e.g. Adaptive Cross Approximation) using hierarchical matrices (H-matrices). The library uses MPI and OpenMP for parallelism, and is interfaced with HPDDM for the solution of linear systems.

URL: https://github.com/htool-ddm/htool

Contact: Pierre Marchand

6.1.4 DataFlowTasks.jl

Keyword: Task scheduling

Functional Description: DataFlowTasks.jl is a Julia package dedicated to parallel programming on multi-core shared memory CPUs. From user annotations (READ, WRITE, READWRITE) on program data, DataFlowTasks.jl automatically infers dependencies between parallel tasks.

URL: https://github.com/maltezfaria/DataFlowTasks.jl

Contact: Luiz Maltez Faria

6.1.5 HMatrices.jl

Keywords: Boundary element method, Hierarchical matrices

Functional Description: This package provides some functionality for assembling as well as for doing linear algebra with hierarchical matrices with a strong focus in applications arising in boundary integral equation methods. It includes shared as well as distributed memory assembly and matrix/vector product, as well as a shared memory LU factorization.

URL: https://github.com/WaveProp/HMatrices.jl

Contact: Luiz Maltez Faria

7 New results

7.1 Wave propagation in metamaterials and dispersive media

Mathematical analysis of metamaterials in time domain

Participants: Patrick Joly, Alex Rosas Martinez.

This topic is the subject of our important collaboration with Maxence Cassier (Institut Fresnel). and corresponds the PhD thesis of Alex Rosas Martinez who started in November 2020.

We have completed the study of large time behaviour of the associated Cauchy problem for dissipative version of generalized Lorentz media. The main result is the polynomial stability of this model. This work has been presented at the CIRM Conference "Fonctions de Herglotz-Nevanlinna et leurs applications aux systèmes dispersifs et aux matériaux composites". A first article on the approach via "frequency dependent" Lyapunov methods has been submitted for publication. A second article based on a spectral approach is currently being written.

A second aspect of our research concerned guided waves by a slab of a non dissipative metamaterial (a Drude material) embedded in the vacuum. When this slab is homogeneous one can compute explicitly the dispersion relation of the modes. A thorough study of this dispersion relation leads to new existence and dispersion results which are quite different from those obtained qwith s sleb of dielectric material. These results have been presented at the Conference Aspect'22 in Oldenburg.

Wave Propagation in Plasmas

Participants: Patrick Ciarlet, Maryna Kachanovska, Etienne Peillon.

This work is a continuation of the research done in collaboration with B. Després et al. on the degenerate elliptic equations describing plasma heating and is a part of the PhD thesis of E. Peillon. Plasma heating is modelled by the Maxwell equations with variable coefficients, which, in the simplest 2D setting can be reduced to the 2D Helmholtz equation, where the coefficient the principal part of the operator changes its sign smoothly along an interface. Such problems are naturally well-posed in a certain weighted Sobolev space; however, the corresponding solutions cannot contribute to the plasma heating, due to their high regularity.

It is possible to demonstrate that plasma heating is induced by singular solutions, which are square integrable but do not longer lie in this weighted Sobolev space. Such solutions can be represented as a product of an L^2-function varying in tangential direction of the interface and a singular function depending on the normal direction. The numerical method suggested in previous works is based on exploiting the plasma heating property, however, leads to the variational formulation which requires high regularity of the tangential component of the singular solution along the interface where the variable coefficient changes its sign.

We were able to alter the existing variational formulation in the way that requires a lower regularity of this tangential component of the solution; this allows in particular to be able to use lower order finite elements in the numerical simulation while preserving the general order of convergence of the scheme. Moreover, our numerical experiments indicate that the original regularity assumption for the tangential component is likely to be too restrictive, and does not seem to hold even in the cases when the coefficients of the problem are analytic in both variables. We are currently investigating this phenomenon, both numerically and theoretically.

The well-posedness of the variational formulation relies on numerical regularization, where the regularization parameter depends on the meshsize; however, it can be shown to be injective, and we are currently working on understanding whether this can be used in the numerical experiments.

Finally, from the theoretical viewpoint, the explanation of the plasma heating phenomenon is based on the limiting absorption principle. No justification was known, but in some special cases (1D or for slab geometries in 2D). We have obtained a proof of the limiting absorption principle that does not rely on such assumptions, and which makes use of the refined studies of the regularity of the limiting solution and the reflection principles.

Optimal control-based numerical method for problems with sign-changing coefficients

Participants: Patrick Ciarlet, Farah Chaaban, Mahran Rihani.

One considers the equation $\mathrm{div}(\varepsilon\nabla u) - \omega^2\mu u = f$ in Ω (plus boundary conditions), where the electric permittivity ε and the magnetic permeability μ are piecewise constant, and ε is strictly positive in part of the domain, and stricly negative elsewhere. When the problem is well-posed in $H^1(\Omega)$, meshing rules have been designed in the past, to ensure convergence of the discrete solution towards the exact solution (the so-called T-conform meshes). Following the Master's thesis of David Lassounon (2021) in which the model with $\mu = 0$ was considered, we are investigating methods based on control techniques, that allow in principle to compute solutions of problems with sign-changing coefficients without having to comply with those meshing rules. The mathematical theory has been successfully extended to the case $\mu \neq 0$. The numerical results obtained in two dimensions are very promising

Towards non-local interface models

Participants: Patrick Ciarlet.

A collaboration with Juan Pablo Borthagaray (DMEL, Universidad de la República, Montevideo, Uruguay). Consider the equation $\mathrm{div}(\sigma\nabla u) = f$ in Ω (plus boundary conditions), where the diffusivity is piecewise constant, and equals σ_i in Ω_i $(i = \{1,2\})$, with $\overline{\Omega_1} \cup \overline{\Omega_2} = \overline{\Omega}$ and $\Omega_1 \cap \Omega_2 = \emptyset$. If σ_1 and σ_2 have different sign, well-posedness in $H^1(\Omega)$ may not hold. This occurs when the ratio σ_2/σ_1 belongs to the so-called *critical interval*. When the interface has a corner, we have observed that this critical interval is shrunk if one replaces the standard H^1-bilinear forms by corresponding H^s-forms $(s \in (0,1))$. However, the cost of computing the nonlocal interactions may be prohibitive in applications. Thus, our long term goal is to confine the non-local model to a neighborhood of the interface, while keeping the standard local model in the rest of the domain. A first step in this direction consisted in considering the numerical solution with Lagrange finite element of the fractional Laplacian of index $s \in (1/2,1)$, whose solution a priori belongs to the fractional order Sobolev space $\widetilde{H}^s(\Omega)$, and to derive error estimates in $H^1(\Omega)$-norm when the solution belongs to this space. The second step is to build a model that couples local and nonlocal diffusion models, with fixed-sign diffusivity everywhere. Mathematical analysis is currently under way.

Maxwell's equations in presence of a conical tip with negative electromagnetic constants

Participants: Anne-Sophie Bonnet-Ben Dhia, Mahran Rihani.

This is the continuation of many works on transmission problems involving negative materials done in collaboration with Lucas Chesnel from INRIA team IDEFIX.

In the PhD of Mahran Rihani, we were interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip, and the problem is no longer well-posed in the classical framework. Previous results using the T-coercivity approach are not applicable.

Our work is inspired by what is known (thanks to ou rprevious works) for the 2D scalar case with critical coefficients: it has been proved hat well-posedness in the classical $H1$ framework is lost. This well-posedness can be recovered by working in an appropriate weighted Sobolev spaces and adding in the space one singular fonction, the so-called outgoing propagating singularity.

For the 3D tip, we established similar results, firstly for the scalar problem: a main difference with the 2D case is the possible existence of several outgoing singularities (the smallest the apex angle, the largest the number of outgoing singularities). These singularities are the eigenfunctions of a Laplace Beltrami spectral problem on the sphere, with sign-changing coefficients. The discreteness of the corresponding spectrum can be proved, for a smooth conical tip, using T-coercivity techniques on the sphere. The so-called critical interval of coefficients, for which propagating singularities exist, has been completely determined for the case of a circular conical tip.

Then we have established a new functional framework for the study of Maxwell's equations in presence of a conical tip of a negative material, when either the dielectric permittivity or the magnetic permeability, or both, belong to critical intervals. The space of electric fields for instance is obtained by adding to some weighted Sobolev space (included in L^2) the gradients of outgoing singularities (which do not belong to L^2). The formulation is proved to be Fredholm, the proof requiring establishing new vector potential decomposition results for non $L2$ vector fields.

Generalized normal modes of a metallic nanoparticle

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard. .

In the context of a collaboration with Matias Ruiz (University of Edinburgh) who spent 3 months at POEMS (from october to december, 2021), we have started a work concerning theoretical and numerical aspects of the so-called *plasmonic eigenvalue problem*. This problem can be formulated as follows, considering time-harmonic electromagnetic scattering (at a fixed frequency) by a metallic particle of given (possibly complex) permittivity ε in vacuum. Thanks to integral equation methods or Dirichlet-to-Neumann map techniques, such a problem can be reduced to a problem set on the particle itself. In the absence of incident wave, the reduced problem can be seen as a *non selfadjoint* eigenvalue problem where ε plays the role of an eigenvalue. The eigenfunctions associated to a possible eigenvalue are called *generalized normal modes*. The questions we are interested in are the following. What can be said about the essential / discrete spectrum? Do the generalized normal modes form a complete family (Riesz basis ?) of the natural energy space? How can we compute these eigenfunctions and associated modes?

In order to deal first with a simple situation for which a dispersion equation can be derived, we have considered the two-dimensional Helmholtz equation in a half uniform waveguide, where the end of the waveguide plays the role of a rectangular metallic particle. The problem has been implemented using the finite element library XLiFE++. Some promising numerical results have already been obtained. They show in particular two categories of modes: bulk modes and plasmonic modes (localized near the interface between the metallic particle and vacuum). A remedy for the occurrence of spurious modes has been tested.

7.2 Methods for unbounded domains, Perfectly Matched Layers, Dirichlet to Neumann maps and Half Space Matching method

On the Halfspace Matching Method for real wavenumber

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

We developed for several years a new method for the solution of 2D scattering problems with complex backgrounds, providing an alternative to Perfectly Matched Layers (PML) or other artificial boundary conditions. This method is called the Half-Space Matching (HSM) method. Based on half-plane representations for the solution, the scattering problem is rewritten as a system coupling (1) a standard finite element discretisation localised around the scatterer and (2) integral equations whose unknowns are traces of the solution on the boundaries of a finite number of overlapping half-planes contained in the domain. While satisfactory numerical results have been obtained for real wavenumbers, well-posedness and equivalence of this HSM formulation to the original scattering problem were established only for complex wavenumbers. Our new results, obtained in collaboration with Simon Chandle-Wilde (Reading University) concern the case of a real wavenumber and a homogeneous background. We proved that the HSM formulation is equivalent to the original scattering problem, and so is well-posed, provided the traces satisfy radiation conditions at infinity analogous to the standard Sommerfeld radiation condition. As a key component of our argument we show that, if the trace on the boundary of a half-plane satisfies our new radiation condition, then the corresponding solution to the half-plane Dirichlet problem satisfies the Sommerfeld radiation condition in a slightly smaller half-plane. We expect that this last result will be of independent interest, in particular in studies of rough surface scattering.

The Half-Space Matching method for elastodynamic scattering problems

Participants: Eliane Bécache, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

In collaboration with Antoine Tonnoir from INSA of Rouenm, we have extended the Half-Space Matching (HSM) method, first introduced for scalar problems, to elastodynamics, to solve time-harmonic 2D scattering problems, in locally perturbed infinite anisotropic homogeneous media. The HSM formulation couples a variational formulation around the perturbations with Fourier integral representations of the outgoing solution in four overlapping half-spaces. These integral representations involve outgoing plane waves, selected according to their group velocity, and evanescent waves. Numerically, the HSM method consists in a finite element discretization of the HSM formulation, together with an approximation of the Fourier integrals. We have performed numerical results, validating the method for different materials, isotropic and anisotropic, and we have compared them to results obtained with the Perfectly Matched Layers (PML) method. For materials for which PMLs are unstable in the time domain, these results highlight the robustness of the HSM method, contrary to the PML method which is very sensitive to the choice of the parameters.

Evaluation of oscillatory integrals in the Half-Space Matching Method

Participants: Amond Allouko, Anne-Sophie Bonnet-Ben Dhia.

This work concerns the Half Space Matching method described just above. A main ingredient of this method is a half-space formula for the outgoing scattered field. This formula is based on a convolution with a half-space Green function, whose partial Fourier transform is known analytically. Then computing this kernel amounts to evaluate Fourier integrals which may become highly oscillating, so that they cannot be evaluated with standard quadrature formulas. The difficulty is aggravated by the fact that oscillations accumulate at the branch points if the function to integrate.

We proposed two ways for improving both the accuracy and the efficiency of the evaluation of this kernel $K(\mathbf{x},\mathbf{y})$. For an evaluation of the kernel far from the point source (large $|\mathbf{x}-\mathbf{y}|$), a simple far-field formula can be used. In other cases, a deformation of the Fourier path in the complex plane allows to get rid of the oscillations and of the singularity at the branch points. The deformation has to be carefully chosen, depending on the orientation of the segment defined by $\mathbf{x}$ and $\mathbf{y}$, with respect to the boundary of the half-space.

Numerical experiments for the 2D isotropic and anisotropic cases confirm that these two ideas dramatically reduce the cost of the method. We are now generalizing this approach to the case of an elastic plate, where the expression of the kernel involves both apartial Fourie-tranform and a modale decomposition on Lamb modes.

Radial PML for anisotropic media

Participants: Maryna Kachanovska, Markus Wess.

In this work, joint with Martin Halla (post-doc at Uni Göttingen), we would like to answer the following question, suggested by Martin Halla some time ago: since anisotropic media does not exhibit backward propagating waves in radial directions, are radial PMLs stable in this case?

We start with the simplest model of this kind: anisotropic wave equation, for which much can be computed analytically. Our numerical experiments were quite inconclusive: in fact we noticed instabilities only on fine discretizations. Again, to analyze the problem, we work in the Laplace domain. The analysis of the complex-scaled fundamental solution and underlying PDE revealed the following:

- the PML problem is well-posed;
- at the continuous level, if the source term is located far enough from the interface between the PML and the physical media, the respective solution can be proven to be stable; the distance between the source and the PML depends on the anisotropy;
- however, the problem itself is not stable, more precisely, the associated spatial operator has essential spectrum in the right-half complex plane.

This resembles very much our previous findings for frequency-domain behaviour of Cartesian PMLs for time-harmonic anisotropic acoustic wave equation, done in the framework of the postdoc of Maria Kazakova.

Although we can ensure stability of the continuous solution by choosing sufficiently large physical domain, this does not seem true for the discrete solution. Therefore, we suggest the following: the radial PMLs can be shown to be stable if instead of the classical Bérenger's scaling $r + \frac{\sigma(r-r_{pml})}{s}$, we make use of the complex frequency shifted scaling $r + \frac{\sigma(r-r_{pml})}{s+\gamma}$, with $\frac{\sigma}{\gamma} < c$, where c depends on the anisotropy of the media. In general, if we truncate such PMLs at the length L, we cannot expect their convergence, unless L depends on the final time (which is what we would like to avoid). Therefore, instead we do not truncate the PMLs and discretize the problem with the help of the infinite elements. We are currently writing the manuscript summarizing our findings.

Stability and convergence of Perfectly Matched Layers in dispersive waveguides

Participants: Eliane Bécache, Maryna Kachanovska, Markus Wess.

During the post-doc of Markus Wess, we have worked on the extension of the stability and convergence analysis of the generalized perfectly matched layers (GPMLs) for dispersive media, proposed by Bécache, Joly, Vinoles. The idea is to extend the techniques developed for non-dispersive waveguides to the dispersive case. Like in the non-dispersive case, we obtain an explicit representation of the solution in the Laplace domain. The point is to estimate the inverse Laplace integral. One of the difficulties compared to the non-dispersive case is the appearance of various propagation regimes (also in the time domain): there exist backward propagating waves, forward propagating waves, evanescent waves depending on the frequency. We demonstrate that surprisingly, despite the complexity of the physical phenomena, the GPMLs for dispersive media converge as fast as the classical PMLs for non-dispersive media. Moreover, our analysis is done in a way that allows to reveal the optimality/non-optimality of the intermediate results, and we hope that it can be used further if any improvement is necessary.

When working on this project, we have realized a rather surprising fact: it appears that the classical PMLs can exhibit instabilities in the presence of heterogeneities inside the physical domain (which do not touch the PML boundary)!

We have submitted the paper summarizing our results.

Construction of transparent conditions for electromagnetic waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Aurélien Parigaux.

This work is done in the framework of the PhD of Aurélien Parigaux, co-advised by Anne-Sophie Bonnet-Ben Dhia and Lucas Chesnel from Inria team IDEFIX.

Electromagnetic waveguide-based components play an important role in many application areas. We are particularly interested in configurations where several semi-infinite guides interact in a bounded zone of space, which is discretized by finite elements. In this context, a major difficulty is to truncate the guides by imposing well-chosen transparent conditions, in order to obtain a well-posed problem corresponding to the initial problem. This question is very well understood for scalar models of acoustic guides, but remains a delicate subject for heterogeneous electromagnetic guides.

Up to now, we have considered the case of an homogeneous electromagnetic waveguide with a simply connected cross-section. It is well-known in this case that a transparent boundary condition which links the tangential electric field to the tangential magnetic field on the artifical boundary, can be written using a modal expansion on Tranverse Electric and Transverse Magnetic modes. Our contributions are the following ones.

- We have proved that the problem with a truncated modal expansion, keeping N modes in the expansion, is well-posed if N is large enough, and that the error produced by the truncation decays exponentially with N, and with L, where L is the distance of the artificial boundary to the sources.
- We have also proposed a new formulation where the transparent boundary condition links the tangential electric field on an internal cross-section to the tangential magnetic field on the artificial boundary. The advantage is the compactness of the non-local term coming from the transparent boundary condition.

PML-BIE methods for unbounded interfaces

Participants: Anne-Sophie Bonnet-Ben Dhia, Luiz Faria.

There are several important applications where one must solve a scattering problem in a domain with infinite boundaries; e.g. seismic waves in a stratified medium, or water waves in the ocean. In order to handle such infinite interfaces in a boundary integral equation context, a few options are available. For simple geometries, one can construct a problem specific Green function which incorporates the imposed boundary condition on all but a bounded portion of the interface, thus reducing the problem again to integrals over bounded curves/surfaces. This has the advantage of being conceptually simple provided such problem-specific Green function can be efficiently computed. Unfortunately, that is usually not the case, and the computation of problem-specific Green functions involves challenging integrals which must be approximated numerically.

An alternative approach consists of utilizing the free-space Green function — readily available for many PDEs of physical relevance — in conjunction with a truncation technique. For non-dissipative problems, the slow (algebraic) decay — or even logarithmic growth — of the Green function makes the choice of truncation technique an important aspect which needs to be considered in order to reduce the errors associated with the domain's truncation. An easy-to-implement solution, the so-called windowed Green function approach, has been proposed and validated in several configurations.

We are currently investigating the interest of using instead a complex-scaled Green function, which amounts to combine the method of perfectly-matched-layers (PMLs) and boundary integral equations.

We have applied this idea to the 2D linear time-harmonic water wave problem, in finite or infinite depth, writing a complex-scaled integral equation on the free surface. The formulation uses only simple function evaluations (e.g. complex logarithms and square roots). Let us mention that because the water waves are surface waves, the windowed Green function approach does not work for this problem.

Numerical results show that the error decays exponentially with respect to the distance of truncation.

Another advantage of our method is that the formulation has a simple quadratic dependence with respect to the frequency, and is well-suited for computing complex scattering frequencies.

Convolution quadrature for the PMLized boundary integral formulation for scattering by infinite rough surfaces

Participants: Luiz Faria, Maryna Kachanovska.

This work was an M1 internship of Tiphaine George, and here we present some preliminary results.

The numerical simultation of time-domain scattering by infinite rough surfaces is often done with the help of boundary-integral equations. However, unlike in the classical case, when the use of the BIE formalism allows to work on the finite computational domains, in the rough surface scattering one has to deal with integral operators defined on infinite domains. One can do so by using special quadratures, or, alternatively, by applying the idea of Lu et al. '18, originally formulated in the time-harmonic case, by combining the PMLs and the boundary integral equations. In the time domain, the first approach had been studied numerically before, however not much is known for the second approach.

We have formulated the PMLized BIE for our problem. We realized that one of the difficulties of this approach is that the fundamental solution is known in the Laplace domain, however, no closed time-domain formulation, at least in 3D, is available. Therefore it is natural to semi-dicsretize the corresponding formulation in time using the convolution quadrature method. This amounts to replacing the original time-domain integral kernel depending on t by its discrete counterpart at the discrete times $n\Delta t$. Our numerical experiments indicate that for all $n \geq 2$, these new kernels decay exponentially fast in space inside the PMLs, and thus we should be able to truncate the corresponding spatial integrals. However, for $n = 1$, this does not seem to be the case because of the singularity of the kernel; this phenomenon requires a deeper investigation. Our future efforts are dedicated in particular to the theoretical justification and numerical investigation of the proposed method.

7.3 Fast solution of boundary integral equations

General-purpose kernel regularization of boundary integral equations via density interpolation

Participants: Luiz Faria, Marc Bonnet.

This research is done in collaboration with Carlos Pérez-Arancibia (University of Twente, Netherlands) and Thomas Anderson (Univ. of Michigan, USA). We initially developed a general high-order kernel regularization technique applicable to all four integral operators of Calderón calculus associated with linear elliptic PDEs in two and three spatial dimensions. The proposed technique relies on interpolating the density function around the kernel singularity in terms of solutions of the underlying homogeneous PDE, so as to recast singular and nearly singular integrals in terms of bounded (or more regular) integrands. We developed a simple interpolation strategy which, unlike previous approaches, does not entail explicit computation of high-order derivatives of the density function along the surface. Furthermore, the proposed approach is kernel- and dimension-independent, thus making the procedure applicable, in principle, to any PDE with known Green's function. In the initial work we have focused on Nyström methods for the (scalar) Laplace and Helmholtz equations and the (vector) elastostatic and time-harmonic elastodynamic equations. We currently work on extending this approach to the evaluation of volume potentials of the kind involved in e.g. Lippmann-Schwinger volume integral equations.

Windowed Green function method for wave scattering by periodic arrays of 2D obstacles

Participants: Luiz Faria.

This research is done in collaboration with Carlos Pérez-Arancibia (University of Twente, Netherlands), Thomas Strauszer-Caussade (PUC, Chile), and Augustín Fernandez-Lado (Intel Coorporation, Oregon, USA). This work introduces a novel boundary integral equation (BIE) method for the numerical solution of problems of planewave scattering by periodic line arrays of two-dimensional penetrable obstacles. Our approach is built upon a direct BIE formulation that leverages the simplicity of the free-space Green function but in turn entails evaluation of integrals over the unit-cell boundaries. Such integrals are here treated via the window Green function method. The windowing approximation together with a finite-rank operator correction—used to properly impose the Rayleigh radiation condition—yield a robust second-kind BIE that produces superalgebraically convergent solutions throughout the spectrum, including at the challenging Rayleigh–Wood anomalies. The corrected windowed BIE can be discretized by means of off-the-shelf Nyström and boundary element methods, and it leads to linear systems suitable for iterative linear algebra solvers as well as standard fast matrix–vector product algorithms. A variety of numerical examples demonstrate the accuracy and robustness of the proposed methodology.

High-order Boundary Integral Equations on implicitly defined surfaces

Participants: Luiz Faria, Dongchen He.

This research is being done in collaboration with Aline Lefebvre-Lepot (CMAP), and in the context of the Ph.D. thesis of Dongchen He. We are developing a method for accurately solving boundary integral equations on implicitly defined surfaces in $\mathbb{R}^d$. The method relies on combining a dimension-indepent technique for generating a high-order surface quadrature on level-set surfaces, with the general-purpose density interpolation method for handling the singular and nearly-singular integrals ubiquitous in boundary integral formulations. The proposed methodology, based on a Nystrom discretization scheme, bypasses the need for generating a body conforming mesh for the implicit surface, allowing in principle for an efficient coupling between a robust dynamic level-set representation of deforming surfaces, and boundary integral equation solvers. Particular attention is being paid to the computation of singular integrals when only a surface quadrature is available (i.e. in the absence of an actual mesh). We believe such techniques could prove useful in applications involving microscopic flows governed by the Stokes equations; in particular, the simulation of micro-swimmers and droplet microfluidics.

Boundary Element - Finite Element coupling for transient fluid-structure interaction

Participants: Marc Bonnet, Stéphanie Chaillat, Alice Nassor.

This study is done in collaboration with Bruno Leblé (Naval Group). It aims at developing computational strategies for modelling the impact of a far-field underwater explosion shock wave on a structure, in deep water. An iterative transient acoustic-elastic coupling is developed to solve the problem. Two complementary methods are used: the Finite Element Method (FEM), that offers a wide range of tools to compute the structure response; and the Boundary Element Method (BEM), more suitable to deal with large surrounding acoustic fluid domains. We concentrate on developing a transient FEM-BEM coupling algorithm with a fast convergence. Since the fast transient BEM is based on a fast multipole-accelerated Laplace-domain BEM (implemented in the in-house code COFFEE), extended to the time domain by the Convolution Quadrature Method (CQM), we have proposed a proof of convergence of global-in-time iterative solution procedures for the coupled transient fluid-structure problem based on Robin-Robin coupling iterations. This proof itself relies on solvability results for the coupled acoustic-elastic problems, some of which we established in the course of this study, and the initial-boundary value problems involved in coupling iterations. We have initially implemented a 2D version of this algorithm and assessed its sensitivity to various parameters. A variant of the previous coupling algorithm where the coupling interface is immersed in the water, rather than taken as the fluid-structure interface, has in addition been formulated. We proved the convergence of this variant, and validated it on 2D numerical experiments. We are currently implementing this approach into code aster (the open-source

EDF general-purpose structural FEM analysis library) in order to demonstrate its adaptability to existing industry computational platforms.

Modelling the sound radiated by a turbulent flow

Participants: Stéphanie Chaillat, Jean-François Mercier, Louise Pacaut.

This PhD study is done in collaboration with Gilles Serre (Naval Group). The aim is to develop an optimized method to determine numerically the 3D Green's function of the Helmholtz equation in presence of an obstacle of arbitrary shape. The determination of such so-called adapted Green's function is useful to solve the Lighthill's equation, notably giving the hydrodynamic noise radiated by a ship. The case of a rigid obstacle satisfying the Neumann boundary condition at the boundary surfaces has been the subject of a previous PhD. The aim of this second PhD is to extend to the penetrable case. The case of an elastic body is the most interesting but complicated and thus the simpler fluid case is considered first. For this fluid-fluid case, first an integral equation is derived, expressing the adapted Green's function versus the free space Green's functions associated to the two fluids. Then a Boundary Element Method (BEM) implemented in the code COFFEE is used to compute the adapted Green's function. In order to consider realistic geometries in a reasonable amount of time, fast BEMs are used: fast multipole accelerated BEM and hierarchical matrix based BEM. The validity of these two approaches is tested on a simple sphere geometry for which an analytic solution can be determined.

Asymptotic based methods for very high frequency problems.

Participant: Eric Lunéville.

This research is developed in collaboration with Marc Lenoir (retired) and Daniel Bouche (CEA).

It has recently been realized that the combination of integral and asymptotic methods was a remarkable and necessary tool to solve scattering problems, in the case where the frequency is high and the geometry must be finely taken into account.

In order to implement the high-frequency approximations that we are developing as part of these hybrid HF/BF methods, we have introduced new geometric tools into the XLiFE++ library, in particular splines and B-Splines approximations as well as parameterizations to access quantities such as curvature, curvilinear abscissa, etc. We have also achieved the interface between the OpenCascad library and the XLiFE++ library, which allows us to manage complex geometric situations (cylinder and sphere intersection for example). In parallel, we have completed the implementation of 2D HF approximations in the shadow-light transition zone based on the Fock function and the diffraction by a 2D corner using asymptotics approximation. More recently we began to investigate the 3D case. As a first step we developed in XLiFE++ some new tools to compute geodesics on any surfaces (parametrized or only meshed). Now, the work consists in extending the 2D asymptotic expansions along the geodesics.

High-frequency estimates and error bounds on the h-BEM for the Helmholtz exterior Neumann problem

Participants: Pierre Marchand.

This research is done in collaboration with Jeffrey Galkowski (University College London) and Euan Spence (University of Bath).

We studied how to solve the Helmholtz equation posed in the exterior of a smooth obstacle, with Neumann boundary conditions, and using second-kind boundary integral equations (BIEs). In the context of Galerkin discretisation, it is important to understand the behaviour of Galerkin error when the frequency increases, and high-frequency estimates for the considered boundary integral operators are usually necessary to explicit the dependency between the discretisation parameters and the frequency.

In the case of Dirichlet boundary condition, results are well-known in the literature for the standard second-kind BIE, but in the case of Neumann boundary condition, where regularisation techniques are commonly used, such estimates are rare. Thus, we developed new high-frequency estimates for a particular boundary integral equation using a single-layer operator with complex frequency as regularising operator, which led to the first result (that we know of) about frequency-explicit bound for the discretisation error of a second-kind formulation for the Helmholtz equation with Neumann boundary condition.

Diffraction by fractal screens

Participants: Patrick Joly, Maryna Kachanovska, Zoïs Moitier.

This is a new topic that enters the framework of the HyBox project and is the subject of the Post-Doc of Zoïs Moitier.

The goal of the postdoc is to develop efficient numerical method to compute the scattering by fractal screen. The motivation comes from applications to, for example, fractal antenna engineering. A fractal antenna has a shape with self-similar features at different scales which allows the antenna to operate at multiple frequencies or even for a range of frequencies. However, numerical simulation of such device is complicated as it does not enter the framework of regular antennas. Recent methods have been developed to do numerical computations for fractal antennas, but due to their intricate shape, the discretization of the antenna needs to be very fine which translate to high number of degree of freedom and a high computational cost. The idea is to adapt to fractal antenna known techniques of matrix compression used in the context of scattering by obstacle such as H-matrix to reduce the computational cost. Those numerical methods boil down to solving a linear system that involve a matrix describing a boundary integral equation on the fractal antenna. This matrix compression technique aims to replace the matrix by a close enough matrix for which solving the linear system is cheaper.

Eddy-current asymptotics of the Maxwell PMCHWT formulation for multiple bodies and conductivity levels

Participants: Marc Bonnet.

(Work done in collaboration with Edouard Demaldent from CEA)

In eddy current (EC) testing applications, currents are induced in tested metal parts by a low-frequency (LF) source idealized as a closed current loop in air. In the case of highly conducting (HC) parts, a boundary integral equation (BIE) of the first kind under the magneto-quasi-static approximation - which neglects the displacement current - was shown in a previous work to coincide with the leading order of an asymptotic expansion of the Maxwell BIE in a small parameter reflecting both LF and HC assumptions. The main goal of this work is to generalize the latter approach by establishing a unified asymptotic framework that is applicable to configurations that may involve multiple moderately-conducting and non-conducting objects in addition to (possibly multiply-connected) HC objects. Leading-order approximations of the quantities relevant to EC testing, in particular the impedance variation, are then found to be computable from a reduced set of primary unknowns (three on HC objects and two on other objects, instead of four per object for the Maxwell problem). Moreover, when applied to the Maxwell BIE, the scalings suggested by the asymptotic approach stabilize the condition number at low frequencies and remove the low-frequency breakdown effect. The established asymptotic properties are confirmed on 3D numerical examples for simple geometries as well as two EC testing configurations, namely a classical benchmark and a steam generator tube featured in pressurized water reactors of nuclear power plants.

7.4 Accelerated finite element solvers and domain decomposition methods

GPU-accelerated DG finite element solver for unsteady acoustics

Participants: Axel Modave, Rose-Cloé Meyer.

This research topic is developed in collaboration with Hadrien Bériot (SIEMENS), Thijs van Putten (SIEMENS) and Gwénaël Gabard (LAUM). It corresponds to the post-doctoral work of Rose-Cloé Meyer, funded in part by the french "Plan de relance" program and SIEMENS.

In this project, we are interested in accelerating solution procedures for unsteady wave problems with discontinuous Galerkin (DG) finite element methods. These methods are well suited for acoustic and aeroacoustic studies thanks to e.g. adjusted meshes and high-order finite elements. In addition, the algorithmic structure obtained after discretization makes it possible to effectively take advantage of the power of parallel computing architectures, and in particular graphics cards (GPU). In applicative cases, the complexity of the phenomena and structures to be modelled, as well as the growing needs for precision, lead to an increase in the costs associated with numerical simulations (e.g. runtime, memory storage, ...), which motivates researches on more efficient numerical methods and implementations. During the first year of this project, Rose-Cloé Meyer has extended and studied a GPU version of a C++ DG finite element code developed by SIEMENS. Some implementation strategies were well known, but a difficulty was to apply these strategies while preserving as much as possible the structure of the existing code. We are currently working on a numerical extension of the method with wave-type basis functions, in the spirit of the Trefftz methods.

A Trefftz DG finite element formulation with high-order absorbing boundary conditions for Helmholtz problems

Participants: Axel Modave, Simone Pescuma.

This research topic is developed in collaboration with Gwénaël Gabard (LAUM). It corresponds to the M2 internship of Simone Pescuma, funded by the WavesDG ANR project.

In this project, we have considered a Trefftz discontinuous Galerkin (DG) finite element method with plane-wave basis functions for solving Helmholtz problems set in unbounded domains. In practice, truncated domains must be used, and non-reflecting boundary treatments must be prescribed at the artificial boundaries. During his internship, Simone Pescuma developed an Ultra Weak Variational Formulation (UWVF) with high-order absorbing boundary conditions. This formulation has been implemented and studied in a 2D dedicated MATLAB code. In particular, we have considered scattering problems with square/circular truncated domains and we have investigated p-refinement strategies. We have observed that the proposed strategies improve the accuracy of the numerical solution in comparison with standard low-order absorbing boundary condition.

A hybridizable DG method with characteristic variables for Helmholtz problems

Participants: Axel Modave.

This research topic is developed in collaboration with Théophile Chaumont-Frelet (INRIA, Atlantis), within the framework of the WavesDG ANR project.

We have proposed a new hybridizable discontinuous Galerkin method, named the CHDG method, for solving time-harmonic scalar wave propagation problems. This method relies on a standard discontinuous Galerkin scheme with upwind numerical fluxes and high-order polynomial bases. Auxiliary unknowns corresponding to characteristic variables are defined at the interface between the elements, and the physical fields are eliminated to obtain a reduced system. The reduced system can be written as a fixed-point problem that can be solved with stationary iterative schemes. Numerical results with 2D benchmarks have been proposed to study the performance of the approach. Compared to the standard HDG approach, the properties of the reduced system are improved with CHDG, which is more suited for iterative solution procedures. The condition number of the reduced system is smaller with CHDG than with the standard HDG method. Iterative solution procedures with CGN or GMRES required smaller numbers of iterations with CHDG.

Non-overlapping Domain Decomposition for wave propagation problems

Participants: Patrick Joly, Jérémy Héleine.

This work is a continuation of the PhD thesis of É. Parolin on non-local and contitutes the subject of the short post-doctoral stay of Jérémy Heleine in the framework of the RAPID Project Hybox. A first contribution was to write and analyse a dual version of the multi-trace formulation of Claeys-Parolin at the continuous level. This formulation is adapted to the use of midex hybrid finite elements for the discretization of the local Helmholtz problems.

The interest of the multi-trace formulation is that its leads to an exponential convergence of the iterative domain decomposition algorithm even in the presence of junction points. A drawback is that the communication process between subdomains couples subdomains which are not neighbour the one from the other. To avoid this, J. Heleine has proposed a first "quasi-localized" multi-trace formulation based ofn the use of a partition of unity wirh cut-off functions. The analysis and the implementation of the method remain to be done.

7.5 Inverse Problems, Invisibility and Optimization

Imaging junctions of waveguides

Participants: Laurent Bourgeois, Jean-François Fritsch, Christophe Hazard.

A new activity recently started concerning forward and inverse scattering problems in junctions of waveguides. It corresponds to the PHD of Jean-François Fritsch and is a collaboration with the CEA-List, in particular Arnaud Recoquillay. The last year we have considered the challenging case of an elastic waveguide which is embedded in water, which corresponds to a solid-fluid interaction problem. A typical situation in the oil industry is the case of a metallic tube which is partially embedded into a fluid. It may happen that some defects within the embedded part of the tube or at the interface between the tube and the surrounding medium have to be retrieved from measurements located on the only accessible part of the tube, that is its free part. We have chosen to use Perfectly Matched Mayers in the transverse direction of the open part of the structure in order to transform the actual structure into a closed stratified waveguide which consists of the elastic part, the fluid part, and the PMLs. First, we have analyzed the different types of modes in such closed waveguide, and then extended the notion of variables (X,Y) introduced by Maurel and Pagneux in the case of the elastic waveguide, which mix some components of the stress tensor and some components of the displacement vector, to the case of our closed stratified waveguide. Such variables (X,Y) extended to the solid-fluid structure enable us, with the help of a biorthogonality relationship, to project the different fields on the transverse sections. Secondly, we have used those (X,Y) variables to introduce artificial boundary conditions in the longitudinal direction with the help of a $Y - to - X$ operator. A weak formulation in a bounded domain was then derived to solve forward scattering problems, and we have compared the obtained results with those obtained with another PML in the longitudinal direction instead of using the $Y - to - X$ operator, in the absence of backward modes. Lastly, we extended the modal formulation of the Linear Sampling Method in order to solve the inverse obstacle problems for our fluid-structure interaction case, by using artificial data provided by the previous forward computations. Despite such inverse problem is particularly difficult because we are in a back-scattering situation (data are known on only one side of the defects) and because the leaky modes are evanescent in the longitudinal direction (they are responsible for a loss of information), the numerical experiments show very satisfactory identification results.

Computation of the interior transmission eigenvalues in presence of strongly oscillating singularities

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard, Florian Monteghetti.

In the context of time-harmonic scattering by a bounded penetrable scatterer, interior transmission eigenvalues correspond, when they are real, to discrete frequencies for which there exists an incident wave which does not scatter. At such frequencies, inversion algorithms such as the linear sampling method fail. Real interior transmission eigenvalues are a part of a larger spectrum made of complex values, which has been largely studied in the case where the difference between the parameters in the scatterer and outside does not change sign on the boundary. In collaboration with Lucas Chesnel (INRIA team IDEFIX), we obtained some years ago some results for a 2D configuration where such sign-change occurs. The main idea was that, due to very strong singularities that can occur at the boundary, the problem may lose Fredholmness in the natural H^1 framework. Using Kondratiev theory, we proposed a new functional framework where the Fredholm property is restored. This is very similar (while more intricate) to what happens for the plasmonic eigenvalue problem in presence of a corner of negative material.

This explains why we decided to extend the numerical method we used for plasmonic eigenvalues to interior transmission eigenvalues. It has been already checked that a naive finite element computation does not converge, and that the convergence is restored by using some complex scaling near the singular point.

Mixed formulations of the Tikhonov regularization

Participant: Laurent Bourgeois.

Work done in collaboration with Jérémi Dardé, from IMT Toulouse.

For a quite long time now, we have developed the notion of mixed formulations of the Tikhonov regularization in collaboration with Jérémi Dardé (IMT Toulouse). This notion has connections with the old concept of quasi-reversibility introduced by Lattès and Lions (67) and is intended to regularize ill-posed PDE problems. Essentially, our mixed formulations enable us to adapt this concept to a friendly Finite Element Method context using simple conforming elements. We adapted the notion of mixed formulations of the Tikhonov regularization to the context of data assimilation problems, in particular by applying the Morozov's discrepancy principle to select the regularization parameter as a function of the amplitude of noise. A specific problem in data assimilation problems is that the underlying operator has not a dense range, which required to extend some well-known results to that situation. A dual problem was introduced in order to compute the regularized solution corresponding to such regularization parameter, possibly by enforcing the solution to satisfy some constraints.

Shape optimization problems involving slow viscous fluids

Participant: Marc Bonnet.

Work done in collaboration with Shravan Veerapaneni and his group, University of Michigan, USA.

This collaboration addresses the design and implementation of computational methods for solving shape optimization problems involving slow viscous fluids modelled by the Stokes equations. We have developed a new boundary integral equation (BIE) approach for finding optimal shapes of peristaltic pumps that transport viscous fluids, as well as dedicated formulas for computing the shape derivatives of the relevant cost functionals and constraints, expressed in boundary-only form. By employing these formulas in conjunction with a BIE approach for solving forward and adjoint problems, we completely avoid the issue of volume remeshing when updating the pump shape as the optimization proceeds. This is especially useful when the fluid carries objects (e.g. particles, deformable vesicles) whose motion is not known beforehand. Significant cost savings are achieved as a result, and we demonstrate the performance on several numerical examples. We also currently investigate methods allowing the numerical evaluation of negative fractional Sobolev norms on arbitrary domains, which in our context are involved in the quantification of the quality of mixing for concentrations carried by fluid flows. We formulated and demonstrated a treatment based on Padé approximants, and are working on an alternative approach based on volume integral equations associated with fractional-order PDEs.

7.6 Asymptotic analysis, homogenization and effective models

Propagation of ultrasounds in complex biological media

Participants: Laure Giovangigli, Quentin Goepfert.

This is a joint work with Josselin Garnier (X-CMAP) and Pierre Millien (Institut Langevin). This project aims at modelling and studying the propagation and diffusion of ultrasounds in complex multi-scale media in order to obtain quantitive images of physical parameters of these media.

The propagation of ultrasounds in biological tissues is a complex multi-scale phenomenon : the scattered wave is produced by small (compared to the wavelength) inhomogeneities randomly distributed throughout the medium. In order to characterize the response of this medium to an incident plane wave, we perform an asymptotic expansion of the scattered wave with respect to the size of the inhomogeneities using stochastic homogenisation techniques. The difficulties lie in the transmission conditions at the boundary of the medium. We derive quantitative error estimates given that the random distribution of inhomogeneities verifies mixing properties. Finally we present numerical simulations to illustrate and validate our results.

Effective dielectric properties of bio-materials in humid environment

Participants: Laure Giovangigli.

This is a joint work with Pierre Millien (Institut Langevin), Hatem Zaag (Paris 13), Amina Gharbi (Sfax) and Nabil Derbel (Sfax). In humid environment bio-materials exhibit a change from resistive to capacitive electrical properties. The goal of this work is to model the influence of the adsorption process at the nano-scale on the effective electric properties at the macroscopic scale. The adsorption process is modeled by random alterations of the surface conductivity of the material. Effective properties are then derived using stochastic homogenization techniques. This work has applications in the design of humidity bio-sensors.

Modelling a thin layer of randomly distributed nano-particles

Participants: Sonia Fliss, Laure Giovangigli, Amandine Boucart.

This is a joint work with Bruno Stupfel from CEA-CESTA. We study the time-harmonic scattering by a heterogeneous object covered with a thin layer of nanoparticles. The size of the particles, their distance between each other and the layer's thickness are all of the same order but small compared to the wavelength of the incident wave. Solving numerically Maxwell's equation in this context is very costly. To circumvent this, we propose, via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles is replaced by an equivalent boundary condition.

We consider two configurations : - a two-dimensional random distribution of particles invariant in the tangential direction. Maxwell's equations reduce then to the Helmholtz equation with either Neumann or Dirichlet boundary conditions on the particles. The coefficients that appear in the equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. Under the assumption that the particles are distributed given a stationary and ergodic random point process, we prove that those problems admit a unique solution in the proper spaces with either homogeneous Dirichlet or Neumann boundary conditions on the inclusions. - a three-dimensional periodic distribution of particles. We take advantage here of the periodicity to restrict our domain of study to an infinite vertical strip with periodic boundary conditions on the lateral boundaries.

For both configurations we establish quantitative error estimates for the effective model and present numerical simulations that illustrate our theoretical results.

Enriched homogenized model in the presence of boundaries for wave equations

Galerkin Time-Domain Foldy-Lax models

Participants: Maryna Kachanovska, Adrian Savchuk.

The Foldy-Lax model is an asymptotic model used to compute the solution to the problem of scattering by small obstacles. While this subject had been fairly well-studied in the frequency-domain, its time-domain analysis is still in its infancy stage. In our previous work, we have suggested a construction of an asymptotic model as a Galerkin spatial semi-discretization of associated boundary integral formulations. The main idea is to choose the basis functions in a way that the convergence of the method is ensured not by increasing the cardinality of the Galerkin basis, but rather by decreasing the size of the obstacles. We have submitted a manuscript where we presented and analyzed the method for the asymptotic sound-soft scattering by circles.

Currently we are working in two directions. First of all, it is a generalization of the method to arbitrary obstacles. We use basically the same Galerkin Foldy-Lax method, however, the analysis has to be altered, and we no longer obtain (at least theoretically) the same convergence order as for the circles. The implementation of the method to validate the theoretical results is currently on the way.

Second, and more importantly, we are working on the construction of the method for the Maxwell equations (this is a part of the M2 internship and of the PhD of Adrian Savchuk). We base our considerations on the single layer boundary integral formulation, and the difficulty here is that the corresponding densities are no longer scalars but vectors. This makes the choice of the basis functions quite subtle, especially for general domains. Our asymptotic analysis of scattering by a sphere revealed that a good candidate for the Galerkin basis is a set of tangential traces of constant vector-valued functions (whose dimension is 3), and tangential traces of linear vector-valued functions (whose dimension is a priori at least 9). The number of the 'linear' basis functions can be further be reduced to 3, if one considers asymptotic behaviour of their Helmholtz decomposition on the boundary of the domain. This idea was validated for scattering by a sphere, and currently we are working on the numerical experiments in order to verify whether it can be extended to more general domains.

Let us remark that our analysis on a sphere led to another interesting conclusion. The convergence analysis of existing asymptotic models in the frequency domain is quite technical, and relies on the Helmholtz decomposition on the boundary of the domain. We realized that the use of the correct space framework, with the norms depending on the asymptotic parameter, may elevate the technical difficulties in the analysis.

Asymptotic modeling of thin periodic layers with high-contrast inclusions for electromagnetic waves

Participants: Sonia Fliss, Patrick Joly, Florian Monteghetti.

This is the subject of the Post Doc of F. Monteghetti which is part of the HyBox project (task T1.1).

Metasurfaces are thin composite materials that are engineered to control incident electromagnetic waves at wavelengths larger than their thickness. They have promising applications in both microwave and optical regimes, such as flat antennas for radar cross section reduction or flat lenses for head-up displays. However the design of metasurfaces using numerical simulation is challenging due to the existence of multiple scales leading to prohibitively expensive meshes.

The broad objective of this study is to model metasurfaces as effective boundary or transmission conditions for Maxwell's equations. Mathematically, we assume that the metasurface: (i) consists of a periodic array of sub-wavelength structures; (ii) is a *thin layer*, i.e. the ratio between the thickness L and the incident wavelength λ is small; (iii) the ratio between the spatial period l and λ is small. These assumptions enable to use techniques from asymptotic analysis and homogenization theory.

We have begun to investigate two metasurfaces that are of interest to the industrial partners involved in HyBox:

- Array of lossy thin wires, as encountered in e.g. the Faraday cages used to protect buildings from lightning. Our intent is to bridge the gap between the physical literature, where the dominant model is the so-called "Casey law" whose accuracy has not been studied, and the mathematical literature, where only perfectly-conducting wires have been rigorously considered with Maxwell's equations.
- Array of high-contrast inclusions above a perfectly-conducting surface. This type of metasurfaces have only been studied under a transverse approximation. To avoid topological effects, we first consider inclusions that are both compactly embedded in the unit cell and simply connected.

For both cases, the numerical validation of the derived effective conditions will be studied.

Wave diffraction by thin finite periodic layers

Participants: Cédric Baudet, Sonia Fliss, Patrick Joly.

This is the subject of the PhD thesis of C. Baudetwhich is part of the HyBox project (task T1.1).

In this work, we consider the diffraction of waves by an object partially covered by a periodic layer whose thickness tends to 0. This situation can model industrial applications where the layer is often made of a metamaterial with unusual wave propagation properties. The aim is, first, to made an asymptotic analysis of the solution when the layer's thickness tends to 0. This uses a technique of domain splitting in coupled subdomains, mainly the zone near the layer and the rest of the exterior domain. Secondly, we aim to design approximated models that replace the layer with an effective boundary condition inspired by the asymptotic development found in step 1. Finally we will set up numerical methods to solve the diffraction problem using the previous models, thus reducing a lot the computational cost compared to a naive method that implies finely meshing the layer. For layers covering the whole object, this problem already has known solutions. So in our case where the covering is only partial, the difficulty is to treat the limits of the layer where no model is known so far.

Enriched homogenized model in the presence of boundaries for wave equations

Participants: Sonia Fliss, Corentin Kilque.

We study the time-dependent scalar wave equation in presence of a periodic medium when the period is small compared to the wavelength. The classical homogenization theory enables to derive an effective model which provides an approximation of the solution. But this effective model does not take into account the long time dispersive effects which appears naturally in periodic media. This is well known since the works of Santosa and Symes in the 90s and high order effective models involving high order differential operators of higher orders (at least 4) have been proposed for infinite periodic media. Dealing with boundaries and proposing boundary conditions for these models were open questions. Note that one of the difficulty is that one to derive variational conditions for differential operators of order 4 from original variational conditions for operators of order 2. The past few years, we have proposed a new asymptotic expansion which takes into account the microscopic phenomena near the boundaries. Our approach enables to propose appropriate boundary conditions for these models. The well-posedness of such models is under study. We want then to tackle similar questions for Maxwell's and elastodynamics equations. This work is done in collaboration with Bruno Lombard (LMA Marseille) and Remi Cornaggia (Institut d'Alembert, Sorbonne Université).

7.7 Waves in quasi 1D or 2D domains

Heat and momentum losses effect on hydrogen detonations

Participants: Luiz Faria.

This is work in collaboration with Josue Melguizo-Gavilanes and Ferando Veiga-Lopez (institute PPrime) in the context of the ANR-JCJC FASTD.

The steady propagation of hydrogen-fueled detonation waves with momentum and heat losses is analyzed including detailed kinetics by means of the detonation velocity - friction coefficient ($D-c_f$) curves. We show that for undiluted H_2-O_2 mixtures the heat losses ($c_q = \alpha c_f$ with α a similarity factor) yield strong changes on the $D-c_f$ steady solutions, moving the critical point, $c_f^\star$, towards faster detonations propagating in smoother tubes (i.e., lower c_f), which also limits their propagation at low velocities. Thus we found that for a high enough sim. factor $\alpha > \alpha^\star$ no detonations may propagate at the *choking* regime. For $\alpha < \alpha^\star$, we found several solutions given a fixed D, confirming the existence of set-valued solutions only for *choking* detonations with a realistic chemistry model and a popular mixture. We also assessed the effect of the mixture composition (nitrogen and argon were added) on the $D-c_f$ curves choosing different similarity factors: (i) nitrogen strongly reduces $c_f^\star$ and moves it to faster D; no detonations were found at the *choking* regime for very low α, limiting the appearance of the set-values; (ii) argon presents a peculiar non-linear effect on the $c_f^\star$ but always moves it to slower D; the set-valued regions widen and higher α are required to avoid them. We finally relate our results to realistic configurations, comparing them to previous works: good agreements were found. We asses the uncertainties that appear due to the models applied; the detailed chemical schemes used strongly modify the predictions.

Mathematical modelling of thin coaxial cables

Participants: Patrick Joly, Akram Beni Hamad.

This topic is the subject of a long term collaboration with Sébastien Imperiale (M3disim) and the unerics for this problem constituted the subject of the PhD thesis of Akram Beni Hamad, defended last September.

The part of the thesis dedicated to the case of cylindrical cables and more precisely to the design an original approach condining Nédélec's edge elements on elongated prismatic meshes with a hybrid time discretization procedure which is explicit in the longitudinal directions and implicit in the transverse ones, has been accepted for publication in Computational methods in Applied Mathematics and presented at the Conference at the 10th Workshop on Numerical Methods for Evolution Equations in Heraklion. This includes a quantitative comparison of 3D results with the results of the simplified 1D model proposed previously.

In the non cylindrical case, the development of another hybrid method combining a conforming discretization in the longitudinal variable and a discontinuous Galerkin method in the transverse ones is still under way.

7.8 From periodic to random media

Guided modes in a hexagonal periodic graph-like domain: the zigzag and the armchair cases

Participant: Sonia Fliss.

In collaboration with Bérangère Delourme (LAGA, Paris 13), we have studied the spectrum of periodic operators in thin graph-like domains: more precisely Neumann Laplacian defined in periodic media which are close to quantum graphs. Moreover, we exhibit situations where the introduction of lineic defects into the geometry of the domain leads to the appearance of guided modes. We dealt with rectangular lattices few years ago and more recently we are studying hexagonal lattices. In this last case, we have shown that the dispersion curves have conical singularities called Dirac points. Their presence is linked to the invariance by rotation, symmetry and conjugation of the model. We have also observed that the direction of the line defect leads to very different properties of the guided modes. Finally, we have also proven the stability of the guided modes when the position of the edge varies (keeping the same direction). We want now to (1) open gaps (around the Dirac points) in the spectrum of the periodic operator by breaking one of the invariance of the problem (2) study the effect on guided waves.

Discrete honeycombs, rational edges and edge states

Participant: Sonia Fliss.

This work is done in collaboration with C.L. Fefferman (Princeton University) and M.I. Weinstein (Columbia University). We Consider the tight binding model of graphene terminated along a *rational edge*, *i.e.* an arbitrary line in a direction of periodicity of the structure. We present a comprehensive rigorous study of zero energy / flat band edge states; all zigzag-type edges support zero energy / flat bands and armchair-type edges support no zero energy / flat bands. We also perform a careful numerical investigations showing very strong evidence for the existence of non-zero energy (dispersive / non-flat) edge states. We are investigating now the existence of states which are bounded and oscillatory parallel to an irrational edge and which decay into the bulk. The idea is to construct an « edge » state for irrational termination as the limit of a sequence of edge state wave-packets (superpositions of edge states) of rationally terminated structures.

Wave propagation in quasi periodic media

Participants: Sonia Fliss, Patrick Joly, Pierre Amenoagbadji.

This is the subject of the PhD thesis of P. Amenoagbadji. Our main objective is to develop original numerical methods for the solution of the time-harmonic wave equation where some quasi-periodicity arises in the heterogeneity or in the geometry of the propagation medium. This includes two situations:

- 1D quasi-periodic media: we developed in the past years an adapted numerical method based on the so-called lifting approach that was first studied and implemented in the case with absorption. The idea is to interpret the solution of the 1D Helmholtz equation as the trace along the same line of the solution of an augmented degenerate PDE in higher dimensions, with periodic coefficients. The key point is characterize the transparent DtN operator associated to the augmented problem through a propagation operator which is the solution of a Riccati equation. The corresponding article has been accepted for publication in *Communications in Optimization Theory*.

  The most recent development concerns the non-absorbing case, for which we proposed a heuristic method based on the ideas of the limiting absorption principle. For reasons described in the previous activity report, one first needs to replace the DtN operator by a so-called RtR operator which associates an incoming Robin trace to an outgoing one. The second difficulty consists in selecting the good physical solution of the Riccati equation for the corresponding propagation operator. This has led us to look at the spectral theory of weighted shift operators, a class of operators to which the propagation operator belongs. This helped us in simplifying and in improving the initally designed method, by computing the principal “eigenpair”. The first numerical results are quite promising.

- Transmission between two 2D periodic media: the interesting case is when the two structures are not periodic in the direction of the interface, or when their periods along the interface are not commensurate. However, in this situation, the problem presents some quasi-periodic structure with respect to the coordinate along the interface, in such a way that they fall into the scope of the lifting approach. In a first step, we have considered situations where the structures could be lifted in 3D, that is

  1. the case where the two media are periodic along the interface, but with non-commensurate periods;
  2. the case where one medium is constant, while the other is not periodic with respect to the variable along the interface.

  In each case, the full method couples the DtN (or RtR) approach similar to the 1D case, with the use of Floquet-Bloch transform with respect to the variable of the interface. The implementation of this method produces satisfactory results, and an article is being written.

  We are currently working on the generalization to a junction between two arbitrary periodic media, which appears treatable with the lifting method but with an augmented problem in dimension 5.

7.9 Coupled phenomena for waves in fluids and solids

Variational methods for acoustic radiation in a duct with a shear flow and an absorbing boundary

Participants: Jean-François Mercier.

The well-posedness of the acoustic radiation in a 2D duct in presence of both a shear flow and an absorbing wall described by the Myers boundary condition is studied thanks to variational methods. Without flow the problem is found well-posed for any impedance value. The presence of a flow complicates the results. With a uniform flow the problem is proven to be always of the Fredholm type but is found well-posed only when considering a dissipative radiation problem. With a general shear flow, the Fredholm property is recovered for a weak enough shear and the dissipative radiation problem requires to introduce extra conditions to be well-posed: enough dissipation, a large enough frequency and non-intuitive conditions on the impedance value.

Modelling fluid injection in seismic cycles

Participants: Laura Bagur, Stéphanie Chaillat.

This work is done in collaboration with JF Semblat (ENSTA Paris) and I Stefanou (Ecole Centrale Nantes). Earthquakes due to either natural or anthropogenic sources cause important human and material damage. In both cases, the presence of pore fluid influence the triggering of seismic instabilities. Preliminary results in the team of I. Stefanou show that the earthquake instability could be avoided by active control of the fluid pressure. We work on the ability of Fast Boundary Element Methods (Fast BEMs) to provide a multi-physic large-scale robust model required for modeling earthquake processes, human induced seismicity and their control. The main challenges concern :

- the modelling of a realistic on-fault behaviour as well as hydro-mechanical couplings;
- the extension of Fast Boundary Element methods to fault mechanic problems incorporating the effect of fluid injection of the on-fault behaviour;
- the simulation of both small and large time scales corresponding to earthquakes and fluid diffusion respectively by using a single advance in time algorithm.

The main methods used for numerical modeling of earthquake ruptures at a planar interface between two elastic half-spaces are spectral BEMs. As a first step, we have considered this method for a simple problem in crustal faulting. A rate-and-state friction law is considered and different adaptive time stepping algorithms available in the literature are tested to take into account both small and large time scales with the correct resolution in time. These methods are compared on different benchmarks and convergence studies are conducted on each of them. We have then considered poro-elastodynamics effects. To this aim, a dimensional analysis of generic poroelastodynamic equations has been performed. It allows to determine which of the poroelastodynamics effects are predominant depending on the observation time of the fault. The obtained equations corroborate and justify simplified multiphysics models from the literature. A first multi-physics test using Fast BEMs to solve a simplified crustal faulting problem with fluid injection is considered.

Modelling non-spherical bubbles of gaz generated by airguns

Participants: Stéphanie Chaillat.

This work is done in collaboration with E. Dunham (Stanford university) and Shuki Ronen (company Sercel). The company Sercel develops airgun-type seismic sources. These are compressed air sources that generate an underwater acoustic wave to identify the nature of the underwater ground depending on its acoustic reflection. It is an ultrasound seismic imaging technique.

In this work, we model the gas bubbles that are generated by the source and determine their acoustic signatures. The difficulty in the context of seismic sources is that the bubble cannot be assumed to remain spherical. This greatly complicates the physical modeling because one does not simply have to solve an ordinary differential equation but a coupled vector problem. To model this problem we assume that the liquid near the source is incompressible and inviscid, and an irrotational flow. We can describe the flow velocity as the gradient of the velocity potential. The flow being incompressible, the velocity potential satisfies the Laplace equation. With this modeling, the variables in time and in space are decoupled. The Laplace equation is solved to determine the evolution of the geometry of the bubble whereas the Bernoulli equation is used to update the boundary conditions at each time step. As the bubble is in an infinite space (the ocean), it is natural to consider the BEM to solve the Laplace equation. But this problem is much more complicated than it seems. There are two difficulties. The first difficulty is that the solution is very expensive because it requires to solve a Laplace problem on an evolving geometry at each time step. Even if fast BEMs are very effective they become too expensive in this context as soon as that the time of a solve exceeds one second, which limits the precision. The second difficulty is that the problem is subject to numerical instabilities due to approximations at each BEM solve. We have worked on (i) the development of a filter to stabilize the simulations and avoid the accumulation of numerical errors which lead to numerical instabilities and we have derived (ii) a mesh adaptation method in order to determine the optimal mesh for all time steps.

8 Bilateral contracts and grants with industry

8.1 Bilateral Contracts with Industry

- Contract with DGA and Naval Group on *transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects*

  Participants: Marc Bonnet, Stéphanie Chaillat, Alice Nassor.

  Start: 10/2020. End: 09/2023. Administrator: CNRS

- Contract and CIFRE PhD with CEA on *Modelling of thin layers of randomly distributed nanoparticles for electromagnetic waves*

  Participants: Amandine Boucart, Sonia Fliss, Laure Giovangigli.

  Start: 10/2019. End: 09/2022. Administrator: ENSTA Paris

- Contract with SIEMENS on *GPU accelerated discontinuous Galerkin finite element solver for aeroacoustics*

  Participants: Rose-Cloe Meyer, Axel Modave.

  Start: 01/2022. End: 01/2024. Administrator: ENSTA Paris

- Contract and CIFRE PhD with Naval Group on flow noise prediction

Participants: Stéphanie Chaillat, Jean-Francois Mercier, Laure Pacaut.

Start: 02/2022. End: 01/2025. Administrator: CNRS

9 Partnerships and cooperations

9.1 International initiatives

9.1.1 STIC/MATH/CLIMAT AmSud projects

NoLoCE

Participants: Patrick Ciarlet, Maryna Kachanovska.

Title: nonlocal and local coupled equations: analysis, computation, and probability

Partner Institution(s): ENSTA Paris, Inria POEMS, Univ. de la República, Montevideo

Program: MATH-AmSud

Duration: January 1, 2022 – December 31, 2023

Partners:

- Acosta (Argentine)
- Otárola (Chili)
- Borthagaray (Uruguay)

Coordinators: J.P. Borthagaray (PI) ; Patrick Ciarlet (French Coordinator)

Summary: The design of accurate mathematical models and the development of efficient numerical algorithms for their resolution are topics of paramount importance in applied mathematics and engineering. Good models capture the underlying mechanisms, and their inspection may lead to new questions in pure mathematics. In this project, we propose to study problems involving systems of coupled partial differential and integro-differential equations. Our research concerns the modeling, analysis, and simulation of such problems. We deal with questions related to materials science, such as the study of interface models in elasticity and electromagnetism and the development of novel formulations in nanophotonics. We propose to study approaches to coupled systems through game theory. Furthermore, we aim to compare local and nonlocal diffusion either in a mixing environment and through the analysis of population dispersal. External optimal control is a salient feature of nonlocal formulations that we propose to analyze. Motivated by their application in the modeling of the respiratory system, we shall also study problems on fractal and random infinite trees.

9.2 International research visitors

9.2.1 Visits of international scientists

Christophe Geuzaine

Status researcher

Institution of origin: University of Liège

Country: Belgique

Dates: from October to December 2022

Context of the visit: scientific collaboration with Axel Modave

Mobility program/type of mobility: research stay - call “invited professor” for the Inria Saclay Centre

Juan Pablo Borthagaray

Status researcher

Institution of origin: Universidad de la República, Montevideo

Country: Uruguay

Dates: 31.01.2022 - 04.02.2022

Context of the visit: scientific collaboration with Patrick Ciarlet

Mobility program/type of mobility: IFUMI

Stefan Sauter

Status researcher

Institution of origin: Universität Zürich

Country: Switzerland

Dates: 01.09.2022 - 31.01.2023

Context of the visit: scientific collaboration with Patrick Ciarlet

Mobility program/type of mobility: Sabbatical

9.2.2 Visits to international teams

Research stays abroad

Stéphanie Chaillat

Visited institution: Stanford

Country: United States

Dates: 15/02/2022 - 22/12/2022

Context of the visit: Collaboration with Eric Dunham

Mobility program/type of mobility: research stay

9.3 National initiatives

ANR
ANR JCJC project FAsTD

Participants: Luiz Maltez Faria.

Title: ANR JCJC project FAsTD *(Flame Acceleration and Transition to Detonation in Narrow Channels)*

Partner Institution(s): INRIA (POEMS), CNRS (Institut Pprime)

Duration: Start: 12/2020. End: 12/2024.

Coordinator: J. Melguizo Gavilanes (Institut Pprime)

Administrator: CNRS

ANR JCJC project WavesDG

Participants: Axel Modave, Patrick Ciarlet.

Title: ANR JCJC project WavesDG *(Wave-specific Discontinuous Galerkin Finite Element Methods for Time-Harmonic Problems)*

Partner Institution(s): POEMS (CNRS, INRIA, ENSTA Paris), Atlantis (INRIA), LAUM (U. Le Mans), U. Liège

Duration: Start: 10/2021. End: 12/2025.

Coordinator: Axel Modave (POEMS, CNRS)

Administrator: CNRS

ANR project DynImplant

Participants: Stéphanie Chaillat.

Title: Model-based ultrasound characterization of the bone-implant interface

Partner Institution(s): Laboratoire Analyse, Géométrie et Applications de l'université Paris 8, start-up Wave Implant (waveimplant.com) et le CHU de Nantes.

Duration: Start 10/2022. End: 11/2026.

Coordinator: Vu-Hieu Nguyen (MSME)

DGA / AID
Projet RAPID HyBOX

Participants: Patrick Joly, Sonia Fliss, Maryna Kachanovska, Axel Modave, Pierre Marchand.

Title: Projet RAPID HyBOX *(Hybridization toolbox for complex materials and metamaterials)*

Partner Institution(s): IMACS, ARIANEGROUP, ENSTA Paris

Duration: Start: 10/2020. End: 09/2023.

Administrator: ENSTA Paris

Projet CIEDS ElectroMath

Participants: Anne-Sophie Bonnet-Ben Dhia, Patrick Ciarlet, Sonia Fliss, Pierre Marchand, Axel Modave.

Title: Projet CIEDS ElectroMath

Partner Institution(s): ENSTA Pari ,Inria POEMS, Inria IDEFIX

Duration: 01.10.2022 - 01.04.2026.

Coordinators: Patrick Ciarlet et Axel Modave

Administrator: ENSTA Paris

Plan de relance

Projet "WavesDG - GPU"

Participants: Rose-Cloe Meyer, Axel Modave.

Title: Plan de préservation des emplois R& D - *Projet WavesDG - GPU*

Partner Institution(s): POEMS (CNRS, Inria, ENSTA Paris), SIEMENS

Duration: Start: 01/2022. End: 01/2024.

Coordinators: Coordinator: Axel Modave (POEMS, CNRS)

Administrator: ENSTA Paris

Action Exploratoire Inria

Action exploratoire OptiGPR3D

Participants: Pierre Marchand.

Title: Action exploratoire OptiGPR3D (*Modélisations directe et inverse optimales pour l'imagerie GPR 3D en milieu complexe*)

Partner Institution(s): POEMS (CNRS, Inria, ENSTA Paris), IDEFIX (Inria, EDF)

Duration: Start: 05/2022.

Coordinators: Marcella Bonazzoli (IDEFIX, Inria), Pierre Marchand (POEMS, Inria)

Administrator: Inria

10 Dissemination

10.1 Promoting scientific activities

10.1.1 Scientific events: organisation

Member of the organizing committees

- Most of the permanent members of POEMS, as well as PhD students and post-docs, were involved in the organization (chaired by Sonia Fliss and Christophe Hazard) of WAVES 2022 (15th International Conference on Mathematical and Numerical Aspects of Wave Propagation). The conference, initially scheduled for July 26-30, 2021, was postponed due to the Covid situation. It was held from July 24 to 29, 2022 at ENSTA Paris, Palaiseau. See the section *Highlights of the year* for more details.
- S. Chaillat is a co-animator of the topic *"Modeling and simulation"* of the GDR Ondes (gdr-ondes.cnrs.fr).
- J.-F. Mercier is a co-animator of the topic *"Effective dynamics of microstructured media"* of the GDR MecaWave (mecawave.cnrs.fr).
- A.-S. Bonnet-Ben Dhia is a member of the Organization Committee of the Scientific Committee of the hybrid conference on *"Herglotz-Nevanlinna Functions and their Applications to Dispersive Systems and Composite Materials"* which was held in May 2022 in CIRM.
- J. Heleine and P. Marchand were members of the organizing committee for the third *"rencontres Jeunes Chercheuses Jeunes Chercheurs"* about the analysis and simulation of wave propagation (JCJC Ondes 2022) at Sophia-Antipolis (website).
- S. Chaillat was a member of the organizing committee of the hybrid workshop *"Outstanding Challenges in Computational Methods for Integral Equations"* which was held at la Casa Matematica Oaxaca, Mexico in May 2022 (website).
- S. Fliss was a member of the organizing committee for the workshop *"Scattering and propagation of waves: theoretical and computational chalenges"* celebrating Simon Chandler-Wilde 60th birthday, which was held in June 2022 at the University of Reading.
- S. Chaillat and L. Faria were members of the organizing committee for the workshop celebrating Marc Bonnet 60th birthday which was held in July 2022 at ENSTA Paris in Palaiseau.
- POEMS organizes, under the responsability of M. Kachanovska, a monthly seminar. One occurrence each semester is co-organized with two other inria teams, IDEFIX and M3DISIM.

10.1.2 Journal

Member of the editorial boards

- A. S. Bonnet-Ben Dhia is a member of the Editorial Board of the SIAM journal of applied mathematics.
- M. Bonnet is a member in the Editorial Board of Computational Mechanics (Comput. Mech.), Engineering Analysis with Boundary Elements (EABE), J. of Integral Equations and Applications (JIE), J. Optimization Theory and Applications (JOTA), and Inverse Problems.
- L. Bourgeois is a member in the Editorial Board of IMA Journal of Applied Mathematics.
- P. Ciarlet is a member in the Editorial Board of ESAIM:M2AN (Mathematical Modeling and Numerical Analysis).
- S. Chaillat is a member in the Editorial Board of Advances in Computational Mathematics (ACOM).
- P. Joly is a member of the Editorial Board of the Book series "Scientific Computing" of Springer.

10.1.3 Research administration

- E. Bécache is a deputy chair of the Doctoral School EDMH.
- M. Bonnet is a member of the COMEVAL, a committee of the Ministry of Ecological and Inclusive Transition (MEIT) similar to a CNRS National Committee section and tasked with the recruiting and career overseeing of the cadre of junior and senior scientists managed by MEIT.
- A.-S. Bonnet-Ben Dhia is deputy-chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris. She is a member of the Scientific Council of the Doctoral School EDMH, and of the BCEP (Bureau du Comité des Equipes Projets) at INRIA Saclay from 2018. She is a member of the exterior scientific councils of Institut Fresnel and Laboratoire de Mécanique et d'Acoustique, both in Marseille.
- A.-S. Bonnet-Ben Dhia and S. Chaillat are members of the Academic Council of IP Paris (Institut Polytechnique de Paris).
- S. Chaillat is a member of the Scientific Council of CNRS from October 2021.
- P. Ciarlet is coordinator of the *Mathematics in Computational Science and Engineering Program* of the Mathematics Hadamard Labex (LMH).
- A. Modave is a member of the scientific committee of the mesocenter of IP Paris (Institut Polytechnique de Paris), and a member of the board of directors of ENSTA Paris.
- M. Kachanovska is a member of the INRIA Scientific Committee for PhD and Postdoctoral Positions, from 2017.

10.2 Teaching - Supervision - Juries

10.2.1 Administration

Permanent members of POEMS are involved in the management of the engineering program at ENSTA Paris and the master program in applied mathematics at IP Paris and Université Paris-Saclay.

- L. Bourgeois: coordinator of the 2nd year Maths Program at ENSTA; co-head of the M1 Applied Mathematics common to IP Paris and Université Paris-Saclay;
- P. Ciarlet: coordinator of the master program in applied mathematics at IP Paris;
- S. Fliss: coordinator of the 3nd year ENSTA programs on modelling and simulation; co-head of the M2 AMS *(Analyse, Modélisation et Simulation)* common to IP Paris and Université Paris-Saclay;
- L. Giovangigli: coordinator of the 3nd year ENSTA programs on finance and mathematics for life sciences.

10.2.2 Courses taught

All permanent members of POEMS, as well as most PhD students and post-docs, are involved in teaching activities. A large fraction of these activities is included in the curriculum of the engineering school ENSTA Paris that hosts POEMS team. The 3rd year of this curriculum is coupled with various research masters, in particular the master *Analysis, Modelization and Simulation* (denoted below M2 AMS) common to Institut Polytechnique de Paris and Université Paris-Saclay.

Teaching activities of the permanent members of POEMS

- Eliane Bécache
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse et approximation par éléments finis d'EDP*, ENSTA (2nd year) and Master Applied Math (M1)

    - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)

- Marc Bonnet
    - *Méthodes numériques matricielles avancées: analyse et expérimentation*, ENSTA (2nd year) and Master Applied Math (M1).
- Anne-Sophie Bonnet-Ben Dhia
    - *Fonctions de variable complexe*, ENSTA (1st year)
    - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)
    - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
    - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)
- Laurent Bourgeois
    - *Outils élémentaires pour l'analyse des équations aux dérivées partielles*, ENSTA (1st year)
    - *Fonctions de variable complexe*, ENSTA (1st year)
    - *Problèmes inverses pour des systèmes gouvernés par des EDPs*, ENSTA (3rd year) and Master AMS (M2)
- Stéphanie Chaillat
    - *Techniques numériques et algorithmiques pour les équations intégrales*, ENSTA (3rd year) and Master AMS (M2)
    - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)
- Colin Chambeyron
    - *Remise à niveau en maths*, Licence (1st year), Paris-Dauphine University
    - *Outils mathématiques*, Licence (L1), Paris-Dauphine University
    - *Analyse - Optimisation*, Licence (L1), Paris-Dauphine University
    - *Algèbre linéaire*, Licence (L2), Paris-Dauphine University
- Patrick Ciarlet
    - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
    - *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)
- Luiz Faria
    - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
    - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
    - *Méthodes numériques matricielles avancées: analyse et expérimentation*, ENSTA (2nd year) and Master Applied Math (M1)
    - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)
- Sonia Fliss
    - *La méthode des éléments finis*, ENSTA (2nd year) and Master Applied Math (M1)

  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Homogénéisation périodique*, ENSTA (3rd year), ENSTA (3rd year) and Master AMS (M2)

- Laure Giovangigli
  - *Introduction aux probabilités et aux statistiques*, ENSTA (1st year)
  - *Martingales et algorithmes stochastiques*, ENSTA (2nd year)
  - *Calcul stochastique*, ENSTA (3rd year) and Master MMMEF (M2)
  - *Introduction à l'imagerie médicale*, ENSTA (3rd year) and Master AMS and MSV (M2)
  - *Homogénéisation stochastique*, ENSTA (3rd year) and Master AMS and MSV (M2)

- Christophe Hazard
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)

- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse fonctionnelle*, ENSTA (2nd year) and Master AMS (M2)
  - *Techniques de discrétisation avancées pour les problèmes d'évolution*, ENSTA (3rd year) and Master AMS (M2)

- Maryna Kachanovska
  - *Analyse fonctionnelle*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)
  - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)

- Nicolas Kielbasiewicz
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)

- Eric Lunéville
  - *Introduction au calcul scientifique*, ENSTA (2nd year).
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)

- Pierre Marchand
  - *Introduction à MATLAB*, ENSTA (1st year)
  - *Quadratic optimization*, ENSTA (1st year)
  - *La méthode des éléments finis*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Cours accéléré de programmation*, Master AMS (M2)

- Jean-François Mercier
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)

  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)

- Axel Modave
  - *Initiation au calcul haute performance*, ENSTA (2rd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)
  - *Éléments finis et éléments de frontière : parallélisation, couplage et performance*, ENSTA (3rd year) and Master AMS (M2)

10.2.3 Supervision

• PhD : Mahran Rihani,"Maxwell's equations in presence of metamaterials", defended in February 2022, A.-S. Bonnet-Ben Dhia and L. Chesnel

• PhD : Akram Beni Hamad, "Numerical modelling and simulation of electromagnetic wave propagation in coaxial cables.", defended in Septembre 2022, S. Imperiale, P. Joly and M. Khenissi

• PhD in progress : Jean-François Fritsch,"Imaging in burried waveguides", started Octobre2019,L. Bourgeois and C. Hazard

• PhD in progress : Amandine Boucart, "Modeling of a thin layer of randomly distributed nanoparticles for electromagnetic waves", started Octobre 2019, S. Fliss and L. Giovangigli

• PhD in progress : Amond Allouko, "A hybrid semi-analytical method for the integrated health control of composite plates", started September 2020, A.-S. Bonnet-Ben Dhia and A. Lhemery

• PhD in progress : Laura Bagur, "Three dimensional modeling of seismic and aseismic slip using Fast Boundary Element Methods", started September 2020, S. Chaillat, J.-F. Semblat and I. Stéfanou

• PhD in progress : Pierre Amenoagbadji,"Wave propagation in quasi-periodic media", started October 2020, S. Fliss and P. Joly

• PhD in progress : Etienne Peillon, "Justification and mathematical analysis of plasma models", started October 2020, P. Ciarlet and M. Kachanovska

• PhD in progress : Alice Nassor, "Transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects", started Octobre 2020, S. Chaillat and M. Bonnet

• PhD in progress : Luis Alejandro Rosas Martinez,"Dispersive electromagnetic media : mathematical and numerical analysis", started November 2020, M. Cassier and P. Joly

• PhD in progress : Quentin Goepfert, "Inverse problems in ultrasonic imaging", October 2021, J. Garnier, L. Giovangigli and P. Millien

• PhD in progress : Cédric Baudet, "Modelisation of partial coatings in electromagnetism", started October 2022, S. Fliss and P. Joly

• PhD in progress : Aurélien Parigaux, "Construction of transparent boundary conditions for electromagnetic waveguides, analysis and applications", started October 2022, A.-S. Bonnet-Ben Dhia and L. Chesnel

• PhD in progress : Farah Chaaban, "An optimization-based numerical method for diffusion problems with sign-changing coefficients ", started October 2022, P. Ciarlet and M. Rihani

• PhD in progress : Simone Pescuma, "Novel Discontinuous Finite Elements Methods for Time-Harmonic Wave Propagation"", started October 2022, G. Gabard and A. Modave

• PhD in progress : Louise Pacaut, "Development of an accelerated numerical BEM/BEM method to determine the Green function of a fluid-structure problem.", started October 2022, S. Chaillat and J. F. Mercier

• PhD in progress : Mario Gervais, "A posteriori estimators of a nonconforming domain decomposition method", started October 2022, P. Ciarlet and F. Madiot

• PhD in progress : Adrian Savchuk, "Asymptotic modelling of time-domain electromagnetic scattering by small particles", started October 2022, M. Kachanovska and E. Bécache

• PhD in progress : Dongshen He, "Boundary integral methods for Stokes flows with deformable implicit surfaces", started October 2022, L. Faria

• PostDoc : Jérémy Héleine, "Domain decomposition in time-harmonic regime", Mars-Juillet 2022, P. Joly

• PostDoc : Corentin Kilque : "Long time homogenization for the waves equation and the Maxwell equation" started October 2022, S. Fliss

• PostDoc : Zoïs Moitier : "Fast methods for the solution of boundary integral equations on fractal antennas", started October 2022, P. Joly and M. Kachanovska

• PostDoc : Florian Monteghetti : "Asymptotic modeling of thin periodic layers with high-contrast inclusions for Maxwell's equations" started October 2022, S. Fliss and P. Joly

• PostDoc : Rose-Cloé Meyer : "GPU-accelerated DG finite element solver for unsteady acoustics", started October 2022, A. Modave

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11 Scientific production

11.1 Publications of the year

International journals

[1] P. Amenoagbadji, S. Fliss and P. Joly. 'Wave propagation in one-dimensional quasiperiodic media'. In: *Communications in Optimization Theory* (2023). DOI: 10.48550/arXiv.2301.01159. URL: https://hal.archives-ouvertes.fr/hal-03786730.

[2] T. G. Anderson, M. Bonnet and S. Veerapaneni. 'Quantifying mixing in arbitrary fluid domains: A Padé approximation approach'. In: *Numerical Algorithms* (2022). URL: https://hal.archives-ouvertes.fr/hal-03698941.

[3] L. Bagur, S. Chaillat and P. Ciarlet. 'Improvement of hierarchical matrices for 3D elastodynamic problems with a complex wavenumber'. In: *Advances in Computational Mathematics* 48.9 (15th Feb. 2022). DOI: 10.1007/s10444-021-09921-3. URL: https://hal.archives-ouvertes.fr/hal-03367755.

[4] A. Beni-Hamad, G. Beck, S. Imperiale and P. Joly. 'An efficient numerical method for time domain electromagnetic wave propagation in co-axial cables'. In: *Computational Methods in Applied Mathematics* (9th June 2022). URL: https://hal.archives-ouvertes.fr/hal-03408400.

[5] M. Bonnet. 'On the justification of topological derivative for wave-based qualitative imaging of finite-sized defects in bounded media'. In: *Engineering Computations* 39.1 (2022), pp. 313–336. DOI: 10.1108/EC-08-2021-0471. URL: https://hal.archives-ouvertes.fr/hal-03319821.

[6] M. Bonnet, R. Liu, S. Veerapaneni and H. Zhu. 'Shape optimization of peristaltic pumps transporting rigid particles in Stokes flow'. In: *SIAM Journal on Scientific Computing* (2022). URL: https://hal.archives-ouvertes.fr/hal-03364866.

[7] A.-S. Bonnet-Ben Dhia, S. N. Chandler-Wilde and S. Fliss. 'On the Half-Space Matching Method for Real Wavenumber'. In: *SIAM Journal on Applied Mathematics* 82.4 (2022). DOI: 10.1137/21M1459216. URL: https://hal.archives-ouvertes.fr/hal-03427382.

[8] A.-S. Bonnet-Ben Dhia, S. N. Chandler-Wilde, S. Fliss, C. Hazard, K.-M. Perfekt and Y. Tjandrawidjaja. 'The Complex-Scaled Half-Space Matching Method'. In: *SIAM Journal on Mathematical Analysis* 54.1 (2022), pp. 512–557. DOI: 10.1137/20M1387122. URL: https://hal.inria.fr/hal-03087232.

[9] A.-S. Bonnet-Ben Dhia, L. Chesnel and M. Rihani. 'Maxwell's equations with hypersingularities at a conical plasmonic tip'. In: *Journal de Mathématiques Pures et Appliquées* 161 (7th Mar. 2022), pp. 70–110. DOI: 10.1016/j.matpur.2022.03.001. URL: https://hal.science/hal-02969739.

[10] L. Bourgeois and J. Dardé. 'The Morozov's principle applied to data assimilation problems'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* (2022). DOI: 10.1051/m2an/2022061. URL: https://hal.inria.fr/hal-03344087.

[11] C. Carvalho, P. Ciarlet and C. Scheid. 'Limiting amplitude principle and resonances in plasmonic structures with corners: numerical investigation'. In: *Computer Methods in Applied Mechanics and Engineering* (2022). DOI: 10.1016/j.cma.2021.114207. URL: https://hal.science/hal-03160574.

[12] M. Cassier, C. Hazard and P. Joly. 'Spectral theory for Maxwell's equations at the interface of a metamaterial. Part II: Limiting absorption, limiting amplitude principles and interface resonance'. In: *Communications in Partial Differential Equations* 47.6 (25th Apr. 2022), pp. 1217–1295. DOI: 10.1080/03605302.2022.2051188. URL: https://hal.archives-ouvertes.fr/hal-03379054.

[13] S. Chaillat, B. Cotté, J.-F. Mercier, G. Serre and N. Trafny. 'Efficient evaluation of three-dimensional Helmholtz Green's functions tailored to arbitrary rigid geometries for flow noise simulations'. In: *Journal of Computational Physics* 452 (3rd Jan. 2022). DOI: 10.1016/j.jcp.2021.110915. URL: https://hal.science/hal-03370256.

[14] D. Chicaud and P. Ciarlet. 'Analysis of time-harmonic Maxwell impedance problems in anisotropic media'. In: *SIAM Journal on Mathematical Analysis* (Nov. 2022). URL: https://hal.inria.fr/hal-03613183.

[15] P. Ciarlet. 'On the approximation of electromagnetic fields by edge finite elements. Part 4: analysis of the model with one sign-changing coefficient'. In: *Numerische Mathematik* 152 (2022), pp. 223–257. DOI: 10.1007/s00211-022-01315-x. URL: https://hal.inria.fr/hal-03273264.

[16] P. Ciarlet, M. H. Do and F. Madiot. 'A posteriori error estimates for mixed finite element discretizations of the Neutron Diffusion equations'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.1 (Jan. 2023), pp. 1–27. DOI: 10.1051/m2an/2022078. URL: https://hal.science/hal-03936904.

[17] P. Ciarlet and M. Kachanovska. 'A mathematical study of a hyperbolic metamaterial in free space'. In: *SIAM Journal on Mathematical Analysis* 54.2 (2022), pp. 2216–2250. DOI: 10.1137/21M1404223. URL: https://hal.science/hal-03164619.

[18] X. Claeys and E. Parolin. 'Robust treatment of cross points in Optimized Schwarz Methods'. In: *Numerische Mathematik* 151.2 (22nd May 2022), pp. 405–442. DOI: 10.1007/s00211-022-01288-x. URL: https://hal.archives-ouvertes.fr/hal-03118695.

[19] R. Dai, A. Modave, J.-F. Remacle and C. Geuzaine. 'Multidirectional sweeping preconditioners with non-overlapping checkerboard domain decomposition for Helmholtz problems'. In: *Journal of Computational Physics* 453 (15th Mar. 2022), p. 110887. DOI: 10.1016/j.jcp.2021.110887. URL: https://hal.archives-ouvertes.fr/hal-03240042.

[20] A. Dansou, S. Mouhoubi, C. Chazallon and M. Bonnet. ‘Modelling of the fatigue cracking resistance of grid reinforced asphalt concrete by coupling fast BEM and FEM’. In: *Road Materials and Pavement Design* (2022). DOI: 10.1080/14680629.2022.2029755. URL: https://hal.archives-ouvertes.fr/hal-03524856.

[21] C. L. Fefferman, S. Fliss and M. I. Weinstein. ‘Edge states in rationally terminated honeycomb structures’. In: *Proceedings of the National Academy of Sciences of the United States of America* 119.47 (22nd Nov. 2022), e2212310119. DOI: 10.1073/pnas.2212310119. URL: https://hal.archives-ouvertes.fr/hal-03877091.

[22] C. L. Fefferman, S. Fliss and M. Weinstein. ‘DISCRETE HONEYCOMBS, RATIONAL EDGES AND EDGE STATES’. In: *Communications on Pure and Applied Mathematics* (2022). URL: https://hal.archives-ouvertes.fr/hal-03606004.

[23] P. Joly and M. Kachanovska. ‘Local transparent boundary conditions for wave propagation in fractal trees (ii): error and complexity analysis’. In: *SIAM Journal on Numerical Analysis* (2022). DOI: 10.1137/20M1357524. URL: https://hal.archives-ouvertes.fr/hal-02909750.

[24] A. Royer, C. Geuzaine, E. Béchet and A. Modave. ‘A non-overlapping domain decomposition method with perfectly matched layer transmission conditions for the Helmholtz equation’. In: *Computer Methods in Applied Mechanics and Engineering* 395 (15th May 2022), p. 115006. DOI: 10.1016/j.cma.2022.115006. URL: https://hal.archives-ouvertes.fr/hal-03416187.

[25] T. Strauszer-Caussade, L. M. Faria, A. Fernandez-Lado and C. Pérez-arancibia. ‘Windowed Green function method for wave scattering by periodic arrays of 2D obstacles’. In: *Studies in Applied Mathematics* 150.1 (1st Oct. 2022), pp. 277–315. DOI: 10.1111/sapm.12540. URL: https://hal.science/hal-03940468.

[26] F. Veiga-Lopez, L. Maltez Faria and J. Melguizo-Gavilanes. ‘Influence of chemistry on the steady solutions of hydrogen gaseous detonations with friction losses’. In: *Combustion and Flame* 240 (2022), p. 112050. DOI: 10.1016/j.combustflame.2022.112050. URL: https://hal.archives-ouvertes.fr/hal-03748612.

Doctoral dissertations and habilitation theses

[27] M. Rihani. ‘Maxwell’s equations in presence of metamaterials’. Institut Polytechnique de Paris, 17th Feb. 2022. URL: https://theses.hal.science/tel-03670420.

Reports & preprints

[28] M. Barré and P. Ciarlet. *The T-coercivity approach for mixed problems.* 19th Oct. 2022. URL: https://hal.archives-ouvertes.fr/hal-03820910.

[29] E. Bécache, A.-S. Bonnet-Ben Dhia, S. Fliss and A. Tonnoir. *The Half-Space Matching method for elastodynamic scattering problems in unbounded domains.* 30th Sept. 2022. URL: https://hal.archives-ouvertes.fr/hal-03793031.

[30] E. Bécache, M. Kachanovska and M. Wess. *Convergence analysis of time-domain PMLs for 2D electromagnetic wave propagation in dispersive waveguides.* 22nd Dec. 2022. URL: https://hal.science/hal-03910611.

[31] M. Bonnet and E. Demaldent. *Eddy-current asymptotics of the Maxwell PMCHWT formulation for multiple bodies and conductivity levels.* 15th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03696173.

[32] L. Bourgeois, S. Fliss, J.-F. Fritsch, C. Hazard and A. Recoquillay. *Scattering in a partially open waveguide: the forward problem.* 20th Dec. 2022. URL: https://hal.inria.fr/hal-03407434.

[33] M. Cassier, P. Joly and L. A. Rosas Martínez. *Long time behaviour of the solution of Maxwell’s equations in dissipative generalized Lorentz materials (I) A frequency dependent Lyapunov function approach.* 13th Nov. 2022. URL: https://hal.inria.fr/hal-03850472.

[34] S. Chaillat, R. Cottereau and R. Sevilla. *A high-order discontinuous Galerkin Method using a mixture of Gauss-Legendre and Gauss-Lobatto quadratures for improved efficiency*. 15th June 2022. URL: https://hal.archives-ouvertes.fr/hal-03695573.

[35] L. Chesnel, J. Heleine, S. A. Nazarov and J. Taskinen. *Acoustic waveguide with a dissipative inclusion*. 1st July 2022. URL: https://hal.science/hal-03711258.

[36] J. Galkowski, P. Marchand, J. Wang and M. Zworski. *The scattering phase: seen at last*. 19th Oct. 2022. URL: https://hal.inria.fr/hal-03820689.

[37] M. Kachanovska. *A new class of uniformly stable time-domain Foldy-Lax models for scattering by small particles. Acoustic sound-soft scattering by circles*. 11th May 2022. URL: https://hal.archives-ouvertes.fr/hal-03664569.

[38] A. Modave and T. Chaumont-Frelet. *A hybridizable discontinuous Galerkin method with characteristic variables for Helmholtz problems*. 21st Dec. 2022. URL: https://hal.science/hal-03909368.

Other scientific publications

[39] L. Bourgeois, J.-F. Fritsch and A. Recoquillay. 'Scattering in a partially open waveguide: the inverse problem'. In: *Inverse Problems and Imaging* (2022). DOI: 10.3934/ipi.2022052. URL: https://hal.inria.fr/hal-03619606.

[40] P. Ciarlet, D. Lassounon and M. Rihani. 'An optimal control-based numerical method for scalar transmission problems with sign-changing coefficients'. In: *SIAM Journal on Numerical Analysis* (2023). URL: https://hal.science/hal-03666913.

11.2 Other

Educational activities

[41] A.-S. Bonnet-Ben Dhia and P. Ciarlet. 'Notes de cours sur les méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs'. Master. France, 1st Sept. 2022. URL: https://hal.archives-ouvertes.fr/hal-03764649.

[42] P. Ciarlet. 'Notes de cours sur les équations de Maxwell et leur approximation'. Master. France, 17th Nov. 2022. URL: https://hal.science/hal-03153780.