RESEARCH CENTRE

Inria Saclay Centre
at Institut Polytechnique de
Paris

IN PARTNERSHIP WITH:

CNRS, Institut Polytechnique de
Paris

2023 ACTIVITY REPORT

Project-Team GEOMERIX

Geometry-driven Numerics

IN COLLABORATION WITH: Laboratoire d'informatique de
l'école polytechnique (LIX)

DOMAIN

Perception, Cognition and
Interaction

THEME

Interaction and visualization

Inria

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Creation of the Project-Team: 2022 September 01

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2 Overall objectives

Historical context. Geometry has been a unifying formalism for science: predictive models of the world around us have often been derived using geometric notions which formalize observable symmetries and experimental invariants. Tools such as differential geometry and tensor calculus quickly became invaluable in describing the complexity of natural phenomena and mechanical systems through concise equations, condensing local and global properties into simple relations between measurable quantities. Today, geometry (be it Euclidean or not) is at the core of many current physical theories: general relativity, electromagnetism (E&M), gauge theory, quantum mechanics, as well as solid and fluid mechanics, all have strong underlying structures that are best described and elucidated through geometric notions like differential forms, curvatures, vector bundles, connections, and covariant derivative. Geometry also creeps up in unexpected fields such as number theory and functional analysis, offering new insights and even breakthroughs, e.g., the use of algebraic geometry to address Fermat's last theorem.

Geometry in Digital Sciences. In sharp contrast, the role of geometry was mostly ignored at the inception of computer science. Yet, it has now become clear that digital sciences are imbued with an overwhelming amount of fundamentally geometric and topological concepts. Some are rather obvious, when dealing with the modeling of Euclidean shapes in computer graphics or the analysis of images in computer vision for instance; some are more subtle, such as the "manifold hypothesis" underlying a number of supervised or unsupervised learning techniques; and some are only nascent, such as the fields of Information Geometry (basically, the geometry used to understand probability distributions), Geometric Statistics (new statistical methodology for non-Euclidean entities), and Topological Data Analysis (where algebraic topology is used as a tool to enhance data analysis pipelines). In fact, even the discretization of physical theories needed to offer fast numerical simulation has brought geometry back to the forefront after it was understood that the loss of numerical fidelity in standard numerical methods is due to a fundamental failure to preserve geometric or topological structures of the underlying continuous models: partial differential equations (PDEs) modeling our physical world are typically encoding invariants and structures that are independent of the choice of coordinates used to express the equations and the tensors involved in them; but invariance to the choice of basis is often lost during discretization, as numerical approximations will in general not capture, let alone preserve, the key geometric structures that exist in the continuous case. Seeing these numerical issues through the lens of geometry is thus not just of academic interest: failure to maintain geometric invariants has serious consequences for the accuracy and stability of solutions.

Rationale. Given the unusual reach of geometry and its rich literature, there is an opportunity to assemble a team of experts in geometry and its vernacular, to help broadly impact digital science and technology. We thus propose the creation of a **new project-team whose core scientific mission is to use geometry as a bedrock for the development of numerical tools and algorithms**: we wish to exploit the properties of infinite-dimensional and finite-dimensional spaces that are related with distance, shape, size, and relative position, and bringing them to bear on *computational discretizations and algorithms for analysis, processing, and simulation.* Adhering to geometric structures and invariants as a guiding principle for computations is a rich source of both theoretical and practical challenges, allowing to combine concepts and results from different areas of geometry broadly construed to produce new computational tools with solid mathematical foundations. While our team will be very focused in terms of the mathematical foundations and tools upon which it builds, it will also be very broad in terms of applications given the pervasiveness of geometry in sciences and technology. This makes for an unusual, yet powerful scientific setup that will facilitate interdisciplinary projects through the common use of geometric foundations and their specialized terminology. It will also allow us to contribute sporadically to pure and computational mathematics when appropriate in order to push our scientific mission forward.

Positioning. We see GeomeriX as first and foremost Inria Saclay's graphics team, but with wider objectives afforded by the broad relevance of geometry. It is worth noting that graphics has evolved to the point where it often intersects with applied mathematics, machine learning, vision, and computational science in some of its efforts, and GeomeriX intends to continue this trend.

Objectives. Our project-team's overall scientific objective is to contribute, through a geometric perspective, both foundational and practical methods for geometric data processing. In particular, we seek the development of predictive computational tools by drawing from the many facets of geometry and topology: whether it be *discrete geometry, basic differential geometry or exterior calculus, symplectic geometry, persistent homology or sheaf theory, optimal transport, Riemannian or conformal geometry*, these topics of geometry inform and guide both our discretizations and algorithmic designs towards computing. Note that we do not plan to merely adapt and exploit geometric concepts and understanding for numerical purposes, as our focus on digital data may even result in contributions to these mathematical fields, extending the current body of knowledge. While we intentionally leave the range of our mathematical foundations open so as not to restrict our potential team-wide explorations, **we concentrate our research on four concrete themes, which we believe can be most significantly impacted by a geometric approach to developing new numerical tools**:

① **Euclidean shape processing**: from computer graphics to geometry processing and vision, the analysis and manipulation of low-dimensional shapes (2D and 3D) is an important endeavor with applications covering a wide range of areas from entertainment and classical computer-aided design, to reverse engineering and biomedical engineering. Our project-team intends to lead efforts in this competitive field, with key contributions in shape matching, geometric analysis, and discrete calculus on meshes.

② **Simulation**: traditional finite-element treatments of various physical models have had tremendous success. Recently, a number of geometric integrators have upended the field, either through structure-preserving integration which offers improved statistical predictability by respecting the geometric properties of the exact flow of the differential equations, or through novel discretizations of the state space. We intend to continue introducing novel integration methods for increasingly complex multiphysics systems, as well as exploiting the use of learning methods to accelerate simulation.

③ **Dynamical systems**: we intend to leverage the geometric nature of dynamical systems to investigate and promote high-dimensional data analysis for dynamics: the study of dynamical systems from a limited number of observations of the state of a given system (for example, time series or a sparse set of trajectories) offers a unique opportunity to develop scalable computational tools to detect or characterize unusual features and coherent structures. Meanwhile, the study of dynamical systems from a combinatorial point of view opens up the possibility of characterizing their invariant sets and assessing their stability.

④ **Data science**: finally, we are intent on exploring the underlying role of geometry in machine learning and statistical analysis. This role has been put forward in the recent years, with the emergence of approaches such as geometric deep learning or topological data analysis, whose aim is to leverage the underlying geometry or topology of the data to enhance the performance, robustness, or explainability of the methods used for their analysis. We will pursue investigations toward this goal, concentrating our efforts on topics related to explainable feature design, geometric feature learning, geometry-driven learning, and geometry for categorical and mixed data types.

Evidently, our research efforts may at times lie across multiple of these themes given our multi-disciplinary objectives, and it is our hope that we will all eventually participate in the four themes.

3 Research program

Below we introduce the details of our four research themes, in four separate subsections. In each subsection, we first present the scientific focus and research objectives of the corresponding theme, then we detail the research topics we intend to address and how we plan to leverage topology and geometry for each one of them. For each theme, we list the most likely contributors, and organize the various subtopics within each theme from short to long-term goals, based on our current expectations and focus.

3.1 Geometry for Euclidean shape processing

Euclidean space is the default setting of classical geometry in two or three dimensions. Shapes in 3D space are of particular interest as they represent the typical objects we interact with. **Geometry processing** is an area of research focusing on these low-dimensional shapes in Euclidean space, with the goal to design algorithms, data structures, as well as analysis tools for their digital acquisition, reconstruction, analysis, manipulation, synthesis, classification, transmission, and animation. Digital shapes are typically discretized through either point clouds, triangle meshes, or polygonal meshes for surfaces, and through tetrahedron or polytopal meshes for volumes. Analyzing and manipulating these digital representations already involve fundamental difficulties in terms of efficiency, scalability, and robustness to arbitrary sampling, for which computational geometry and computer graphics have generated a number of key algorithms. Simple surface meshes in 3D also offer a simple context in which to define discrete notions of basic topological properties (quantities preserved through arbitrary stretching, such as Euler characteristic, genus, Betti numbers, etc) and relevant geometric properties (normal, curvatures, covariant derivatives, parallel transport, etc). Yet the digital counterpart of the low-dimensional case of Euclidean geometry is far from being settled or complete: it remains obviously relevant in a number of scientific fields on which we plan to focus. A few research directions of particular interest are described below.

Operator-based methods for shape analysis We plan to develop novel approaches for representing and manipulating geometric concepts as *linear functional operators*. Specifically we will focus on tools for shape matching, design and analysis of differential quantities such as vector fields or cross fields, shape deformation and shape comparison, where functional approaches have recently been shown to provide a natural and discretization-agnostic representation [110, 43, 44, 120]. This "functional" point of view is classical in many scientific areas, including **dynamical systems** (where the pullback with respect to a map is closely related to the Koopman or composition operator, allowing the study ergodicity or mixing property of non-linear maps through the spectral properties of a linear operator), **differential geometry** (where vector fields are often defined by their action on real-valued functions) and **representation theory** among others. However, it has only recently been adopted in geometry processing with tremendous and constantly growing potential in both axiomatic or even learning-based approaches [98, 88, 71]. We will continue developing efficient and robust algorithms by considering shapes as functional spaces and by representing various geometric operations as linear operators acting on appropriate real-valued functions. In addition to the efficiency and robustness of methods obtained by considering this linear operator point of view of geometry processing and dynamical systems, another very significant advantage of these techniques is that they allow to express many different geometric operations in a common language. This means, for example, that it makes it easy to define the pushforward of a vector field with respect to a map by simply considering a composition of appropriate discrete operators. Despite the significant recent success of tools within this area, especially related to the functional map framework [111], there does not exist a unified coherent theoretical framework in which different geometric concepts can be represented and manipulated via their functional equivalents. Our main long-term goal therefore would be to establish a novel field within geometry processing by creating both a computational framework and a coherent theoretical formalism in which *all* of the different basic geometric operations can be expressed, and thus in which different concepts can "communicate" with one another. We believe that such a formalism and associated computational tools, already quite well developed, will not only greatly extend the scope of applicability of many existing geometry processing pipelines, but will also help expand this language to novel concepts, and ultimately help pave the way towards representation-agnostic geometric data manipulation.

Discrete metrics and applications. While three-dimensional shapes are often encoded via their Euclidean embedding, numerous research efforts have focused on studying and discretizing their intrinsic metric. Regge calculus [118], an early approach to numerical relativity without coordinates, proposed the use of edge lengths to encode a piecewise-Euclidean metric per simplex, from which the Riemann curvature tensor can be easily computed to derive local areas or curvatures. This early work led to a series of alternative metric representations: tip angles, for instance, are known to encode the intrinsic geometry of a triangle mesh up to a scaling, while local measurements (an angle [119] or a length cross-ratio [101] per edge) later formed the basis of circle patterns [47, 93] as well as conformal representations [125]; the discrete Laplace-Beltrami cotan formula [114] also determines the edge lengths of a triangle mesh (and thus its discrete metric) up to a global scaling [137]. More recently, generalized notions of metrics were proposed; for instance, [85] presented a characterization of an augmented discrete metric resulting from the orthogonal primal-dual structure of weighted triangulations. Common to many of these various metric characterizations is the existence of convex energies which allow to efficiently compute these metrics from various boundary conditions. We intend to investigate the discrete treatment of metric for low-dimensional manifolds as a counterpart to the discretization of antisymmetric tensors (differential forms), which is far less studied — and a discrete theory unifying symmetric and anti-symmetric tensors remains elusive despite recent advances [84]. Moreover, the metric of a surface is known in the continuous realm to induce Hodge stars and a canonical torsion-free Levi-Civita connection (or parallel transport), but this picture is far less clear for discrete manifolds, even if the construction of arbitrary-order discrete Hodge stars and metric connections are well understood by now. A few research directions on generalized metrics seem particularly interesting due to their likelihood of resulting in novel algorithmic and computational frameworks:

- *Metric-dependent meshing:* Given a set of metric-based operators, optimized mesh structures can be designed to offer optimal accuracy akin to Hodge-star mesh optimization for the augmented weighted metric proposed in [107]. Another interesting research question is the existence and construction of intrinsic Delaunay triangulation, the most common discrete shape representation, with respect to a particular metric [48].
- *Metric-aware sampling:* Metric-dependent descriptors such as the pair correlation function are particularly efficient in characterizing statistical properties of point distributions for texture synthesis [72]. Extending this framework to arbitrary non-flat domains through Multi-Dimensional Scaling (MDS) seem particularly promising.
- *Shape characterization:* Highly convoluted embeddings like the cortical surface of the brain and its functional connectivity graph are naturally hyperbolic in nature [53]. However, investigating a link between cortical folding and the volumetric fiber bundle structure from a pure geometric viewpoint through a hyperbolic metric characterization has surprisingly not be done in brain analysis, despite striking visual similarities between brain folding and geometric realizations of the hyperbolic plane (see [130] and Taimiņa's crochet model). We are hoping that this intrinsic metric characterization can be investigated through recent discrete hyperbolic parametrization tools [80], which may also lead to other shape classification techniques in more general contexts.
- *Piecewise-linear maps:* We also wish to study the classification of the deformation of a triangle mesh through its induced metric change in the embedding space. Developing an approach to

decompose such a diffeomorphic piecewise-linear map into canonical geometric transformations through either linear algebra or convex minimization could offer new discrete equivalences for conformal, equiareal, and curvature-preserving maps between triangulations, with direct applications to mesh parameterization and more general processing of discrete meshes.

- *Geodesic abstractions:* curve-network representations [83] based on a few geodesics to describe a shape provide a compact encoding of surfaces. While it is increasingly useful for artistic depictions, we also want to study its relevance as a compact compression scheme from which the shape and its metric can be derived with controllable precision.
- *Metric-dependent cage*: Finally, we also want to understand how to define optimized metric-dependent cages for intuitive & expressive deformation and animation of complex shapes [128], and how these cages can be understood as polygonal or polyhedral cells to locally simplify a simplicial complex.

Discrete differential and tensor calculus. When working on low-dimensional spaces, the use of meshes (as opposed to just point clouds) pays dividends as it allows for the development of discrete versions of Exterior Calculus (see DEC [67] or FEEC [41]), where k-dimensional integrals can be directly evaluated in k-cells, and differentiation can formally achieved through the boundary operator: the concept of chains and cochains from algebraic topology forms the basis of a discrete analog of Cartan's exterior calculus of differential forms, providing crucial numerical tools such as a discrete de Rham cohomology and a discrete Helmholtz-Hodge decomposition that precisely mimick their continuous counterparts. Moreover, finite elements of arbitrary order can be associated with these discrete forms through subdivision [82] to provide a powerful Isogeometric Analysis (IGA). Recent developments [99, 81] have offered also a discrete approach to tangent vector fields. While DEC encodes vector fields as 1-forms, processing tangent vectors and, more generally, directional fields sampled at vertices of discrete surfaces requires the development of *discrete (metric) connections* [64, 99] (which can be seen as discrete equivalent to the Christoffel symbols) to handle the non-linearity of non-flat domains. From these connections can be derived the usual continuous notions of covariant derivatives or Killing operator, and these discrete operators demonstrate the same intimate link between geometry and topology as exemplified by the hairy ball theorem (Hopf index theorem). While these operators apply equally well on discrete three-manifolds, much remains to do: properly defining the notion of curvature matrix-valued 2-form or torsion vector-valued 2-form in 3D and checking that these definitions provide consistent Bianchi identities (i.e., there exists an exterior covariant derivative satisfying fundamental geometric and topological properties) is an exciting research direction. Not only will it allow to deal with the line singularities in hexahedral meshing *robustly*, but it will also provide a Bochner Laplacian (also called the vector Laplacian) in 3D devoid of the type of spurious modes that discrete Laplacians over flat domains can introduce if one does not enforce a proper discrete deRham complex. Such a tensor calculus for three-manifolds may allow us to explore possible applications in the context of general relativity in the longer term. Finally, the design of simplicial or cell meshes that guarantee accurate computations while approximating a given domain well remains an important endeavor for practical applications.

3.2 Geometry for simulation

Mathematical models of the evolution in time of mechanical systems generally involve systems of differential equations. Simulating a physical system consists in figuring out how to move the system forward in time from a set of initial conditions, allowing the computation of an actual trajectory through classical methods such as fourth-order Runge-Kutta or Newmark schemes. However, a geometric — instead of a traditional numerical-analytic — approach to the problem of time integration is particularly pertinent [86]: the very essence of a mechanical system is indeed characterized by its symmetries and invariants (e.g., momenta), thus preserving these geometric notions into the discrete computational setting is of paramount importance if one wants discrete time integration to properly capture the underlying continuous motion. Considering mechanics from a variational point of view goes back to Euler, Lagrange and Hamilton [74], and Poincaré famously stated that geometry and physics are “indissociable”. The variational principle most important for continuous mechanics is due to Hamilton, and is often called **Hamilton's principle** or the *least action principle*: it states that a dynamical system always finds an optimal course from one position to another. One consequence is that we can recast the traditional way of thinking about an object accelerating in response to applied forces, into a geometric viewpoint: the path followed by the object between two space-time positions has optimal geometric properties, analogous to the notion of geodesics on curved surfaces. This point of view is equivalent to Newton's laws in the context of classical mechanics, but is broad enough to encompass physical models ranging to E&M and quantum mechanics [104]. While the idea of discretizing variational formulations of mechanics is standard for elliptic problems using Galerkin Finite Element methods for instance, only recently did it get used to derive variational time-stepping algorithms for mechanical systems [103]. These variational integrators have been shown to be remarkably versatile, powerful, and general for simulations of physical phenomena when compared to traditional numerical time stepping methods: the symplectic character of variational integrators guarantees good statistical predictability through accurate preservation of the geometric properties of the exact flow of the differential equations. We endeavor to continue contributing to this particular application of geometry and extend it further, as we foresee a number of interesting scientific developments and industrial applications.

State-space discretization of statistical physics. Kinetic equations are used to describe a variety of phenomena in various scientific fields, ranging from rarefied gas dynamics and plasma physics to biology and socio-economics, and appear naturally when one considers a statistical description of a large particle system evolving in time. In incompressible fluid simulation, kinetic solvers based on the lattice Boltzmann method (LBM) have generated growing interest due to their use of the Boltzmann transport equation and to its unusual *state-space discretization* based on a computationally-efficient lattice [123]: compared to macroscopic solvers directly integrating Navier-Stokes equations, LBM totally bypasses the difficult issue of discretizing advection to high order, and absence of global pressure solves makes for extremely efficient parallel implementations, which are now surpassing alternative discretizations [96]. However, the numerical treatment of the *collision operator* of the Boltmann equation has not reached maturity; most surprising is the *complete absence of geometric approaches to deal with Boltzmann equations.* One should be able to *formulate a variational approach to LBM* based on Hamilton's principle to derive a systematic integrator with guaranteed accuracy and structure-preserving properties. Moreover, while dealing with isothermal and incompressible flows is a good starting point, the kinetic standpoint of fluid dynamics is not theoretically restricted to this case: far more complex physical systems, from compressible flow (with shocks), to thermal conductivity, to even acoustics for example, can be handled; but far less is known on how to handle these more involved cases computationally, because no systematic numerical approach to handle Boltzmann equations is known. Success in our geometric approach to LBM should offer a much better handle to deal with these difficult cases: between new Hermite regularization tools [49, 63] and the recent introduction of variational integrators for non-equilibrium thermodynamical systems mentioned above should provide the necessary theoretical foundations to establish a geometric solver for this generalized case.

Learning-aided simulation. Computational physics is experiencing a tectonic shift as data-driven approaches are quickly becoming mainstream. While we do not adhere to the idea being floated that numerical integration could be simply "learned" to improve current solvers, the fact is that many machine learning tools may have profound influence in practical applications using simulation. Long standing problems such as the design of perfectly matched layers (PML, an artificial absorbing layer for transport equations used to reduce the domain of simulation without suffering from reflected waves [61]) or flux limiters in high resolution schemes [132] (to avoid the spurious oscillations (wiggles) that would otherwise occur due to shocks or sharp changes) could be found through training, and applied at very low numerical cost. We are curious to see if geometry can help design better architectures or approaches for this type of learning-aided simulation, by helping with better loss functions (with soft constraints) or better architectures (to enforce hard constraints) that account for the importance of structure preservation. Learning the highly non-linear and chaotic dynamics of fluids is also an interesting direction: we believe that one can infer predictive high-frequency details of a turbulent flow from a low-resolution simulation as it is an attractive alternative to non-linear turbulence modeling, extending the computationally-expensive Reynolds-Averaged Navier-Stokes (RANS [39]), Large-Eddy Simulation (LES [91]), or Detached-Eddy Simulation (DES [124]) models used in CFD. Many other learning efforts in the domain of simulation are being explored, in particular towards the goal of allowing real-time design of shapes that satisfy some physical properties, such as lowest drag for improved aerodynamics or highest stiffness for a light cantilever.

Geometric integration of physical systems and multiphysics. Although the use of geometric integrators for differential equations in computational physics has recently brought off many numerical improvements, the large body of knowledge in differential geometric mechanics remains vastly under-utilized in discrete mechanics. Many mechanical systems require geometric objects such as diffeomorphisms, vector fields, or (principal) connections for which no structure-preserving discretization exists. Hydrodynamics, for instance, has well established and rich differential geometric foundations, but rare are the numerical methods that take advantage of this rich body of knowledge as yet. Yet, satisfying a form of "particle relabeling" symmetry [104] on a discrete level could directly enforce Kelvin's circulation theorem, a momentum preservation as important as angular momentum preservation for rigid bodies. Relativity is another example, albeit much more involved, where structure-preserving numerics would strongly impact the scientific community: having discretizations automatically enforcing Bianchi's identities would not only simplify the numerical procedures involved in gravitational theory (as spectral accuracy would no longer be required to avoid spurious modes), but could in fact result in conservation of energy and angular momentum. Moreover, multiphysics (coupled mechanical systems involving more than one simultaneously occurring physical field) can be consistently described through constrained variational principles: a simple, yet already interesting example is the case of the equations of motion for the garden hose, where rod dynamics coupled with fluid motion was only fully modeled (along with its nonlinear solutions of traveling-wave type) a few years back [116] through such a geometric treatment. Now that a variational formulation of nonequilibrium thermodynamics extending Hamilton's principle to include irreversible processes has been proposed [78], we are particularly interested in advancing further the arsenal of computational methods for physical simulation.

3.3 Geometry for dynamical systems

Dynamical systems – whether physical, biological, chemical, or social – are ubiquitous in nature, and their study deals with the concept of change, rate of change, rate of rate of change, etc. Dynamical systems are often better elucidated and modeled through *topology and geometry.* Whether we consider a continuous-time dynamical system (flow) or discrete-time dynamical system (map), the geometric theory of dynamical systems studies phase portraits: on the state-space manifold (a geometric model for the set of all possible states of the system), the global behavior of the dynamical system is determined by a *cellular structure of basins enclosed by separatrices*, each basin being dominated by a different specific behavior or fate. A system's trajectories on the state-space manifold determine velocity vectors by differentiation; conversely, velocity vectors determine trajectories by integration. Bifurcations can also be understood as geometric models for the controlled change of one system into another, while the rate of divergence of trajectories in phase space measures a system's stability. Given this overwhelming relevance of geometry in dynamical systems, we intend to dedicate some of our activities to develop geometry-based computational tools to study time series and dynamical systems: while classic dynamical systems theory has established solid foundations to study structures in steady and time-periodic flows and maps, new tools are needed to analyze the complexity of time series or aperiodic large-scale flows from sampled trajectories, and to automatically generate a simplified skeleton of the overall dynamics of a system from input data. We discuss a few directions we are interested in further impacting next.

Time series. Geometric methods play an important part in the study of time series. Of particular interest are time-delay embeddings, which are generically able to capture the underlying state space and dynamics from which the time series data have been acquired, by the Takens embedding theorem [127]. Such embeddings transform discrete time series into point clouds in Euclidean space, so that the underlying geometry of the point cloud reflects the geometry of the phase space the data originate from. By doing so, questions related to the seasonality or anomalous behavior of the time series are naturally reformulated into questions about the geometry or topology of their embeddings [113]. Beside this approach, other more direct methods apply geometric or topological tools in the original physical or frequency domain, which, despite its simplicity, has proven to be relevant in various contexts [66, 70]. A common thread to all these developments is their restriction to numerical time series, including (but not restricted to) data for which geometry plays an obvious role—e.g. inertial or gyroscopic sensor data. With potential medical applications in mind, one of our main long-term goals will be to adapt and extend these approaches to handle *categorical data*, in connection to the item *Geometry for categorical and mixed data types* in the *Geometry for data science* theme. We also plan to find principled methods to tuning the various parameters involved in the techniques, e.g. the window size in time-delay embeddings: we will seek to optimize or learn these parameters automatically, in connection to the item *Geometry-driven learning* in the *Geometry for data science* theme. We will also seek to make these parameters adaptive, e.g. using time-varying window sizes in time-delay embeddings of irregular time series, in order to obtain more accurate data representations and improved learning performance.

Coherent structures. Another interesting area in need of new numerical methods concerns coherent structures, i.e., persisting features of a flow over long periods that tend to favor or inhibit material transport between distinct flow regions. While there is no universally agreed-upon definition for coherent structures (there exist ergodicity-based [52], observer-based [105], and probabilistic [76] approaches to their definition), most variants and associated computational methods assume a fine knowledge of the Eulerian velocity field in space and time to deduce a good approximation of the flow. However, flows are often known only as a set of sparse particle trajectories in time (an example is the trajectory of buoys in the ocean). Such a sparse sampling of the dynamical system does not lend itself well to a geometric analysis of transport, so topological methods have recently been proposed to extract structures from a sparse set of trajectories by measuring their entanglement [129, 40, 136] based on the theory of *braid groups*, a classical area of topology. Coherent regions can then be defined as containing particles that possibly mix with other particles within the region itself but do not mix with particles outside the region; the set of trajectories arising from the particles within a coherent region forms a *coherent bundle*. Even if the use of braid groups offers sound foundations and numerical tools for the definition of coherent structures in 2D, there has been only limited efforts in developing practical and scalable computational tools for the efficient analysis of flow structures in 3D, offering a clear opportunity for us to try new geometric insights.

Invariant sets. Much of the theory of dynamical systems revolves around the existence and structure of invariant sets, which by definition are subsets of the state space that are invariant under the action of the dynamics. Invariant sets come in many different forms (stationary solutions, periodic orbits, connecting orbits, chaotic invariant sets, etc), and their structure can be very complicated and can undergo drastic changes under perturbations of the system, thus making their study difficult. This is all the more true in practical applications, where the systems are only known through space and/or time discretizations. *Conley index theory* [62] overcomes these issues by restricting the focus to invariant sets that admit an isolating neighborhood, and by introducing a topological invariant—the Conley index—that characterizes whether such isolated invariant sets are attracting, repelling, or saddle-like. It is defined as the homotopy type of a pair of compact subsets of the neighborhood, and it is proven to be independent of the choice of neighborhood—thus characterizing the invariant set itself. We are interested in the study of invariant sets in the discrete space and continuous time setting, where the space is typically described by a simplicial complex and the dynamics by a combinatorial vector (or multivector) field. Building upon Forman's seminal work in combinatorial dynamical systems [73], recent advances [45, 97] have shown that isolated invariant sets and their Conley indices can be properly defined even in this setting, and that they can be related to the dynamics of some upper semicontinuous acyclic multivalued map defined on the geometric realization of the simplicial complex; in simpler terms, not only can Conley index theory be adapted to the combinatorial setting, but it also connects to its classical analog in the underlying space. Two important questions for applications arise from this line of work: (1) how to compute the invariant sets and their Conley indices (including choosing relevant isolating neighboroods) efficiently? (2) how do they behave under perturbations of the input vector field or simplicial complex? These questions have just started to be addressed [68, 69], mostly through the lens of single-parameter topological persistence theory, developed in the context of topological data analysis. We intend to push this direction further, notably using multi-parameter persistence theory to cope with some of the key difficulties such as the choice of isolating neighborhoods.

3.4 Geometry for data science

The last decade has seen the advent of machine learning (ML), and in particular deep learning (DL), in a large variety of fields, including some directly connected to geometry. For instance, DL-based approaches have become increasingly popular in geometry processing [117] due to their ability to outperform state-of-the-art, domain-specific methods by leveraging the ever-increasing amounts of available labeled data. On the downside, DL approaches suffer from a general lack of explainability. Moreover, their performances can be disappointing on small data due to their large numbers of parameters; this is especially true with end-to-end learning pipelines, which tend to require humongous amounts of training data to learn the right data representation. Finally, DL is by essence tied to Euclidean data representations, and as such it requires intermediate transforms in order to be applicable to non-Euclidean data types such as graphs or probability measures. Because of these limitations, we are seeing a rise of geometric and topological methods for data science in general, and for ML and DL in particular, whose aim is to help address the aforementioned challenges as well as others. For instance, geometric deep learning [50] tries to generalize deep neural models to non-Euclidean domains. This includes for instance using information geometry to apply deep neural models in probability spaces. Topological data analysis (TDA) [109] is another popular approach to enhance ML and DL methods. It contributes to data science in at least three different ways: first, by providing data mining tools that can help users uncover hidden structures in data; second, by providing generic descriptors for geometric data that can be turned into features for ML and DL with provable stability properties; third, by integrating itself deeply into existing ML methods or DL architectures to enhance their performances or to analyze their behavior [58, 100]. Other contributions of geometry to data science at large include: the use of Forman's Ricci curvature and its corresponding Ricci flow in networks, to understand the networks' properties and growth [133]; the application of the Hodge-Hemholtz decomposition to statistical ranking problems with sparse response data, with theoretical connections to both PageRank and LASSO [90]; the use of Reeb graphs or Morse-Smale complexes in statistical inference [60] as well as in data visualization [131]. These important developments reinforce our argument that geometry and topology have their role to play in the elaboration of the next-generation data analysis tools. We plan to focus on a few research directions related to these developments, which are of particular interest in our view.

Deep learning for large-scale 3D geometric data analysis. We first propose to develop efficient algorithms and mathematical tools for analyzing large geometric data collections using Deep Learning techniques. This includes 3D shapes represented as triangle or quad meshes, volumetric data, point clouds possibly embedded in high-dimensions, and graphs representing geometric (e.g. proximity) data. Our project is motivated by the fact that large annotated collections of geometric models have recently become available [57, 135], and that machine learning algorithms applied to such collections have shown promising initial results, both for data analysis as well as synthesis. We believe that these results can be significantly extended by building on recent advances in geometry processing, optimization and learning. Our ultimate goal is to design novel deep learning techniques capable both of handling geometric data directly and of combining and integrating different data sources into a unified analysis pipeline. A key challenge in this project is the fact that geometric data can come in a myriad different representations, such as point clouds and meshes among others, with variable sampling and discretization. Furthermore, geometric shapes can undergo both rigid and non-rigid deformations. Unfortunately, most existing deep learning approaches focus only on a particular type of representations and deformation classes (e.g., considering purely extrinsic or purely intrinsic methods). Instead we propose to place special focus on designing learning techniques capable of handing *diverse* multimodal data sources undergoing arbitrary deformations, in a coherent theoretical and practical framework. Moreover we propose to develop novel powerful *interactive* tools for analysis and annotation, to help harness user input, and also provide better mechanisms for exploration of variability in the data [120, 112].

Explainable geometric and topological features for data. Another of our goals is to design geometric and topological features that can capture richer content from the data, while maintaining the robustness and stability properties that the existing features enjoy. If we can make our features rich enough so that they characterize the input data (or their underlying geometric structures, assuming such structures exist) completely, then we will be able to leverage them in the context of explainable AI, to compute pre-images with guarantees on the corresponding interpretations. In cases where our features cannot completely describe the data, we will study the geometry of the fibers of the feature extraction step, in order to quantify the discrepancy that may appear between different interpretations of the same feature. We envision two complementary approaches for this:

- The first approach relies on feature aggregation. In the context of TDA for instance, one may consider using multiple filtrations (or filter functions on a fixed simplicial complex), computing their corresponding topological descriptors, then aggregating these descriptors together to form a feature vector.
- The second approach relies on more elaborate geometric and topological tools to design the features. The idea is to encode the joint effect of multiple geometric and topological constructions on the data, in a more integrated way than just by aggregating the corresponding features. By encoding more complex effects, we hope to extract a richer content using smaller constructions.

Research on the first approach in TDA started with [65, 79], who proved that, in the special case where the data are sampled from some subanalytic compact sets in Euclidean space $\mathbb{R}^n$, the compact sets themselves are fully described by the aggregated features obtained by orthogonal projections onto lines. This follows from a more fundamental result on the invertibility of the Radon transforms of constructible functions [122], to which the above aggregated features belong. This initial result has sparked a thriving new direction of research, exploring larger and larger classes of compact sets [89, 102, 108]. Many important questions arise from this line of work, some of which have been partially addressed, including: what kind of stability or robustness properties do these aggregated features enjoy? Can the size of the collection of filter functions used be reduced, to become finite and (more importantly) independent of the compact set under consideration? Can the aggregated features be computed efficiently? Can non-Euclidean compact sets, such as manifolds or length spaces, be considered as well, with similar guarantees?

The second approach is related to the development of *multi-parameter persistence* [54], which is undeniably the most widely open and long-standing research topic in TDA today. The core challenge is to define computationally tractable algebraic invariants that can capture as much of the joint structure of multiple topological constructions as possible. The notorious difficulty of this question comes from the fact that the algebraic objects underlying multi-parameter topological constructions are significantly more complicated than the ones underlying single-parameter constructions. The question also connects to notoriously hard problems in other areas of pure mathematics, such as the classification of isomorphism classes of indecomposable poset representations in quiver representation theory for instance. It can benefit from these connections, as mathematical tools that have been developed for those problems can be imported into the TDA literature—several promising such imports have been made in the recent past, including from representation theory [46] and from sheaf theory [92]. In turn, mathematical and algorithmic advances made in multi-parameter persistence may benefit these other areas of mathematics as well. This is clearly a high-risk and long-term research topic, but if successful, it may eventually have an enormous impact on TDA and related areas.

Geometric feature learning. Geometry and topology have played a key role in the design of feature extraction pipelines for certain types of data. The numerous existing geometric features for geometry processing (shape contexts [75], differential and integral invariants [115], heat or wave kernel signatures [42, 126], etc.) are a sign of the importance of this topic for the computer graphics community. Meanwhile, the TDA community has developed generic feature extraction pipelines, based on combinatorial constructions and their algebraic invariants, which have proven to be useful in a variety of application domains [109]. All these approaches are, however, handcrafted, with hyperparameters being tuned via manual, grid, or random search. Our goal is to make these approaches transition from a paradigm of feature engineering to that of feature learning, in order to set up end-to-end learning pipelines for improved performances and adaptability. Two complementary directions are considered:

- designing piecewise-smooth variants of the existing pipelines, with a fine control over the underlying stratification. This will make it possible to apply variational optimization methods, typically stochastic (sub-)gradient descent, and to optimize the gradient sampling steps for improved convergence rates.
- designing novel pipelines based on a combination of geometric/topological tools and deep learning, in order to get the best out of both worlds.

Research in the first direction is still in its infancy. Promising theoretical advances were made recently, towards understanding the piecewise differentiability of the basic *topological persistence* operator in full generality [95], as well as towards optimizing its parameters using classical stochastic gradient descent [55]. Can the knowledge gained in these studies about the underlying stratification of the operator be leveraged to optimize the gradient sampling step and thus improve the convergence rates? Can these results be extended to more advanced pipelines, such as the one for Mapper or for zigzags and multi-parameter persistence?

The idea behind the second direction is to integrate topological or geometric layers into neural network architectures such as auto-encoders or GANs for feature extraction — the challenge being to determine how to do it in the appropriate way, so that we can make the most of this combination. This question connects to the research topic *Geometry-driven learning* described further down in this section.

Geometry-driven learning. Most of the contributions of geometry and topology to machine learning until recently have been to the design of pre-processing steps (e.g. feature extraction) to enhance the performances of the learning pipeline. There is now a thriving effort of the community toward integrating geometric and/or topological computations deeper into the core of the pipeline. This includes for instance: *ToMATo* [58], which integrates a TDA-based feedback loop into density based algorithms to improve their stability and robustness; *topological regularizers* [59, 87], which add topology-based regularization terms to the loss in supervised statistical learning; *topological layers* [56, 77, 94], which are meant to be incorporated into neural networks. Meanwhile, geometry and topology have been used to analyze the behavior of neural networks [121, 51]. This exciting line of work is just emerging, and our intent is to push this direction further, in particular to address the following important questions:

- How can we generalize the use of topological layers in neural networks? This question is connected to the differentiability of the TDA pipeline, addressed in the research topic *Geometric feature learning.* Inded, generalizing the current (nascent) framework for differential calculus and optimization with the TDA pipeline will be key to designing both generic and effective topological layers. Another more practical aspect of the question is to evaluate the contribution of topological layers as initial or intermediate layers, depending on the neural network architecture that they are combined with and on the data they are applied to.
- The same question arises for topological regularizers, with similar theoretical and practical challenges.
- The development of richer families of geometric and topological descriptors, undertaken in the item *Richer geometric and topological features for data*, will eventually lead to the question of

generalizing the current differentiable framework to these new descriptors, in order to make them as widely applicable as the current descriptors, and also to the practical question of determining how to best combine them with existing loss functions, regularizers, or neural network architectures.

- The aforementioned contributions and research directions concern mostly supervised learning. Can we contribute as well to unsupervised learning problems, including clustering (as ToMATo does already for density-based clustering), dimensionality reduction, or unsupervised feature learning? This question connects also to the research topic *Geometric feature learning* described previously. One direction we may explore is the design of geometric or topological layers to be inserted in unsupervised neural network architectures such as auto-encoders or GANs.
- Finally, as TDA is concerned primarily with topology, an obvious (yet still wide open) question to ask is whether it can contribute to the current effort towards generating neural network architectures automatically.

Geometry for categorical and mixed data types. Categorical data types are notoriously hard to deal with in the context of ML and AI. Indeed, most of the existing ML toolbox has been designed specifically to work with numerical variables, usually sitting in some vector or metric space. By contrast, spaces of categorical data do not naturally come equipped with a linear structure nor a metric. More importantly, these spaces are discrete by nature, so choices of metrics or (dis-)similarity measures can be scarce, with limited effects on the learning efficiency. To make things worse, categorical variables are often mixed with numerical variables, and choosing a proper weighting for them is a challenge in its own right. Meanwhile, categorical variables play an important part in many applications: for instance, in precision medicine, where the monitoring of patients relies on collected longitudinal data that include not only numerical variables such as temperature or blood pressure, but also categorical variables such as illness antecedents or symptoms lists. Thus, handling categorical and mixed data types represents an important challenge today. Unfortunately, with very few exceptions [134], it has been mostly overlooked so far in the development of topological methods for ML and AI, so our goal will be to help fix this situation. The standard approach for handling categorical variables is to define a proper vector representation, then to apply—either off-the-shelf or with minor adaptations—an analysis method designed for numerical variables to the new data representation. A prototypical instance of this approach is Multiple Correspondance Analysis for dimensionality reduction [38], which applies classical PCA to the one-hot encoding matrix of the input data. A variant of the approach replaces the vector representation by a suitable metric or (dis-)similarity measure on the initial categorical variables or on some transformed version of those. For instance, in clustering, one can define a metric on the input data, e.g. Jaccard or Hamming distance, then apply a hierarchical bottom-up clustering algorithm such as single-linkage to the resulting distance matrix. This variant seems quite appropriate for geometric or topological methods, since the latter typically work with metric or (dis-)similarity spaces. The challenge is to determine with which metrics or (dis-)similarity measures, and on which data types, geometric or topological methods will be provably better.

A more refined version of the approach learns the new data representation instead of engineering it, which is particularly relevant when end-to-end learning pipelines are sought for. The methods are usually taylored to a specific data type, for instance word2vec [106] computes word embeddings for text data using a two-layer neural network. Our developments in the research topic *Geometry-driven learning* will make it possible to combine TDA layers with such networks, and thus to benefit from the most recent advances on representation learning for these data types. The challenge will be to understand when and how to make the most of this combination.

4 Application domains

Our work aims at a wide range of applications covering 3D shape analysis and processing, simulation, and data science in general. While we typically focus on contributions that are of a fundamental, mathematical and algorithmic nature, we seek collaborations with academics and industrial from applied fields, who can use our tools on practical and concrete problems. Here are a few examples of collaborations:

- In the context of 3D geometry processing, we collaborate with Dassault Systèmes for a) the PhD of Lucas Brifault on the design of novel geometric representations for shapes through measure theory and b) the PhD of Mariem Mezghanni on the design of physical simulation layers for 3D modeling.
- In the context of personalized medicine, we collaborate with statisticians and medical doctors to incorporate our geometric and topological features into learning pipelines to design better dynamic treatment regimens (AEx PreMediT).
- In a collaboration with the French Ministry of Defense, we seek to develop tools to analyze multimodal time series data in order to predict the appearance of G-LOCs among fighter jet pilots in training or in operation (PhD of Julie Mordacq).

Beside these few illustrative examples, GeomeriX also maintains regular collaborations with Sanofi, EDF, Danone R&D, Immersion Tools, as well as with several key players in the world-wide tech industry, including Ansys, Adobe Research, Disney/Pixar, NVidia.

5 Highlights of the year

5.1 Awards

- The paper "Functional Maps: A Flexible Representation of Maps Between Shapes" co-authored by Maks Ovsjanikov and colleagues, has received the SIGGRAPH 2023 Test-of-Time award.
- Maks Ovsjanikov has received an ERC Consolidator Grant in 2023.
- The paper "Implicit fairing of irregular meshes using diffusion and curvature flow" co-authored by Mathieu Desbrun and colleagues has been included in the ACM list of "Seminal Graphics Papers: Pushing the Boundaries".

5.2 Distinctions

- S. Oudot was a CAS fellow at the Norwegian Academy of Science and Letters for the academic year 2022-2023.
- Maks Ovsjanikov became a fellow of ELLIS, the European Laboratory for Learning and Intelligent Systems, bringing together top AI researchers in Europe.
- Mathieu Desbrun received an INRIA Chair on Geometry and AI.

6 New results

We list our new results for each of the four themes that our team is articulated around.

6.1 Geometry for Euclidean shape processing

6.1.1 Patternshop: Editing Point Patterns by Image Manipulation

Participants: Pooran Memari.

In collaboration with Xingchang Huang, Hans-Peter Seidel and Gurprit Singh (MPI Saarbrucken) and Tobias Ritschel (UCL).

Point patterns are characterized by their density and correlation. While spatial variation of density is well-understood, analysis and synthesis of spatially-varying correlation is an open challenge. No tools are available to intuitively edit such point patterns, primarily due to the lack of a compact representation for spatially varying correlation. In this work [13], we propose a low-dimensional perceptual embedding for point correlations. This embedding can map point patterns to common three-channel raster images, enabling manipulation with off-the-shelf image editing software. To synthesize back point patterns, we propose a novel edge-aware objective that carefully handles sharp variations in density and correlation. The resulting framework allows intuitive and backward-compatible manipulation of point patterns, such as recoloring, relighting to even texture synthesis that have not been available to 2D point pattern design before. Effectiveness of our approach is tested in several user experiments. Our proposed framework was patented under the reference M35655EP. The corresponding code is however publicly available at https://github.com/xchhuang/patternshop.

6.1.2 Robust Pointset Denoising of Piecewise-Smooth Surfaces through Line Processes

Participants: Jiayi Wei, Jiong Chen, Pooran Memari, Mathieu Desbrun.

In collaboration with Damien Rohmer (LIX).

Denoising is a common, yet critical operation in geometry processing aiming at recovering high-fidelity models of piecewise smooth objects from noise-corrupted pointsets. Despite a sizable literature on the topic, there is a dearth of approaches capable of processing very noisy and outlier-ridden input pointsets for which no normal estimates and no assumptions on the underlying geometric features or noise type are provided. In this paper [21], we propose a new robust-statistics approach to denoising pointsets based on line processes to offer robustness to noise and outliers while preserving sharp features possibly present in the data. While the use of robust statistics in denoising is hardly new, most approaches rely on prescribed filtering using data-independent blending expressions based on the spatial and normal closeness of samples. Instead, our approach deduces a geometric denoising strategy through robust and regularized tangent plane fitting of the initial pointset, obtained numerically via alternating minimizations for efficiency and reliability. Key to our variational approach is the use of line processes to identify inliers vs. outliers, as well as the presence of sharp features. We demonstrate that our method can denoise sampled piecewise-smooth surfaces for levels of noise and outliers at which previous works fall short.

6.1.3 Feature-Sized Sampling for Vector Line Art

Participants: Pooran Memari.

In collaboration with Stefan Ohrhallinger (TU Wien) and Amal D. Parakkat (Telecom Patis).

In this work [27], by introducing a first-of-its-kind quantifiable sampling algorithm based on feature size, we present a fresh perspective on the practical aspects of planar curve sampling. Following the footsteps of ϵ-sampling, which was originally proposed in the context of curve reconstruction to offer provable topological guarantees (Crust algorithm) under quantifiable bounds, we propose an arbitrarily precise ϵ-sampling algorithm for sampling smooth planar curves (with a prior bound on the minimum feature size of the curve). This paper not only introduces the first such algorithm which provides user-control and quantifiable precision but also highlights the importance of such a sampling process under two key contexts: 1) To conduct a first study comparing theoretical sampling conditions with practical sampling requirements for reconstruction guarantees that can further be used for analysing the upper bounds of ϵ for various reconstruction algorithms with or without proofs, 2) As a feature-aware sampling of vector line art that can be used for applications such as coloring and meshing.

6.1.4 Bio-Sketch: A new medium for interactive storytelling, illustrated by the phenomenon of infection

Participants: Pooran Memari.

In collaboration with Pauline Olivier, Renaud Chabrier, Marie-Paule Cani (LIX), Jean-Luc Coll (INSERM, CNRS, Institute for Advanced Biosciences, Grenoble).

In the field of biology, digital illustrations play a crucial role in conveying complex phenomena, allowing for idealized shapes and motion, in contrast to data visualization. In the absence of suitable media, scientists often rely on oversimplified 2D figures or have to call in professional artists to create better illustrations, which can be limiting. In this work [28] we introduce Bio-Sketch, a novel progressive sketching system designed to ease the creation of animated illustrations, as exemplified here in the context of the infection phenomenon. Our solution relies on a new progressive sketching paradigm that seamlessly combines 3D modeling and pattern-based shape distribution to create background volume and temporal animation control. The elements created can be assembled into a complex scenario, enabling narrative design experiments for educational applications in biology. Our results and first feedback from experts in illustration and biology demonstrate the potential of Bio-Sketch to assist communication on the infection phenomenon, helping to bridge the gap between expert and non-expert audiences.

6.1.5 Somigliana Coordinates

Participants: Jiong Chen, Mathieu Desbrun.

In collaboration with Fernando de Goes (Pixar).

In this work [24], we present a novel cage deformer based on elasticity-derived matrix-valued coordinates. In order to bypass the typical shearing artifacts and lack of volume control of existing cage deformers, we promote a more elastic behavior of the cage deformation by deriving our coordinates from the Somigliana identity, a boundary integral formulation based on the fundamental solution of linear elasticity. Given an initial cage and its deformed pose, the deformation of the cage interior is deduced from these Somigliana coordinates via a corotational scheme, resulting in a matrix-weighted combination of both vertex positions and face normals of the cage. Our deformer thus generalizes Green coordinates, while producing physically-plausible spatial deformations that are invariant under similarity transformations and with interactive bulging control. We demonstrate the efficiency and versatility of our method through a series of examples in 2D and 3D.

6.2 Geometry for simulation

6.2.1 Building a Virtual Weakly-Compressible Wind Tunnel Testing Facility

Participants: Wei Li, Mathieu Desbrun.

In collaboration with Chaoyang Lyu, Kai Bai, Yiheng Wu, and Xiaopei Liu (all from Shanghaitech University, China), and Changxi Zheng (Columbia University, USA).

Virtual wind tunnel testing is a key ingredient in the engineering design process for the automotive and aeronautical industries as well as for urban planning: through visualization and analysis of the simulation data, it helps optimize lift and drag coefficients, increase peak speed, detect high pressure zones, and reduce wind noise at low cost prior to manufacturing. In this paper [18], we develop an efficient and accurate virtual wind tunnel system based on recent contributions from both computer graphics and computational fluid dynamics in high-performance kinetic solvers. Running on one or multiple GPUs, our massively-parallel lattice Boltzmann model meets industry standards for accuracy and consistency while exceeding current mainstream industrial solutions in terms of efficiency Ð especially for unsteady turbulent flow simulation at very high Reynolds number (on the order of 10^7) – due to key contributions in improved collision modeling and boundary treatment, automatic construction of multiresolution grids for complex models, as well as performance optimization. We demonstrate the efficacy and reliability of our virtual wind tunnel testing facility through comparisons of our results to multiple benchmark tests, showing an increase in both accuracy and efficiency compared to state-of-the-art industrial solutions. We also illustrate the fine turbulence structures that our system can capture, indicating the relevance of our solver for both VFX and industrial product design.

6.2.2 Fluid-Solid Coupling in Kinetic Two-Phase Flow Simulation

Participants: Wei Li, Mathieu Desbrun.

Real-life flows exhibit complex and visually appealing behaviors such as bubbling, splashing, glugging and wetting that simulation techniques in graphics have attempted to capture for years. While early approaches were not capable of reproducing multiphase flow phenomena due to their excessive numerical viscosity and low accuracy, kinetic solvers based on the lattice Boltzmann method have recently demonstrated the ability to simulate water-air interaction at high Reynolds numbers in a massively-parallel fashion. However, robust and accurate handling of fluid-solid coupling has remained elusive: be it for CG or CFD solvers, as soon as the motion of immersed objects is too fast or too sudden, pressures near boundaries and interfacial forces exhibit spurious oscillations leading to blowups. Built upon a phase-field and velocity-distribution based lattice-Boltzmann solver for multiphase flows, this paper [16] spells out a series of numerical improvements in momentum exchange, interfacial forces, and two-way coupling to drastically reduce these typical artifacts, thus significantly expanding the types of fluid-solid coupling that we can efficiently simulate. We highlight the numerical benefits of our solver through various challenging simulation results, including comparisons to previous work and real footage.

6.2.3 High-Order Moment-Encoded Kinetic Simulation of Turbulent Flows

Participants: Wei Li, Mathieu Desbrun.

In collaboration with Tongtong Wang, Zherong Pang, Xifeng Gao, and Kui Wu (Tencent Lightspeed Studios, China).

Kinetic solvers for incompressible fluid simulation were designed to run efficiently on massively parallel architectures such as GPUs. While these lattice Boltzmann solvers have recently proven much faster and more accurate than the macroscopic Navier-Stokes-based solvers traditionally used in graphics, it systematically comes at the price of a very large memory requirement: a mesoscopic discretization of statistical mechanics requires over an order of magnitude more variables per grid node than most fluid solvers in graphics. In order to open up kinetic simulation to gaming and simulation software packages on commodity hardware, we propose a High-Order Moment-Encoded Lattice-Boltzmann-Method solver which we coined HOME-LBM, requiring only the storage of a few moments per grid node [17], with little to no loss of accuracy in the typical simulation scenarios encountered in graphics. We show that our lightweight and lightspeed fluid solver requires three times less memory and runs ten times faster than state-of-the-art kinetic solvers, for a nearly-identical visual output.

6.3 Geometry for data science

6.3.1 Stable Vectorization of Multiparameter Persistent Homology using Signed Barcodes as Measures

Participants: Steve Oudot.

In collaboration with David Loiseaux (Inria, Datashape team), Luis Scoccola (Northeastern University), Magnus Botnan (Vrije Universiteit Amsterdam), and Mathieu Carrière (Inria, Datashape team).

Persistent homology (PH) provides topological descriptors for geometric data, such as weighted graphs, which are interpretable, stable to perturbations, and invariant under, e.g., relabeling. Most applications of PH focus on the one-parameter case – where the descriptors summarize the changes in topology of data as it is filtered by a single quantity of interest – and there is now a wide array of methods enabling the use of one-parameter PH descriptors in data science, which rely on the stable vectorization of these descriptors as elements of a Hilbert space. Although the multiparameter PH (MPH) of data that is filtered by several quantities of interest encodes much richer information than its one-parameter counterpart, the scarceness of stability results for MPH descriptors has so far limited the available options for the stable vectorization of MPH. In this work [25] we aim to bring together the best of both worlds by showing how the interpretation of signed barcodes – a recent family of MPH descriptors – as signed measures leads to natural extensions of vectorization strategies from one parameter to multiple parameters. The resulting feature vectors are easy to define and to compute, and provably stable. While, as a proof of concept, we focus on simple choices of signed barcodes and vectorizations, we already see notable performance improvements when comparing our feature vectors to state-of-the-art topology-based methods on various types of data.

6.3.2 Local characterizations for decomposability of 2-parameter persistence modules

Participants: Vadim Lebovici, Steve Oudot.

In collaboration with Magnus Botnan (Vrije Universiteit Amsterdam).

In this work [12] we investigate the existence of sufficient local conditions under which poset representations decompose as direct sums of indecomposables from a given class. In our work, the indexing poset is the product of two totally ordered sets, corresponding to the setting of 2-parameter persistence in topological data analysis. Our indecomposables of interest belong to the so-called interval modules, which by definition are indicator representations of intervals in the poset. While the whole class of interval modules does not admit such a local characterization, we show that the subclass of rectangle modules does admit one and that it is, in some precise sense, the largest subclass to do so.

6.3.3 A Gradient Sampling Algorithm for Stratified Maps with Applications to Topological Data Analysis

Participants: Steve Oudot.

In collaboration with Jacob Leygonie (University of Oxford), Mathieu Carrière (Inria, Datashape team), and Théo Lacombe (Université de Marne la Vallée).

In this work [15] we introduce a novel gradient descent algorithm refining the well-known Gradient Sampling algorithm on the class of stratifiably smooth objective functions, which are defined as locally Lipschitz functions that are smooth on some regular pieces—called the strata—of the ambient Euclidean space. On this class of functions, our algorithm achieves a sub-linear convergence rate. We then apply our method to objective functions based on the (extended) persistent homology map computed over lower-star filters, which is a central tool of Topological Data Analysis. For this, we propose an efficient exploration of the corresponding stratification by using the Cayley graph of the permutation group. Finally, we provide benchmarks and novel topological optimization problems that demonstrate the utility and applicability of our framework.

6.3.4 Shape Non-rigid Kinematics (SNK): A Zero-Shot Method for Non-Rigid Shape Matching via Unsupervised Functional Map Regularized Reconstruction

Participants: Souhaib Attaiki, Maks Ovsjanikov.

In this work [32], we present Shape Non-rigid Kinematics (SNK), a novel zero-shot method for non-rigid shape matching that eliminates the need for extensive training or ground truth data. SNK operates on a single pair of shapes, and employs a reconstruction-based strategy using an encoder-decoder architecture, which deforms the source shape to closely match the target shape. During the process, an unsupervised functional map is predicted and converted into a point-to-point map, serving as a supervisory mechanism for the reconstruction. To aid in training, we have designed a new decoder architecture that generates smooth, realistic deformations. SNK demonstrates competitive results on traditional benchmarks, simplifying the shape matching process without compromising accuracy.

6.3.5 Zero-Shot 3D Shape Correspondence

Participants: Ahmed Abdelreheem, Abdelrahman Eldesokey, Maks Ovsjanikov, Peter Wonka.

In collaboration with Ahmed Abdelreheem, Abdelrahman Eldesokey, and Peter Wonka, from King Abdullah University of Science and Technology (KAUST), Saudi Arabia.

In this work [29] we propose a novel zero-shot approach to computing correspondences between 3D shapes. Existing approaches mainly focus on isometric and near-isometric shape pairs (e.g., human vs. human), but less attention has been given to strongly non-isometric and inter-class shape matching (e.g., human vs. cow). To this end, we introduce a fully automatic method that exploits the exceptional reasoning capabilities of recent foundation models in language and vision to tackle difficult shape correspondence problems. Our approach comprises multiple stages. First, we classify the 3D shapes in a zero-shot manner by feeding rendered shape views to a language-vision model (e.g., BLIP2) to generate a list of class proposals per shape. These proposals are unified into a single class per shape by employing the reasoning capabilities of ChatGPT. Second, we attempt to segment the two shapes in a zero-shot manner, but in contrast to the co-segmentation problem, we do not require a mutual set of semantic regions. Instead, we propose to exploit the in-context learning capabilities of ChatGPT to generate two different sets of semantic regions for each shape and a semantic mapping between them. This enables our approach to match strongly non-isometric shapes with significant differences in geometric structure. Finally, we employ the generated semantic mapping to produce coarse correspondences that can further be refined by the functional maps framework to produce dense point-to-point maps. Our approach, despite its simplicity, produces highly plausible results in a zero-shot manner, especially between strongly non-isometric shapes.

6.3.6 VoroMesh: Learning Watertight Surface Meshes with Voronoi Diagrams

Participants: Nissim Maruani, Roman Klokov, Maks Ovsjanikov, Mathieu Desbrun.

In collaboration with Pierre Alliez (Inria).

In this work [26], we present VoroMesh, a novel and differentiable Voronoi-based representation of watertight 3D shape surfaces. From a set of 3D points (called generators) and their associated occupancy, we define our boundary representation through the Voronoi diagram of the generators as the subset of Voronoi faces whose two associated (equidistant) generators are of opposite occupancy: the resulting polygon mesh forms a watertight approximation of the target shape's boundary. To learn the position of the generators, we propose a novel loss function, dubbed VoroLoss, that minimizes the distance from groundtruth surface samples to the closest faces of the Voronoi diagram which does not require an explicit construction of the entire Voronoi diagram. A direct optimization of the Voroloss to obtain generators on the Thingi32 dataset demonstrates the geometric efficiency of our representation compared to axiomatic meshing algorithms and recent learning-based mesh representations. We further use VoroMesh in a learning-based mesh prediction task from input SDF grids on the ABC dataset, and show comparable performance to state-of-the-art methods while guaranteeing closed output surfaces free of self-intersections.

6.3.7 SATR: Zero-Shot Semantic Segmentation of 3D Shapes

Participants: Maks Ovsjanikov.

In collaboration with Ahmed Abdelreheem, Ivan Skorokhodov, and Peter Wonka, from from King Abdullah University of Science and Technology (KAUST), Saudi Arabia.

In this work [30], we explore the task of zero-shot semantic segmentation of 3D shapes by using large-scale off-the-shelf 2D image recognition models. Surprisingly, we find that modern zero-shot 2D object detectors are better suited for this task than contemporary text/image similarity predictors or even zero-shot 2D segmentation networks. Our key finding is that it is possible to extract accurate 3D segmentation maps from multi-view bounding box predictions by using the topological properties of the underlying surface. For this, we develop the Segmentation Assignment with Topological Reweighting (SATR) algorithm and evaluate it on ShapeNetPart and our proposed FAUST benchmarks. SATR achieves state-of-the-art performance and outperforms a baseline algorithm by 1.3% and 4% average mIoU on the FAUST coarse and fine-grained benchmarks, respectively, and by 5.2% average mIoU on the ShapeNetPart benchmark.

6.3.8 Spatially and Spectrally Consistent Deep Functional Maps

Participants: Maks Ovsjanikov.

Joint work with Mingze Sun (Tsinghua Shenzhen International Graduate School, China) , Shiwei Mao (Tsinghua Shenzhen International Graduate School, China) , Puhua Jiang (Tsinghua Shenzhen International Graduate School, China, Peng Cheng Laboratory, China), Ruqi Huang (Tsinghua Shenzhen International Graduate School, China).

In this paper [34], we investigate the utility of cycle consistency in Deep Functional Maps. We first justify that under certain conditions, the learned maps, when represented in the spectral domain, are already cycle consistent. Furthermore, we identify the discrepancy that spectrally consistent maps are not necessarily spatially, or point-wise, consistent. In light of this, we present a novel design of unsupervised Deep Functional Maps, which effectively enforces the harmony of learned maps under the spectral and the point-wise representation. By taking advantage of cycle consistency, our framework produces state-of-the-art results in mapping shapes even under significant distortions. Beyond that, by independently estimating maps in both spectral and spatial domains, our method naturally alleviates over-fitting in network training, yielding superior generalization performance and accuracy within an array of challenging tests for both near-isometric and non-isometric datasets.

6.3.9 TIDE: Time Derivative Diffusion for Deep Learning on Graphs

Participants: Maysam Behmanesh, Maks Ovsjanikov.

In collaboration with Maximilian Krahn (Aalto University, Finland).

In this paper [33], we present a novel method based on time derivative graph diffusion (TIDE) to overcome the structural limitations of the message-passing framework in graph neural networks. Our approach allows for optimizing the spatial extent of diffusion across various tasks and network channels, thus enabling medium and long-distance communication efficiently. Furthermore, we show that our architecture design also enables local message-passing and thus inherits from the capabilities of local message-passing approaches. We show that on both widely used graph benchmarks and synthetic mesh and graph datasets, the proposed framework outperforms state-of-the-art methods by a significant margin.

6.3.10 ReVISOR: ResUNets with visibility and intensity for structured outlier removal

Participants: Maxime Kirgo, Maks Ovsjanikov.

In collaboration with Guillaume Terrasse and Guillaume Thibault (EDF R&D).

In this paper [14], we make several contributions to address the problem of reflection-induced outlier detection. First, to overcome the scarcity of annotated data, we introduce a new dataset tailored for this task. Second, to capture non-local dependencies, we study and demonstrate, for the first time, the utility of deep learning based semantic segmentation architectures for reflection-induced outlier detection. By doing so, we bring together the fields of shape denoising/repair and semantic segmentation. Third, we demonstrate that additional non-local cues in the form of laser intensity and a computed visibility signal help boost the performance considerably. We denote our pipeline as ResUNets with Visibility and Intensity for Structured Outlier Removal, or ReVISOR, and demonstrate its superior performance against existing baselines on real-world data.

6.3.11 Assessing craniofacial growth and form without landmarks: A new automatic approach based on spectral methods

Participants: Robin Magnet, Maks Ovsjanikov.

In collaboration with Kevin Bloch (Institut Necker Enfants-Malades), Maxime Taverne (Institut Necker Enfants-Malades) , Simone Melzi (University of Milano-Bicocca), Maya Geoffroy (Institut Necker Enfants-Malades), Roman Khonsari (Institut Necker Enfants-Malades)

In this paper [19] we present a novel method for the morphometric analysis of series of 3D shapes, and demonstrate its relevance for the detection and quantification of two craniofacial anomalies: trigonocephaly and metopic ridges, using CT-scans of young children. Our approach is fully automatic, and does not rely on manual landmark placement and annotations. Our approach furthermore allows to differentiate shape classes, enabling successful differential diagnosis between trigonocephaly and metopic ridges, two related conditions characterized by triangular foreheads. These results were obtained using recent developments in automatic non-rigid 3D shape correspondence methods and specifically spectral approaches based on the functional map framework. Our method can capture local changes in geometric structure, in contrast to methods based, for instance, on global shape descriptors. As such, our approach allows to perform automatic shape classification and provides visual feedback on shape regions associated with different classes of deformations. The flexibility and generality of our approach paves the way for the application of spectral methods in quantitative medicine.

6.3.12 Affection: Learning Affective Explanations for Real-World Visual Data

Participants: Maks Ovsjanikov.

In collaboration with Panos Achlioptas (Snap Inc.), Leonidas Guibas (Stanford University), Sergey Tulyakov (Snap Inc.)

In this work [31], we explore the space of emotional reactions induced by real-world images. For this, we first introduce a large-scale dataset that contains both categorical emotional reactions and free-form textual explanations for 85,007 publicly available images, analyzed by 6,283 annotators who were asked to indicate and explain how and why they felt when observing a particular image, with a total of 526,749 responses. Although emotional reactions are subjective and sensitive to context (personal mood, social status, past experiences)-we show that there is significant common ground to capture emotional responses with a large support in the subject population. In light of this observation, we ask the following questions: i) Can we develop neural networks that provide plausible affective responses to real-world visual data explained with language? ii) Can we steer such methods towards producing explanations with varying degrees of pragmatic language, justifying different emotional reactions by grounding them in the visual stimulus? Finally, iii) How to evaluate the performance of such methods for this novel task? In this work, we take the first steps in addressing all of these questions, paving the way for more human-centric and emotionally-aware image analysis systems.

6.3.13 Generalizable Local Feature Pre-training for Deformable Shape Analysis

Participants: Souhaib Attaiki, Lei Li, Maks Ovsjanikov.

In this paper [22], we analyze the link between feature locality and transferability in tasks involving deformable 3D objects, while also comparing different backbones and losses for local feature pre-training. We observe that with proper training, learned features can be useful in such tasks, but, crucially, only with an appropriate choice of the receptive field size. We then propose a differentiable method for optimizing the receptive field within 3D transfer learning. Jointly, this leads to the first learnable features that can successfully generalize to unseen classes of 3D shapes such as humans and animals. Our extensive experiments show that this approach leads to state-of-the-art results on several downstream tasks such as segmentation, shape correspondence, and classification.

6.3.14 Understanding and Improving Features Learned in Deep Functional Maps

Participants: Souhaib Attaiki, Maks Ovsjanikov.

In this paper [23], we show that under some mild conditions, the features learned within deep functional map approaches can be used as point-wise descriptors and thus are directly comparable across different shapes, even without the necessity of solving for a functional map at test time. Furthermore, informed by our analysis, we propose effective modifications to the standard deep functional map pipeline, which promote structural properties of learned features, significantly improving the matching results. Finally, we demonstrate that previously unsuccessful attempts at using extrinsic architectures for deep functional map feature extraction can be remedied via simple architectural changes, which encourage the theoretical properties suggested by our analysis. We thus bridge the gap between intrinsic and extrinsic surface-based learning, suggesting the necessary and sufficient conditions for successful shape matching.

6.3.15 Scalable and Efficient Functional Map Computations on Dense Meshes

Participants: Robin Magnet, Maks Ovsjanikov.

In this paper [20], we propose a new scalable version of the functional map pipeline that allows to efficiently compute correspondences between potentially very dense meshes. Unlike existing approaches that process dense meshes by relying on ad-hoc mesh simplification, we establish an integrated end-to-end pipeline with theoretical approximation analysis. In particular, our method overcomes the computational burden of both computing the basis, as well the functional and pointwise correspondence computation by approximating the functional spaces and the functional map itself. Errors in the approximations are controlled by theoretical upper bounds assessing the range of applicability of our pipeline. With this construction in hand, we propose a scalable practical algorithm and demonstrate results on dense meshes, which approximate those obtained by standard functional map algorithms at the fraction of the computation time. Moreover, our approach outperforms the standard acceleration procedures by a large margin, leading to accurate results even in challenging cases.

7 Bilateral contracts and grants with industry

7.1 Bilateral contracts with industry

7.1.1 Contract with Sanofi Inc.

Participants: Maks Ovsjanikov.

Title: Machine learning approaches for cryo-EM

Partner Institution(s): Sanofi Inc.

Date/Duration: 2023-2024

Additionnal info/keywords: Cryogenic electron microscopy (cryo-EM) allows the structure determination of antibody fragments bound to pharmaceutically relevant targets to accelerate drug discovery. The process of cryo-EM data analysis is time consuming and requires user input. To accelerate the rate of structure solution by cryo-EM, this project investigates machine learning approachesto fit and model the atomic coordinates of antibody fragments into the cryo-EM density.

The project funds one post-doctoral researcher for 2 years, jointly between Sanofi Inc., and Ecole Polytechnique (the employer of Maks Ovsjanikov).

7.1.2 Contract with DASSAULT SYSTEMES

Participants: Maks Ovsjanikov.

Title: Generative Models for the Guided Synthesis of Complex and Functional 3D Scenes

Partner Institution(s): DASSAULT SYSTEMES

Date/Duration: 2023-2026

Additionnal info/keywords: This thesis focuses on machine learning applied to 3D computer vision, specifically addressing challenges related to the automatic synthesis of 3D environments.

The project funds one PhD student for 3 years.

7.1.3 MEDITWIN with DASSAULT SYSTEMES

Participants: Maks Ovsjanikov, Mathieu Desbrun.

Title: MEDITWIN: Virtual human twins for medical applications

Partner Institution(s): DASSAULT SYSTEMES

Date/Duration: 2023-2028

Additionnal info/keywords: In the context of IPCEI on Health called MEDITWIN, Geomerix has started working on geometric measure theory and reduced models (Desbrun) and non-rigid registration (Ovsjanikov), with one student and two postdocs to be hired soon.

8 Partnerships and cooperations

8.1 International research visitors

8.1.1 Visits of international scientists

Other international visits to the team

Shreyas Samaga

Status PhD

Institution of origin: Purdue University

Country: USA

Dates: May-July 2023

Context of the visit: collaboration with Steve Oudot

Mobility program/type of mobility: research stay

8.1.2 Visits to international teams

Research stays abroad

Steve Oudot

Visited institution: Center for Advanced Study

Country: Norway

Dates: January-February 2023

Context of the visit: participation in the program *Representation Theory: Combinatorial Aspects and Applications*

Mobility program/type of mobility: research stay as a fellow

Pooran Memari

Visited institution: Technische Universität Berlin

Country: Germany

Dates: February-June 2023

Context of the visit: Sabbatical leave, Visiting TU Berlin CG group,

Mobility program/type of mobility: research stay for collaboration initiation

8.2 European initiatives

8.2.1 Horizon Europe

Participants: Maks Ovsjanikov.

Title: Exploring Relations in Structured Data with Functional Maps

Partner Institution(s):
- European Research Commission (ERC) Starting Grant

Date/Duration: 2018-2023

Additionnal info/keywords: We propose to lay the theoretical foundations and design efficient computational methods for analyzing, quantifying and exploring relations and variability in structured data sets, such as collections of geometric shapes, point clouds, and large networks or graphs, among others. In particular, we propose to depart from the standard representations of objects as collections of primitives, such as points or triangles, and instead to treat them as functional spaces that can be easily manipulated and analyzed. Since real-valued functions can be defined on a wide variety of data representations and as they enjoy a rich algebraic structure, such an approach can provide a completely novel unified framework for representing and processing different types of data. Key to our study is the exploration of relations and variability between objects, which can be expressed as operators acting on functions and thus treated and analyzed as objects in their own right using the vast number of tools from functional analysis in theory and numerical linear algebra in practice.

Participants: Maks Ovsjanikov.

Title: VEGA: Universal Geometric Transfer Learning

Partner Institution(s):
- European Research Commission (ERC) Consolidator Grant

Date/Duration: 2024-2028

Additionnal info/keywords: In this project, we propose to develop a theoretical and practical framework for transfer learning with geometric 3D data. Most existing learning-based approaches, aimed at analyzing 3D data, are based on training neural networks from scratch for each data modality and application. Our main goal will be to develop universally-applicable methods by combining powerful pre-trainable modules with effective multi-scale analysis and fine-tuning, given minimal task-specific data. The overall key to our study will be analyzing rigorous ways, both theoretically and in practice, in which solutions can be transferred and adapted across problems, semantic categories and geometric data types.

8.2.2 H2020 projects

Participants: Pooran Memari.

Title: Creating Lively Interactive Populated Environments

Partner Institution(s):
- University of Cyprus, Universitat Politecnica de Catalunya, University College London, Trinity College Dublin, Max Planck Institute for Intelligent Systems, KTH Royal Institute of Technology.

Date/Duration: 2020-2024

Additionnal info/keywords: This project designs new techniques to create and control interactive virtual worlds and characters, benefiting from opportunities open by the wide availability of emergent technologies in the domains of human digitization and artificial intelligence.

8.3 National initiatives

AEx PreMediT

Participants: Steve Oudot.

Title: Precision Medicine using Topology

Partner Institution(s):
- CRESS, Hôtel-Dieu, France

Date/Duration: 2022-2025

Additionnal info/keywords: While recent advances in machine learning are opening promising prospects for precision medicine, the sometimes small size, sparsity, or partly categorical nature of the data involved pose some crucial challenges. The goal of PreMediT is to address these challenges by integrating information about the geometric and topological structure of the data into the machine learning pipelines.

ANR AI Chair AIGRETTE

Participants: Maks Osjanikov.

Title: Analyzing Large Scale Geometric Data Collections

Partner Institution(s):
- ANR

Date/Duration: 2020-2024

Additionnal info/keywords: Motivated by the deluge of 3D data using geometric representations (point clouds, triangle, quad meshes, graphs...) that are ill-suited for modern applications, we are developing efficient algorithms and mathematical tools for analyzing diverse geometric data collections.

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

General chair, scientific chair

- Mathieu Desbrun organized the Workshop *Machine Learning for Geometry* at the Institut Poincaré (see https://ml4geo.sciencesconf.org/).

Member of the organizing committees

- Steve Oudot co-organized the Mini-Symposium *Recent Developments in Multi-Parameter Persistence* at the SIAM Conference on Applied Algebraic Geometry (`https://meetings.siam.org/sess/dsp_programsess.cfm?SESSIONCODE=77710`).

9.1.2 Scientific events: selection

Chair of conference program committees

- Pooran Memari served as Program Co-Chair of Eurographics Symposium on Geometry Processing (SGP) 2023.

Member of the conference program committees

- Pooran Memari was a member of the Program Committee of Eurographics 2023 Conference.
- Mathieu Desbrun was a member of the Program Committee of Technical Papers for ACM SIGGRAPH Asia 2023.

9.1.3 Journal

Member of the editorial boards

- Steve Oudot is a member of the Editorial Board of the Journal of Computational Geometry.
- Maks Ovsjanikov is a member of the Editorial Board of the IEEE Transactions on Visualization and Computer Graphics journal.
- Pooran Memari is a member of the Editorial Board of the Journal of Computer Graphics Forum (CGF).
- Pooran Memari is also a member of the Editorial Board of Graphical Models Journal, Elsevier.
- Mathieu Desbrun ss a member of the Editorial Board of the Journal of Geometric Mechanics.

9.1.4 Invited talks

- Maks Ovsjanikov gave a keynote talk at the Symposium on 3D Object Retrieval 2023 (3DOR'23) held in Lille, France.
- Maks Ovsjanikov gave a keynote talk at Pacific Graphics 2023, held in Daejeon, South Korea.
- Maks Ovsjanikov gave a keynote talk at the Machine Learning for Geometry Workshop, held in Paris, France.
- Steve Oudot gave an invited talk in the special session *The ubiquity of quivers and their representations* at the 29th Nordic Congress of Mathematicians, Aalborg, Denmark.
- Pooran Memari gave an invited talk at the LIB colloquium (Laboratoire d'Informatique de Bourgogne) in Dijon, France, Nov. 2023.
- Mathieu Desbrun gave an invited talk at Ecole Normale Supérieure, France, Nov. 2023.
- Mathieu Desbrun gave an invited talk for the "Journée scientifique du groupe SMAI-SIGMA" at Jussieu, France, Dec. 2023.

9.1.5 Leadership within the scientific community

- Pooran Memari is the local coordinator for the GT-MG (Modélisation Géométrique).

9.1.6 Research administration

- Steve Oudot is vice-president of the Commission Scientifique at Inria Saclay.
- Steve Oudot is a member of the Comité de Département in the CS Department of École Polytechnique (DIX).
- Pooran Memari is a member of the comité de Web du LIX, École Polytechnique.
- Pooran Memari is a deputy member of the conseil de laboratoire du LIX, École Polytechnique.
- Mathieu Desbrun is a member of the Comité du Labo in the CS lab of École Polytechnique (LIX)

9.2 Teaching - Supervision - Juries

9.2.1 Teaching

- Master: Steve Oudot, Computational Geometry and Topology, 18h eq-TD, M2, MPRI;
- Master: Steve Oudot, Topological data analysis, 45h eq-TD, M1, École polytechnique, France;
- Master: Mathieu Desbrun, Digital Representation and Analysis of Shapes, M2, École polytechnique, France;
- Master: Pooran Memari, Artificial Intelligence and Advanced Visual Computing, and Digital Representation and Analysis of Shapes, M2, École polytechnique, France;
- Master: Maks Ovsjanikov, Artificial Intelligence and Advanced Visual Computing, École polytechnique, France;
- Undergrad-Master: Steve Oudot, Algorithms for data analysis in C++, 22.5h eq-TD, L3/M1, École Polytechnique, France.
- Master-PhD: Pooran Memari is a member of the Jury d'admission Masters & PhD Track IGD (Interaction, Graphics & Design), IP-Paris (2020-2023).

9.2.2 Supervision

- PhD: Vadim Lebovici, Deux Approches Complémentaires de la Persistance Multiparamétrique: Décompositions en Intervalles et Fonctions Constructibles, Université Paris-Saclay. Defended in September 2023. Steve Oudot and François Petit (CRESS).
- PhD: Jiayi Wei, Robust Statistics for Geometry Processing – Detecting and Handling Discontinuities and Dissimilarities in 3D Pointset Denoising and Mesh Parameterization. Defended in December 2023. Pooran Memari and Damien Rohmer.
- PhD in progress: Julie Mordacq, Analyse Topologique des Données et Apprentissage Machine pour analyser et prédire des transitions de phase en n-dimensions, Institut Polytechnique de Paris. Started Sept. 2022. Steve Oudot and Vicky Kalogeiton (Vista, LIX).
- PhD in progress: Jingyi Li, Invariants algébriques effectifs pour la persistance multi-paramètre, Institut Polytechnique de Paris. Started Nov. 2023. Steve Oudot.
- PhD: Mariem Mezghanni, Structural and Functional Learning for Industrial Design Automatization. Defended in February 2023. Institut Polytechnique de Paris. Maks Ovsjanikov with Malika Boulkenafed (Dassault Systèmes).
- PhD: Nicolas Donati, Robust representations for supervised and unsupervised 3D shape matching. Defended in January 2023. Institut Polytechnique de Paris. Maks Ovsjanikov with Etienne Corman (CNRS).
- PhD in progress: Souhaib Attaiki, 3D shape analysis with methods based on Deep Learning, Institut Polytechnique de Paris. Started Nov. 2020. Maks Ovsjanikov
- PhD in progress: Robin Magnet, Robust Spectral Methods for Shape Analysis and Deformation Assessment, Institut Polytechnique de Paris. Started February 2021. Maks Ovsjanikov
- PhD in progress: Souhail Hadgi, Transfer learning for 3D data, Institut Polytechnique de Paris. Started January 2023. Maks Ovsjanikov
- PhD in progress: Ramana S Sundararaman, Analysis of large scale 3D shape collection with learning based approaches, Institut Polytechnique de Paris. Started October 2021. Maks Ovsjanikov

- PhD in progress: Tim Scheller, Capturing 4D Plant Growth, Institut Polytechnique de Paris. Started October 2021. Maks Ovsjanikov with Marie-Paule Cani (Ecole Polytechnique).
- PhD in progress: Nissim Maruani, Machine Learning for Geometric Modeling; Started October 2022. Mathieu Desbrun amd Pierre Alliez (INRIA Sophia-Antipolis).
- PhD in progress: Lucas Brifault, Geometric Measure Theory for Geometric Modeling; Started April 2022. Mathieu Desbrun and David Cohen-Steiner (INRIA Sophia-Antipolis).
- PhD in progress: Theo Braune, Discrete Bundled-valued Exterior Calculus; Started October 2022. Mathieu Desbrun.

9.2.3 Juries

- Pooran Memari was an examiner for the Ph.D. defense of Guillaume Coiffier, LORIA, Université de Lorraine, Dec. 2023.
- Mathieu Desbrun was an examiner for the PhD defense of Loï Paulin, LIRIS, Lyon, Apr. 2023.

10 Scientific production

10.1 Major publications

[1] K. Bai, C. Wang, M. Desbrun and X. Liu. 'Predicting high-resolution turbulence details in space and time'. In: *ACM Transactions on Graphics* 40.6 (Dec. 2021), p. 200. DOI: 10.1145/3478513.3480492. URL: https://hal.inria.fr/hal-03551723.

[2] J. Chen, F. Schäfer, J. Huang and M. Desbrun. 'Multiscale Cholesky Preconditioning for Ill-conditioned Problems'. In: *ACM Transactions on Graphics* 40.4 (19th July 2021), Art. 91. DOI: 10.1145/3450626.3459851. URL: https://hal.archives-ouvertes.fr/hal-03277277.

[3] N. Donati, A. Sharma and M. Ovsjanikov. 'Deep Geometric Functional Maps: Robust Feature Learning for Shape Correspondence'. In: CVPR. Seattle (virtual), United States, 14th June 2020. URL: https://hal-polytechnique.archives-ouvertes.fr/hal-03001057.

[4] P. Ecormier-Nocca, P. Memari, J. Gain and M.-P. Cani. 'Accurate Synthesis of Multi-Class Disk Distributions'. In: *Computer Graphics Forum* 38.2 (2019). URL: https://hal.inria.fr/hal-02064699.

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[6] T. Lacombe, M. Cuturi and S. Oudot. 'Large Scale computation of Means and Clusters for Persistence Diagrams using Optimal Transport'. In: NIPS. Montreal, Canada, 2018. URL: https://hal.inria.fr/hal-01966674.

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[9] P. Mullen, P. Memari, F. de Goes and M. Desbrun. 'Hodge-optimized triangulations'. In: *ACM Transactions on Graphics* 30.4 (Aug. 2011), pp. 103, 1–12. URL: https://hal.telecom-paris.fr/hal-02286600.

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[11] J.-M. Roufosse, A. Sharma and M. Ovsjanikov. 'Unsupervised Deep Learning for Structured Shape Matching'. In: ICCV. SEOUL, South Korea, 29th Oct. 2019. URL: https://hal.science/hal-02305166.

10.2 Publications of the year

International journals

[12] M. B. Botnan, V. Lebovici and S. Oudot. 'Local characterizations for decomposability of 2-parameter persistence modules'. In: *Algebras and Representation Theory* (2023). DOI: 10.1007/s10468-022-10189-4. URL: https://hal.science/hal-02995822.

[13] X. Huang, T. Ritschel, H.-P. Seidel, P. Memari and G. Singh. 'Patternshop: Editing Point Patterns by Image Manipulation'. In: *ACM Transactions on Graphics* 42.4 (26th July 2023), pp. 1–14/53. DOI: 10.1145/3592418. URL: https://hal.science/hal-04235133.

[14] M. Kirgo, G. Terrasse, G. Thibault and M. Ovsjanikov. 'ReVISOR: ResUNets with visibility and intensity for structured outlier removal'. In: *ISPRS Journal of Photogrammetry and Remote Sensing* 202 (2023), pp. 184–204. DOI: 10.1016/j.isprsjprs.2023.05.027. URL: https://hal.science/hal-04352356.

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[18] C. Lyu, K. Bai, Y. Wu, M. Desbrun, C. Zheng and X. Liu. 'Building a Virtual Weakly-Compressible Wind Tunnel Testing Facility'. In: *ACM Transactions on Graphics* 42.4 (Aug. 2023), pp. 1–20. DOI: 10.1145/3592394. URL: https://inria.hal.science/hal-04174295.

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[20] R. Magnet and M. Ovsjanikov. 'Scalable and Efficient Functional Map Computations on Dense Meshes'. In: *Computer Graphics Forum* 42.2 (23rd May 2023), pp. 89–101. DOI: 10.1111/cgf.14746. URL: https://hal.science/hal-04352328.

[21] J. Wei, J. Chen, D. Rohmer, P. Memari and M. Desbrun. 'Robust Pointset Denoising of Piecewise-Smooth Surfaces through Line Processes'. In: *Computer Graphics Forum* 42.2 (2023), pp. 175–189. DOI: 10.1111/cgf.14752. URL: https://polytechnique.hal.science/hal-04027983.

International peer-reviewed conferences

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