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ACTIVITY REPORT

Project-Team MINGUS

MultI-scale Numerical Geometric Schemes

IN COLLABORATION WITH: Institut de recherche mathématique de Rennes (IRMAR)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Project-Team MINGUS

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Other research topics and application domains

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2 Overall objectives

2.1 Presentation

In applications involving complex physics, such as plasmas and nanotechnologies, numerical simulations serve as a prediction tool supplementing real experiments and are largely endorsed by engineers or researchers. Their performances rely not only on computational power, but also on the efficiency of the underlying numerical method and the complexity of the underlying models. The contribution of applied mathematics is then required, on the one hand for a better understanding of qualitative properties and a better identification of the different regimes present in the model, and on the other hand, for a more sounded construction of new models based on asymptotic analysis. This mathematical analysis is expected to greatly impact the design of *multiscale* numerical schemes.

The proposed research group MINGUS will be dedicated to the mathematical and numerical analysis of (possibly stochastic) partial differential equations (PDEs), originating from plasma physics and nanotechnologies, with emphasis on *multiscale* phenomena either of **highly-oscillatory**, of **dissipative** or

stochastic types. These equations can be also encountered in applications to rarefied gas dynamics, radiative transfer, population dynamics or laser propagation, for which the multiscale character is modelled by a scale physical parameter ε.

Producing accurate solutions of multiscale equations is extremely challenging owing to severe restrictions to the numerical methods imposed by fast (or stiff) dynamics. *Ad-hoc* numerical methods should aim at capturing the slow dynamics solely, instead of resolving finely the stiff dynamics at a formidable computational cost. At the other end of the spectrum, the separation of scales -as required for numerical efficiency- is envisaged in asymptotic techniques, whose purpose is to describe the model in the limit where the small parameter ε tends to zero. MINGuS aspires to accommodate sophisticated tools of mathematical analysis and heuristic numerical methods in order to produce simultaneously rich asymptotic models and efficient numerical methods.
To be more specific, MINGuS aims at finding, implementing and analysing new multiscale numerical schemes for the following physically relevant multiscale problems:

- **Highly-oscillatory Schrödinger equation for nanoscale physics:** In quantum mechanics, the Schrödinger equation describes how the quantum state of some physical system changes with time. Its mathematical and numerical study is of paramount importance to fundamental and applied physics in general. We wish to specifically contribute to the mathematical modeling and the numerical simulation of confined quantum mechanical systems (in one or more space dimensions) possibly involving stochastic terms. Such systems are involved in quantum semi-conductors or atom-chips, as well as in cold atom physics (Bose-Einstein condensates) or laser propagation in optical fibers.

  The prototypical equation is written

$$\boxed{i\varepsilon\partial_t\psi^\varepsilon = \frac{\varepsilon^2}{\beta}\Delta\psi^\varepsilon + |\psi^\varepsilon|^2\psi^\varepsilon + \psi^\varepsilon\xi} \tag{1}$$

  where the function $\psi^\varepsilon = \psi^\varepsilon(t,x) \in \mathbb{C}$ depends on time $t \geq 0$ and position $x \in \mathbb{R}^3$, $\xi = \xi(x,t)$ is a white noise and where the small parameter ε is the Planck's constant describing the microscopic/macroscopic ratio. The limit $\varepsilon \to 0$ is referred to as the semi-classical limit. The regime $\varepsilon = 1$ and $\beta \to 0$ (this can be for instance the relative length of the optical fiber) is highly-oscillatory. The noise ξ acts as a potential, it may represent several external perturbations. For instance temperature effects in Bose-Einstein condensation or amplification in optical fibers. The highly oscillatory regime combined with noise introduces new challenges in the design of efficient schemes.

- **Highly-oscillatory or highly-dissipative kinetic equations:** Plasma is sometimes considered as the fourth state of matter, obtained for example by bringing a gas to a very high temperature. A globally neutral gas of neutral and charged particles, called plasma, is then obtained and is described by a kinetic equation as soon as collective effects dominate as compared to binary collisions. A situation of major importance is magnetic fusion in which collisions are not predominant. In order to confine such a plasma in devices like tokamaks (ITER project) or stellarators, a large magnetic field is used to endow the charged particles with a cyclotronic motion around field lines. Note that kinetic models are also widely used for modeling plasmas in earth magnetosphere or in rarefied gas dynamics.

  Denoting $f^\varepsilon = f^\varepsilon(t,x,v) \in \mathbb{R}^+$ the distribution function of charged particles at time $t \geq 0$, position $x \in \mathbb{R}^3$ and velocity $v \in \mathbb{R}^3$, a typical kinetic equation for f^ε reads

$$\boxed{\partial_t f^\varepsilon + v\cdot\nabla_x f^\varepsilon + \left(E + \frac{1}{\varepsilon}(v\times B)\right)\cdot\nabla_v f^\varepsilon = \frac{1}{\beta}Q(f^\varepsilon) + f^\varepsilon m^\varepsilon} \tag{2}$$

where (E,B) is the electro-magnetic field (which may itself depend on f through Maxwell's equations), m^ε is a random process (which may describe absorption or creation of particles) and Q is a collision operator. The dimensionless parameters ε, β are related to the cyclotronic frequency and the mean free path. Limits $\varepsilon \to 0$ and $\beta \to 0$ do not share the same character (the former is oscillatory and the latter is dissipative) and lead respectively to gyrokinetic and hydrodynamic models. The noise term m^ε is correlated in space and time. At the limit $\varepsilon \to 0$, it converges formally to a white noise and stochastic PDEs are obtained.

The objective of MINGUS is twofold: the construction and the analysis of numerical schemes for multiscale (S)PDEs originating from physics. In turn, this requires (i) a deep mathematical understanding of the (S)PDEs under consideration and (ii) a strong involvement into increasingly realistic problems, possibly resorting to parallel computing. For this aspect, we intend to benefit from the Inria Selalib software library which turns out to be the ideal complement of our activities.

During the last period, some results have been obtained by the members of the team [1, 2, 3, 4, 5].

3 Research program

The MINGUS project is devoted to the mathematical and numerical analysis of models arising in plasma physics and nanotechnology. The main goal is to construct and analyze numerical methods for the approximation of PDEs containing multiscale phenomena. Specific multiscale numerical schemes will be proposed and analyzed in different regimes (namely highly-oscillatory and dissipative). The ultimate goal is to dissociate the physical parameters (generically denoted by ε) from the numerical parameters (generically denoted by h) with a uniform accuracy. Such a task requires mathematical prerequisite of the PDEs.

Then, for a given stiff (highly-oscillatory or dissipative) PDE, the methodology of the MINGUS team will be the following

- Mathematical study of the asymptotic behavior of multiscale models.
  This part involves averaging and asymptotic analysis theory to derive asymptotic models, but also long-time behavior of the considered models.
- Construction and analysis of multiscale numerical schemes.
  This part is the core of the project and will be deeply inspired from the mathematical prerequisite. In particular, our ultimate goal is the design of *Uniformly Accurate* (UA) schemes, whose accuracy is independent from ε.
- Validation on physically relevant problems.
  The last goal of the MINGUS project is to validate the new numerical methods, not only on toy problems, but also on realistic models arising in physics of plasmas and nanotechnologies. We will benefit from the Selalib software library which will help us to scale-up our new numerical methods to complex physics.

3.1 Dissipative problems

In the dissipative context, the asymptotic analysis is quite well understood in the deterministic case and multiscale numerical methods have been developed in the last decades. Indeed, the so-called Asymptotic-Preserving schemes has retained a lot of attention, in particular in the context of collisional kinetic equations. But, there is still a lot to do if one is interested in deriving high order asymptotic models, which enable to capture the original solution for all time. Moreover, this analysis is still misunderstood when more complex systems are considered, involving non homogeneous relaxation rates or stochastic terms for instance. Following the methodology we aim at using, we first address the mathematical analysis before deriving multiscale efficient numerical methods.

A simple model of dissipative systems is governed by the following differential equation

$$\left\{\begin{array}{rcll} \dfrac{dx^\varepsilon(t)}{dt} & = & \mathscr{G}(x^\varepsilon(t),y^\varepsilon(t)), & x^\varepsilon(0)=x_0, \\ \dfrac{dy^\varepsilon(t)}{dt} & = & -\dfrac{y^\varepsilon(t)}{\varepsilon}+\mathscr{H}(x^\varepsilon(t),y^\varepsilon(t)), & y^\varepsilon(0)=y_0, \end{array}\right. \tag{3}$$

for given initial condition $(x_0,y_0)\in\mathbb{R}^2$ and given smooth functions $\mathscr{G},\mathscr{H}$ which possibly involve stochastic terms.

3.1.1 Asymptotic analysis of dissipative PDEs (*F. Castella, A. Debussche, E. Faou*)

Derivation of asymptotic problems
Our main goal is to analyze the asymptotic behavior of dissipative systems of the form (3) when ε goes to zero. The *center manifold theorem* [37] is of great interest but is largely unsatisfactory from the following points of view

- a constructive approach of h and x_0^ε is clearly important to identify the high-order asymptotic models: this would require expansions of the solution by means of B-series or word-series [39] allowing the derivation of error estimates between the original solution and the asymptotic one.
- a better approximation of the transient phase is strongly required to capture the solution for small time: extending the tools developed in averaging theory, the main goal is to construct a suitable change of variable which enables to approximate the original solution for all time.

Obviously, even at the ODE level, a deep mathematical analysis has to be performed to understand the asymptotic behavior of the solution of (3). But, the same questions arise at the PDE level. Indeed, one certainly expects that dissipative terms occurring in collisional kinetic equations (2) may be treated theoretically along this perspective. The key new point indeed is to see the center manifold theorem as a change of variable in the space of unknowns, while the standard point of view leads to considering the center manifold as an asymptotic object.

Stochastic PDEs
We aim at analyzing the asymptotic behavior of stochastic collisional kinetic problems, that is equation of the type (2). The noise can describe creation or absorption (as in (2)), but it may also be a forcing term or a random magnetic field. In the parabolic scaling, one expects to obtain parabolic SPDEs at the limit. More precisely, we want to understand the fluid limits of kinetic equations in the presence of noise. The noise is smooth and non delta correlated. It contains also a small parameter and after the rescaling, it converges formally to white noise. Thus, this adds another scale in the multiscale analysis. Following the pioneering work by Debussche and Vovelle [40], substantial progresses have been done in this topic.

More realistic problems may be addressed such as high field limit describing sprays, or even hydrodynamic limit. The full Boltzmann equation is a very long term project and we wish to address simpler problems such as convergence of BGK models to a stochastic Stokes equation.

The main difficulty is that when the noise acts as a forcing term, which is a physically relevant situation, the equilibria are affected by the noise and we face difficulties similar to that of high field limit problems. Also, a good theory of averaging lemma in the presence of noise is lacking. The methods we use are generalization of the perturbed test function method to the infinite dimensional setting. We work at the level of the generator of the infinite dimensional process and prove convergence in the sense of the martingale problems. A further step is to analyse the speed of convergence. This is a prerequisite if one wants to design efficient schemes. This requires more refined tools and a good understanding of the Kolmogorov equation.

3.1.2 Numerical schemes for dissipative problems (*All members*)

The design of numerical schemes able to reproduce the transition from the microscopic to macroscopic scales largely matured with the emergence of Asymptotic Preserving schemes which have been developed initially for collisional kinetic equations (actually, for solving (2) when $\beta\to 0$). Several techniques have flourished in the last decades. As said before, AP schemes entail limitations which we aim at overcoming by deriving

- AP numerical schemes whose numerical cost diminishes as $\beta \to 0$,
- Uniformly accurate numerical schemes, whose accuracy is independent of β.

Time diminishing methods
The main goal consists in merging Monte-Carlo techniques [35] with AP methods for handling *automatically* multiscale phenomena. As a result, we expect that the cost of the so-obtained method decreases when the asymptotic regime is approached; indeed, in the collisional (i.e. dissipative) regime, the deviational part becomes negligible so that a very few number of particles will be generated to sample it. A work in this direction has been done by members of the team.

We propose to build up a method which enables to realize the transition from the microscopic to the macroscopic description without domain decomposition strategies which normally requires to fix and tune an interface in the physical space and some threshold parameters. Since it will permit to go over domain decomposition and AP techniques, this approach is a very promising research direction in the numerical approximation of multiscale kinetic problems arising in physics and engineering.

Uniformly accurate methods
To overcome the accuracy reduction observed in AP schemes for intermediate regimes, we intend to construct and analyse multiscale numerical schemes for (3) whose error is uniform with respect to ε. The construction of such a scheme requires a deep mathematical analysis as described above. Ideally one would like to develop schemes that preserve the center manifold (without computing the latter!) as well as schemes that resolve numerically the stiffness induced by the fast convergence to equilibrium (the so-called transient phase). First, our goal is to extend the strategy inspired by the central manifold theorem in the ODE case to the PDE context, in particular for collisional kinetic equations (2) when $\beta \to 0$. The design of Uniformly Accurate numerical schemes in this context would require to generalize two-scale techniques introduced by members of the team in the framework of highly-oscillatory problems [38].

Multiscale numerical methods for stochastic PDEs
AP schemes have been developed recently for kinetic equations with noise in the context of Uncertainty Quantification UQ [43]. These two aspects (multiscale and UQ) are two domains developed in separate communities. UQ has drawn a lot of attention recently to control the propagation of data pollution; undoubtedly UQ has a lot of applications and one of our goals will be to study how sources of uncertainty are amplified or not by the multiscale character of the model. We also wish to go much further in developing AP schemes when the noise is also rescaled and the limit is a white noise driven SPDE, as described in section (3.1.1). For simple nonlinear problem, this should not present much difficulties but new ideas will definitely be necessary for more complicated problems when noise deeply changes the asymptotic equation.

3.2 Highly-oscillatory problems

As a generic model for highly-oscillatory systems, we will consider the equation

$$\frac{du^\varepsilon(t)}{dt} = \mathscr{F}(t/\varepsilon, u^\varepsilon(t)), \quad u^\varepsilon(0) = u_0, \tag{4}$$

for a given u_0 and a given periodic function $\mathscr{F}$ (of period P w.r.t. its first variable) which possibly involves stochastic terms. Solution u^ε exhibits high-oscillations in time superimposed to a slow dynamics. Asymptotic techniques -resorting in the present context to *averaging* theory [47]- allow to decompose

$$u^\varepsilon(t) = \Phi_{t/\varepsilon} \circ \Psi_t \circ \Phi_0^{-1}(u_0), \tag{5}$$

into a fast solution component, the εP-periodic change of variable $\Phi_{t/\varepsilon}$, and a slow component, the flow Ψ_t of a non-stiff *averaged* differential equation. Although equation (5) can be satisfied only up to a small remainder, various methods have been recently introduced in situations where (4) is posed in $\mathbb{R}^n$ or for the Schrödinger equation (1).

In the asymptotic behavior $\varepsilon \to 0$, it can be advantageous to replace the original singularly perturbed model (for instance (1) or (2)) by an approximate model which does not contain stiffness any longer. Such reduced models can be derived using asymptotic analysis, namely averaging methods in the case

of highly-oscillatory problems. In this project, we also plan to go beyond the mere derivation of limit models, by searching for better approximations of the original problem. This step is of mathematical interest *per se* but it also paves the way of the construction of multiscale numerical methods.

3.2.1 Asymptotic analysis of highly-oscillatory PDEs (*All members*)

Derivation of asymptotic problems
We intend to study the asymptotic behavior of highly-oscillatory evolution equations of the form (4) posed in an infinite dimensional Banach space.

Recently, the stroboscopic averaging has been extended to the PDE context, considering nonlinear Schrödinger equation (1) in the highly-oscillatory regime. A very exciting way would be to use this averaging strategy for highly-oscillatory kinetic problem (2) as those encountered in strongly magnetized plasmas. This turns out to be a very promising way to re-derive gyrokinetic models which are the basis of tokamak simulations in the physicists community. In contrast with models derived in the literature (see [36]) which only capture the average with respect to the oscillations, this strategy allows for the complete recovery of the exact solution from the asymptotic (non stiff) model. This can be done by solving companion transport equation that stems naturally from the decomposition (5).

Long-time behavior of Hamiltonian systems
The study of long-time behavior of nonlinear Hamiltonian systems have received a lot of interest during the last decades. It enables to put in light some characteristic phenomena in complex situations, which are beyond the reach of numerical simulations. This kind of analysis is of great interest since it can provide very precise properties of the solution. In particular, we will focus on the dynamics of nonlinear PDEs when the initial condition is close to a stationary solution. Then, the long-time behavior of the solution is studied through mainly three axes

- *linear stability*: considering the linearized PDE, do we have stability of a stationary solution ? Do we have linear Landau damping around stable non homogeneous stationary states?
- *nonlinear stability*: under a criteria, do we have stability of a stationary solution in energy norm like in [44], and does this stability persist under numerical discretization? For example one of our goals is to address the question of the existence and stability of discrete travelling wave in space and time.
- do we have existence of damped solutions for the full nonlinear problem ? Around homogeneous stationary states, solutions scatter towards a modified stationary state (see [45, 41]). The question of existence of Landau damping effects around non homogeneous states is still open and is one of our main goal in the near future.

Asymptotic behavior of stochastic PDEs
The study of SPDEs has known a growing interest recently, in particular with the fields medal of M. Hairer in 2014. In many applications such as radiative transfer, molecular dynamics or simulation of optical fibers, part of the physical interactions are naturally modeled by adding supplementary random terms (the noise) to the initial deterministic equations. From the mathematical point of view, such terms change drastically the behavior of the system.

- In the presence of noise, highly-oscillatory dispersive equations present new problems. In particular, to study stochastic averaging of the solution, the analysis of the long time behavior of stochastic dispersive equations is required, which is known to be a difficult problem in the general case. In some cases (for instance highly-oscillatory Schrödinger equation (1) with a time white noise in the regime $\varepsilon \ll 1$), it is however possible to perform the analysis and to obtain averaged stochastic equations. We plan to go further by considering more difficult problems, such as the convergence of a stochastic Klein-Gordon-Zakharov system to a stochastic nonlinear Schrödinger equation.
- The long-time behavior of stochastic Schrödinger equations is of great interest to analyze mathematically the validity of the Zakharov theory for wave turbulence (see [46]). The problem of wave turbulence can be viewed either as a deterministic Hamiltonian PDE with random initial data or a randomly forced PDEs where the stochastic forcing is concentrated in some part of the spectrum

(in this sense it is expected to be a hypoelliptic problem). One of our goals is to test the validity the Zakharov equation, or at least to make rigorous the spectrum distribution spreading observed in the numerical experiments.

3.2.2 Numerical schemes for highly-oscillatory problems (*All members*)

This section proposes to explore numerical issues raised by highly-oscillatory nonlinear PDEs for which (4) is a prototype. Simulating a highly-oscillatory phenomenon usually requires to adapt the numerical parameters in order to solve the period of size ε so as to accurately simulate the solution over each period, resulting in an unacceptable execution cost. Then, it is highly desirable to derive numerical schemes able to advance the solution by a time step independent of ε. To do so, our goal is to construct *Uniformly Accurate* (UA) numerical schemes, for which the numerical error can be estimated by Ch^p (h being any numerical parameters) with C independent of ε and p the order of the numerical scheme.

Recently, such numerical methods have been proposed by members of the team in the highly-oscillatory context [38]. They are mainly based on a separation of the fast and slow variables, as suggested by the decomposition (5). An additional ingredient to prove the uniformly accuracy of the method for (4) relies on the search for an appropriate initial data which enables to make the problem smooth with respect to ε.

Such an approach is assuredly powerful since it provides a numerical method which enables to capture the high oscillations in time of the solution (and not only its average) even with a large time step. Moreover, in the asymptotic regime, the potential gain is of order $1/\varepsilon$ in comparison with standard methods, and finally averaged models are not able to capture the intermediate regime since they miss important information of the original problem. We are strongly convinced that this strategy should be further studied and extended to cope with some other problems. The ultimate goal being to construct a scheme for the original equation which degenerates automatically into a consistent approximation of the averaged model, without resolving it, the latter can be very difficult to solve.

- Space oscillations:
  When rapidly oscillating coefficients in **space** (*i.e.* terms of the form $a(x, x/\varepsilon)$) occur in elliptic or parabolic equations, homogenization theory and numerical homogenization are usually employed to handle the stiffness. However, these strategies are in general not accurate for all $\varepsilon \in]0,1]$. Then, the construction of numerical schemes which are able to handle both regimes in an uniform way is of great interest. Separating fast and slow *spatial* scales merits to be explored in this context. The delicate issue is then to extend the choice of suitable initial condition to an *appropriate choice of boundary conditions* of the augmented problem.

- Space-time oscillations:
  For more complex problems however, the recent proposed approaches fail since the main oscillations cannot be identified explicitly. This is the case for instance when the magnetic field B depends on t or x in (2) but also for many other physical problems. We then have to deal with the delicate issue of space-time oscillations, which is known to be a very difficult problem from a mathematical and a numerical points of view. To take into account the space-time mixing, a periodic motion has to be detected together with a phase S which possibly depends on the time and space variables. These techniques originate from **geometric optics** which is a very popular technique to handle high-frequency waves.

- Geometrical properties:
  The questions related to the geometric aspects of multiscale numerical schemes are of crucial importance, in particular when long-time simulations are addressed (see [42]). Indeed, one of the main questions of geometric integration is whether intrinsic properties of the solution may be passed onto its numerical approximation. For instance, if the model under study is Hamiltonian, then the exact flow is symplectic, which motivates the design of symplectic numerical approximation. For practical simulations of Hamiltonian systems, symplectic methods are known to possess very nice properties (see [42]). It is important to combine multiscale techniques to geometric numerical integration. All the problems and equations we intend to deal with will be addressed with a view to preserve intrinsic geometric properties of the exact solutions and/or to approach

the asymptotic limit of the system in presence of a small parameter. An example of a numerical method developed by members of the team is the multi-revolution method.

- Quasi-periodic case:
  So far, numerical methods have been proposed for the periodic case with single frequency. However, the quasi-periodic case [1] is still misunderstood although many complex problems involve multi-frequencies. Even if the quasi-periodic averaging is doable from a theoretical point of view in the ODE case, (see [47]), it is unclear how it can be extended to PDEs. One of the main obstacle being the requirement, usual for ODEs like (4), for $\mathscr{F}$ to be analytic in the periodic variables, an assumption which is clearly impossible to meet in the PDE setting. An even more challenging problem is then the design of numerical methods for this problem.

- Extension to stochastic PDEs:
  All these questions will be revisited within the stochastic context. The mathematical study opens the way to the derivation of efficient multiscale numerical schemes for this kind of problems. We believe that the theory is now sufficiently well understood to address the derivation and numerical analysis of multiscale numerical schemes. Multi-revolution composition methods have been recently extended to highly-oscillatory stochastic differential equations The generalization of such multiscale numerical methods to SPDEs is of great interest. The analysis and simulation of numerical schemes for highly-oscillatory nonlinear stochastic Schrödinger equation under diffusion-approximation for instance will be one important objective for us. Finally, an important aspect concerns the quantification of uncertainties in highly-oscillatory kinetic or quantum models (due to an incomplete knowledge of coefficients or imprecise measurement of data). The construction of efficient multiscale numerical methods which can handle multiple scales as well as random inputs have important engineering applications.

4 Application domains

4.1 Application domains

The MINGuS project aims at applying the new numerical methods on realistic problems arising for instance in physics of nanotechnology and physics of plasmas. Therefore, in addition to efforts devoted to the design and the analysis of numerical methods, the inherent large size of the problems at hand requires advanced mathematical and computational methods which are hard to implement. Another application is concerned with population dynamics for which the main goal is to understand how the spatial propagation phenomena affects the demography of a population (plankton, parasite fungi, ...). Our activity is mostly at an early stage in the process of transfer to industry. However, all the models we use are physically relevant and all have applications in many areas (ITER, Bose-Einstein condensate, wave turbulence, optical tomography, transport phenomena, population dynamics, ...). As a consequence, our research aims at reaching theoretical physicists or computational scientists in various fields who have strong links with industrial applications. In order to tackle as realistic physical problems as possible, a fundamental aspect will consist in working on the realization of numerical methods and algorithms which are able to make an efficient use of a large number of processors. Then, it is essential for the numerical methods developed in the MINGuS project to be thought through this prism. We will benefit from the strong expertise of P. Navaro in scientific computing and more precisely on the Selalib software library (see description below). Below, we detail our main applications: first, the modeling and numerical approximation of magnetized plasmas is our major application and will require important efforts in terms of software developments to scale-up our multiscale methods; second, the transport of charged particles in nanostructures has very interesting applications (like graphene material), for which our contributions will mainly focus on dedicated problems; lastly, applications on population dynamics will be dedicated to mathematical modeling and some numerical validations.

[1] replacing t/ε by $t\omega/\varepsilon$ in (4), with $\omega \in \mathbb{R}^d$ a vector of non-resonant frequencies

4.2 Plasmas problems

The SeLaLib (Semi-Lagrangian Library) software library is a modular library for kinetic and gyrokinetic simulations of plasmas in fusion energy devices. Selalib is a collection of fortran modules aimed at facilitating the development of kinetic simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities and modules to help parallelization (both MPI and OpenMP), as well as pre-packaged simulations. Its main objective is to develop a documented library implementing several numerical methods for the numerical approximation of kinetic models. Another objective of the library is to provide physicists with easy-to-use gyrokinetic solvers. It has been originally developed by E. Sonnendrücker and his collaborators in the past CALVI Inria project, and has played an important role in the activities of the IPL FRATRES. P. Navaro is one of the main software engineer of this library and as such he played an important daily role in its development and its portability on supercomputers. Though Selalib has reached a certain maturity, additional work is needed to make it available to the community. There are currently discussions for a possible evolution of Selalib, namely the writing of a new release which will be available for free download. At the scientific level, Selalib is of great interest for us since it provides a powerful tool with which we can test, validate and compare our new methods and algorithms (user level). Besides numerical algorithms the library provides low-level utilities, input-output modules as well as parallelization strategies dedicated to kinetic problems. Moreover, a collection of simulations for typical test cases (of increasing difficulties) with various discretization schemes supplements the library. This library turns out to be the ideal complement of our activities and it will help us to scale-up our numerical methods to high-dimensional kinetic problems. During the last years, several experiments have been successfully performed in this direction (especially with PhD students) and it is important for us that this approach remains thorough. Then, we intend to integrate several of the numerical methods developed by the team within the Selalib library, with the strong help of P. Navaro (contributor level). This work has important advantages: (i) it will improve our research codes (in terms of efficiency software maintenance); (ii) it will help us to promote our research by making our methods available to the research community.

4.3 Quantum problems

Nowadays, a great challenge consists in the downscaling at the nanometer scale of electronic components in order to improve speed and efficiency of semiconductor materials. In this task, modeling and numerical simulations play an important role in the determination of the limit size of the nanotransistors. At the nanoscale, quantum effects have to be considered and the Schrödinger equation is a prominent equation in this context. In the so-called semiclassical regime or when the transport is strongly confined, the solution endows space-time high oscillations which are very difficult to capture numerically. An important application is the modeling of charged particles transport in graphene. Graphene is a sheet of carbone made of a single layer of molecule, organized in a bidimensional honeycomb crystal. The transport of charged particles in this structure is usually performed by Dirac equation (which is the relativistic counterpart of the Schrödinger equation). Due to the unusual properties of graphene -at room temperature, electrons moving in graphene behave as massless relativistic particles- physicists and compagnies are nowadays actively studying this material. In this context, predicting how the material properties are affected by the uncertainties in the hexagonal lattice structure or in external potentials, is a major issue.

4.4 Population dynamics

The main goal is to characterize how spatial propagation phenomena (diffusion, transport, advection, ...) affect the time evolution of the demography of a population. In collaboration with Y. Lagadeuc (ECOBIO, Rennes), this question has been studied for plankton. In this context, mathematical models have been proposed and it has been shown that the spatial dynamic (in this context, due to the marine current) which is fast compared to demographic scales, can strongly modify the demographic evolution of the plankton. In collaboration with Ecole d'Agronomie de Rennes, a mathematical study on the demography of a parasite fungi of plants has been performed. In this context, the demography is specific: the fungi can proliferate through sexual reproduction or through parthenogenesis. These two ways of reproduction

give rise mathematically to quadratic and linear growth rates with respect to the population variable. The demography is then coupled with transport (transport of fungi spore by wind). Here, the goal is to characterize the propagation of the fungi population by finding travelling waves solutions which are well adapted to describe the evolution of invasive fronts. Moreover, this approach enables to recover with a good agreement realistic examples (infection of ash or banana tree) for which experimental data are available. In these contexts, mathematical models are a powerful tool for biologists since measurements are very complicated to obtain and laboratory experiments hardly reproduce reality. The models derived are multiscale due to the nature of the underlying phenomena and the next step is to provide efficient numerical schemes.

5 Social and environmental responsibility

5.1 Footprint of research activities

A group called ECO-IRMAR has been created in the IRMAR laboratory to inform about the footprint of research activities at the level of the laboratory. The members of the team follow the advices proposed by this group.

6 Highlights of the year

- The MINGUS team hired Adrien Laurent as CRCN.
- A. Debussche was a main speaker at the 13th AIMS conference.
- The paper entitled "Second order perturbation theory of two-scale systems in fluid dynamics" has been accepted in Journal of the European Math. Soc. (A. Debussche and U. Pappalettera).
- The ANR BOURGEONS has been granted.

7 New software, platforms, open data

7.1 New software

7.1.1 Selalib

Name: SEmi-LAgrangian LIBrary

Keywords: Plasma physics, Semilagrangian method, Parallel computing, Plasma turbulence

Scientific Description: The objective of the Selalib project (SEmi-LAgrangian LIBrary) is to develop a well-designed, organized and documented library implementing several numerical methods for kinetic models of plasma physics. Its ultimate goal is to produce gyrokinetic simulations.

Another objective of the library is to provide to physicists easy-to-use gyrokinetic solvers, based on the semi-lagrangian techniques developed by Eric Sonnendrücker and his collaborators in the past CALVI project. The new models and schemes from TONUS are also intended to be incorporated into Selalib.

Functional Description: Selalib is a collection of modules conceived to aid in the development of plasma physics simulations, particularly in the study of turbulence in fusion plasmas. Selalib offers basic capabilities from general and mathematical utilities and modules to aid in parallelization, up to pre-packaged simulations.

URL: https://selalib.github.io

Contact: Philippe Helluy

Participants: Edwin Chacon Golcher, Pierre Navaro, Sever Hirstoaga, Eric Sonnendrücker, Michel Mehrenberger

Partners: Max Planck Insitute - Garching, Université de Strasbourg, CNRS, Université de Rennes 1

7.1.2 HOODESolver.jl

Name: Julia package for solving numerically highly-oscillatory ODE problems

Keywords: Ordinary differential equations, Numerical solver

Functional Description: Julia is a programming language for scientists with a syntax and functionality similar to MATLAB, R and Python. HOODESolver.jl is a julia package allowing to solve ordinary differential equations with sophisticated numerical techniques resulting from research within the MINGUS project team. To use it, just install Julia on your workstation.

Release Contributions: This is the first version of the package. It will evolve further because we want to have a better integration with the Julia organization on differential equations. This one already includes a lot of methods to numerically solve differential equations. This integration will allow us to have a larger audience and thus more feedback and possibly external collaborations.

Contact: Nicolas Crouseilles

Participants: Yves Mocquard, Pierre Navaro, Nicolas Crouseilles

Partners: Université de Rennes 1, CNRS

7.1.3 PhaseLifting

Keyword: Python

Functional Description: Performs the phase bearing of a complex function defined on a grid, taking into account two criteria based on a non uniform weight map.

Author: Yoann Le Henaff

Contact: Erwan Faou

8 New results

Our results of the year are presented in the following three sections:

- multiscale numerical schemes (for dissipative or highly-oscillatory problems),
- numerical schemes for Hamiltonian problems,
- analysis of PDEs and SPDEs.

8.1 Multiscale numerical schemes

Participants: G. Beck, N. Crouseilles, E. Faou, Y. Le Hénaff, D. Prel.

Multiscale (ie highly oscillatory or dissipative) ordinary differential equations (ODEs) have a long history since they are ubiquitous to describe dynamical multiscale physical phenomena in physics or chemistry. They can be obtained by appropriate spatial discretization of the partial differential equation or can directly describe the behavior of dynamical quantities. In addition to the standard difficulties coming from their numerical resolution, multiscale ODEs involve a stiffness (characterized by a parameter $\varepsilon \in]0,1]$). Creating strong gradients in the solution. Hence, to capture these small scales, conventional

methods have to consider a time step smaller than ε leading to unacceptable computational cost. The team members proposed several strategies to overcome this stiffness.

In [18], high order asymptotic preserving schemes are constructed and analysed for kinetic equations under a diffusive scaling. The framework enables to consider different cases: the diffusion equation, the advection-diffusion equation and the presence of inflow boundary conditions. Starting from the micro-macro reformulation of the original kinetic equation, high order time integrators are introduced. This class of numerical schemes enjoys the Asymptotic Preserving (AP) property for arbitrary initial data and degenerates when ε goes to zero into a high order scheme which is implicit for the diffusion term, which makes it free from the usual diffusion stability condition. The space discretization is also discussed and high order methods are also proposed based on classical finite differences schemes. The Asymptotic Preserving property is analysed and numerical results are presented to illustrate the properties of the proposed schemes in different regimes.

In [21], we deal with wave propagation into a coaxial cable, which can be modeled by the 3D Maxwell equations or 1D simplified models. The usual one, called the telegrapher's model, is a 1D wave equation on the electrical voltage and current. We derived a more accurate model from the Maxwell equations that takes into account dispersive effects. These two models aim to be a good approximation of the 3D electromagnetic fields in the case where the thinness of the cable is small. We perform some numerical simulations of the 3D Maxwell equations and of the 1D simplified models in order to validate the usual model and the new one. Moreover, we show that while the usual telegrapher model is of order one with respect to the thinness of the cable, the dispersive 1D model is of order two.

The goal of this work [22] is to study waves interacting with partially immersed objects allowed to move freely in the vertical direction, and in a regime in which the propagation of the waves is described by the one dimensional Boussinesq-Abbott system. The problem can be reduced to a transmission problem for this Boussinesq system, in which the transmission conditions between the components of the domain at the left and at the right of the object are determined through the resolution of coupled forced ODEs in time satisfied by the vertical displacement of the object and the average discharge in the portion of the fluid located under the object. We propose a new extended formulation in which these ODEs are complemented by two other forced ODEs satisfied by the trace of the surface elevation at the contact points. The interest of this new extended formulation is that the forcing terms are easy to compute numerically and that the surface elevation at the contact points is furnished for free. Based on this formulation, we propose a second order scheme that involves a generalization of the MacCormack scheme with nonlocal flux and a source term, which is coupled to a second order Heun scheme for the ODEs. In order to validate this scheme, several explicit solutions for this wave-structure interaction problem are derived and can serve as benchmark for future codes. As a byproduct, our method provides a second order scheme for the generation of waves at the entrance of the numerical domain for the Boussinesq-Abbott system.

The aim of this paper [25] is to investigate a deterministic particle method for a model containing a Fokker-Planck collision operator in velocity and strong oscillations (characterized by a small parameter ε) induced by a space and velocity transport operator. First, we investigate the properties (collisional invariants and equilibrium) of the asymptotic model obtained when $\varepsilon \to 0$. Second a numerical method is developed to approximate the solution of the multiscale Fokker-Planck model. To do so, a deterministic particle method (recently introduced for the Landau equation in [8]) is proposed for Fokker-Planck type operators. This particle method consists in reformulating the collision operator in an advective form and in regularizing the advection field in such a way that it conserves the geometric bracket structure. In the Fokker-Planck homogeneous case, the properties of the resulting method are analysed. In the non homogeneous case, the particle method is coupled with a uniformly accurate time discretization in ε that enables to capture numerically the solution of the asymptotic model. Numerous numerical results are displayed, illustrating the behavior of the method.

Based on recent ideas, stemming from the use of bubbles, we discuss in [29] an algorithm for the numerical simulation of the cubic nonlinear Schrödinger equation with harmonic potential in any dimension, which could be easily extended to other polynomial nonlinearities. For the linear part of the equation, the algorithm consists in discretizing the initial function as a sum of modulated complex functions, each one having its own set of parameters, and then updating the parameters exactly so that the modulated function remains a solution to the equation. When cubic interactions are introduced, the Dirac-Frenkel-MacLachlan principle is used to approximate the time evolution of parameters. We then

obtain a grid-free algorithm in any dimension, and it is compared to a spectral method on numerical examples.

Phenotypic plasticity has important ecological and evolutionary consequences. In particular, behavioural phenotypic plasticity such as adaptive foraging (AF) by consumers, may enhance community stability. Yet little is known about the ecological conditions that favor the evolution of AF, and how the evolutionary dynamics of AF may modulate its effects on community stability. In order to address these questions, we constructed in [33] an eco-evolutionary model in which resource and consumer niche traits underwent evolutionary diversification. Consumers could either forage randomly, only as a function of resources abundance, or adaptatively, as a function of resource abundance, suitability and consumption by competitors. AF evolved when the niche breadth of consumers with respect to resource use was large enough and when the ecological conditions allowed substantial functional diversification. In turn, AF promoted further diversification of the niche traits in both guilds. This suggests that phenotypic plasticity can influence the evolutionary dynamics at the community-level. Faced with a sudden environmental change, AF promoted community stability directly and also indirectly through its effects on functional diversity. However, other disturbances such as persistent environmental change and increases in mortality, caused the evolutionary regression of the AF behaviour, due to its costs. The causal relationships between AF, community stability and diversity are therefore intricate, and their outcome depends on the nature of the environmental disturbance, in contrast to simpler models claiming a direct positive relationship between AF and stability.

In [10], we study the construction of multiscale numerical schemes efficient in the finite Larmor radius approximation of the collisional Vlasov equation. Following the paper of Bostan and Finot (2019), the system involves two different regimes, a highly oscillatory and a dissipative regimes, whose asymptotic limits do not commute. In this work, we consider a Particle-In-Cell discretization of the collisional Vlasov system which enables to deal with the multiscale characteristics equations. Different multiscale time integrators are then constructed and analysed. We prove asymptotic properties of these schemes in the highly oscillatory regime and in the collisional regime. In particular, the asymptotic preserving property towards the modified equilibrium of the averaged collision operator is recovered. Numerical experiments are then shown to illustrate the properties of the numerical schemes.

8.2 Geometric numerical schemes

Participants: I. Almuslimani, N. Crouseilles, X. Hong, A. Laurent.

The MINGuS team has a long history in the design and study of numerical schemes for Hamiltonian PDEs. The main examples are Schroedinger or Vlasov equations.

In [6], a novel second order family of explicit stabilized Runge-Kutta-Chebyshev methods for advection-diffusion-reaction equations is introduced. The new methods outperform existing schemes for relatively high Peclet number due to their favorable stability properties and explicitly available coefficients. The construction of the new schemes is based on stabilization using second kind Chebyshev polynomials first used in the construction of the stochastic integrator SK-ROCK. An adaptive algorithm to implement the new scheme is proposed. This algorithm is able to automatically select the suitable step size, number of stages, and damping parameter at each integration step. Numerical experiments that illustrate the efficiency of the new algorithm are presented.

In [11], we aim at constructing numerical schemes, that are as efficient as possible in terms of cost and conservation of invariants, for the Vlasov-Fokker-Planck system coupled with Poisson or Ampère equation. Splitting methods are used where the linear terms in space are treated by spectral or semi-Lagrangian methods and the nonlinear diffusion in velocity in the collision operator is treated using a stabilized Runge–Kutta–Chebyshev (RKC) integrator, a powerful alternative of implicit schemes. The new schemes are shown to exactly preserve mass and momentum. The conservation of total energy is obtained using a suitable approximation of the electric field. An H-theorem is proved in the semi-discrete case, while the entropy decay is illustrated numerically for the fully discretized problem. Numerical experiments that include investigation of Landau damping phenomenon and bump-on-tail instability are performed to illustrate the efficiency of the new schemes.

Aromatic B-series were introduced as an extension of standard Butcher-series for the study of volume-preserving integrators. It was proven with their help that the only volume-preserving B-series method is the exact flow of the differential equation. The question was raised whether there exists a volume-preserving integrator that can be expanded as an aromatic B-series. In this work [16], we introduce a new algebraic tool, called the aromatic bicomplex, similar to the variational bicomplex in variational calculus. We prove the exactness of this bicomplex and use it to describe explicitly the key object in the study of volume-preserving integrators: the aromatic forms of vanishing divergence. The analysis provides us with a handful of new tools to study aromatic B-series, gives insights on the process of integration by parts of trees, and allows to describe explicitly the aromatic B-series of a volume-preserving integrator. In particular, we conclude that an aromatic Runge-Kutta method cannot preserve volume.

The aromatic bicomplex is an algebraic tool based on aromatic Butcher trees and used in particular for the explicit description of volume-preserving affine-equivariant numerical integrators. The present work [15] defines new tools inspired from variational calculus such as the Lie derivative, different concepts of symmetries, and Noether's theory in the context of aromatic forests. The approach allows to draw a correspondence between aromatic volumepreserving methods and symmetries on the Euler-Lagrange complex, to write Noether's theorem in the aromatic context, and to describe the aromatic B-series of volume-preserving methods explicitly with the Lie derivative.

Exotic aromatic B-series were originally introduced for the calculation of order conditions for the high order numerical integration of ergodic stochastic differential equations in $\mathbb{R}^d$ and on manifolds. We prove in [32] that exotic aromatic B-series satisfy a universal geometric property, namely that they are characterised by locality and orthogonal-equivariance. This characterisation confirms that exotic aromatic B-series are a fundamental geometric object that naturally generalises aromatic B-series and B-series, as they share similar equivariance properties. In addition, we classify with stronger equivariance properties the main subsets of the exotic aromatic B-series, in particular the exotic B-series. Along the analysis, we present a generalised definition of exotic aromatic trees, dual vector fields, and we explore the impact of degeneracies on the classification.

In [31], we propose an Eulerian-Lagrangian (EL) Runge-Kutta (RK) discontinuous Galerkin (DG) method for linear hyperbolic system. The method is designed based on the EL DG method for transport problems [J. Comput. Phy. 446: 110632, 2021], which tracks solution along approximations to characteristics in the DG framework, allowing extra large time stepping sizes with stability with respect to the classical RK DG method. Considering each characteristic family, a straightforward application of EL DG for hyperbolic system will be to transform to the characteristic variables, evolve them on associated characteristic related space-time regions, and transform them back to the original variables. However, the conservation could not be guaranteed in a general setting. In this paper, we formulate a conservative semi-discrete EL DG method by decomposing each variable into two parts, each of them associated with a different characteristic family. As a result, four different quantities are evolved in EL fashion and recombined to update the solution. The fully discrete scheme is formulated by using method-of-lines RK methods, with intermediate RK solutions updated on the background mesh. Numerical results for 1D and 2D wave equations are presented to demonstrate the performance of the proposed ELDG method. These include the high order spatial and temporal accuracy, stability with extra large time stepping size, and conservative property.

In [17], we study a grid-free particle method based on following the evolution of the characteristics of the Vlasov-Poisson system, and we show that it converges for smooth enough initial data. This method is built as a combination of well-studied building blocks-mainly time integration and integral quadratures-, hence allows to obtain arbitrarily high orders. By making use of the Non-Uniform Fast Fourier Transform (NUFFT), the overall computational complexity is $\mathcal{O}(P + K^d \log K^d)$, where P is the total number of particles and where we only keep the Fourier modes $k \in \mathbb{Z}^d$ such that $k_1^2 + \cdots + k_d^2 \leq K^2$. Some numerical results are given for the Vlasov-Poisson system in the one-dimensional case.

In [20], a two-species Vlasov-Poisson model is described together with some numerical simulations, permitting to exhibit the formation of a plasma sheath. The numerical simulations are performed with two different methods: a first order classical finite difference (FD) scheme and a high order semi-Lagrangian (SL) scheme with Strang splitting; for the latter one, the implementation of (non-periodic) boundary conditions is discussed. The codes are first evaluated on a one-species case, where an analytical solution is known. For the two-species case, cross comparisons and the influence of the numerical parameters for the SL method are performed in order to have an idea of a reference numerical simulation.

In [12], we present a numerical method to solve the Vlasov-Maxwell equations for spin-1/2 particles, in a semiclassical approximation where the orbital motion is treated classically while the spin variable is fully quantum. Unlike the spinless case, the phase-space distribution function is a 2×2 matrix, which can also be represented, in the Pauli basis, as one scalar function $f_0 \in \mathbb{R}$ and one three-component vector function $\vec{f} \in \mathbb{R}^3$. The relationship between this "vectorial" representation and the fully scalar representation on an extended phase space first proposed by Brodin et al. [Phys. Rev. Lett. 101, 245002 (2008)] is analyzed in detail. By means of suitable approximations and symmetries, the vectorial spin-Vlasov-Maxwell model can be reduced to two-dimensions in the phase space, which is amenable to numerical solutions using a high-order grid-based Eulerian method. The vectorial model enjoys a Poisson structure that paves the way to accurate Hamiltonian split-time integrators. As an example, we study the stimulated Raman scattering of an electromagnetic wave interacting with an underdense plasma, and compare the results to those obtained earlier with the scalar spin-Vlasov-Maxwell model and a particle-in-cell code.

In [13], an exponential Discontinuous Galerkin (DG) method is proposed to solve numerically Vlasov type equations. The DG method is used for space discretization which is combined exponential Lawson Runge-Kutta method for time discretization to get high order accuracy in time and space. In addition to get high order accuracy in time, the use of Lawson methods enables to overcome the stringent condition on the time step induced by the linear part of the system. Moreover, it can be proved that a discrete Poisson equation is preserved. Numerical results on Vlasov-Poisson and Vlasov Maxwell equations are presented to illustrate the good behavior of the exponential DG method.

8.3 Analysis of PDEs and SPDEs

Participants: G. Beck, F. Castella, A. Debussche, E. Faou.

In view of the construction of efficient multiscale numerical schemes, the study and analysis of PDEs or SPDEs is of great importance. Below is a list of some contributions of the team on this aspect.

In [8], we study the Boltzmann equation with external forces, not necessarily deriving from a potential, in the incompressible Navier-Stokes perturbative regime. On the torus, we establish local-in-time, for any time, Cauchy theories that are independent of the Knudsen number in Sobolev spaces. The existence is proved around a time-dependent Maxwellian that behaves like the global equilibrium both as time grows and as the Knudsen number decreases. We combine hypocoercive properties of linearized Boltzmann operators with linearization around a time-dependent Maxwellian that catches the fluctuations of the characteristics trajectories due to the presence of the force. This uniform theory is sufficiently robust to derive the incompressible Navier-Stokes-Fourier system with an external force from the Boltzmann equation. Neither smallness, nor time-decaying assumption is required for the external force, nor a gradient form, and we deal with general hard potential and cutoff Boltzmann kernels. As a by-product the latest general theories for unit Knudsen number when the force is sufficiently small and decays in time are recovered.

In [7], we characterize geometrically the regularizing effects of the semigroups generated by accretive non-selfadjoint quadratic differential operators. As a byproduct, we establish the subelliptic estimates enjoyed by these operators, being expected to be optimal. These results prove conjectures by M. Hitrik, K. Pravda-Starov and J. Viola. The proof relies on a new representation of the polar decomposition of these semigroups. In particular, we identify the selfadjoint part as the evolution operator generated by the Weyl quantization of a time-dependent real-valued nonnegative quadratic form for which we prove a sharp anisotropic lower bound.

In [9], we analyze a new asynchronous rumor spreading protocol to deliver a rumor to all the nodes of a large-scale distributed network. This protocol relies on successive pull operations involving k different nodes, with $k \geq 2$, and called k-pull operations. Specifically during a k-pull operation, an uninformed node a contacts $k-1$ other nodes at random in the network, and if at least one of them knows the rumor, then node a learns it. We perform a detailed study in continuous-time of the total time $\Theta_{k,n}$ needed for all the n nodes to learn the rumor. These results extend those obtained in a previous paper which dealt with the discrete-time case. We obtain the mean value, the variance and the distribution of $\Theta_{k,n}$ together with their asymptotic behavior when the number of nodes n tends to infinity.

In [14], we prove that the stochastic Nonlinear Schrödinger (NLS) equation is the limit of NLS equation with random potential with vanishing correlation length. We generalize the perturbed test function method to the context of dispersive equations. Apart from the difficulty of working in infinite dimension, we treat the case of random perturbations which are not assumed uniformly bounded.

In [23], we consider the moments and the distribution of the hitting and cover times of a random walk in the complete graph. We study both the time needed to reach any subset of states and the time needed to visit all the states of a subset at least once. We obtain recurrence relations for the moments of all orders and we use these relations to analyze the asymptotic behavior of the hitting and cover times distributions when the number of states tends to infinity.

In [24], we consider systems of damped wave equations with a state-dependent damping coefficient and perturbed by a Gaussian multiplicative noise. Initially, we investigate their well-posedness, under quite general conditions on the friction. Subsequently, we study the validity of the so-called Smoluchowski-Kramers diffusion approximation. We show that, under more stringent conditions on the friction, in the small-mass limit the solution of the system of stochastic damped wave equations converges to the solution of a system of stochastic quasi-linear parabolic equations. In this convergence, an additional drift emerges as a result of the interaction between the noise and the state-dependent friction. The identification of this limit is achieved by using a suitable generalization of the classical method of perturbed test functions, tailored to the current infinite dimensional setting.

In [26], we are concerned with a slow-fast system of coupled three dimensional Navier-Stokes equations where the fast component is perturbed by an additive Brownian noise. By means of the rough path theory we establish the convergence in law of the slow component towards a Navier-Stokes system with an Itô-Stokes drift and a rough path driven transport noise. This gives an alternative, more general and direct proof to Debussche, Pappalettera, 2022. In particular, the limit driving rough path is identified as a geometric rough path which does not necessarily coincide with the Stratonovich lift of the Brownian motion.

In [27], we consider the nonlinear Schrödinger equation with multiplicative spatial white noise and an arbitrary polynomial nonlinearity on the two-dimensional full space domain. We prove global well-posedness by using a gauge-transform introduced by Hairer and Labbé (2015) and constructing the solution as a limit of solutions to a family of approximating equations. This paper extends a previous result by Debussche and Martin (2019) with a sub-quadratic nonlinearity.

In [28], we study slow-fast systems of coupled equations from fluid dynamics, where the fast component is perturbed by additive noise. We prove that, under a suitable limit of infinite separation of scales, the slow component of the system converges in law to a solution of the initial equation perturbed with transport noise, and subject to the influence of an additional Itô-Stokes drift. The obtained limit equation is very similar to turbulent models derived heuristically. Our results apply to the Navier-Stokes equations in dimension $d = 2,3$; the Surface Quasi-Geostrophic equations in dimension $d = 2$; and the Primitive equations in dimension $d = 2,3$.

In [30], we start from the remark that in wave turbulence theory, exemplified by the cubic twodimensional Schrödinger equation (NLS) on the real plane, the regularity of the resonant manifold is linked with dispersive properties of the equation and thus with scattering phenomena. In contrast with classical analysis starting with a dynamics on a large periodic box, we propose to study NLS set on the real plane using the dispersive effects, by considering the time evolution operator in various time scales for deterministic and random initial data. By considering periodic functions embedded in the whole space by gaussian truncation, this allows explicit calculations and we identify two different regimes where the operators converge towards the kinetic operator but with different form of convergence.

In [34], we consider the moments and the distribution of hitting times on the lollipop graph which is the graph exhibiting the maximum expected hitting time among all the graphs having the same number of nodes. We obtain recurrence relations for the moments of all order and we use these relations to analyze the asymptotic behavior of the hitting time distribution when the number of nodes tends to infinity.

Turbulent cascades characterize the transfer of energy injected by a random force at large scales towards the small scales. In hydrodynamic turbulence, when the Reynolds number is large, the velocity field of the fluid becomes irregular and the rate of energy dissipation remains bounded from below even if the fluid viscosity tends to zero. A mathematical description of the turbulent cascade is a very active research topic since the pioneering work of Kolmogorov in hydrodynamic turbulence and that

of Zakharov in wave turbulence. In both cases, these turbulent cascade mechanisms imply power-law behaviors of several statistical quantities such as power spectral densities. For a long time, these cascades were believed to be associated with nonlinear interactions, but recent works have shown that they can also take place in a dynamics governed by a linear equation with a differential operator of degree 0. In this spirit, we construct in [19] a linear equation that mimics the phenomenology of energy cascades when the external force is a statistically homogeneous and stationary stochastic process. In the Fourier variable, this equation can be seen as a linear transport equation, which corresponds to an operator of degree 0 in physical space. Our results give a complete characterization of the solution: it is smooth at any finite time, and, up to smaller order corrections, it converges to a fractional Gaussian field at infinite time.

9 Bilateral contracts and grants with industry

Participants: E. Faou.

9.1 Bilateral contracts with industry

Participants: E. Faou.

- Contract with the Cailabs compagny.
  A long standing collaboration has emerged between MINGuS and the company CAILABS whose main aim is the conception and construction of optical fibers. Most of the main objectives of this collaboration are strictly confidential. However they have strong common point with the scientific goals of the MINGuS project, for instance the development of efficient numerical methods for quantum simulation and many aspects of mathematical and physical analysis of quantum systems. The impact of this collaboration are very important both from the transfer of technological points of view and from the interaction with a very active startup providing very practical problems that are often very close to hot academic topics. We believe that this interaction will last long and continue to feed the scientific activity of the whole project with problem directly coming from the industrial and economical worlds.

- Agence Lebesgue.

  Participants: E. Faou.

  Since 2019 and up to August 2023, E. Faou was head of the *Agence Lebesgue pour les mathématiques* whose role is precisely to increase the role of mathematics in the socio-economic world by facilitating contact between mathematicians and companies or institute working in distant sector of activity.

  An important activity of the Agence Lebesgue is formation where mathematicians go to industries, companies of the private sector or other institutes to organize some crash course in some hot topics in mathematics, or on demand depending on the requirement of the partners.

- CIFRE Phd with Naval Group: Tom Laborde defended his PhD in december 2023. The advisors were members of the team but they left in 2021.

10 Partnerships and cooperations

10.1 International initiatives

10.1.1 Inria associate team not involved in an IIL or an international program

- Associated team BUBBLES between MINGUS (E. Faou) and university of Cambridge (P. Raphaël).

10.2 International research visitors

10.2.1 Visits of international scientists

- We received the visits of two indian colleagues: M. Anandan (PhD, Bangalore) and S. Samantaray (postdoc, Chenai).

10.2.2 Visits to international teams

- N. Crouseilles visited L. Einkemmer (university of Innsbruck) and E. Sonnendrücker (Max Planck IPP).
- E. Faou and Y. Le Hénaff visited P. Raphaël (university of Cambridge).
- E. Faou visited P. Raphaël (university of Cambridge).

10.3 International initiatives

E. Faou is PI of the Simons collaboration on wave turbulence gathering mathematicians and physicists from New-York University, ENS Paris, ENS Lyon and Torino.

10.4 National initiatives

- 2018-2023: participation IPL SURF headed by A. Vidard (Airsea team).

  Participants: Arnaud Debussche, Erwan Faou.

  This project aims at the modelling and simulation of coastal and littoral ocean circulation problems, including quantification. This project involves 7 Inria teams and Ifremer, BRGM and SHOM.

- 2023: project funded by Fédération de Recherche Fusion par Confinement Magnétique, headed by N. Crouseilles. 5000 euros.

  This project is focused on the design of numerical schemes for tokamak plasmas and involve members of the team but also colleagues from university of Nantes.

  Participants: Nicolas Crouseilles.

- 2019-2023: A. Debussche is the local coordinator of the ANR project ADA, headed by J. Vovelle (ENS Lyon). 160000 euros

Participants: Arnaud Debussche.

This project focuses on multiscale models which are both infinite-dimensional and stochastic with a theoretic and computational approach. The project involved a group in Lyon and MINGuS members.

- 2019-2024 GdR TRAG on rough path theory.

Participants: Arnaud Debussche.

The goal of the TRAG GDR is to gather french mathematicians who work on the rough path theory. GDR TRAG.

- 2023-2027: E. Faou is the PI of the ANR project KEN (Kinetic, PDE and Numerics). The partners are R. Krikorian (Ecole Polytechnique) and B. Grébert (Univeristy Nantes). Budget total 391000 euros

Participants: Nicolas Crouseilles, Erwan Faou.

The project involved a group in Nantes, Ecole Polytechnique and some MINGuS members. It gathers people from the Dynamical system community, specialists of the analysis of Partial Differential Equations, as well as people coming from the numerical analysis and scientific computing worlds.

- 2023-2027: G. Beck is a member of the ANR project BOURGEONS, headed by A.-L. Dalibard (Sorbonne university).

Participants: Geoffrey Beck.

This project focuses on some theoretical challenges and environmental applications on boundary, congestion and vorticity in fluids.

10.5 Regional initiatives

- X. Hong was granted by the Région Bretagne council. ’

11 Dissemination

Participants: Beck Geoffrey, Castella François, Crouseilles Nicolas, Debussche Arnaud, Faou Erwan, Laurent Adrien, Navaro Pierre.

11.1 Promoting scientific activities

11.1.1 Scientific events: organisation

International conference

- A. Debussche organized the conference "Mean Field Models" in Rennes (June 2023).

Julia events

- P. Navaro organized the event "café Julia: GraphNeuralNetworks.jl" (September 2023).
- P. Navaro organized the event Atelier Dask in Grenoble (September 2023).
- P. Navaro organized the event Journées Julia et Optimisation (October 2023).
- P. Navaro organized the event "café Julia: DataFlowTasks.jl (November 2023).
- P. Navaro organized the event "café Julia: Exploring Deep Learning through Flux.jl (December 2023).

Member of scientific committees

- E. Faou is member of the scientific council of the SCICADE-2024 conference.

11.1.2 Journal

Member of the editorial boards

- A. Debussche: Editor in chief of "Stochastics and Partial Differential Equations: Analysis and Computations" (2013-)
- A. Debussche: Member of the editorial committee of ESAIM: PROCS (2012-).
- A. Debussche: Member of the editorial committee of Annales de l'IHP Probabilités et Statisques (2020-).
- A. Debussche: Member of the editorial committee of Journal of Evolution equation (2014-).
- A. Debussche: Member of the editorial committee of Annales Henri Lebesgue (2018-).
- A. Debussche: Member of the editorial committee of the collection de monographie : Mathématiques and Applications, sous l'égide de la SMAI.

Reviewer - reviewing activities The members of MINGuS reviewed papers from journals in which they usually publish their works.

11.1.3 Invited talks

International events

- I. Almulimani gave a talk at the third Conference of Young Applied Mathematicians (YAMC), Sienne, Italie (September 2023).
- G. Beck gave a talk at Journées de Modélisation des Vagues à Phases Résolues, Ile d'Aix, France (October 2023).
- N. Crouseilles participated to the NUMKIN workshop, Munich, Germany (November 2023).
- A. Debussche was main speaker at the 13th AIMS conference, Wilmington, USA (June 2023).
- A. Debussche gave a talk at the conference "SPDEs, optimal control and mean field games - analysis, numerics and applications", Bielefeld, Germany (July 2023).
- A. Debussche gave a talk at the conference "Colloque Analyse Appliquée et Modélisation", Monastir, Tunisie (November 2023).
- E. Faou gave a talk at the conference "Nonlinear Waves and Hamiltonian PDEs", La Thuile, Italy (February 2023).

- E. Faou gave a talk at the conference "Harmonic Analysis and Differential Equations: new questions and challenges in honor of Luis Vega", Bilbao, Spain (September 2023).
- P. Navaro participated to the NUMKIN workshop, Munich, Germany (November 2023).
- P. Navaro participated to the CEMRACS 2023 "Scientific Machine Learning" in the project "Physics Informed Neural Network to reduce data storage of gyrokinetic plasma turbulence simulations", CIRM Marseille (July-August 2023).

National events

- I. Almuslimani gave a talk at the "Congrès des Jeunes Chercheurs en Mathématiques et Applications", Gif-sur-Yvette (September 2023).
- I. Almuslimani gave a seminar at CMAP école polytechnique, Paris (October 2023).
- G. Beck gave a lecture (with D. Lannes) on Wave-structure interactions (4.5 hours), at the summer school New Trends In Mathematical Fluid Dynamics, University Grenoble Alpes (June 2023).
- G. Beck gave the PDE seminar "Modélisation et Calcul Scientifique de Lyon-Saint Etienne" (October 2023).
- N. Crouseilles gave a seminar at university of Lille (June 2023).
- Y. Le Hénaff gave a talk at the "Congrès des Jeunes Chercheurs en Mathématiques et Applications", Gif-sur-Yvette (September 2023).
- Y. Le Hénaff gave a talk at the kick-off meeting of the ANR KEN, Nantes (September 2023).
- P. Navaro gave the seminar Infomath, Paris (February 2023).
- P. Navaro gave the Webinar Quarto (January 2023).
- P. Navaro gave the Webinar "Interroger les données" (March 2023).
- P. Navaro gave a Webinar at Journées de statistique (April 2023).
- P. Navaro gave a Webinar "l'heure ProDev" (April 2023).
- P. Navaro gave a seminar at INRIA HPC (June 2023).
- P. Navaro gave a seminar at devtech INRIA (June 2023).
- P. Navaro gave a seminar at Journées Julia et Optimisation (October 2023).
- P. Navaro gave a seminar at Journées MATHRICE (November 2023).

11.1.4 Leadership within the scientific community

- N. Crouseilles is head of the MINGuS Inria team.
- A. Debussche is co-head of the Labex Lebesgue (since July 2023).
- E. Faou is co-head of the Labex Lebesgue (up to August 2023).
- E. Faou is head of the Agence Lebesgue pour l'innovation en mathématiques (up to August 2023).

11.1.5 Scientific expertise

- N. Crouseilles is a member of the Commission d'Evaluation Inria (up to August 2023): member of the hiring committee for the Inria young researcher position at Lille (June 2023), member of the promotion committee for the Inria senior researcher position, coordinator of the evaluation of some Inria teams.
- N. Crouseilles was member of the hiring committee of the professor position PR852 at university of Rennes.
- A. Debussche was member of the hiring committee of the professor position PR852 at university of Rennes.
- A. Debussche was member of the hiring committee of the chaire professeur junior at ENS-Rennes (science de l'environnement department).
- E. Faou was member and coordinator of the hiring committee of the professor position PR852 at university of Rennes.
- E. Faou was member of the committee for the ICIAM Maxwell prize.
- P. Navaro was member of the hiring committee of CNRS research engineer position (number 69) for the mathematical laboratory of Rouen.

11.1.6 Research administration

- N. Crouseilles is member of the IRMAR council.
- N. Crouseilles is member of the "comité des projets" of Inria Rennes.
- N. Crouseilles is responsible for the university of Rennes of the Fédération de recherche pour la fusion confinement magnétique.
- N. Crouseilles is member of the "comité de centre Inria de Rennes" and of the bureau.
- A. Debussche is member of the administration council of ENS Paris Saclay.
- A. Debussche was member of the HCERES committe for the evaluation of the Laboratory Jacques Louis Lions, Sorbonne University.
- A. Debussche is member of the scientific council of the Institut Denis Poisson (Orléans, Tours and LAMFA- Amiens).
- A. Debussche is member of the steering comittee of the EUR "Digisport", Rennes.
- A. Debussche is member of the steering comittee of the cofund "MathPhDInFrance".
- P. Navaro animates the Julia french community at CNRS (see newsletters).
- P. Navaro is member of the CNRS working group on datas (see guide and website).
- P. Navaro is coordinator of the "Groupe Calcul" dedicated to people using scientific computing for their research.

11.2 Teaching - Supervision - Juries

11.2.1 Teaching

The members of team are involved in teaching activities. We list below the Master courses only.

- F. Castella, Heat, Schrodinger and Laplace equations methods, 40 hours, Master 1, University of Rennes.
- F. Castella, Numerical analysis for PDEs, 36 hours, Master 2, University of Rennes.
- F. Castella, Finite elements, 18 hours, Master 2, University of Rennes.
- N. Crouseilles, Numerical methods for PDEs, 24 hours, Master 1, ENS Rennes.
- N. Crouseilles, Numerical methods for kinetic equations, 36 hours, Master 2, University of Rennes.
- A. Debussche, Calcul stochastique, 36 hours, Master 2, University Rennes.
- P. Navaro, Python courses, 20 hours, Master 2 Smart Data, ENSAI.

11.2.2 Supervision

- I. Almuslimani (postdoc 2021-2023, SNSF grant) advised by N. Crouseilles.
- Y. Fotso-Fotso (postdoc 2023-2025, PEPR AgroEcoNum) co-advised by F. Castella and F. Hamelin (agrocampus, Rennes).
- Y. Le Hénaff (PhD 2021-2024, Univ Rennes grant) co-advised by N. Crouseilles and E. Faou. Title: High order modulated particle methods: theoretical and numerical studies.
- X. Hong (postdoc 2022-2023, Brittany council and Labex grant) advised by N. Crouseilles.
- T. Laborde (PhD 2020-2023, Naval Group) co-advised by former members of the team. Due to the contract with Naval Group, his name still appears as a member of the team. The defense was in December 2023.
- L. Martaud (postdoc 2023-2024, Inria grant) co-advised by G. Beck and N. Crouseilles.
- H. Martin (postdoc (2022-2025, chair Modélisation Mathématiques et Biodiversité from Ecole Polytechnique), co-advised by F. Castella and F. Hamelin (agrocampus, Rennes).
- A. Moneyron, (PhD 2023-2026, ERC Stuod grant), co-advised by A. Debussche and E. Mémin (Odyssey Inria team).
- A. Mouzard (postdoc 2021-2023, Simons grant) advised by E. Faou. He left the team in September 2023 for a postdoc position at ENS Ulm (DMA) granted by FMSP.
- R. Nader (postdoc 2023, ANR ADA grant) advised by A. Debussche.
- D. Prel (PhD 2021-2024, Université Nantes grant) co-advised by A. Crestetto and N. Crouseilles. Title: Multi-scale numerical methods for tokamak plasma simulations.

11.2.3 Juries

- F. Castella was president of the HDR committee of M. Andraud, Anses Rennes (September 2023).
- N. Crouseilles was referee of the PhD thesis of C. Guillet, Toulouse (June 2023).
- N. Crouseilles was president of the PhD thesis committee of L. Martaud, Nantes (September 2023).
- N. Crouseilles was member of the HDR committee of B. Delourme, Sorbonne Paris Nord (September 2023).

- N. Crouseilles was referee of the PhD thesis of L. Bois, Strasbourg (December 2023).
- A. Debussche was referee of the PhD thesis of Y. Zine, Edinburgh (June 2023).
- A. Debussche was member of the PhD thesis committee of M. Douaihy, Montpellier (December 2023).
- E. Faou was president of the PhD thesis committee of O. Cosserat, La Rochelle (September 2023).

11.3 Popularization

- G. Beck gave a talk at "5min Lebesgue": Turbulence I : les cascades de Kolmogorov comme théorie du ruissellement des "échelles".
- N. Crouseilles gave a conference for the master students of the university of Nantes (October 2023).
- N. Crouseilles collaborated with T. Menuet to create an enigma combining mathematics and music.
- Y. Le Hénaff participated to the event on the interaction between music and mathematic proposed by T. Menuet.
- Y. Le Hénaff participated to C2+.
- A. Laurent participated to the event "Rennes en Science" (November 2023).
- P. Navaro organized the following workshops at IRMAR: Julia (for statistics, optimization, create packages), git, PLM.
- D. Prel participated to Science Dating pour Nuit Blanche des Chercheurs.
- D. Prel participated to "Fête de la science" at Nantes.

12 Scientific production

12.1 Major publications

[1] F. Casas, N. Crouseilles, E. Faou and M. Mehrenberger. 'High-order Hamiltonian splitting for Vlasov-Poisson equations'. In: *Numerische Mathematik* 135.3 (2017), pp. 769–801. DOI: 10.1007/s00211-016-0816-z. URL: https://hal.inria.fr/hal-01206164.

[2] P. Chartier, N. Crouseilles, M. Lemou, F. Méhats and X. Zhao. 'Uniformly accurate methods for three dimensional Vlasov equations under strong magnetic field with varying direction'. In: *SIAM Journal on Scientific Computing* 42.2 (2020), B520–B547. DOI: 10.1137/19M127402X. URL: https://hal.archives-ouvertes.fr/hal-02179534.

[3] N. Crouseilles, P.-A. Hervieux, Y. Li, G. Manfredi and Y. Sun. 'Geometric Particle-in-Cell methods for the Vlasov-Maxwell equations with spin effects'. In: *Journal of Plasma Physics* 87.3 (28th May 2021), article n° 825870301. DOI: 10.1017/S0022377821000532. URL: https://hal.inria.fr/hal-03148534.

[4] A. Debussche and J. Vovelle. 'Diffusion-approximation in stochastically forced kinetic equations'. In: *Tunisian Journal of Mathematics* 3.1 (2021), pp. 1–53. DOI: 10.2140/tunis.2021.3.1. URL: https://hal.archives-ouvertes.fr/hal-01567138.

[5] E. Faou. 'Linearized wave turbulence convergence results for three-wave systems'. In: *Communications in Mathematical Physics* 378.2 (Sept. 2020), pp. 807–849. DOI: 10.1007/s00220-020-03799-w. URL: https://hal.inria.fr/hal-01801810.

12.2 Publications of the year

International journals

[6] I. Almuslimani. 'A fully adaptive explicit stabilized integrator for advection-diffusion-reaction problems'. In: *BIT Numerical Mathematics* 63.1 (2023), article n°3. DOI: 10.1007/s10543-023-00945-3. URL: https://hal.science/hal-03542086.

[7] P. Alphonse and J. Bernier. 'Polar decomposition of semigroups generated by non-selfadjoint quadratic differential operators and regularizing effects'. In: *Annales Scientifiques de l'École Normale Supérieure* 56.2 (1st Mar. 2023), pp. 323–382. DOI: 10.24033/asens.2536. URL: https://hal.science/hal-02280971.

[8] M. Briant, A. Debussche and J. Vovelle. 'The Boltzmann equation with an external force on the torus: Incompressible Navier-Stokes-Fourier hydrodynamical limit'. In: *Pure and Applied Analysis* 4.4 (2023), pp. 597–628. DOI: 10.2140/paa.2022.4.597. URL: https://hal.science/hal-02150286.

[9] F. Castella, B. Sericola, E. Anceaume and Y. Mocquard. 'Continuous-Time Stochastic Analysis of Rumor Spreading with Multiple Operations'. In: *Methodology and Computing in Applied Probability* 25.4 (23rd Oct. 2023), p. 82. DOI: 10.1007/s11009-023-10058-7. URL: https://cnrs.hal.science/hal-04255487.

[10] A. Crestetto, N. Crouseilles and D. Prel. 'Multiscale numerical schemes for the collisional Vlasov equation in the finite Larmor radius approximation regime'. In: *Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal* 21 (14th Sept. 2023), pp. 1–24. DOI: 10.1137/22M1496839. URL: https://hal.science/hal-03690427.

[11] N. Crouseilles and I. Almuslimani. 'Conservative stabilized Runge-Kutta methods for the Vlasov-Fokker-Planck equation'. In: *Journal of Computational Physics* 488 (2023), pp. 1–34. DOI: 10.1016/j.jcp.2023.112241. URL: https://hal.science/hal-03911417.

[12] N. Crouseilles, P.-A. Hervieux, X. Hong and G. Manfredi. 'Vlasov-Maxwell equations with spin effects'. In: *Journal of Plasma Physics* 89.2 (2023), article n° 905890215, PII S0022377823000314. DOI: 10.1017/S0022377823000314. URL: https://inria.hal.science/hal-03960201.

[13] N. Crouseilles and X. Hong. 'Exponential DG methods for Vlasov equations'. In: *Journal of Computational Physics* 498 (2024). DOI: 10.1016/j.jcp.2023.112682. URL: https://hal.science/hal-04389308.

[14] A. Debussche, M. Tusseau, A. Debussche and G. Barrué. 'Approximation diffusion for the nonlinear Schrödinger equation with a random potential'. In: *Asymptotic Analysis* (2024). URL: https://hal.science/hal-04383170.

[15] A. Laurent. 'The Lie derivative and Noether's theorem on the aromatic bicomplex for the study of volume-preserving numerical integrators'. In: *Journal of Computational Dynamics* (2023), pp. 1–14. DOI: 10.3934/jcd.2023011. URL: https://hal.science/hal-04347291.

[16] A. Laurent, R. I. Mclachlan, H. Z. Munthe-Kaas and O. Verdier. 'The aromatic bicomplex for the description of divergence-free aromatic forms and volume-preserving integrators'. In: *Forum of Mathematics, Sigma* 11 (26th Jan. 2023), E69. DOI: 10.1017/fms.2023.63. URL: https://hal.science/hal-04347717.

[17] Y. Le Henaff. 'Grid-free Weighted Particle method applied to the Vlasov-Poisson equation'. In: *Numerische Mathematik* 155 (2023), pp. 289–344. DOI: 10.1007/s00211-023-01378-4. URL: https://hal.science/hal-03736227.

Reports & preprints

[18] M. Anandan, B. Boutin and N. Crouseilles. *High order asymptotic preserving scheme for linear kinetic equations with diffusive scaling*. 2023. URL: https://hal.science/hal-04103000.

[19] G. B. Apolinário, G. Beck, L. Chevillard, I. Gallagher and R. Grande. *A Linear Stochastic Model of Turbulent Cascades and Fractional Fields*. 2023. URL: https://hal.science/hal-03919233.

[20] V. Ayot, M. Badsi, Y. Barsamian, A. Crestetto, N. Crouseilles, M. Mehrenberger, A. Prost and C. Tayou-Fotso. *High order numerical methods for Vlasov-Poisson models of plasma sheaths*. 6th Jan. 2023. URL: https://inria.hal.science/hal-03926305.

[21] G. Beck and A. Beni Hamad. *Electromagnetic waves propagation in thin heterogenous coaxial cables. Comparaison between 3D and 1D models*. Mar. 2023. URL: https://hal.science/hal-04147142.

[22] G. Beck, D. Lannes and L. Weynans. *A NUMERICAL METHOD FOR WAVE-STRUCTURE INTERACTIONS IN THE BOUSSINESQ REGIME*. July 2023. URL: https://hal.science/hal-04151128.

[23] F. Castella and B. Sericola. *Random walk in the complete graph : hitting and cover times*. Centre Inria, 27th Dec. 2023. URL: https://inria.hal.science/hal-04365696.

[24] S. Cerrai and A. Debussche. *Smoluchowski-Kramers diffusion approximation for systems of stochastic damped wave equations with non-constant friction*. 2023. DOI: 10.48550/arXiv.2312.08925. URL: https://hal.science/hal-04383799.

[25] A. Crestetto, N. Crouseilles and D. Prel. *Deterministic particle method for Fokker-Planck equation with strong oscillations*. 13th Dec. 2023. URL: https://hal.science/hal-04343011.

[26] A. Debussche and M. Hofmanová. *Rough analysis of two scale systems*. 2023. DOI: 10.48550/arXiv.2306.15781. URL: https://hal.science/hal-04383808.

[27] A. Debussche, R. Liu, N. Tzvetkov and N. Visciglia. *Global well-posedness of the 2D nonlinear Schrödinger equation with multiplicative spatial white noise on the full space*. 2023. DOI: 10.48550/arXiv.2301.10825. URL: https://hal.science/hal-04383816.

[28] A. Debussche and U. Pappalettera. *Second order perturbation theory of two-scale systems in fluid dynamics*. 2022. DOI: 10.48550/arXiv.2206.07775. URL: https://hal.science/hal-04383854.

[29] E. Faou, Y. Le Henaff and P. Raphaël. *Modulation algorithm for the nonlinear Schrödinger equation*. 22nd Mar. 2023. URL: https://hal.science/hal-04041787.

[30] E. Faou and A. Mouzard. *Scattering, random phase and wave turbulence*. 4th July 2023. URL: https://hal.science/hal-04151046.

[31] X. Hong and J. Qiu. *A conservative Eulerian-Lagrangian Runge-Kutta discontinuous Galerkin method for linear hyperbolic system with large time stepping size*. 2023. URL: https://hal.science/hal-03920844.

[32] A. Laurent and H. Z. Munthe-Kaas. *The universal equivariance properties of exotic aromatic B-series*. 18th May 2023. URL: https://hal.science/hal-04349836.

[33] L. Ledru, J. Garnier, O. Guillot, E. Faou, C. Noûs and S. Ibanez. *the evolutionary dynamics of plastic foraging and its ecological consequences: a resource-consumer model*. 8th Nov. 2023. URL: https://hal.science/hal-04047341.

[34] B. Sericola and F. Castella. *Hitting times on the lollipop graph*. Centre Inria de l'université de Rennes, 27th June 2023, pp. 1–27. URL: https://inria.hal.science/hal-04143403.

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[39] P. Chartier, A. Murua and J. Sanz-Serna. 'Higher-order averaging, formal series and numerical integration III: error bounds'. In: *Foundation of Comput. Math.* 15 (2015), pp. 591–612.

[40] A. Debussche and J. Vovelle. 'Diffusion limit for a stochastic kinetic problem'. In: *Commun. Pure Appl. Anal.* 11 (2012), pp. 2305–2326.

[41] E. Faou and F. Rousset. 'Landau damping in Sobolev spaces for the Vlasov-HMF model'. In: *Arch. Ration. Mech. Anal.* 219 (2016), pp. 887–902.

[42] E. Hairer, C. Lubich and G. Wanner. *Geometric Numerical Integration. Structure-Preserving Algorithms for Ordinary Differential Equations, Second edition.* Springer Series in Computational Mathematics 31. Berlin: Springer, 2006.

[43] S. Jin and H. Lu. 'An Asymptotic-Preserving stochastic Galerkin method for the radiative heat transfer equations with random inputs and diffusive scalings'. In: *J. Comp. Phys.* 334 (2017), pp. 182–206.

[44] M. Lemou, F. Méhats and P. Raphaël. 'Orbital stability of spherical galactic models'. In: *Invent. Math.* 187 (2012), pp. 145–194.

[45] C. Mouhot and C. Villani. 'On Landau damping'. In: *Acta Math.* 207 (2011), pp. 29–201.

[46] S. Nazarenko. *Wave turbulence.* Springer-Verlag, 2011.

[47] L. Perko. 'Higher order averaging and related methods for perturbed periodic and quasi-periodic systems'. In: *SIAM J. Appl. Math.* 17 (1969), pp. 698–724.