RESEARCH CENTRE

Inria Saclay Centre at Institut Polytechnique de Paris

IN PARTNERSHIP WITH:

CNRS, Ecole nationale supérieure des techniques avancées

2023 ACTIVITY REPORT

Project-Team POEMS

Wave propagation: mathematical analysis and simulation

IN COLLABORATION WITH: Propagation des ondes : étude mathématique et simulation (POEMS)

DOMAIN

Applied Mathematics, Computation and Simulation

THEME

Numerical schemes and simulations

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Creation of the Project-Team: 2019 November 01

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2 Overall objectives

The propagation of waves is one of the most common physical phenomena in nature. From the human scale (sounds, vibrations, water waves, telecommunications, radar) to the scales of the universe (electromagnetic waves, gravity waves) and of the atoms (spontaneous or stimulated emission, interferences between particles), the emission and the reception of waves are our privileged way to understand the world that surrounds us. The study and the simulation of wave propagation phenomena constitute a very broad and active field of research in various domains of physics and engineering sciences. The variety and the complexity of the underlying problems, their scientific and industrial interest, the existence of a common mathematical structure to these problems from different areas altogether justify a research project in applied mathematics and scientific computing devoted to this topic.

3 Research program

3.1 Expertises

The activity of the team is oriented towards the design, the analysis and the numerical approximation of mathematical models for all types of problems involving wave propagation phenomena, in mechanics, physics and engineering sciences. Let us briefly describe our core business and current expertise, in order to clarify the new challenges that we want to address in the short and long terms.

Typically, our works are based on *boundary value problems* established by physicists to model the propagation of waves in various situations. The basic ingredient is a partial differential equation of the hyperbolic type, whose prototype is the scalar wave equation, or the Helmholtz equation if time-periodic solutions are considered. More generally, we systematically consider both the transient problem, in the time domain, and the time-harmonic problem, in the frequency domain. Let us mention that, even if different waves share a lot of common properties, the transition from the scalar acoustic equation to the vectorial electromagnetism and elastodynamics systems raises a lot of mathematical and numerical difficulties, and requires a specific expertise.

A notable particularity of the problems that we consider is that they are generally set in *unbounded domains*: for instance, for radar applications, it is necessary to simulate the interaction of the electromagnetic waves with the airplane only, without any complex environment perturbing the wave phenomena. This raises an intense research activity, both from a theoretical and a numerical point of view. There exist several approaches which all consist in rewriting the problem (or an approximation of it) in a bounded domain, the new formulation being well-suited for classical mathematical and numerical techniques.

One class of methods consists in applying an appropriate condition on some boundary enclosing the zone of interest. In the frequency domain, one can use a non-local transparent condition, which can be expressed by a convolution with a Green function like in integral equation techniques, or by a modal decomposition when a separation of variables is applicable. But for explicit schemes in the time domain, local radiation conditions at a finite distance are generally preferred (constructed as local approximations at various orders of the exact non-local condition). A second class of methods consists in surrounding the computational domain by so called *Perfectly Matched absorbing Layers* (PML), which are very popular because they are easy to implement. POEMS members have provided several contributions to these two classes of methods for more than twenty-five years. Among them, on can mention the understanding of the instability of PMLs in anisotropic media and in dispersive media, the derivation of transparent boundary conditions in periodic media or the improvement of Fast Multipole techniques for elastodynamic integral equations.

In addition to more classical domains of applied mathematics that we are led to use (variational analysis and functional analysis, interpolation and approximation theory, linear algebra of large systems, etc...), we have acquired a deep expertise in *spectral theory*. Indeed, the analysis of wave phenomena is intimately linked to the study of some associated spectral problems. Acoustic resonance frequencies of a cavity correspond to the eigenvalues of a selfadjoint Laplacian operator, modal solutions in a waveguide correspond to a spectral problem set in the cross section. In these two examples, if the cavity or the cross-section is unbounded, a part of the spectrum is a continuum. Again, POEMS has produced several contributions in this field. In particular, a large number of significant results have been obtained for the existence or non-existence of guided modes in open waveguides and of trapped modes in infinite domains.

To end this far from exhaustive presentation of our main expertise domains, let us mention the *asymptotic techniques* with respect to some small scale appearing in the model: it can be the wavelength compared to the size of the scatterer, or on the contrary, the scale of the scatterer compared to the wavelength, it can be the scale of some microstructure in a composite material or the width of a thin layer or a thin tube. In each case, the objective, in order to avoid the use of costly meshes, is to derive effective simplified models. Our specificity here is that we can combine skills in physics, mathematics and numerics: in particular, we take care of the mathematical properties of the effective model, which are used to ensure the robustness of the numerical method, and also to derive error estimates with respect to the small parameter. There has been a lot of contributions of POEMS to this topic, going from the modeling of electromagnetic coatings to the justification of models for piezoelectric sensors. Let us mention that effective models for small scatterers and thin coatings have been used to improve imaging techniques that we are developing (topological gradient, time reversal or sampling techniques).

3.2 Recent evolutions

In order to consider more and more challenging problems (involving non-deterministic, large-scale and more realistic models), we decided recently to enlarge our domain of expertise in three directions.

Firstly, we want to reinforce our activity on *efficient solvers for large-scale wave propagation problems*. Since its inception, POEMS has frequently contributed to the development and the analysis of numerical methods that permit the fast solution of large-scale problems, such as high-order finite element methods, boundary elements methods and domain decomposition methods. Nevertheless, implementing these methods in parallel programming environments and dealing with large-scale benchmarks have generally not been done by the team. We want to continue our activities on these methods and, in a more comprehensive approach, we will incorporate modern algebraic strategies and high-performance computing (HPC) aspects in our methodology. In collaboration with academic or industrial partners, we would like to address industrial-scale benchmarks to assess the performance of our approaches. We believe that taking all these aspects into consideration will allow us to design more efficient wave-specific computational tools for large-scale simulations.

Secondly, up to now, *probabilistic methods* were outside the expertise of POEMS team, restricting us to deterministic approaches for wave propagation problems. We however firmly believe in the importance and usefulness of addressing uncertainty and randomness inherent to many propagation phenomena. Randomness may occur in the description of complex propagation media (for example in the modeling of ultrasound waves in concrete for the simulation of non-destructive testing experiments) or of data uncertainties. To quantify the effect of such uncertainties on the design, behavior, performance or reliability of many systems is then a natural goal in diverse fields of application.

Thirdly and lastly, we wish to develop and strengthen collaborations allowing a *closer interaction between our mathematical, modeling and computing activities and physical experiments*, where the latter may either provide reality checks on existing models or strongly affect the choice of modeling assumptions. Within our typical domain of activities, we can mention four areas for which such considerations are highly relevant. One is musical acoustics, where POEMS has made several well-recognized contributions dealing with the simulation of musical instruments. Another area is inverse problems, whose very purpose is to extract useful information from actual measurements with the help of (propagation) models. This is a core of our partnership with CEA on ultrasonic Non Destructive Testing. A third area is the modelling of effective (acoustic or electromagnetic) metamaterials, where predictions based on homogenized models have to be confirmed by experiments. Finally, a fourth area of expertise is the modeling and simulations of waves in reactive media, where the development of simple mathematical models is of great importance in order to better understand the complex dynamics of reactive flows.

4 Application domains

Our research finds applications in many fields where acoustic, elastic, electromagnetic and aquatic waves are involved. Topics that have given rise to industrial partnerships include aircraft noise reduction (aeroacoustics), ultrasonic non-destructive testing of industrial structures, and seismic wave simulations in the subsoil, for the oil exploration.

Nowadays, the numerical techniques for solving the basic academic problems are well mastered, and significant progress has been made during the last twenty years for handling problems closer to real applications. But several bottlenecks remain, among which one can mention the high-frequency problems for radar applications, the multiscale problems that arise for instance in nanotechnologies or the multi-physics couplings, like in *aeroacoustics*. Moreover, in the recent period, new challenges have emerged, related to new discoveries in physics (like negative index metamaterials) or to the fantastic development of information and communication techniques. For example, the growing development of increasingly connected objects (internet of things) and the forthcoming availability of autonomous vehicles depend crucially on electromagnetic waves, raising important issues about radar performance, sensor reliability, component miniaturization and electromagnetic compatibility. Generally, there are a lot of application domains which could benefit from advanced research on waves phenomena. Enhancing ultrasound-based methods for *detection* and *imaging*, which are already intensively used in e.g. medicine, could permit real-time health monitoring of aircrafts or nuclear plants. Guarding against seismic risks still requires considerable advances in the simulation of elastic waves in large and complex media. And many other applications motivating our research and our prospects could be added to this far-from-comprehensive list.

5 New software, platforms, open data

5.1 New software

5.1.1 COFFEE

Functional Description: COFFEE is an adapted fast BEM solver to model acoustic and elastic wave propagation (full implementation in Fortran 90). The 3-D acoustic or elastodynamic equations are solved with the boundary element method accelerated by the multi-level fast multipole method or a hierarchical-matrices based representation of the system matrix. The fundamental solutions for the infinite space are used in this implementation. A boundary element-boundary element coupling strategy is also implemented so multi-region problems (strata inside a valley for example) can be solved. In order to accelerate the convergence of the iterative solver, various analytic or algebraic preconditioners are available. Finally, an anisotropic mesh adaptation strategy is used to further reduce the computational times.

URL: https://uma.ensta-paris.fr/soft/COFFEE/

Author: StÉphanie Chaillat Loseille

Contact: StÉphanie Chaillat Loseille

5.1.2 XLiFE++

Name: eXtended Library of Finite Elements in C++

Keywords: Numerical simulations, Finite element modelling, Boundary element method

Functional Description: XLiFE++ is an FEM-BEM C++ library developed by POEMS laboratory, that can solve 1D/2D/3D, scalar/vector, transient/stationary/harmonic problems.

URL: https://uma.ensta-paris.fr/soft/XLiFE++/

Contact: Eric Lunéville

5.1.3 HTool

Keyword: Hierarchical matrices

Functional Description: HTool is a C++ header-only library implementing compression techniques (e.g. Adaptive Cross Approximation) using hierarchical matrices (H-matrices). The library uses MPI and OpenMP for parallelism, and is interfaced with HPDDM for the solution of linear systems.

URL: https://github.com/htool-ddm/htool

Contact: Pierre Marchand

5.1.4 DataFlowTasks.jl

Keyword: Task scheduling

Functional Description: DataFlowTasks.jl is a Julia package dedicated to parallel programming on multi-core shared memory CPUs. From user annotations (READ, WRITE, READWRITE) on program data, DataFlowTasks.jl automatically infers dependencies between parallel tasks.

URL: https://github.com/maltezfaria/DataFlowTasks.jl

Contact: Luiz Maltez Faria

5.1.5 HMatrices.jl

Keywords: Boundary element method, Hierarchical matrices

Functional Description: This package provides some functionality for assembling as well as for doing linear algebra with hierarchical matrices with a strong focus in applications arising in boundary integral equation methods. It includes shared as well as distributed memory assembly and matrix/vector product, as well as a shared memory LU factorization.

URL: https://github.com/WaveProp/HMatrices.jl

Contact: Luiz Maltez Faria

6 New results

6.1 Wave propagation in metamaterials and dispersive media

Mathematical analysis of metamaterials in time domain

Participants: Patrick Joly, Alex Rosas Martinez.

This topic is the subject of our important collaboration with Maxence Cassier (Institut Fresnel). and corresponds the PhD thesis of Alex Rosas Martinez who started in November 2020 and has been defended in October 2023.

A first part of the thesis has been devoted to the large time behaviour of the associated Cauchy problem for dissipative version of generalized Lorentz media. The main result is the polynomial stability of this model. A first article on the approach via "frequency dependent" Lyapunov methods has been accepted for publication. More precise results have then been obtaines via a (more involved) spectral approach : the corresponding article has been submitted.

A second part of the thesis concernes guided waves by a slab of a non dissipative metamaterial (a Drude material) embedded in the vacuum. When this slab is homogeneous one can compute explicitly the dispersion relation of the modes. A thorough study of this dispersion relation leads to new existence and dispersion results which are quite different from those obtained with a slab of dielectric material. An article is in preparation.

Wave Propagation in Plasmas

Participants: Patrick Ciarlet, Maryna Kachanovska, Etienne Peillon.

This work is a continuation of the research done in collaboration with B. Després et al. on the degenerate elliptic equations describing plasma heating and is a part of the PhD thesis of E. Peillon. Plasma heating is modelled by the Maxwell equations with variable coefficients, which, in the simplest 2D setting can be reduced to the 2D Helmholtz equation, where the coefficient the principal part of the operator changes its sign smoothly along an interface. Such problems are naturally well-posed in a certain weighted Sobolev space; however, the corresponding solutions cannot contribute to the plasma heating, due to their high regularity.

It is possible to demonstrate that plasma heating is induced by singular solutions, which are square integrable but do not longer lie in this weighted Sobolev space. From the theoretical viewpoint, the explanation of the plasma heating phenomenon is based on the limiting absorption principle. No justification was known, except in some special cases. We have obtained a proof of the limiting absorption principle which makes use of the refined studies of the regularity of the limiting solution.In addition, in some situations, we proved that the corresponding singular solutions can be represented as a product of an H^1-function varying in tangential direction along the interface where the variable coefficient changes its sign, and a singular function depending on the normal direction.

The numerical method suggested in previous works is based on exploiting the plasma heating property, however it relies on a variational formulation which requires H^2-regularity of the tangential part. We were able to alter the existing variational formulation in the way that requires a regularity of this tangential part that conforms with the theory; interestingly, this allows to use lower order finite elements in the numerical simulation while preserving the general order of convergence of the scheme. Convergence has been observed on many numerical experiments when the mesh is refined. Finally, the numerical algorithms can be drastically simplified to take into account jump conditions at the interface.

Optimal control-based numerical method for problems with sign-changing coefficients

Participants: Patrick Ciarlet, Farah Chaaban, Mahran Rihani.

We started from the scalar equation $\mathrm{div}(\sigma\nabla u) - \omega^2\eta u = f$ in a domain Ω (plus boundary conditions), where σ and η are real-valued, and piecewise constant. Specifically, σ is strictly positive in part of the domain, and stricly negative elsewhere. When the problem is well-posed in $H^1(\Omega)$, meshing rules have been designed in the past to solve the problem with the finite element method (the so-called T-conform meshes), that ensure convergence of the discrete solution towards the exact solution. Following the Master's thesis of David Lassounon (2021) in which the model with $\eta = 0$ was addressed, we investigated a method based on control techniques, that allows in principle to compute solutions without having to comply with those meshing rules. The mathematical theory has been completed, and the numerical results confirm theory. Currently, we are investigating similar issues for the 2D Maxwell equation $\mathbf{curl}\,(\mu^{-1}\,\mathrm{curl}\,\mathbf{E}) - \omega^2\varepsilon\mathbf{E} = \mathbf{f}$ in Ω (plus b.c.). The mathematical analysis is under way.

Towards non-local interface models

Participants: Patrick Ciarlet.

A collaboration with Juan Pablo Borthagaray (DMEL, Universidad de la República, Montevideo, Uruguay). The long term goal is to better take into account interface transmission conditions between a classical material and and metamaterial. The purely local models have limitations, in the sense that they are not well-posed mathematically in some configurations. On the other hand, nonlocal models allow to take transmission conditions in a more flexible manner but, on the downside, they are much more expensive to solve numerically. So, the thread currently investigated with Juan Pablo Borthagaray is to build a global model that couples local to nonlocal models, the former for their low numerical cost, the latter for their flexibility. As a first step, we focus on the design of a global diffusion model that couples local and nonlocal models, with fixed-sign diffusivity everywhere: there exist several choices to realize the coupling, not all of them being equivalent. Mathematical analysis of the most promising is currently under way.

Generalized normal modes of a metallic nanoparticle

Participants: Anne-Sophie Bonnet-Ben Dhia, Christophe Hazard. .

In the context of a collaboration with Matias Ruiz (University of Edinburgh) who spent 3 months at POEMS (from october to december, 2021), we have started a work concerning theoretical and numerical aspects of the so-called *plasmonic eigenvalue problem*. This problem can be formulated as follows, considering time-harmonic electromagnetic scattering (at a fixed frequency) by a metallic particle of given (possibly complex) permittivity ε in vacuum. Thanks to integral equation methods or Dirichlet-to-Neumann map techniques, such a problem can be reduced to a problem set on the particle itself. In the absence of incident wave, the reduced problem can be seen as a *non selfadjoint* eigenvalue problem where ε plays the role of an eigenvalue. The eigenfunctions associated to a possible eigenvalue are called *generalized normal modes*. The questions we are interested in are the following. What can be said about the essential / discrete spectrum? Do the generalized normal modes form a complete family (Riesz basis ?) of the natural energy space? How can we compute these eigenfunctions and associated modes?

In order to deal first with a simple situation for which a dispersion equation can be derived, we have considered the two-dimensional Helmholtz equation in a half uniform waveguide, where the end of the waveguide plays the role of a rectangular metallic particle. The problem has been implemented using the finite element library XLiFE++. Some promising numerical results have already been obtained. They show in particular two categories of modes: bulk modes and plasmonic modes (localized near the interface between the metallic particle and vacuum).

6.2 Methods for unbounded domains, Perfectly Matched Layers, Dirichlet to Neumann maps and Half Space Matching method

On the Halfspace Matching Method for real wavenumber

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

We developed for several years a new method for the solution of 2D scattering problems with complex backgrounds, providing an alternative to Perfectly Matched Layers (PML) or other artificial boundary conditions. This method is called the Half-Space Matching (HSM) method. Based on half-plane representations for the solution, the scattering problem is rewritten as a system coupling (1) a standard finite element discretisation localised around the scatterer and (2) integral equations whose unknowns are traces of the solution on the boundaries of a finite number of overlapping half-planes contained in the domain. While satisfactory numerical results have been obtained for real wavenumbers, well-posedness and equivalence of this HSM formulation to the original scattering problem were established only for complex wavenumbers. Our new results, obtained in collaboration with Simon Chandle-Wilde (Reading University) concern the case of a real wavenumber and a homogeneous background. We proved that the HSM formulation is equivalent to the original scattering problem, and so is well-posed, provided the traces satisfy radiation conditions at infinity analogous to the standard Sommerfeld radiation condition. As a key component of our argument we show that, if the trace on the boundary of a half-plane satisfies our new radiation condition, then the corresponding solution to the half-plane Dirichlet problem satisfies the Sommerfeld radiation condition in a slightly smaller half-plane. We expect that this last result will be of independent interest, in particular in studies of rough surface scattering.

We are presently working, still in collaboration with Simon Chandler-Wilde, on the application of the HSM method to solve the diffraction by a 2D perfect infinite wedge. The difficulty is that we have to take into account non-homogeneous data (possibly non-decaying) on the infinite boundaries of the wedge.

The Half-Space Matching method for elastodynamic scattering problems

Participants: Eliane Bécache, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

In collaboration with Antoine Tonnoir from INSA of Rouen, we have extended the Half-Space Matching (HSM) method, first introduced for scalar problems, to elastodynamics, to solve time-harmonic 2D scattering problems, in locally perturbed infinite anisotropic homogeneous media. The HSM formulation couples a variational formulation around the perturbations with Fourier integral representations of the outgoing solution in four overlapping half-spaces. These integral representations involve outgoing plane waves, selected according to their group velocity, and evanescent waves. Numerically, the HSM method consists in a finite element discretization of the HSM formulation, together with an approximation of the Fourier integrals. We have performed numerical results, validating the method for different materials, isotropic and anisotropic, and we have compared them to results obtained with the Perfectly Matched Layers (PML) method. For materials for which PMLs are unstable in the time domain, these results highlight the robustness of the HSM method, contrary to the PML method which is very sensitive to the choice of the parameters.

Evaluation of oscillatory integrals in the Half-Space Matching Method

Participants: Amond Allouko, Anne-Sophie Bonnet-Ben Dhia.

This work, realized during the PhD of Amond Allouko (funded by the European training Network ENHAnCE, in partnership with CEA-List) concerns the Half Space Matching method described just above. A main ingredient of this method is a half-space formula for the outgoing scattered field. This formula is based on a convolution with a half-space Green function, whose partial Fourier transform is known analytically. Then computing this kernel amounts to evaluate Fourier integrals which may become highly oscillating, so that they cannot be evaluated with standard quadrature formulas. The difficulty is aggravated by the fact that oscillations accumulate at the branch points if the function to integrate.

We proposed two ways for improving both the accuracy and the efficiency of the evaluation of this kernel $K(\mathbf{x},\mathbf{y})$. For an evaluation of the kernel far from the point source (large $|\mathbf{x}-\mathbf{y}|$), a simple far-field formula can be used. In other cases, a rotation of the Fourier path in the complex plane allows to get rid of the oscillations and of the singularity at the branch points. For an angle of rotation α, the formula is valid everywhere except when the segment defined by $\mathbf{x}$ and $\mathbf{y}$ makes an angle less than α with the boundary of the half-space. Numerical experiments for the 2D isotropic and anisotropic cases confirm that these two ideas dramatically reduce the cost of evaluation of the kernel.

The second approach, contrary to the far-field formula, can be used to get a factorization of the dense matrices appearing in the HSM method, which could be used to accelerate a GMRES resolution of the system.

This idea has been extended to the case of an isotropic elastic plate, where the expression of the kernel involves both a partial Fourier-tranform, parallel to the plate, and a modal decomposition on the so-called ξ-Lamb modes, in the thickness. We have shown that the components of the Green tensors can be expressed as sums of scalar Fourier integrals, with properties similar to the ones studied in the scalar case. An interesting point is that the method can be adapted to the treatment of inverse modes (with phase and group velocities of opposite signs), by simply changing the sign of the angle of rotation α. This new treatment of the Fourier integrals, combined with other improvments of the implementation, allowed to reduce drammatically the time of execution of the HSM method for a 3D scattering problem in an isotropic elastic plate. A similar treatment of the Fourier integrals for an anisotropic plate still raises many open questions.

The Half-Space Matching method for the junction of open waveguides

Participants: Sarah Al Humaikani, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss.

During the internship and then during the beginning of the PhD of Sarah Al Humaikani, we have been interested by the application of the HSM method to 2D heterogeneous media that could represent a junction of several open waveguides. More precisely, we consider configurations where each half-plane of the HSM is stratified, with a stratification orthogonal to the boundary of the half-plane. This allows to exploit the generalized Fourier transform to derive, for any given trace, an analytic representation of the solution in each half-space.

Up to now, we considered the case where each half-space is either homogeneous, either a planar dioptra (which means that the coefficients of the equation are constant on each side of a planar interface). We generalized to this case the HSM software developed previously for homogeneous half-planes in the XLiFE++ library.

A Rellich type theorem for a class of Helmholtz equations with non-constant coefficients

Participants: Sarah Al Humaikani, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Christophe Hazard.

We proved some years ago the following result : there aren't non trivial square integrable solutions to the Helmholtz equation in a 2D conical domain with opening angle larger than π. We proved this year that this result can be generalized to some Helmholtz equations with non-constant coefficients. More precisely, the conical domain must be a union of half-planes, such that, as in the previous paragraph, each half-plane is either homogeneous, either a planar dioptra.

Radial PML for anisotropic media

Participants: Maryna Kachanovska.

In this work, joint with Martin Halla (post-doc at Uni Göttingen) and Markus Wess (post-doc at TU Wien) we have answered the following question : since anisotropic media does not exhibit backward propagating waves in radial directions, are radial PMLs stable in this case?

We start with the simplest model of this kind: anisotropic wave equation, for which much can be computed analytically. Our numerical experiments were quite inconclusive: in fact we noticed instabilities only on fine discretizations. Again, to analyze the problem, we work in the Laplace domain. The analysis of the complex-scaled fundamental solution and underlying PDE revealed the following:

- the PML problem is well-posed;
- at the continuous level, if the source term is located far enough from the interface between the PML and the physical media, the respective solution can be proven to be stable; the distance between the source and the PML depends on the anisotropy;
- however, the problem itself is not stable, more precisely, the associated spatial operator has essential spectrum in the right-half complex plane.

This resembles very much our previous findings for frequency-domain behaviour of Cartesian PMLs for time-harmonic anisotropic acoustic wave equation, done in the framework of the postdoc of Maria Kazakova.

Although we can ensure stability of the continuous solution by choosing sufficiently large physical domain, this does not seem true for the discrete solution. Therefore, we suggest the following: the radial

PMLs can be shown to be stable if instead of the classical Bérenger's scaling $r + \frac{\sigma(r-r_{pml})}{s}$, we make use of the complex frequency shifted scaling $r + \frac{\sigma(r-r_{pml})}{s+\gamma}$, with $\frac{\sigma}{\gamma} < c$, where c depends on the anisotropy of the media. In general, if we truncate such PMLs at the length L, we cannot expect their convergence, unless L depends on the final time (which is what we would like to avoid). Therefore, instead we do not truncate the PMLs and discretize the problem with the help of the infinite elements. We have almost finished the manuscript summarizing our findings.

Construction of transparent conditions for electromagnetic waveguides

Participants: Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Aurélien Parigaux.

This work is done in the framework of the PhD of Aurélien Parigaux, co-advised by Anne-Sophie Bonnet-Ben Dhia and Lucas Chesnel from Inria team IDEFIX.

Electromagnetic waveguide-based components play an important role in many application areas. We are particularly interested in configurations where several semi-infinite guides interact in a bounded zone of space, which is discretized by finite elements. In this context, a major difficulty is to truncate the guides by imposing well-chosen transparent conditions, in order to obtain a well-posed problem corresponding to the initial problem. This question is very well understood for scalar models of acoustic guides, but remains a delicate subject for heterogeneous electromagnetic guides.

Up to now, we have considered the case of an homogeneous electromagnetic waveguide with a simply connected cross-section. It is well-known in this case that a transparent boundary condition which links the tangential electric field to the tangential magnetic field on the artificial boundary, called EtM condition, can be written using a modal expansion on Transverse Electric and Transverse Magnetic modes. Our contributions are the following ones.

- We have proved that the problem with a truncated modal expansion, keeping N modes in the expansion, is well-posed if N is large enough, and that the error produced by the truncation decays exponentially with N, and with L, where L is the distance of the artificial boundary to the sources.
- We have also proposed a new formulation whith a transparent boundary condition that we call EtM with overlap, because it links the tangential electric field on an internal cross-section to a Robin type quantity, linear combination of the tangential electric and magnetic fields on the artificial boundary. The advantage is the compactness of the non-local term coming from the transparent boundary condition.
- Finally, we have proposed another transparent condition with overlap, linking both the tangential electric and magnetic fields on the internal boundary to the same Robin quantity as previously on the artificial boundary. One main advantage of this second approach is that it works also for tranversely heterogeneous waveguides.

PML-BIE methods for unbounded interfaces

Participants: Anne-Sophie Bonnet-Ben Dhia, Luiz Faria.

There are several important applications where one must solve a scattering problem in a domain with infinite boundaries; e.g. seismic waves in a stratified medium, or water waves in the ocean. In order to handle such infinite interfaces in a boundary integral equation context, a few options are available. For simple geometries, one can construct a problem specific Green function which incorporates the imposed boundary condition on all but a bounded portion of the interface, thus reducing the problem again to integrals over bounded curves/surfaces. This has the advantage of being conceptually simple provided such problem-specific Green function can be efficiently computed. Unfortunately, that is usually not the case, and the computation of problem-specific Green functions involves challenging integrals which must be approximated numerically.

An alternative approach consists of utilizing the free-space Green function — readily available for many PDEs of physical relevance — in conjunction with a truncation technique. For non-dissipative problems, the slow (algebraic) decay — or even logarithmic growth — of the Green function makes the choice of truncation technique an important aspect which needs to be considered in order to reduce the errors associated with the domain's truncation. An easy-to-implement solution, the so-called windowed Green function approach, has been proposed and validated in several configurations.

We are currently investigating the interest of using instead a complex-scaled Green function, which amounts to combine the method of perfectly-matched-layers (PMLs) and boundary integral equations.

We have applied this idea to the 2D linear time-harmonic water wave problem, in finite or infinite depth, writing a complex-scaled integral equation on the free surface. The formulation uses only simple function evaluations (e.g. complex logarithms and square roots). Let us mention that because the water waves are surface waves, the windowed Green function approach does not work for this problem.

Numerical results show that the error decays exponentially with respect to the distance of truncation.

Another advantage of our method is that the formulation has a simple quadratic dependence with respect to the frequency, and is well-suited for computing complex scattering frequencies.

An article is currently under review concerning this work.

6.3 Fast solution of boundary integral equations

Boundary Element - Finite Element coupling for transient fluid-structure interaction

Participants: Marc Bonnet, Stéphanie Chaillat, Alice Nassor.

This study is done in collaboration with Bruno Leblé (Naval Group). It aims at developing computational strategies for modelling the impact of a far-field underwater explosion shock wave on a structure, in deep water. An iterative transient acoustic-elastic coupling is developed to solve the problem. Two complementary methods are used: the Finite Element Method (FEM), that offers a wide range of tools to compute the structure response; and the Boundary Element Method (BEM), more suitable to deal with large surrounding acoustic fluid domains. We concentrate on developing a transient FEM-BEM coupling algorithm with a fast convergence. Since the fast transient BEM is based on a fast multipole-accelerated Laplace-domain BEM (implemented in the in-house code COFFEE), extended to the time domain by the Convolution Quadrature Method (CQM), we have proposed global-in-time Robin-Robin coupling iterations (the coupling surface being either the fluid-solid interface or immersed in the fluid), proved the convergence of this iterative process, and demonstrated their effectiveness on various spatially 2D simulations. Next, this approach has been implemented in non-intrusive fashion into `code_aster` (the open-source EDF general-purpose structural FEM analysis library) in order to demonstrate its adaptability to existing industry computational platforms, and extended that implementation to the case of partially-immersed elastic structures representative of configurations from the naval industry.

General-purpose kernel regularization of boundary integral equations via density interpolation

Participants: Luiz Faria, Marc Bonnet.

This research is done in collaboration with Carlos Pérez-Arancibia (University of Twente, Netherlands) and Thomas Anderson (Univ. of Michigan, USA). We initially developed a general high-order kernel regularization technique applicable to all four integral operators of Calderón calculus associated with linear elliptic PDEs in two and three spatial dimensions. The proposed technique relies on interpolating the density function around the kernel singularity in terms of solutions of the underlying homogeneous

PDE, so as to recast singular and nearly singular integrals in terms of bounded (or more regular) integrands. We developed a simple interpolation strategy which, unlike previous approaches, does not entail explicit computation of high-order derivatives of the density function along the surface. Furthermore, the proposed approach is kernel- and dimension-independent, thus making the procedure applicable, in principle, to any PDE with known Green's function. In the initial work we have focused on Nyström methods for the (scalar) Laplace and Helmholtz equations and the (vector) elastostatic and time-harmonic elastodynamic equations. We have extended this approach to the evaluation of volume potentials of the kind involved in e.g. Lippmann-Schwinger volume integral equations, and an article is currently under review.

Windowed Green function method for wave scattering by periodic arrays of 2D obstacles

Participants: Luiz Faria.

This research is done in collaboration with Carlos Pérez-Arancibia (University of Twente, Netherlands), Thomas Strauszer-Caussade (PUC, Chile), and Augustín Fernandez-Lado (Intel Coorporation, Oregon, USA). This work introduces a novel boundary integral equation (BIE) method for the numerical solution of problems of planewave scattering by periodic line arrays of two-dimensional penetrable obstacles. Our approach is built upon a direct BIE formulation that leverages the simplicity of the free-space Green function but in turn entails evaluation of integrals over the unit-cell boundaries. Such integrals are here treated via the window Green function method. The windowing approximation together with a finite-rank operator correction—used to properly impose the Rayleigh radiation condition—yield a robust second-kind BIE that produces superalgebraically convergent solutions throughout the spectrum, including at the challenging Rayleigh–Wood anomalies. The corrected windowed BIE can be discretized by means of off-the-shelf Nyström and boundary element methods, and it leads to linear systems suitable for iterative linear algebra solvers as well as standard fast matrix–vector product algorithms. A variety of numerical examples demonstrate the accuracy and robustness of the proposed methodology.

High-order Boundary Integral Equations on implicitly defined surfaces

Participants: Luiz Faria, Dongchen He.

This research is being done in collaboration with Aline Lefebvre-Lepot (CMAP), and in the context of the Ph.D. thesis of Dongchen He. We are developing a method for accurately solving boundary integral equations on implicitly defined surfaces in $\mathbb{R}^d$. The method relies on combining a dimension-indepent technique for generating a high-order surface quadrature on level-set surfaces, with the general-purpose density interpolation method for handling the singular and nearly-singular integrals ubiquitous in boundary integral formulations. The proposed methodology, based on a Nystrom discretization scheme, bypasses the need for generating a body conforming mesh for the implicit surface, allowing in principle for an efficient coupling between a robust dynamic level-set representation of deforming surfaces, and boundary integral equation solvers. Particular attention is being paid to the computation of singular integrals when only a surface quadrature is available (i.e. in the absence of an actual mesh). We believe such techniques could prove useful in applications involving microscopic flows governed by the Stokes equations; in particular, the simulation of micro-swimmers and droplet microfluidics.

Modelling the sound radiated by a turbulent flow

Participants: Stéphanie Chaillat, Jean-François Mercier, Louise Pacaut.

The aim of this PhD study, done in collaboration with Gilles Serre (Naval Group), is to develop an optimized method to determine numerically the 3D Green's function of the Helmholtz equation in presence of an obstacle of arbitrary shape. The determination of such so-called adapted Green's function is required to solve the Lighthill's equation, notably giving the hydrodynamic noise radiated by a ship. The case of a rigid obstacle satisfying a Neumann boundary condition has been treated in a first PhD and this second PhD extends the study to the penetrable case. In the fluid case, for two fluids separated by interface, first an integral equation is derived, expressing the adapted Green's function versus the two free space Green's functions associated to both fluids. Then a Boundary Element Method (BEM) implemented in the code COFFEE is used to compute the adapted Green's function. In order to consider realistic geometries in a reasonable amount of time, fast BEMs are used: fast multipole accelerated BEM and hierarchical matrix based BEM. The validity of these two approaches is tested on a simple sphere geometry for which an analytic solution can be determined. Moreover the numerically costly application to the screen effect of a bubble curtain, a vertical layer of many air bubbles in the ocean, is successfully implemented. Extension to an elastic body surrounded by a fluid is under consideration. The difficulty is then to use the tensorial elastic Green's function.

Asymptotic based methods for very high frequency problems.

Participant: Eric Lunéville.

This research is developed in collaboration with Marc Lenoir (retired) and Daniel Bouche (CEA).

It has recently been realized that the combination of integral and asymptotic methods was a remarkable and necessary tool to solve scattering problems, in the case where the frequency is high and the geometry must be finely taken into account.

In order to implement the high-frequency approximations that we are developing as part of these hybrid HF/BF methods, we have introduced new geometric tools into the XLiFE++ library, in particular splines and B-Splines approximations as well as parameterizations to access quantities such as curvature, curvilinear abscissa, etc. We have also achieved the interface between the OpenCascad library and the XLiFE++ library, which allows us to manage complex geometric situations (cylinder and sphere intersection for example). In parallel, we have completed the implementation of 2D HF approximations in the shadow-light transition zone based on the Fock function and the diffraction by a 2D corner using asymptotics approximation. More recently we began to investigate the 3D case. As a first step we developed in XLiFE++ some new tools to compute geodesics on any surfaces (parametrized or only meshed). Now, the work consists in extending the 2D asymptotic expansions along the geodesics.

High-frequency estimates and error bounds on the h-BEM for the Helmholtz exterior Neumann problem

Participants: Pierre Marchand.

This research is done in collaboration with Jeffrey Galkowski (University College London) and Euan Spence (University of Bath).

We studied how to solve the Helmholtz equation posed in the exterior of a smooth obstacle, with Neumann boundary conditions, and using second-kind boundary integral equations (BIEs). In the context of Galerkin discretisation, it is important to understand the behaviour of Galerkin error when the frequency increases, and high-frequency estimates for the considered boundary integral operators are usually necessary to explicit the dependency between the discretisation parameters and the frequency.

In the case of Dirichlet boundary condition, results are well-known in the literature for the standard second-kind BIE, but in the case of Neumann boundary condition, where regularisation techniques are commonly used, such estimates are rare. Thus, we developed new high-frequency estimates for a particular boundary integral equation using a single-layer operator with complex frequency as regularising operator, which led to the first result (that we know of) about frequency-explicit bound for the discretisation error of a second-kind formulation for the Helmholtz equation with Neumann boundary condition.

Diffraction by fractal screens

Participants: Patrick Joly, Maryna Kachanovska, Zoïs Moitier.

This topic, in the framework of the HyBox project, is the subject of the Post-Doc of Zoïs Moitier.

The goal of the postdoc is to develop efficient numerical method to compute the scattering by fractal screen. The motivation comes from applications to, for example, fractal antenna engineering. A fractal antenna has a shape with self-similar features at different scales which allows the antenna to operate at multiple frequencies or even for a range of frequencies.

Recently, integral equations for the solution of the Helmholtz scattering problems with Dirichlet conditions have been proposed in the literature. The specificity of these equations is that they involve integrals that use the standard Helmholtz Green's function but the (non standard) Haussdorf measure on fractal sets, instead of the Lebesgue's one. This makes the resolution of these equations particularly intricate. The discretization approach we adopt is to construct a fractal based mesh by dividing the original fractal set into small subfractals (whose size is the equivalent of a mesh stepsize for usual finite elements) and to use a Galerkin approximation with piecewise constant functions. Our contributions have been developed in two directions

- Numerical computation of fractal integrals. Our goal has been to propose new (high order) quadrature formula for such integrals. The leading idea is to exploit the self-similar structure on the integration domains and the corresponding homogeneity properties of the Hausdorff measure. This permits, for N given quadrature points, to characterize the corresponding weights as the components of a normalized eigenvector assiociated to the eigenvalue 1 of a $N \times N$ matrix that can be computed explicitly.
- Efficients resolution of the resulting linear systems. For this, our approachis to adapt the idea of H matrices, based on block low-rank approximations (via compression) of the dense matrix issued from the finite dimensional discretization of the integral equation. Here again, the construction of the compressible blocks is guided by the self-similarity of the fractal, which moreover permits to accelerate the construction of the compressed blocks.

6.4 Accelerated finite element solvers and domain decomposition methods

FEM-BEM coupling in the context of domain decomposition methods

Participants: Antonin Boisneault, Pierre Marchand.

As part of the exploratory action OptiGPR3D, our aim is to combine coupling the Finite Element Method (FEM) and the Boundary Element Method (BEM) with domain decomposition methods. This integration is intended to provide a rapid iterative solver with guaranteed convergence, even when dealing with complex media. In Antonin Boisneault's internship, we first explored diverse formulations for FEM-BEM coupling formulations, incorporating various transmission conditions. Our objective was to assess their effectiveness when solved with an iterative solver such as GMRES, along with different types of preconditioners. We observed how the formulations using combined operators were more efficient in terms of number of iterations, and more robust to restart. We also compared the properties of a classical FEM-BEM formulations with generalized impedance transmission conditions, with a less classical formulations proposed in the PhD thesis of Boris Caudron. In the context of the PhD thesis of Antonin, in collaboration with Marcella Bonazzoli (Inria Idefix) and Xavier Claeys (Sorbonne University), we plan to extend this study of FEM-BEM couplings to the multitrace framework, which is a domain decomposition approach robust to junction point with exponential convergence.

Coupling of discontinuous Galerkin and pseudo-spectral methods for time-dependent acoustic problems

Participants: Axel Modave, Rose-Cloé Meyer.

This research topic is developed in collaboration with Hadrien Bériot (SIEMENS) and Gwénaël Gabard (LAUM). It corresponds to the post-doctoral work of Rose-Cloé Meyer, funded in part by the french "Plan de relance" program and SIEMENS. In this project, we have studied the coupling of a pseudo-spectral (PS) method with a nodal discontinuous Galerkin (DG) finite element method to solve time-domain acoustic wave problems. The PS method is very efficient, but is limited to rectangular shaped domains. In contrast, the DG method is easily applied to complex geometries, but can become costly when considering large-scale problems. The idea is to combine the strengths of these two methods: the PS method is used on the part of the domain without geometric constraint, while the DG method is used around the PS region to represent the geometry accurately. An overlap is introduced between the two domains to implement the coupling between the two numerical methods. A windowing in the spectral domain improves the convergence of the overall model. The exchange of information between the two domains is made possible even with non-conforming meshes, through an interpolation of the solution. We have validated the coupling method with one and two-dimensional numerical results, and we have studied the influence of different parameters on the accuracy of the coupling.

A hybridizable DG method with transmission variables for time-harmonic problems

Participants: Axel Modave, Simone Pescuma.

This research topic is developed in collaboration with Théophile Chaumont-Frelet (Inria, Rapsodi) and Gwénaël Gabard (LAUM), within the framework of the WavesDG ANR project. We have proposed a new hybridizable discontinuous Galerkin method, named the CHDG method, for solving time-harmonic scalar wave propagation problems with homogeneous media. This method relies on a standard discontinuous Galerkin scheme with upwind numerical fluxes and high-order polynomial bases. Auxiliary unknowns corresponding to characteristic variables are defined at the interface between the elements, and the physical fields are eliminated to obtain a reduced system. The reduced system can be written as a fixed-point problem that can be solved with stationary iterative schemes. Numerical results with 2D benchmarks have been proposed to study the performance of the approach. Compared to the standard HDG approach, the properties of the reduced system are improved with CHDG, which is more suited for iterative solution procedures. In particular, iterative solution procedures with CGNR or GMRES required smaller numbers of iterations with CHDG. We are currently extending the CHDG method to scalar problems with heterogeneous media. In particular, we are considering standard numerical upwind fluxes as well as other standard fluxes and non-standard higher-order fluxes. Theoretical results obtained in the homogeneous case can be extended to the heterogeneous case for some of these fluxes. Preliminary 1D and 2D results show that the extended CHDG method is still competitive with the standard HDG method.

Convergence study of the iterative finite element solution of Helmholtz problems with near-resonance phenomena

Participants: Axel Modave, Pierre Marchand, Timothée Raynaud.

In this research topic, we have considered the iterative finite element solution of Helmholtz problems with near-resonance phenomena. Our goal was to interpret the convergence of GMRES for solving the resulting linear systems, and to do connection with the original physical problems. In practice, it is difficult to interpret the convergence of Krylov methods. In the context of the internship of Timothée, we have studied convergence results based on harmonic Ritz values. Superlinear convergence behavior can be interpreted thanks to this approach. We have used this technique to understand the GMRES convergence for a 2D cavity benchmark implemented in a MATLAB finite element code. We observed that the superlinear convergence behavior is related to the approximation of the small eigenvalues of the matrix by the small harmonic Ritz values computed during the iterations. Finally, we have studied the influence of preconditions. In the context of the PhD thesis of Timothée, in collaboration with Victorita Dolean (TU/e, The Netherlands), we plan to extend this study, and to investigate novel preconditioning strategies.

Non-overlapping Domain Decomposition for wave propagation problems

Participants: Patrick Joly, Jérémy Héleine.

This work is a continuation of the PhD thesis of É. Parolin on non-local and contitutes the subject of the short post-doctoral stay of Jérémy Heleine in the framework of the RAPID Project Hybox. A first contribution was to write and analyse a dual version of the multi-trace formulation of Claeys-Parolin at the continuous level. This formulation is adapted to the use of midex hybrid finite elements for the discretization of the local Helmholtz problems.

The interest of the multi-trace formulation is that its leads to an exponential convergence of the iterative domain decomposition algorithm even in the presence of junction points. A drawback is that the communication process between subdomains couples subdomains which are not neighbour the one from the other. To avoid this, J. Heleine has proposed a first "quasi-localized" multi-trace formulation based ofn the use of a partition of unity wirh cut-off functions. The analysis and the implementation of the method remain to be done.

6.5 Inverse Problems, Invisibility and Optimization

Scattered wavefield in the stochastic homogenization regime

Participants: Laure Giovangigli.

This is a joint work with Josselin Garnier (X-CMAP) and Pierre Millien (Institut Langevin). In the context of providing a mathematical framework for the propagation of ultrasound waves in a random multiscale medium, we consider the scattering of classical waves (modeled by a divergence form scalar Helmholtz equation) by a bounded object with a random composite micro-structure embedded in an unbounded homogeneous background medium. Using quantitative stochastic homogenization techniques, we provide asymptotic expansions of the scattered field in the background medium with respect to a scaling parameter describing the spatial random oscillations of the micro-structure. Introducing a boundary layer corrector to compensate the breakdown of stationarity assumptions at the boundary of the scattering medium, we prove quantitative L^2 and H^1 error estimates for the asymptotic first-order expansion. The theoretical results are supported by numerical experiments.

Computation of the interior transmission eigenvalues in presence of strongly oscillating singularities

Participants: Anne-Sophie Bonnet-Ben Dhia, Florian Monteghetti.

In the context of time-harmonic scattering by a bounded penetrable scatterer, interior transmission eigenvalues correspond, when they are real, to discrete frequencies for which there exists an incident wave which does not scatter. At such frequencies, inversion algorithms such as the linear sampling method fail. Real interior transmission eigenvalues are a part of a larger spectrum made of complex values, which has been largely studied in the case where the difference between the parameters in the scatterer and outside does not change sign on the boundary. In collaboration with Lucas Chesnel (INRIA team IDEFIX), we obtained some years ago some results for a 2D configuration where such sign-change occurs. The main idea was that, due to very strong singularities that can occur at the boundary, the problem may lose Fredholmness in the natural H^1 framework. Using Kondratiev theory, we proposed a new functional framework where the Fredholm property is restored. This is very similar (while more intricate) to what happens for the plasmonic eigenvalue problem in presence of a corner of negative material.

This explains why we decided to extend the numerical method we used for plasmonic eigenvalues to interior transmission eigenvalues. It has been already checked that a naive finite element computation does not converge, and that the convergence is restored by using some complex scaling near the singular point.

Inverse problems in oceanography

Participant: Laurent Bourgeois, Raphael Terrine.

This work is devoted to two different inverse problems which arise in oceanography. The first one is the identification of a tsunami from measurements of the free surface deformation, such tsunami being characterized by a brutal displacement of the bottom surface of the ocean. The second one is the bathymetry problem, which consists in recovering the underwater depth by using the same measurements as in the previous problem. In these two problems, the goal is to retrieve some sea bottom parameters from surface data, but while the loading is passive in the first problem (the tsunami is a natural phenomenon), it is active in the second one (a source generated artificially is required). These two problems are severely ill-posed. At the beginning of the PHD thesis of Raphael Terrine, for simplicity we have first considered some potential models in the frequency regime and in two dimensions. The strategy we use is composed of two steps. Firstly, we retrieve the volumic 2d solution with the help of a mixed formulation of the Tikhonov regularization, secondly we reconstruct the unknown parameter of the sea bottom by solving a 1d problem. We use an iterative method to determine the regularization parameter as a function of the amplitude of noise given by the Morozov principle. One of the goal is to achieve conclusions on the right choice of the model used in the inversion process, depending on the frequency.

Shape optimization problems involving slow viscous fluids

Participant: Marc Bonnet.

Work done in collaboration with Shravan Veerapaneni and his group (University of Michigan, USA) and Thomas Anderson (Rice University, USA).

This collaboration addresses the design and implementation of computational methods for solving shape optimization problems involving slow viscous fluids modelled by the Stokes equations. We have developed a new boundary integral equation (BIE) approach for finding optimal shapes of peristaltic pumps that transport viscous fluids, as well as dedicated formulas for computing the shape derivatives of the relevant cost functionals and constraints, expressed in boundary-only form. By employing these formulas in conjunction with a BIE approach for solving forward and adjoint problems, we completely avoid the issue of volume remeshing when updating the pump shape as the optimization proceeds. This is especially useful when the fluid carries objects (e.g. particles, deformable vesicles) whose motion is not known beforehand. Significant cost savings are achieved as a result, and we demonstrate the performance on several numerical examples. We also investigate methods allowing the numerical evaluation of negative fractional Sobolev norms on arbitrary domains, which in our context are involved in the quantification of the quality of mixing for concentrations carried by fluid flows. We formulated and demonstrated a treatment based on Padé approximants, and are currently working on an alternative approach based on volume integral equations associated with fractional-order PDEs.

Error-in-constitutive-relation (ECR) framework for the wave-based characterization of linear viscoelastic solids

Participant: Marc Bonnet.

Work done in collaboration with Bojan Guzina and Prasanna Salasiya, University of Minnesota, USA.

In this collaboration, we develop an error-in-constitutive-relation (ECR) approach toward the full-field characterization of linear viscoelastic solids described by free energy and dissipation potentials. Assuming the availability of full-field interior kinematic data, the constitutive mismatch between the kinematic quantities and their “stress” counterparts, commonly referred to as the ECR functional, is established with the aid of Legendre-Fenchel gap functionals linking the thermodynamic potentials to their energetic conjugates. We then proceed by introducing the modified ECR (MECR) functional as a linear combination between its ECR parent and the kinematic data misfit, computed for a trial set of constitutive parameters. The resulting stationarity conditions then yield a coupled forward-adjoint evolution problem, which allows us to establish compact expressions for the MECR functional and its gradient with respect to the viscoelastic constitutive parameters. The formulation is established assuming either time-domain or time-harmonic data. These developments have so far been implemented in a two-dimensional time-harmonic setting, and demonstrated on the multi-frequency MECR reconstruction of a piecewise-homogeneous standard linear solid and a smoothly-varying Jeffreys viscoelastic material.

Active design of diffuse acoustic fields in enclosures

Participant: Marc Bonnet.

Work done in collaboration with Wilkins Aquino (Duke University, USA) and Jerry Rouse (Sandia Natl. Lab., USA).

This collaboration is concerned with the definition and demonstration of a numerical framework for designing diffuse fields in rooms of any shape and size, driven at arbitrary frequencies. In particular, we aim at overcoming the Schroeder frequency lower limit for generating diffuse fields in an enclosed space. We formulate the problem as a Tikhonov regularized inverse problem and propose a low-rank approximation of the spatial correlation that results in significant computational gains. Our approximation is applicable to arbitrary sets of target points and allows us to produce an optimal design at a computational cost that grows only linearly with the (potentially large) number of target points. We demonstrate the feasibility of our approach through numerical examples where we approximate diffuse fields at frequencies well below the Schroeder lower limit.

6.6 Asymptotic analysis, homogenization and effective models

Acoustic surface homogenization in presence of a flow

Participants: Jean-François Mercier.

The surface homogenization technique in acoustics applied to model a rigid surface with a periodic shape by an equivalent boundary condition set on a flat surface is well established when the fluid is at rest. We have extended this approach to a fluid in motion along the periodic surface. We have considered a potential mean flow satisfying the Laplace equation and the acoustic perturbations are then found to satisfy a modified Helmholtz equation. A difficulty is that the acoustic perturbations are coupled to the unknown mean flow. Both equations are homogenized and effective boundary conditions on a mean flat surface are derived. These equivalent boundary conditions are validated by comparison with a Finite Element calculation done on the actual periodic geometry.

Generalized impedance boundary conditions with vanishing or sign-changing coefficient

Participants: Laurent Bourgeois.

This work is a collaboration with Lucas Chesnel (INRIA/IDEFIX) and concerns a Laplace type problem with a generalized impedance boundary condition of the form $\partial_\nu u = -\partial_x(g\partial_x u)$ on a flat part Γ of the boundary. Here ν is the outward unit normal vector to $\partial\Omega$, g is the impedance parameter and x is the coordinate along Γ. Such problems appear for example in the modelling of small perturbations of the boundary. In the literature, the cases $g = 1$ or $g = -1$ have been investigated. In this work, we address situations where Γ contains the origin and $g(x) = H(x)x^\alpha$ or $g(x) = -\text{sign}(x)|x|^\alpha$ with $\alpha \geq 0$. Here H and sign denote respectively the Heaviside and the sign functions. In other words, we study cases where g vanishes at the origin and changes its sign. The main message is that the well-posedness in the Fredholm sense of the corresponding problems depends on the value of α. For $\alpha \in [0, 1)$, we show that the associated operators are Fredholm of index zero while it is not the case when $\alpha = 1$. The proof of the first results is based on the reformulation as 1D problems combined with the derivation of compact embedding results for the functional spaces involved in the analysis. The proof of the second results relies on the computation of singularities and the construction of Weyl's sequences. We also discuss the equivalence between the strong and weak formulations, which is not straightforward. Finally, we provide simple numerical experiments which seem to corroborate the theorems.

Propagation of ultrasounds in complex biological media

Participants: Laure Giovangigli, Quentin Goepfert.

This is a joint work with Josselin Garnier (X-CMAP) and Pierre Millien (Institut Langevin). This project aims at modelling and studying the propagation and diffusion of ultrasounds in complex multi-scale media in order to obtain quantitive images of physical parameters of these media.

The propagation of ultrasounds in biological tissues is a complex multi-scale phenomenon : the scattered wave is produced by small (compared to the wavelength) inhomogeneities randomly distributed throughout the medium. In order to characterize the response of this medium to an incident plane wave, we perform an asymptotic expansion of the scattered wave with respect to the size of the inhomogeneities using stochastic homogenisation techniques. The difficulties lie in the transmission conditions at the boundary of the medium. We derive quantitative error estimates given that the random distribution of inhomogeneities verifies mixing properties. Finally we present numerical simulations to illustrate and validate our results.

Effective dielectric properties of bio-materials in humid environment

Participants: Laure Giovangigli.

This is a joint work with Pierre Millien (Institut Langevin), Hatem Zaag (Paris 13), Amina Gharbi (Sfax) and Nabil Derbel (Sfax). In humid environment bio-materials exhibit a change from resistive to capacitive electrical properties and can hence be used in the design of humidity bio-sensors. The goal of this work is to model the influence of the adsorption process at the nano-scale on the effective electric properties. The adsorption process is modeled by random alterations of the conductivity on small thin patches at the surface of the material. Effective properties are then derived using stochastic homogenization techniques. The cylindrical geometry of the bio-material sample and in particular its curvature have to be taken into account in the definition of the first order correctors. In her internship Nour El Haddad derived the equations for those curvature-dependent correctors, and exhibited the effective homogenized surface conductivity. She also conducted numerical simulations in order to confront them with the experimental results obtained by Amina Gharbi (Sfax) and Nabil Derbel (Sfax).

Modelling a thin layer of randomly distributed nano-particles

Participants: Sonia Fliss, Laure Giovangigli, Amandine Boucart.

This is a joint work with Bruno Stupfel from CEA-CESTA. We study the time-harmonic scattering by a heterogeneous object covered with a thin layer of nanoparticles. The size of the particles, their distance between each other and the layer's thickness are all of the same order but small compared to the wavelength of the incident wave. Solving numerically Maxwell's equation in this context is very costly. To circumvent this, we propose, via a multi-scale asymptotic expansion of the solution, an effective model where the layer of particles is replaced by an equivalent boundary condition. We consider two configurations :

- a two-dimensional random distribution of particles invariant in the tangential direction. Maxwell's equations reduce then to the Helmholtz equation with either Neumann or Dirichlet boundary conditions on the particles. The coefficients that appear in the equivalent boundary condition depend on the solutions to corrector problems of Laplace type defined on unbounded random domains. Under the assumption that the particles are distributed given a stationary and ergodic random point process, and when Dirichlet boundary conditions are imposed on the inclusions, we prove that those problems admit a unique solution in the proper spaces. If the random process is mixing then we can establish quantitative error estimates between the effective and exact solutions.
- a three-dimensional periodic distribution of particles. We take advantage here of the periodicity to restrict our domain of study to an infinite vertical strip with periodic boundary conditions on the lateral boundaries.

For both configurations, numerical simulations illustrate our theoretical results.

Enriched homogenized model in the presence of boundaries for wave equations

Participants: Laure Giovangigli, Sonia Fliss.

We study the time-dependent scalar wave equation in presence of a periodic medium when the period is small compared to the wavelength. The classical homogenization theory enables to derive an effective model which provides an approximation of the solution. But this effective model does not take into account the long time dispersive effects which appears naturally in periodic media. This is well known since the works of Santosa and Symes in the 90s and high order effective models involving high order differential operators of higher orders (at least 4) have been proposed for infinite periodic media.

The first question concerns the presence of boundaries or interfaces. Proposing boundary conditions for these models remain open questions. Note that one of the difficulty is that one has to derive variational conditions for differential operators of order 4 from original variational conditions for operators of order 2. The past few years, we have proposed a new asymptotic expansion which takes into account the microscopic phenomena near the boundaries. Our approach enables to propose appropriate boundary conditions for these models. The well-posedness of such models is under study. We want then to tackle similar questions for Maxwell's and elastodynamics equations. This work is done in collaboration with Bruno Lombard (LMA Marseille) and Remi Cornaggia (Institut d'Alembert, Sorbonne Université).

The second question concerns the extension of this long time homogenization to random media. Based on error estimates, one can show that for periodic media, the time scale at which dispersive effects appear is of order (ϵ^{-2}) when ϵ is the period of the medium. This is not so clear for random media where it seems that this timescale seems to be much shorter. In the internship of E. Meddouri and O. Empereur, we have tried to quantify this timescale numerically.

Galerkin Time-Domain Foldy-Lax models

Participants: Maryna Kachanovska, Adrian Savchuk.

The Foldy-Lax model is an asymptotic model used to compute the solution to the problem of scattering by small obstacles. While this subject had been fairly well-studied in the frequency-domain, its time-domain analysis is still in its infancy stage. In our previous work, we have suggested a construction of an asymptotic model as a Galerkin spatial semi-discretization of associated boundary integral formulations. The main idea is to choose the basis functions in a way that the convergence of the method is ensured not by increasing the cardinality of the Galerkin basis, but rather by decreasing the size of the obstacles. We have submitted a manuscript where we presented and analyzed the method for the asymptotic sound-soft scattering by circles.

Recently, we have shown that the same choice of the Galerkin basis as for the sphere case cannot yield convergence for particles of arbitrary shape (in 3D). This was confirmed by our numerical experiments. Therefore, we suggested an alternative choice of the basis, inspired by existing works of Sini et al. Namely, now we choose basis functions as equilibrium densities. The analysis however is more involved in this case, and is based on two ingredients: a direct (but not orthogonal) decomposition of the functional space for the error and handling low- and high-frequencies separately. Nonetheless, we are able to arrive at the relative second-order convergence results. The implementation of the method to validate the theoretical results is currently on the way.

Asymptotic modeling of thin periodic layers with high-contrast inclusions for electromagnetic waves

Participants: Sonia Fliss, Patrick Joly, Florian Monteghetti.

The broad objective of this study is to model classes of composite materials as effective boundary or transmission conditions for the full Maxwell's equations (i.e. we do not consider transverse approximation). Mathematically, we assume that :

(i) the composite material consists of a periodic array of sub-wavelength structures;

(ii) the ratio between the structure's thickness and the incident wavelength is small.

(ii) the period of the array and the thickness of the layer have the same order of magnitude.

These assumptions enable to use techniques from asymptotic analysis and homogenization theory to derive the homogenized model. The motivation behind this work is computational: each derived asymptotic model offers a cheaper alternative to a direct numerical simulation, which is challenging due to the multi-scale nature of the problem leading to prohibitively expansive meshes. Once implemented, the derived asymptotic models would for example enable parametric design studies.

In our work, we have derived first-order transmission conditions for the two following families of materials:

1. Array of penetrable inclusions.
2. Array of perfect electric conductor (PEC) wires.

The main outlook of this work is to derive an asymptotic model for an array of high-contrast and simply-connected inclusions above a perfectly-conducting surface.

Wave diffraction by thin finite periodic layers

Participants: Cédric Baudet, Sonia Fliss, Patrick Joly.

This is the subject of the PhD thesis of C. Baudet which is part of the HyBox project (task T1.1).

In this work, we consider the diffraction of waves by an object partially covered by a periodic layer whose thickness tends to 0. This situation can model industrial applications where the layer is often made of a metamaterial with unusual wave propagation properties. For layers covering the whole object, this problem already has known solutions. In our case where the covering is only partial, the difficulty is to treat the tips of the layer for where no effective model is known so far.

Our approach is made of three steps

Step 1: perform an asymptotic analysis of the solution when the layer's thickness tends to 0. This uses a technique of domain splitting in coupled subdomains, mainly the zone near the layer and the rest of the exterior domain.

Step 2: design approximated models that replace the layer with an effective boundary condition inspired by the asymptotic development found in step 1.

Step 3: set up numerical methods to solve the diffraction problem using the previous models, thus reducing a lot the computational cost compared to a naive method that implies finely meshing the layer.

For now we have completed step 1 for a homogeneous layer, and numerics are in progress.

Enriched homogenized model in the presence of boundaries for wave equations

Participants: Sonia Fliss, Corentin Kilque.

We study the time-dependent scalar wave equation in presence of a periodic medium when the period is small compared to the wavelength. The classical homogenization theory enables to derive an effective model which provides an approximation of the solution. But this effective model does not take into account the long time dispersive effects which appears naturally in periodic media. This is well known since the works of Santosa and Symes in the 90s and high order effective models involving high order differential operators of higher orders (at least 4) have been proposed for infinite periodic media. Dealing with boundaries and proposing boundary conditions for these models were open questions. Note that one of the difficulty is that one to derive variational conditions for differential operators of order 4 from original variational conditions for operators of order 2. The past few years, we have proposed a new asymptotic expansion which takes into account the microscopic phenomena near the boundaries. Our approach enables to propose appropriate boundary conditions for these models. The well-posedness of such models is under study. We want then to tackle similar questions for Maxwell's and elastodynamics equations. This work is done in collaboration with Bruno Lombard (LMA Marseille) and Remi Cornaggia (Institut d'Alembert, Sorbonne Université).

6.7 Waves in quasi 1D or 2D domains

Mathematical modelling of thin coaxial cables

Participants: Patrick Joly, Akram Beni Hamad.

This topic is the subject of a long term collaboration with Sébastien Imperiale (M3disim) and the unerics for this problem constituted the subject of the PhD thesis of Akram Beni Hamad, defended last September.

The part of the thesis dedicated to the case of cylindrical cables and more precisely to the design an original approach condining Nédélec's edge elements on elongated prismatic meshes with a hybrid time discretization procedure which is explicit in the longitudinal directions and implicit in the transverse ones, has been accepted for publication in Computational methods in Applied Mathematics and presented at the Conference at the 10th Workshop on Numerical Methods for Evolution Equations in Heraklion. This includes a quantitative comparison of 3D results with the results of the simplified 1D model proposed previously.

In the non cylindrical case, the development of another hybrid method combining a conforming discretization in the longitudinal variable and a discontinuous Galerkin method in the transverse ones is still under way.

6.8 From periodic to random media

Guided modes in a hexagonal periodic graph-like domain: the zigzag and the armchair cases

Participant: Sonia Fliss.

In collaboration with Bérangère Delourme (LAGA, Paris 13), we have studied the spectrum of periodic operators in thin graph-like domains: more precisely Neumann Laplacian defined in periodic media which are close to quantum graphs. Moreover, we exhibit situations where the introduction of lineic defects into the geometry of the domain leads to the appearance of guided modes. We dealt with rectangular lattices few years ago and more recently we are studying hexagonal lattices. In this last case, we have shown that the dispersion curves have conical singularities called Dirac points. Their presence is linked to the invariance by rotation, symmetry and conjugation of the model. We have also observed that the direction of the line defect leads to very different properties of the guided modes. Finally, we have also proven the stability of the guided modes when the position of the edge varies (keeping the same direction). We want now to (1) open gaps (around the Dirac points) in the spectrum of the periodic operator by breaking one of the invariance of the problem (2) study the effect on guided waves.

Discrete honeycombs, rational edges and edge states

Participant: Sonia Fliss.

This work is done in collaboration with C.L. Fefferman (Princeton University) and M.I. Weinstein (Columbia University). We Consider the tight binding model of graphene terminated along a *rational edge*, *i.e.* an arbitrary line in a direction of periodicity of the structure. We present a comprehensive rigorous study of zero energy / flat band edge states; all zigzag-type edges support zero energy / flat bands and armchair-type edges support no zero energy / flat bands. We also perform a careful numerical investigations showing very strong evidence for the existence of non-zero energy (dispersive / non-flat) edge states. We are investigating now the existence of states which are bounded and oscillatory parallel to an irrational edge and which decay into the bulk. The idea is to construct an « edge » state for irrational termination as the limit of a sequence of edge state wave-packets (superpositions of edge states) of rationally terminated structures.

Wave propagation in quasi periodic media

Participants: Sonia Fliss, Patrick Joly, Pierre Amenoagbadji.

This was the subject of the PhD thesis of P. Amenoagbadji which was defended in December 2023. Our main objective is to develop original numerical methods for the solution of the time-harmonic wave equation where some quasi-periodicity arises in the heterogeneity or in the geometry of the propagation medium. This includes two situations:

- 1D quasi-periodic media: we developed an adapted numerical method based on the so-called lifting approach that was first studied and implemented in the case with absorption. The idea is to interpret the solution of the 1D Helmholtz equation as the trace along the same line of the solution of an augmented degenerate PDE in higher dimensions, with periodic coefficients. The key point is to characterize the transparent boundary condition via the DtN operator associated to the augmented problem through a propagation operator which is the solution of a Riccati equation whose construction is based on the solution of periodicity cell problems.. The corresponding article has been published in *Communications in Optimization Theory*.

  More recent developments concern the non-absorbing case, for which we have proposed a heuristic method based on the ideas of the limiting absorption principle. For this, one first needs to replace the DtN operator by a so-called RtR operator which associates an incoming Robin trace to an outgoing one. The second difficulty consists in selecting the good physical solution of the Riccati equation for the corresponding propagation operator. This has led us to look at the spectral theory of weighted shift operators, a class of operators to which the propagation operator belongs. This helped us in improving the initially designed method, by computing the so-called principal "eigenpair" of the propagation operator. This also allowed us to make significative advances in the theory of limiting absorption, still incomplete. The corresponding article should be submitted soon.

- Transmission between two 2D periodic half-spaces: the interesting case is when the two structures are not periodic in the direction of the interface, or when their periods along the interface are not commensurate. However, in this situation, the problem presents a hidden quasi-periodic structure with respect to the coordinate along the interface, in such a way that they fall into the scope of the lifting approach. In a first step, we have considered situations where the structures could be lifted in 3D, that is

1. the case where the two media are periodic along the interface, but with non-commensurate periods;
2. the case where one medium is constant, while the other is not periodic with respect to the variable along the interface.

In each case, the full method couples the DtN (or RtR) approach similar to the 1D case, with the use of the Floquet-Bloch transform with respect to the variable of the lifted interface. An additional difficulty lies in the resolution of 3D cell problems. We have developed a quasi 2D methods which reduces their resolution to a family of independent 2D problems set in rectangles and a non-local problem for an auxiliary unknown set on (a part of) the boundary of the cubic cell. The implementation of this method produces satisfactory results, and an article is being written.

The most recent and last aspect of the work concerns the generalization of the previous study to a transmission problem between two arbitrary periodic media. A priori this is treatable with the lifting method but with an augmented problem in dimension 5 whose solution is a priori too costly to compute. To overcome this, we have proposed a domain decomposition appraoch in which two different lifting approaches in 3D are applied in each subspace, involving two distinct Floquet-Bloch transforms. This leads to a non-local 1D problem whose unknown is the trace of the solution on the interface.

6.9 Coupled phenomena for waves in fluids and solids

Hybrid approach to the numerical simulation of ultrasonic NDT experiments on layered structures

Participants: Marc Bonnet.

This work is done in collaboration with Eric Ducasse, Marc Beschamps, Romain Kubecki (I2M, University of Bordeaux).

We develop a numerical simulation approach for ultrasonic NDT experiments on layered structures that aims at incorporating models for flaws or other local features (sensors, stiffeners,,,) into a semi-analytical computational framework for the unperturbed, ideal structure. The latter takes the form of the existing in-house code TraFiC developed at I2M by E. Ducasse and based on Laplace transform for the time variable and partial Fourier transforms along translation-independent or circumferential spatial coordinates; this code allows to model long-range wave propagation in undisturbed structures. The various flaws or features are then taken into account by using either small-size asymptotic models (which exploit the Green's tensors already implemented in TraFiC) or local finite element models and a domain decomposition iterative coupling approach. Regarding the latter, we established the convergence of DD iterations based on Robin boundary conditions on each (TraFiC or FE) subdomain having a shared interface. This work is undertaken through the jointly-advised thesis of Romain Kubecki, whose doctoral grant is co-funded by DGA and CEA LIST.

Singular solutions of linear aeroacoustics in recirculating base flows

Participants: Patrick Joly, Jean-François Mercier.

This is the continuation of the PhD thesis of A. Bensalah (Airbus) with whom we pursue our collaboration. We recall that aeroacoustics concerns the propagation of sound in a fluid in stationnary flow (for classical acoustics, the flow is at rest). The PhD of A. Bensalah (defended in 2018) was devoted to the Goldstein model in the time harmonic regime, for both mathematical and numerical issues.

We were able to prove that the model was well posed under the essential assumption that the base flow did not contain any closed streamline (plus additional assumption on the size of the vorticity of this flow). The case of recirculant flows, i.e. with closed streamlines, is much more delicate. During his thesis, A. Bensalah initiated the study of a simple model case : a 2D circular flow in an annulus.

This year, we have completed this work. We use the method of limiting absorption (where $\varepsilon > 0$ is the size of the absorption) and the main technical ingredients for the analysis are

- reduce the problem to a countable family of ODE's (separation of variables in polar coordinates)
- use Fröbenius method and Fuchs theory for passing to the limit $\varepsilon \to 0$.

This approach leads to the apparition of singular solutions that can be fully described. These solutions are outside the functional framework used for the analysis of the non recirculating case. The corresponding article is in preparation.

Time stepping methods for linear Friedrichs systems

Participants: Patrick Joly.

This is a work in collaboration with S. Imperiale (Medisim, Inria) and J. Rodríguez (University of Santiago de Compostela).

This relatively new subject has emerged first as a continuation of a rather old work with J. Rodríguez in the framework of the contract ADNUMO with Airbus about numerical modeling in transient aeroacoustics and more recently in conjonction with the M2 course I initiated with S. Imperiale on Advanced Numerical methods for Evolution Equations.

The question we address is a priori very classical and academic : we want to study the stability of explicit numerical schemes for the time discretization of semi-discrete problems issued from the space discretization of first order hyperbolic Friedrichs systems (which include most of relevant linear wave propagation models in physics, such as aeroacoustics) with Discontinuous Galerkin Methods, using centered fluxes (which are slightly suboptimal in terms of accuracy but preserve the conservation of energy) or off-centered schemes (which restaures the optimal accuracy but introduce numerical dissipation)

Curiously, we made the constant that this a priori innocent question had not completely been solved in the literature. The results we have obtained can be seen an a continuation and hopefully an improvement of previous results obtained by Lévy-Tadmor (1998), Burman-Ern-Fernandez (2012) or Ketcheson-Loczi (2015).

We have first studied the class explicit Runge-Kutta schemes RK_N, where $N > 0$ is the order of the scheme. In the conservative case, the Von Neumann analysis can be used and shows that the scheme is CFL stable if and only if N is congruent to 0 or 3 (modolo 4). In the non conservative case, the Von Neumann analysis cannot be used any longer and we have used an energy approach for $1 \leq N \leq 4$. For $N = 1,2$, which gives instability in the conservatice case, the spatial dissipation may help to reach stability : for $\mathbb{P}_0$ DG-elements when $N = 1$, for $\mathbb{P}_0$ and $\mathbb{P}_1$ elements when $N = 2$. However, looking at the toy problem of the transport equation, we show that $N = 1$ does not work for $\mathbb{P}_1$ elements and $N = 2$ does not work for $\mathbb{P}_3$ elements. For $N = 3,4$, the picture is completely different : both schemes are stable but, curiously, the spatial dissupation slighty deteriorates the stability condition.

Finally, we have looked at two-time steps schemes of leap-frog time. If the classical leap-frop scheme is known for having very good properties in the conservative case, it is unstable in the dissipative case. However, off-centering backwards in time the dissipative term restores the CFL stability without affecting the space-time stability of the method.

These results have been presented in Heraklion (Crete) at the 11th Workshop on Numerical Methods for Evolution Equations and in Concepcion (Chile) at the conference WONAPDE2024.

Modelling fluid injection in seismic cycles

Participants: Laura Bagur, Stéphanie Chaillat.

This work is done in collaboration with JF Semblat (ENSTA Paris) and I Stefanou (Ecole Centrale Nantes). Earthquakes due to either natural or anthropogenic sources cause important human and material damage. In both cases, the presence of pore fluid influences the triggering of seismic instabilities. A timely question in the scientific community is to show that the earthquake instability could be avoided by an active control of the fluid pressure.

We use the capabilities of Fast Boundary Element Methods (Fast BEMs) to provide a multi-physic large-scale robust solver required for modeling earthquake processes, human induced seismicity and their control. Fast BEMs are combined with a rate-and-state friction law and different adaptive time stepping algorithms available in the literature. In a first step, we have checked the capabilities of all these algorithms in terms of accuracy, stability and computational times. These methods have been compared on different benchmarks. We have derived an analytic aseismic solution to perform the convergence study.

Once the optimal solver determined, we have considered poro-elastodynamic effects. Since the use of the complete poro-elastic model would lead to unacceptable computational costs to consider realistic 3D configurations, we have performed a dimensional analysis. It allows to determine which of the poroelastodynamic effects are predominant depending on the observation time of the fault. We have rigorously justified the predominant fluid effects at stake during an earthquake or a seismic cycle. We have shown that at the timescale of the earthquake instability, inertial effects are predominant. On the other hand a combination of diffusion and elastic deformation due to pore pressure change should be privileged at the timescale of the seismic cycle instead of the diffusion model mainly used in the literature. We have illustrated these effects on a simplified crustal faulting problem with fluid-injection at the timescale of the earthquake instability.

Modelling non-spherical bubbles of gaz generated by airguns

Participants: Stéphanie Chaillat.

This work is done in collaboration with E. Dunham (Stanford university) and Shuki Ronen (company Sercel). The company Sercel develops airgun-type seismic sources. These are compressed air sources that generate an underwater acoustic wave to identify the nature of the underwater ground depending on its acoustic reflection. It is an ultrasound seismic imaging technique.

In this work, we model the gas bubbles that are generated by the source and determine their acoustic signatures. The difficulty in the context of seismic sources is that the bubble cannot be assumed to remain spherical. This greatly complicates the physical modeling because one does not simply have to solve an ordinary differential equation but a coupled vector problem. To model this problem we assume that the liquid near the source is incompressible and inviscid, and an irrotational flow. We can describe the flow velocity as the gradient of the velocity potential. The flow being incompressible, the velocity potential satisfies the Laplace equation. With this modeling, the variables in time and in space are decoupled. The Laplace equation is solved to determine the evolution of the geometry of the bubble whereas the Bernoulli equation is used to update the boundary conditions at each time step. As the bubble is in an infinite space (the ocean), it is natural to consider the BEM to solve the Laplace equation. But this problem is much more complicated than it seems. There are two difficulties. The first difficulty is that the solution is very expensive because it requires to solve a Laplace problem on an evolving geometry at each time step. Even if fast BEMs are very effective they become too expensive in this context as soon as that the time of a solve exceeds one second, which limits the precision. The second difficulty is that the problem is subject to numerical instabilities due to approximations at each BEM solve. We have worked on (i) the development of a filter to stabilize the simulations and avoid the accumulation of numerical errors which lead to numerical instabilities and we have derived (ii) a mesh adaptation method in order to determine the optimal mesh for all time steps.

7 Bilateral contracts and grants with industry

7.1 Bilateral Contracts with Industry

- Contract with DGA and Naval Group on *transient fluid-structure coupling caused by remote underwater explosions, including cavitation effects*

  Participants: Marc Bonnet, Stéphanie Chaillat, Alice Nassor.

  Start: 10/2020. End: 09/2023. Administrator: CNRS

- Contract and CIFRE PhD with CEA on *Modelling of thin layers of randomly distributed nanoparticles for electromagnetic waves*

  Participants: Amandine Boucart, Sonia Fliss, Laure Giovangigli.

  Start: 10/2019. End: 09/2022. Administrator: ENSTA Paris

- Contract with SIEMENS on *GPU accelerated discontinuous Galerkin finite element solver for aeroacoustics*

  Participants: Rose-Cloe Meyer, Axel Modave.

  Start: 01/2022. End: 01/2024. Administrator: ENSTA Paris

- Contract and CIFRE PhD with Naval Group on flow noise prediction

  Participants: Stéphanie Chaillat, Jean-Francois Mercier, Laure Pacaut.

  Start: 02/2022. End: 01/2025. Administrator: CNRS

8 Partnerships and cooperations

8.1 International initiatives

8.1.1 STIC/MATH/CLIMAT AmSud projects

NoLoCE

Participants: Patrick Ciarlet, Maryna Kachanovska.

Title: nonlocal and local coupled equations: analysis, computation, and probability

Partner Institution(s): ENSTA Paris, Inria POEMS, Univ. de la República, Montevideo

Program: MATH-AmSud

Duration: January 1, 2022 – December 31, 2023

Partners:

- Acosta (Argentine)
- Otárola (Chili)
- Borthagaray (Uruguay)

Coordinator: J.P. Borthagaray (PI) ; Patrick Ciarlet (French Coordinator)

Summary: The design of accurate mathematical models and the development of efficient numerical algorithms for their resolution are topics of paramount importance in applied mathematics and engineering. Good models capture the underlying mechanisms, and their inspection may lead to new questions in pure mathematics. In this project, we propose to study problems involving systems of coupled partial differential and integro-differential equations. Our research concerns the modeling, analysis, and simulation of such problems. We deal with questions related to materials science, such as the study of interface models in elasticity and electromagnetism and the development of novel formulations in nanophotonics. We propose to study approaches to coupled systems through game theory. Furthermore, we aim to compare local and nonlocal diffusion either in a mixing environment and through the analysis of population dispersal. External optimal control is a salient feature of nonlocal formulations that we propose to analyze. Motivated by their application in the modeling of the respiratory system, we shall also study problems on fractal and random infinite trees.

8.2 International research visitors

8.2.1 Visits of international scientists

Other international visits to the team

Martin Halla

Status Postdoc

Institution of origin: Universität Göttingen

Country: Germany

Dates: 1-week visit in January

Context of the visit: collaboration with Maryna Kachanovska about PMLs for anisotropic media

Mobility program/type of mobility: research stay

Kiyan Naderi

Status PhD

Institution of origin: Carl von Ossietzky Universität Oldenburg

Country: Germany

Dates: 3-weeks wisit in March 2023

Context of the visit: collaboration with Maryna Kachanovska about DtNs for fractal trees

Mobility program/type of mobility: research stay

8.2.2 Visits to international teams

Research stays abroad

Stéphanie Chaillat Loseille

Visited institution: Stanford

Country: United States

Dates: 15/02/2022 - 22/12/2022

Context of the visit: Collaboration with Eric Dunham

Mobility program/type of mobility: research stay

Sonia Fliss

Visited institution: Isaac Newton Institute for Mathematical Sciences, Cambridge

Country: United Kingdom

Dates: January 2023 to June 2023

Context of the visit: programme "Mathematical theory and applications of multiple wave scattering", see www.newton.ac.uk/event/mws/

Mobility program/type of mobility: research stay

Patrick Joly

Visited institution: Isaac Newton Institute for Mathematical Sciences, Cambridge

Country: United Kingdom

Dates: 04/06/2023 to 18/06/2023

Context of the visit: programme "Mathematical theory and applications of multiple wave scattering", see www.newton.ac.uk/event/mws/

Mobility program/type of mobility: research stay

8.3 National initiatives

ANR
ANR JCJC project WavesDG

Participants: Axel Modave, Patrick Ciarlet.

Title: ANR JCJC project WavesDG *(Wave-specific Discontinuous Galerkin Finite Element Methods for Time-Harmonic Problems)*

Partner Institution(s): POEMS (CNRS, INRIA, ENSTA Paris), Rapsodi (INRIA), LAUM (U. Le Mans), U. Liège

Duration: Start: 10/2021. End: 12/2025.

Coordinator: Axel Modave (POEMS, CNRS)

Administrator: CNRS

ANR project DynImplant

Participants: Stéphanie Chaillat.

Title: Model-based ultrasound characterization of the bone-implant interface

Partner Institution(s): Laboratoire Analyse, Géométrie et Applications de l'université Paris 8, start-up Wave Implant (waveimplant.com) et le CHU de Nantes.

Duration: Start 10/2022. End: 11/2026.

Coordinator: Vu-Hieu Nguyen (MSME)

ANR project Reward

Participants: Laure Giovangigli.

Title: Reverse weak formulation for parameters identification

Partner Institution(s): Ecole Central de Lyon - ICJ et Institut Langevin

Duration: 01/01/2022 - 31/12/2025

Coordinator: Laurent Seppecher (École Centrale de Lyon)

DGA / AID
Projet RAPID HyBOX

Participants: Patrick Joly, Sonia Fliss, Maryna Kachanovska, Axel Modave, Pierre Marchand.

Title: Projet RAPID HyBOX *(Hybridization toolbox for complex materials and metamaterials)*

Partner Institution(s): IMACS, ARIANEGROUP, ENSTA Paris

Duration: Start: 10/2020. End: end of 2025.

Administrator: ENSTA Paris

Projet CIEDS ElectroMath

Participants: Patrick Ciarlet, Axel Modave, Anne-Sophie Bonnet-Ben Dhia, Sonia Fliss, Pierre Marchand.

Title: Projet CIEDS ElectroMath

Partner Institution(s): ENSTA Paris, Inria POEMS, Inria IDEFIX

Duration: 01.10.2022 - 01.04.2026.

Coordinators: Patrick Ciarlet et Axel Modave

Administrator: ENSTA Paris

Projet CIEDS PRODIPO

Participants: Laure Giovangigli.

Title: Projet CIEDS PRODIPO *(DIrect and Inverse Problems in Wave Propagation)*

Partner Institution(s): Ecole Polytechnique - CMAP et INIRIA

Duration: 01/05/2021 - 31/10/2024

Coordinators: Josselin Garnier

Administrator: Ecole Polytechnique

Plan de relance
Projet "WavesDG - GPU"

Participants: Rose-Cloe Meyer, Axel Modave.

Title: Plan de préservation des emplois R& D - *Projet WavesDG - GPU*

Partner Institution(s): POEMS (CNRS, Inria, ENSTA Paris), SIEMENS

Duration: Start: 01/2022. End: 01/2024.

Coordinators: Coordinator: Axel Modave (POEMS, CNRS)

Administrator: ENSTA Paris

Action Exploratoire Inria
Action exploratoire OptiGPR3D

Participants: Pierre Marchand.

Title: Action exploratoire OptiGPR3D (*Modélisations directe et inverse optimales pour l'imagerie GPR 3D en milieu complexe*)

Partner Institution(s): POEMS (CNRS, Inria, ENSTA Paris), IDEFIX (Inria, EDF)

Duration: Start: 05/2022.

Coordinators: Marcella Bonazzoli (IDEFIX, Inria), Pierre Marchand (POEMS, Inria)

Administrator: Inria

9 Dissemination

9.1 Promoting scientific activities

9.1.1 Scientific events: organisation

Member of the organizing committees

- S. Chaillat is a co-animator of the topic *"Modeling and simulation"* of the GDR Ondes (gdr-ondes.cnrs.fr).
- S. Fliss organized the workshop *Computational methods for multiple scattering* (Newton Institute, Cambridge UK, April 17-21, 2023)
- J.-F. Mercier is a co-animator of the topic *"Effective dynamics of microstructured media"* of the GDR MecaWave (mecawave.cnrs.fr).
- POEMS organizes, under the responsability of M. Kachanovska, a monthly seminar. One occurrence each semester is co-organized with two other inria teams, IDEFIX and M3DISIM.

9.1.2 Journal

Member of the editorial boards

- A. S. Bonnet-Ben Dhia is a member of the editorial board of the SIAM journal of applied mathematics.
- M. Bonnet is a member of the editorial boards of Computational Mechanics (Comput. Mech.), Engineering Analysis with Boundary Elements (EABE), J. Optimization Theory and Applications (JOTA), and Inverse Problems.
- L. Bourgeois is a member in the editorial board of IMA Journal of Applied Mathematics.
- P. Ciarlet is a member in the editorial board of ESAIM:M2AN (Mathematical Modeling and Numerical Analysis).
- S. Chaillat is a member in the editorial boards of Advances in Computational Mathematics (ACOM) and The Journal of Theoretical, Computational and Applied Mechanics (JTCAM).
- S. Fliss is a member of the editorial board of SIAM Multiscale Modelling and Simulation journal (MMS).
- P. Joly is a member of the editorial board of Results in Applied Mathematics and of the Book series "Scientific Computing" of Springer.

9.1.3 Research administration

- E. Bécache is a deputy chair of the Doctoral School EDMH.
- M. Bonnet is since 2019 an appointed member of the COMEVAL, a committee of the Ministry of Ecological and Inclusive Transition (MEIT) similar to a CNRS National Committee section and tasked with the competitive recruitment and career overseeing of the cadre of junior and senior scientists managed by the MEIT. He joined the steering committee of COMEVAL in Sept. 2023.
- A.-S. Bonnet-Ben Dhia is deputy-chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris. She is a member of the Scientific Council of the Doctoral School EDMH, and of the BCEP (Bureau du Comité des Equipes Projets) at INRIA Saclay from 2018, and of the Scientific Council of CNRS since Oct. 2023.
- A.-S. Bonnet-Ben Dhia and S. Chaillat are members of the Academic Council of IP Paris (Institut Polytechnique de Paris).

- S. Chaillat has been a member of the Scientific Council of CNRS until Sept. 2023. She is a member of the Scientific Council of CNRS since October 2021.
- A. Modave is a member of the scientific committee of the mesocenter of IP Paris (Institut Polytechnique de Paris), a member of the board of directors of ENSTA Paris, and a member of the scientific committee of the Math CSI program at FMJH (Fondation Mathématique Jacques Hadamard).
- M. Kachanovska was, since 2017 and until Sept. 2023, a member of the INRIA Scientific Committee for PhD and Postdoctoral Positions.
- P. Marchand is, since Sept. 2023, a member of the INRIA Scientific Committee for PhD and Postdoctoral Positions.

9.1.4 Research administration

- E. Bécache is a deputy chair of the Doctoral School EDMH.
- M. Bonnet is a member of the COMEVAL, a committee of the Ministry of Ecological and Inclusive Transition (MEIT) similar to a CNRS National Committee section and tasked with the recruiting and career overseeing of the cadre of junior and senior scientists managed by MEIT.
- A.-S. Bonnet-Ben Dhia is deputy-chair of the *Applied Mathematics Department* (UMA) at ENSTA Paris. She is a member of the Scientific Council of the Doctoral School EDMH, and of the BCEP (Bureau du Comité des Equipes Projets) at INRIA Saclay from 2018. She is a member of the exterior scientific councils of Institut Fresnel and Laboratoire de Mécanique et d'Acoustique, both in Marseille.
- A.-S. Bonnet-Ben Dhia and S. Chaillat are members of the Academic Council of IP Paris (Institut Polytechnique de Paris).
- S. Chaillat is a member of the Scientific Council of CNRS from October 2021.
- P. Ciarlet is coordinator of the *Mathematics in Computational Science and Engineering Program* of the Mathematics Hadamard Labex (LMH).
- A. Modave is a member of the scientific committee of the mesocenter of IP Paris (Institut Polytechnique de Paris), and a member of the board of directors of ENSTA Paris.
- M. Kachanovska is a member of the INRIA Scientific Committee for PhD and Postdoctoral Positions, from 2017.

9.2 Teaching - Supervision - Juries

9.2.1 Administration

Permanent members of POEMS are involved in the management of the engineering program at ENSTA Paris and the master program in applied mathematics at IP Paris and Université Paris-Saclay.

- L. Bourgeois: coordinator of the 2nd year Maths Program at ENSTA; co-head of the M1 Applied Mathematics common to IP Paris and Université Paris-Saclay;
- P. Ciarlet: coordinator of the master program in applied mathematics at IP Paris;
- S. Fliss: coordinator of the 3nd year ENSTA programs on modelling and simulation; co-head of the M2 AMS *(Analyse, Modélisation et Simulation)* common to IP Paris and Université Paris-Saclay;
- L. Giovangigli: coordinator of the 3nd year ENSTA programs on finance and mathematics for life sciences.

9.2.2 Courses taught

All permanent members of POEMS, as well as most PhD students and post-docs, are involved in teaching activities. A large fraction of these activities is included in the curriculum of the engineering school ENSTA Paris that hosts POEMS team. The 3rd year of this curriculum is coupled with various research masters, in particular the master *Analysis, Modelization and Simulation* (denoted below M2 AMS) common to Institut Polytechnique de Paris and Université Paris-Saclay.

Teaching activities of the permanent members of POEMS

- Eliane Bécache
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse et approximation par éléments finis d'EDP*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)
- Marc Bonnet
  - *Méthodes numériques matricielles avancées: analyse et expérimentation*, ENSTA (2nd year) and Master Applied Math (M1).
- Anne-Sophie Bonnet-Ben Dhia
  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
  - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)
- Laurent Bourgeois
  - *Outils élémentaires pour l'analyse des équations aux dérivées partielles*, ENSTA (1st year)
  - *Fonctions de variable complexe*, ENSTA (1st year)
  - *Problèmes inverses pour des systèmes gouvernés par des EDPs*, ENSTA (3rd year) and Master AMS (M2)
- Colin Chambeyron
  - *Outils mathématiques*, Licence (L1), Paris-Dauphine University
  - *Analyse - Optimisation*, Licence (L1), Paris-Dauphine University
  - *Algèbre linéaire*, Licence (L2), Paris-Dauphine University
- Patrick Ciarlet
  - *Méthodes variationnelles pour l'analyse et la résolution de problèmes non coercifs*, ENSTA (3rd year) and Master AMS (M2)
  - *Modèles mathématiques et leur discrétisation en électromagnétisme*, ENSTA (3rd year) and Master AMS (M2)
- Luiz Faria
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Méthodes numériques matricielles avancées: analyse et expérimentation*, ENSTA (2nd year) and Master Applied Math (M1)

- Sonia Fliss
  - *La méthode des éléments finis*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Homogénéisation périodique*, ENSTA (3rd year), ENSTA (3rd year) and Master AMS (M2)
- Laure Giovangigli
  - *Introduction aux probabilités et aux statistiques*, ENSTA (1st year)
  - *Martingales et algorithmes stochastiques*, ENSTA (2nd year)
  - *Calcul stochastique*, ENSTA (3rd year) and Master MMMEF (M2)
  - *Introduction à l'imagerie médicale*, ENSTA (3rd year) and Master AMS and MSV (M2)
  - *Homogénéisation stochastique*, ENSTA (3rd year) and Master AMS and MSV (M2)
- Christophe Hazard
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)
- Patrick Joly
  - *Introduction à la discrétisation des équations aux dérivées partielles*, ENSTA (1st year)
  - *Analyse fonctionnelle*, ENSTA (2nd year) and Master AMS (M2)
  - *Techniques de discrétisation avancées pour les problèmes d'évolution*, ENSTA (3rd year) and Master AMS (M2)
- Maryna Kachanovska
  - *Equations intégrales de frontière*, ENSTA (3rd year) and Master AMS (M2)
- Nicolas Kielbasiewicz
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)
- Eric Lunéville
  - *Introduction au calcul scientifique*, ENSTA (2nd year).
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Projet de simulation numérique*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Problèmes de diffraction en domaines non bornés*, ENSTA (3rd year) and Master AMS (M2)
- Pierre Marchand
  - *Introduction à MATLAB*, ENSTA (1st year)
  - *Quadratic optimization*, ENSTA (1st year)
  - *Programmation scientifique en C++*, ENSTA (2nd year) and Master Applied Math (M1)
  - *Cours accéléré de programmation*, Master AMS (M2)
- Jean-François Mercier
  - *Outils élémentaires d'analyse pour les équations aux dérivées partielles*, ENSTA (1st year)
  - *Fonctions de variable complexe*, ENSTA (1st year)

  - *Théorie spectrale des opérateurs autoadjoints*, ENSTA (2nd year) and Master Applied Math (M1)

- Axel Modave
  - *Initiation au calcul haute performance*, ENSTA (2rd year) and Master Applied Math (M1)
  - *Calcul scientifique parallèle*, ENSTA (3rd year) and Master AMS (M2)

9.2.3 Supervision

• PhD: Pierre Amenoagbadji,"Wave propagation in quasi-periodic media", defended in December 2023, S. Fliss and P. Joly

• PhD : Amandine Boucart, "Modeling of a thin layer of randomly distributed nanoparticles for electromagnetic waves", defended in April 2023, S. Fliss and L. Giovangigli

• PhD : Jean-François Fritsch ,"Propagation of waves in partially buried guides: resolution of the forward problem and imaging with a sampling type method", defended in June 2023, L Bourgeois and C. Hazard

• PhD: Alice Nassor, "Domain decomposition method for coupled acoustic-elastic problems in the time domain. Application to underwater explosions.", defended in December 2023, S. Chaillat and M. Bonnet

• PhD: Luis Alejandro Rosas Martinez,"Dispersive electromagnetic media : mathematical and numerical analysis", defended in October 2023, M. Cassier and P. Joly

• PhD in progress : Sarah Al Humaikani « Wave propagation in junction of open waveguides", started October 2023, A.-S. Bonnet-Ben Dhia et S. Fliss

• PhD in progress : Amond Allouko, "A hybrid semi-analytical method for the integrated health control of composite plates", started September 2020, A.-S. Bonnet-Ben Dhia and A. Lhemery

• PhD in progress : Laura Bagur, "Three dimensional modeling of seismic and aseismic slip using Fast Boundary Element Methods", started September 2020, S. Chaillat, J.-F. Semblat and I. Stéfanou

• PhD in progress : Cédric Baudet, "Modelisation of partial coatings in electromagnetism", started October 2022, S. Fliss and P. Joly

• PhD in progress : Antonin Boisneault, « Numerical methods and high performance simulation for 3D imaging in complex media », started October 2023, Marcella Bonazzoli, Xavier Claeys and Pierre Marchand

• PhD in progress : Farah Chaaban, "An optimization-based numerical method for diffusion problems with sign-changing coefficients ", started October 2022, P. Ciarlet and M. Rihani

• PhD in progress : Mario Gervais, "A posteriori estimators of a nonconforming domain decomposition method", started October 2022, P. Ciarlet and F. Madiot

• PhD in progress : Quentin Goepfert, "Inverse problems in ultrasonic imaging", October 2021, J. Garnier, L. Giovangigli and P. Millien

• PhD in progress : Dongshen He, "Boundary integral methods for Stokes flows with deformable implicit surfaces", started October 2022, L. Faria

• PhD in progress: Romain Kubecki, "Development of hybrid numerical methods for the scattering of ultrasonic waves by obstacles on layered structures, and application to nondestructive testing", started March 2023, M. Bonnet

• PhD in progress: Yacine Mohammedi, "Discrete adjoint method applied to the Ffowcs-Williams Hawkings integral equation for aeroacoustic shape optimization", started October 2023, M. Bonnet

• PhD in progress : Louise Pacaut, "Development of an accelerated numerical BEM/BEM method to determine the Green function of a fluid-structure problem.", started October 2022, S. Chaillat and J. F. Mercier

• PhD in progress : Aurélien Parigaux, "Construction of transparent boundary conditions for electromagnetic waveguides, analysis and applications", started October 2022, A.-S. Bonnet-Ben Dhia and L. Chesnel

• PhD in progress : Etienne Peillon, "Justification and mathematical analysis of plasma models", started October 2020, P. Ciarlet and M. Kachanovska

• PhD in progress : Simone Pescuma, "Novel Discontinuous Finite Elements Methods for Time-Harmonic Wave Propagation"", started October 2022, G. Gabard and A. Modave

• PhD in progress : Adrian Savchuk, "Asymptotic modelling of time-domain electromagnetic scattering by small particles", started October 2022, M. Kachanovska and E. Bécache

• PhD in progress : Timothée Raynaud, « Analysis and acceleration of Krylov iterative methods for the numerical solution of time-harmonic wave problems », started October 2023, Victorita Dolean, Pierre Marchand and Axel Modave

• PostDoc : Corentin Kilque : "Long time homogenization for the waves equation and the Maxwell equation", until August 2023, S. Fliss

• PostDoc : Zoïs Moitier : "Fast methods for the solution of boundary integral equations on fractal antennas", started October 2022, P. Joly and M. Kachanovska

• PostDoc : Florian Monteghetti : "Asymptotic modeling of thin periodic layers with high-contrast inclusions for Maxwell's equations" ,until August 2023, S. Fliss and P. Joly

• PostDoc : Rose-Cloé Meyer : "GPU-accelerated DG finite element solver for unsteady acoustics", started January 2022, A. Modave

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10 Scientific production

10.1 Publications of the year

International journals

[1] P. Amenoagbadji, S. Fliss and P. Joly. 'Wave propagation in one-dimensional quasiperiodic media'. In: *Communications in Optimization Theory* (2023). DOI: 10.48550/arXiv.2301.01159. URL: https://hal.science/hal-03786730.

[2] T. G. Anderson, M. Bonnet and S. Veerapaneni. 'Quantifying mixing in arbitrary fluid domains: a Padé approximation approach'. In: *Numerical Algorithms* 93 (2023), pp. 441–458. DOI: 10.1007/s11075-022-01423-7. URL: https://hal.science/hal-03698941.

[3] W. Aquino, J. Rouse and M. Bonnet. 'Active Design of Diffuse Acoustic Fields in Enclosures'. In: *The Journal of the Acoustical Society of America* (2024). URL: https://hal.science/hal-04138504.

[4] M. Barré and P. Ciarlet. 'The T-coercivity approach for mixed problems'. In: *Comptes Rendus. Mathématique* (2023). URL: https://hal.science/hal-03820910.

[5] E. Bécache, M. Kachanovska and M. Wess. 'Convergence analysis of time-domain PMLs for 2D electromagnetic wave propagation in dispersive waveguides'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* (July 2023). URL: https://hal.science/hal-03910611.

[6] É. Bécache, A.-S. Bonnet-Ben Dhia, S. Fliss and A. Tonnoir. 'The Half-Space Matching method for elastodynamic scattering problems in unbounded domains'. In: *Journal of Computational Physics* (30th Sept. 2023), p. 112320. DOI: 10.1016/j.jcp.2023.112320. URL: https://hal.science/hal-03793031.

[7] M. Bonnet, S. Chaillat and A. Nassor. 'Solvability results for the transient acoustic scattering by an elastic obstacle'. In: *Journal of Mathematical Analysis and Applications* (2024). URL: https://hal.science/hal-04128900.

[8] M. Bonnet and E. Demaldent. 'Eddy-current asymptotics of the Maxwell PMCHWT formulation for multiple bodies and conductivity levels'. In: *Computers & Mathematics with Applications* 141 (1st July 2023), pp. 80–101. DOI: 10.1016/j.camwa.2023.03.026. URL: https://hal.science/hal-03696173.

[9] M. Bonnet, R. Liu, S. Veerapaneni and H. Zhu. 'Shape optimization of peristaltic pumps transporting rigid particles in Stokes flow'. In: *SIAM Journal on Scientific Computing* 45.1 (2023), B78–B106. DOI: 10.1137/21M144863X. URL: https://hal.science/hal-03364866.

[10] L. Bourgeois, S. Fliss, J.-F. Fritsch, C. Hazard and A. Recoquillay. 'Scattering in a partially open waveguide: the forward problem'. In: *IMA Journal of Applied Mathematics* (Feb. 2023). DOI: 10.1093/imamat/hxad004. URL: https://inria.hal.science/hal-03407434.

[11] L. Bourgeois, J.-F. Fritsch and A. Recoquillay. 'Scattering in a partially open waveguide: the inverse problem'. In: *Inverse Problems and Imaging* 17.2 (2023), pp. 463–469. DOI: 10.3934/ipi.2022052. URL: https://inria.hal.science/hal-03619606.

[12] M. Cassier, P. Joly and L. A. R. Martínez. 'Long time behaviour of the solution of Maxwell's equations in dissipative generalized Lorentz materials (I) A frequency dependent Lyapunov function approach'. In: *Zeitschrift für Angewandte Mathematik und Physik* 74.3 (15th May 2023), p. 115. DOI: 10.1007/s00033-023-01989-9. URL: https://inria.hal.science/hal-03850472.

[13] D. Chicaud and P. Ciarlet. 'Analysis of time-harmonic Maxwell impedance problems in anisotropic media'. In: *SIAM Journal on Mathematical Analysis* 55.3 (Feb. 2023), pp. 1969–2000. DOI: 10.1137/22M1485413. URL: https://inria.hal.science/hal-03613183.

[14] P. Ciarlet, M. H. Do and F. Madiot. 'A posteriori error estimates for mixed finite element discretizations of the Neutron Diffusion equations'. In: *ESAIM: Mathematical Modelling and Numerical Analysis* 57.1 (Jan. 2023), pp. 1–27. DOI: 10.1051/m2an/2022078. URL: https://hal.science/hal-03936904.

[15] P. Ciarlet and E. Jamelot. 'Variational methods for solving numerically magnetostatic systems'. In: *Advances in Computational Mathematics* (2023). URL: https://inria.hal.science/hal-04152740.

[16] P. Ciarlet, D. Lassounon and M. Rihani. 'An optimal control-based numerical method for scalar transmission problems with sign-changing coefficients'. In: *SIAM Journal on Numerical Analysis* 61.3 (2023), pp. 1316–1339. DOI: 10.1137/22M1495998. URL: https://hal.science/hal-03666913.

[17] A. Dansou, S. Mouhoubi, C. Chazallon and M. Bonnet. 'Modelling of the fatigue cracking resistance of grid reinforced asphalt concrete by coupling fast BEM and FEM'. In: *Road Materials and Pavement Design* 24 (2023), pp. 631–652. DOI: 10.1080/14680629.2022.2029755. URL: https://hal.science/hal-03524856.

[18] L. Faria, C. Pérez-Arancibia and C. Turc. 'Combined field-only boundary integral equations for PEC electromagnetic scattering problem in spherical geometries'. In: *SIAM Journal on Applied Mathematics* (25th Mar. 2023). URL: https://hal.science/hal-04219901.

[19] C. L. Fefferman, S. Fliss and M. Weinstein. 'DISCRETE HONEYCOMBS, RATIONAL EDGES AND EDGE STATES'. In: *Communications on Pure and Applied Mathematics* (22nd Sept. 2023). DOI: 10.1002/cpa.22141. URL: https://hal.science/hal-03606004.

[20] M. Kachanovska. 'A new class of uniformly stable time-domain Foldy-Lax models for scattering by small particles. Acoustic sound-soft scattering by circles. Extended version'. In: *Multiscale Modeling and Simulation: A SIAM Interdisciplinary Journal* (2023). URL: https://hal.science/hal-03664569.

[21] J.-F. Mercier. 'Homogenization of thin-structured surfaces for acoustics in the presence of a two-dimensional low Mach potential flow'. In: *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 479.2274 (June 2023). DOI: 10.1098/rspa.2022.0697. URL: https://inria.hal.science/hal-04239707.

[22] H. Methenni, A. Imperiale and S. Imperiale. 'An implicit–explicit time discretization for elastic wave propagation problems in plates'. In: *International Journal for Numerical Methods in Engineering* (16th Nov. 2023), e7393. DOI: 10.1002/nme.7393. URL: https://inria.hal.science/hal-04291680.

[23] A. Modave and T. Chaumont-Frelet. 'A hybridizable discontinuous Galerkin method with characteristic variables for Helmholtz problems'. In: *Journal of Computational Physics* 493 (15th Nov. 2023), p. 112459. DOI: 10.1016/j.jcp.2023.112459. URL: https://hal.science/hal-03909368.

[24] T. Strauszer-Caussade, L. Faria, A. Fernandez-Lado and C. Pérez-Arancibia. 'Windowed Green function method for wave scattering by periodic arrays of 2D obstacles'. In: *Studies in Applied Mathematics* 150.1 (Jan. 2023), pp. 277–315. DOI: 10.1111/sapm.12540. URL: https://hal.science/hal-04219892.

[25] N. Trafny, G. Serre, B. Cotté and J.-F. Mercier. 'A stochastic volume approach based on tailored Green's functions for airfoil noise prediction at low Mach number'. In: *Journal of Sound and Vibration* 551 (May 2023), p. 117603. DOI: 10.1016/j.jsv.2023.117603. URL: https://inria.hal.science/hal-04239726.

Conferences without proceedings

[26] E. Jamelot, P. Ciarlet and S. Sauter. 'Stability of the $\mathbf{P}^1_{nc} - (P^0 + P^1)$ element'. In: ENUMATH 2023 - The European Conference on Numerical Mathematics and Advanced Applications. Lisbonne, Portugal, 2023. URL: https://hal.science/hal-04414894.

[27] N. M. Ribe, A. Chamolly, G. Gerardi, S. Chaillat and Z.-H. Li. 'Scaling of Free Subduction on a Sphere'. In: 2023 EGU General Assembly. Vienna, Austria, 23rd Apr. 2023. URL: https://hal.science/hal-04420118.

Doctoral dissertations and habilitation theses

[28] A. Boucart. 'Scattering from a thin layer of randomly distributed nanoparticles : asymptotic expansion, effective boundary conditions and simulations.' Institut Polytechnique de Paris, 4th Apr. 2023. URL: https://theses.hal.science/tel-04400789.

[29] J.-F. Fritsch. 'Propagation of waves in partially buried guides : resolution of the forward problem and imaging with a sampling type method'. Institut Polytechnique de Paris, 20th June 2023. URL: https://theses.hal.science/tel-04416921.

[30] L. A. Rosas Martinez. 'Study of two wave propagation problems in electromagnetic dispersive media : 1) Long-time stability analysis in Drude-Lorentz media; 2) Transmission between a slab of metamaterial on a dielectric.' Institut Polytechnique de Paris, 16th Nov. 2023. URL: https://theses.hal.science/tel-04401219.

Reports & preprints

[31] M. Ancellin, P. Marchand and F. Dias. *Towards high-performance linear potential flow BEM solver with low-rank compressions*. 9th Oct. 2023. URL: https://inria.hal.science/hal-04233952.

[32] T. G. Anderson, M. Bonnet, L. M. Faria and C. Pérez-Arancibia. *Construction of polynomial particular solutions of linear constant-coefficient partial differential equations*. 23rd June 2023. URL: https://hal.science/hal-04146453.

[33] T. G. Anderson, M. Bonnet, L. M. Faria and C. Pérez-Arancibia. *Fast, high-order numerical evaluation of volume potentials via polynomial density interpolation.* 15th Aug. 2023. URL: https://hal.science/hal-04181164.

[34] M. Bonnet, P. Salasiya and B. B. Guzina. *Error-in-constitutive-relation (ECR) framework for the characterization of linear viscoelastic solids.* 18th Jan. 2024. URL: https://hal.science/hal-04404569.

[35] A.-S. Bonnet-Ben, L. Chesnel and M. Rihani. *Maxwell's equations with hypersingularities at a negative index material conical tip.* 3rd May 2023. URL: https://hal.science/hal-04087360.

[36] A.-S. Bonnet-Ben Dhia, L. Faria and C. Pérez-Arancibia. *A complex-scaled boundary integral equation for time-harmonic water waves.* 2023. DOI: 10.48550/arXiv.2310.04127. URL: https://hal.science/hal-04286461.

[37] L. Bourgeois and L. Chesnel. *Generalized impedance boundary conditions with vanishing or sign-changing impedance.* 23rd Sept. 2023. URL: https://hal.science/hal-04216044.

[38] P. Ciarlet and E. Jamelot. *The T-coercivity approach for solving Stokes problem: stabilization of finite element pairs.* 22nd Jan. 2024. URL: https://inria.hal.science/hal-04414789.

[39] P. Ciarlet, M. Kachanovska and E. Peillon. *Study of a degenerate non-elliptic equation to model plasma heating.* 5th July 2023. URL: https://inria.hal.science/hal-04152267.

[40] B. Delourme and S. Fliss. *Guided modes in a hexagonal periodic graph like domain.* 31st Aug. 2023. URL: https://inria.hal.science/hal-04193026.

[41] J. Garnier, L. Giovangigli, Q. Goepfert and P. Millien. *Scattered wavefield in the stochastic homogenization regime.* 14th Sept. 2023. URL: https://hal.science/hal-04220919.

[42] M. Halla, M. Kachanovska and M. Wess. *Radial perfectly matched layers and infinite elements for the anisotropic wave equation.* 26th Jan. 2024. URL: https://hal.science/hal-04419377.

10.2 Other

Educational activities

[43] M. Bonnet. 'Lecture notes on numerical linear algebra'. Engineering school. France, 14th Mar. 2023. URL: https://hal.science/hal-03321502.

Softwares

[44] [SW] L. Faria, F. Févotte, V. Sivadon and L. Plagne, *DataFlowTasks.jl*, 24th May 2023. LIC: MIT License. HAL: ⟨hal-03940572⟩, URL: https://hal.science/hal-03940572, VCS: https://github.com/maltezfaria/DataFlowTasks.jl, SWHID: ⟨swh:1:dir:30c9a232600daee74fb57083f2f4d58199721cdc;origin=https://github.com/maltezfaria/DataFlowTasks.jl;visit=swh:1:snp:efc8ebc154716dd1fdd6a499ff5ddd153b0d952a;anchor=swh:1:rev:eecc3db640d532029f8b39af3ca33a5b424a4e7e⟩.